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PREFACE

This is Number 3 of Volume 48 of the Quarterly TRANSACTIONS of the American Institute of Electrical Engineers. It contains those papers presented at the Winter Convention that were not included in the April Quarterly, as well as the papers given at the Regional Meetings held in Cincinnati, Ohio, March 20-23, and at Dallas, Texas, May 7-9, 1929.

Three miscellaneous contributions appear in this number: *Bessel Functions for A-C. Problems*, by Herbert Bristol Dwight, *Some Notes on Transformer Design*, by E. H. Waldo, and *Traveling Waves Due to Lightning*, by L. V. Bewley.

The issue concludes with the Annual Report of the Board of Directors for the fiscal year ending April 30, 1929.

Effect of Transient Voltages on Power
Transformer Design

BY K. K. PALUEFF*
Associate, A. I. E. E.

Synopsis.—When subject to transient voltage excitation, local concentration of voltage takes place in an ordinary transformer in which the capacitance charging current of the coils to ground is supplied through the winding, or the ratio of inductance and capacitance of the various parts throughout the winding is not constant. Calculations and tests of voltage distribution in the winding, caused by the impact of (a) damped high-frequency oscillations, and (b) unidirectional traveling waves, are given. In order to make the analysis clearer, the transformer winding is considered as a network of inductances and capacitances, and this term "network" is used throughout the paper. Certain simplified and typical networks are considered.

Transformers having one terminal grounded, such as are used in three-phase star connection, particularly in high voltage systems, are frequently built with the insulation graded to other windings and ground, in the order of the normal frequency voltage stress. The danger of such a practise is shown in power transformers which are subject to transient overvoltage, since voltage oscillation in the winding may raise the voltage to ground at intermediate points above the terminal voltage, unless the design of the winding eliminates oscillation.

The theoretical and experimental data given show that the distribution and magnitude of voltage stresses existing during recognized standard insulation tests are essentially different stresses created by transient voltages. This permits the construction of transformers that would satisfactorily pass standard insulation tests but at the same time would not be suitable for average service.

A new type of a transformer called "non-resonating," for use on grounded neutral systems, is described.

In transformers of this type, voltages of all frequencies distribute uniformly along the windings, as the possibility of internal voltage resonance is eliminated by a proper balance of distributed capacitance and inductance of the winding.

This is accomplished principally by means of conducting surfaces (shields) placed outside of the winding and connected to its line terminal.

The action of the shields is similar to that of the shielding ring on an insulator string. It neutralizes the effect of the capacitance current from the inside surface of the winding to ground, by supplying to every point of the winding a "charging" current equal to the "discharging" current of that point to ground. The application of the shield in some cases reduces the local stresses to one-eightieth.

Up to the present time, the total capacity of this new type of transformer exceeds half a million kv-a.

If the problem of designing the transformer insulation were limited to the requirements of normal frequency dielectric stress, it would be relatively simple.

It is not, however, the normal voltage stresses which, in high-voltage transformers, require the most careful consideration to predetermine the amount, kind, and arrangement of the insulation, but the transient stresses set up by abnormal conditions on the circuit.

The necessity for extra insulation, above that required to meet the A. I. E. E. test (or any other recognized standard rules) is due to two facts:

1. Transformers, even of high voltages, are subject to transient voltages many times the normal circuit voltage to ground. Records show from 10 to 15 times normal voltage to ground even on 220-kv. systems.

2. Most transient voltages are high-frequency oscillations, or are traveling waves lasting a number of microseconds, and the ordinary transformer does not permit these voltages to be uniformly distributed throughout the winding. This is because the transformer winding is not a pure inductance, but also contains distributed capacitance.

To determine the effect of the above considerations on the transformer design two alternative designs of 20,000-kv-a., 220,000-volt transformer were prepared. Both alternatives were to satisfy identical operating characteristics as efficiency, reactance, potential test, etc.

The first design was made in accordance with the A. I. E. E. Standards, as well as transient voltage requirements developed by the years of experience and incorporated into practise for at least ten years.

The second design was made in accordance with A. I. E. E. Standards only.

The comparison revealed that first transformer cost 40 per cent more than the second and has 75 per cent more of active material than the second.

It is important to note that the second transformer would withstand successfully not only the tests called for by A. I. E. E. Standards but also all the tests called for by any other recognized standard rules including those which specify so called "surge or impulse test." This is because such tests impose on transformers transient voltage of an amplitude negligible in comparison with those experienced in actual service.

In 1919, L. F. Blume and A. Boyajian presented to the A. I. E. E. a paper on *Abnormal Voltages within Transformer Windings.*[1] In this paper, with certain approximations, fundamental laws governing voltage transients in transformers were explained and expressed in mathematical form.

In 1922, J. M. Weed,[2] in a paper entitled *Prevention of Transient Voltage in Windings*, before the A. I. E. E.,

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

[1] A. I. E. E. Trans., Vol. XXXVIII., p. 577.

[2] A. I. E. E. Trans., Vol. XLI., p. 149.

scribed a principle for the construction of non-esonating windings, in which line voltages of all frequencies are distributed uniformly throughout the windings.

With the assistance of these two papers, and much ditional study and experiments, a new type of trans- er has been developed for operation with solidly grounded neutral, which is believed to be better adapted to resist stresses caused by voltage transients than any type used heretofore.

This new type differs from the ordinary concentric winding, core type transformer, by the presence of conducting surfaces enveloping part of the outside (high-voltage) winding. These surfaces are called "shields." They are connected to the line terminal and properly insulated from the rest of the winding and ground. The shields, being on the outside of the winding and some distance away from it, do not affect its construction, and therefore ordinary disk coils, that are typical of power transformers, are used in "non-resonating" transformers. The capacitance of shields to the winding is so proportioned that voltages of all frequencies and wave shapes distribute uniformly throughout the winding. Therefore, the voltage stresses, produced by the transients at various parts of such a transformer, are very materially smaller than those found in transformers of the conventional type. In some cases, application of shields reduced stresses to one-eightieth.

The object of this paper is to describe the effect of transient voltage on ordinary as well as "non-resonating" transformers.

However, in order better to understand the principles involved, first an attempt will be made to describe non-mathematically, the chief characteristics of transient voltage phenomena.

Part I

TRANSIENT VOLTAGE PHENOMENA

A. EQUIVALENT NETWORKS

1. Elementary Circuit. Most engineers, who deal with electrical phenomena of commercial frequencies only (25 to 60 cycles), picture to themselves inductance as a spiral wire, and capacitance as two parallel plates, and therefore are not apt to recognize these characteristics when they are disguised in different geometric forms. Furthermore, again due to everyday experience with low frequencies, they are accustomed to think of some apparatus (such as transformers and choke coils) as pure inductances, and of others as pure capacitances (parallel plates, synchronous condensers), or as resistances. To discard these faulty notions is the first step toward the understanding of transient phenomena.

If $a\,b$ (see M, Fig. 1) is a piece of conductor in air, and direct current is applied across the conductor, a current I will be established in it. If we take any two points on the conductor, like a and b, $a_1\,b_1$, etc., we find that there is a difference of potential between them $e\;e_1\;e_2$, etc.

As the conductor is surrounded by a dielectric (air), these potential differences will establish lines of dielectric stress, as shown in Fig. 1. The current I, flowing through the conductor, will of course create lines of magnetic stress around the conductor. The lines of dielectric stress can be represented by minute condensers $c_1\;c_2\;c_3$ and the magnetic flux by minute inductances. As no conducting material is entirely resistanceless, the resistance must also be included, and it can be shown in a conventional manner.

Thus we see that a straight piece of conductor, no matter how small, and regardless of its material, is not a pure resistor, but is equivalent to a complicated network of elementary condensers, inductances, and resistances.

Even a conducting sphere immersed in a dielectric with potential applied between any two points of its surface is also equivalent to a network of all three elementary constants. Also two parallel plates with terminals $a\,b$ are equivalent to a network of all three elementary circuits.

Depending on the size of the conductor and the nature of the problem, different elementary distributed constants can be lumped together, and thus converted into a simpler (but for a given problem, equivalent) network of a pure condenser shunted with a pure inductance, which inductance is connected in series with a pure resistance (N in Fig. 1).

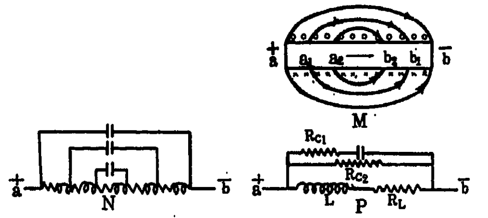

FIG. 1—EQUIVALENT CIRCUIT OF A SIMPLE CONDUCTOR

When it is desired to take losses in the condenser into account, two more pure resistances are added, as shown at P Fig. 1. Their ratio depends on the characteristics of the dielectric and the shape of the condenser plates.

The conclusion is that pure constants do not exist in practise; therefore, whenever a part of a circuit acts as pure inductance, capacitance, or resistance, it simply means that at the frequency of the phenomenon under consideration, the characteristics of this particular constant dominate.

B. SUSTAINED AND DAMPED OSCILLATION

1. Elementary Circuit. To simplify further discussion, let us neglect the resistances of elementary network, Fig. 2, and thus assume the elementary conductor $a\,b$, as represented by a pure concentrated capacitance C in parallel with a pure concentrated inductance L.

With voltage E of a given frequency f impressed across terminals *a b* of elementary circuit Fig. 1, current I will flow through the terminals.

Current $I = i_l - i_c$

where i_l is current in the inductance

where i_c is current in the condenser

As current is proportional to the susceptance of the circuit, I = susceptance of inductance + susceptance of the condenser. Taking the susceptance of the inductance as positive, and of the condenser as negative, we have:

$$\text{Susceptance of inductance} = + \frac{1}{2\pi f l}$$

$$\text{Susceptance of condenser} = - 2\pi f C$$

Fig. 4 shows graphically the change in susceptances due to change in frequency. Curves L and C illustrate susceptance or current in the inductance and condenser respectively. Curve A represents combined susceptance of the two, connected in parallel, or the current through terminals *a b*. This curve is obtained by subtracting the respective values of curves L and C respecting their signs. As reactance is inverse to susceptance, the reactance of the circuit is represented by curve Z.

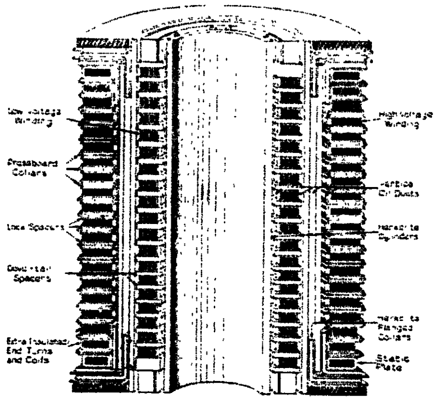

FIG. 2—CROSS-SECTION OF COMPARATIVELY HIGH-VOLTAGE POWER TRANSFORMER WINDING

Judging the circuit by the voltage and current at terminals *a b*, we conclude that at zero frequency (direct current) the entire line current is flowing through inductance and the circuit acts as a pure inductance of value L. With rising frequency, the current in the inductance diminishes and condenser current is created. However, as this condenser current is much smaller than the inductance current, the line current is still inductive but of smaller value than for inductance L alone. Thus the circuit at low frequency acts as pure inductance but greater in value than L. This apparent increase in inductance of the circuit increases with increase in frequency until near a certain frequency f_1 the circuit acts as practically infinite inductance. At frequency f_1 the inductance current is exactly equal to the condenser current, and therefore there is no current in the line and the network acts as an open circuit. At a frequency slightly beyond f_1 the condenser current is greater than the inductance current, therefore the line current is leading and the circuit is in effect a pure condenser but much smaller than condenser C. With further increase in frequency, the inductance current falls down rapidly, while the condenser current steadily increases, and the line current increases as if the circuit were that of a pure condenser, increasing in size with frequency but always remaining smaller than C. Finally at infinite frequency, the current in the inductance becomes zero, and the entire line current flows through the condenser, and the circuit acts as a pure condenser of value C.

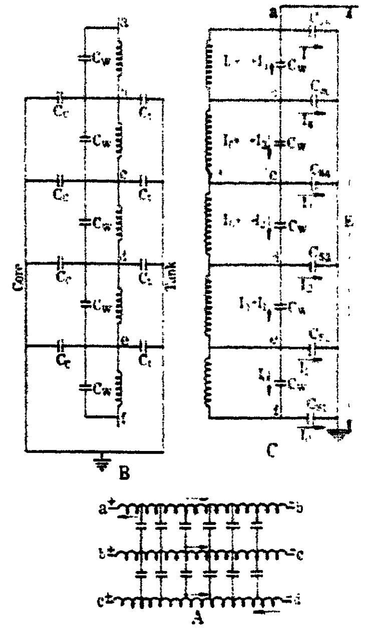

FIG. 3—TRANSFORMER EQUIVALENT NETWORK

A. Network of adjacent turns
B. Network of entire winding.
C. Simplified B.

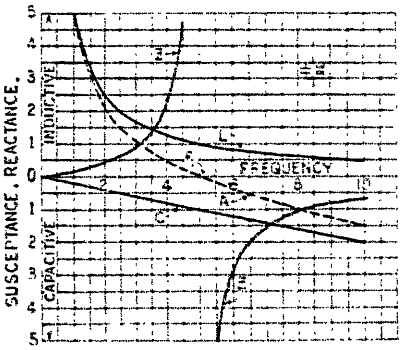

FIG. 4—ELEMENTARY CIRCUIT. CHANGE OF REACTANCE AND ADMITTANCE WITH FREQUENCY

The conclusion is that the elementary circuit, under sine wave excitation at frequencies between zero and f_1, acts as a pure inductance ranging in value from L to infinity and at frequencies between f_1 and infinity acts as a pure condenser ranging in value from zero to C.

It is common knowledge that the frequency f_1, the critical or natural frequency of the circuit, depends only on the product $L\,C$ and is inversely proportional to the square root of this product—

The values of L and C depend on the physical characteristics of the elementary conductor and of the surrounding dielectric medium.

The conclusion is that pure constants do not exist in practise, therefore whenever a part of a circuit acts as pure inductance, capacitance, or resistance, it simply means that at the frequency of the phenomenon under consideration, the characteristics of this particular constant dominate.

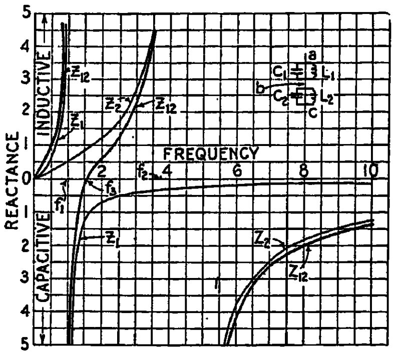

Fig. 5—Two Elementary Circuits in Series. Change of Reactance with Frequency

It follows that one and the same elementary circuit may behave as a short circuit, an inductance, an open circuit, or a capacitance, depending upon the frequency of the applied voltage.

2. Two Elementary Conductors in Series. Let us take two conductors 1 and 2 which differ in L and C as well as in their respective products. The variation of the reactance of each conductor with frequency can be shown as on Fig. 5 (curve Z_1 corresponds to conductor 1 and curve Z_2 to conductor 2).

If these conductors are connected in series then the combined reactance Z_{12} for any frequency will be equal to the algebraic sum of Z_1 and Z_2 for that frequency, and the share of the total voltage E_{ac} that will appear across each conductor will be equal to the proportion that the reactance of this conductor bears to the total reactance Z_{12}.

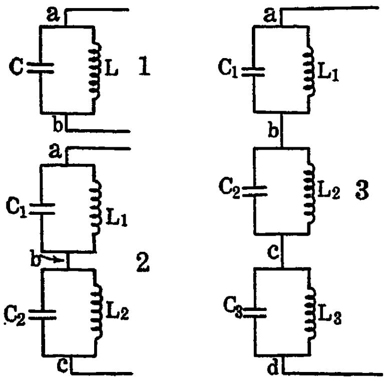

Fig. 6—Elementary Circuits No. 1, 2, 3

Examination of curves Z_1, Z_2, and Z_{12} reveals the following:

At $D\,C$ or low frequency, both conductors act as pure inductances of L_1 and L_2 in values and divide line voltage in direct proportion to these values.

The reactance of circuit No. 2 rises slowly with frequency. The reactance of No. 1 rises very rapidly and becomes infinite at f_1. For these reasons at frequencies approaching f_1 a greater and greater portion of the line voltage appears across conductor No. 1, until at f_1 the entire line voltage E_{ac} concentrates across $a\,b$. This is shown by curve No. 1 of Fig. 7.

Between frequencies f_1 and f_2, conductor No. 1 acts as a condensive reactance, while conductor No. 2 is still an inductive reactance, and the voltage across the two parts of the circuit is 180 deg. out of phase.

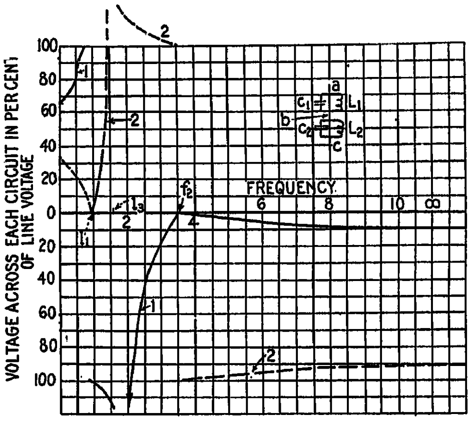

Fig. 7—Variation of Voltage Distribution with Frequency Between Two Elementary Circuits

At a certain frequency f_3, the reactance of both becomes numerically equal, and the total reactance of the circuit becomes zero, therefore any finite voltage will create an infinite voltage across each one. This rise of voltage to infinity, however, is not instantaneous with the closing of the circuit. It is linear with time. The amplitude of each succeeding half cycle increases by the amplitude of the line voltage.

At any frequency above f_2 both conductors act as pure condensers, gradually increasing in size with increasing frequency until at infinite (or very high) frequency they reach their actual values C_1 and C_2.

The line voltage divides between them in inverse ratio of the values of their effective capacities for the corresponding frequency.

As at f_2 and somewhat higher frequency, conductor No. 2 acts as an extremely small condenser in comparison with No. 1, the whole voltage concentrates across it. (Curve No. 2, Fig. 7.)

For example, taking a network where:

$$\frac{L_1}{L_2} = 2; \quad \frac{C_1}{C_2} = 0.1$$

The dielectric stress from *a* to *b* and *b* to *c*, at different frequencies, will be as follows:

Frequency range	Dielectric stress in per cent of line voltage	
	a to *b*	*b* to *c*
Direct current or very low......	66.6	33.3
Natural frequency of *a b* = f_1...	100	0
Natural frequency of *b c* = f_2...	0	100
Natural frequency of *a c* = f_3...	infinity	infinity
Infinite or very high...........	9	91

This tabulation shows that to design apparatus properly it is necessary to know the range of frequency to which it may be subjected as otherwise its life may be unduly short.

3. Three Dissimilar Elements in Series. A circuit composed of three dissimilar conductors in series has characteristics similar to that of two conductors. It has, however, three frequencies of current resonance f_1, f_2, and f_3, corresponding to three products $L_1\,C_1$, $L_2\,C_2$, and $L_3\,C_3$. At each of these frequencies the entire line voltage concentrates across the corresponding part of the network.

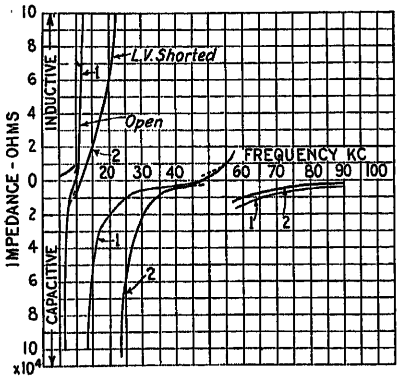

Fig. 8—High Frequency Impedance Test on Power Transformer *B* with Shield Removed

No. 1 L. V. open
No. 2 L. V. shorted

The network also has two frequencies of voltage resonance f_4 and f_5 (f_4 is between f_1 and f_2 and f_5 between f_2 and f_3). At f_4 voltages *a b* and *b d* are infinite and opposite in phase. At f_5 voltages *a c* and *c d* are infinite and opposite in phase.

The very important similarity to a two-conductor network lies in the fact that at very low frequency the line voltage is distributed in direct proportion to inductances L_1 L_2 and L_3 as if condensers were not present, while at very high frequency it is distributed in inverse proportion to capacitances C_1 and C_2 and C_3 as if inductances were not present.

It is easy to foresee that each addition of a different element to the circuit will bring one new frequency of current resonance and one new frequency of voltage resonance.

C. TRAVELING WAVES

1. General. The effect of a traveling wave on the circuits considered above, is so essentially different from the effect of a sustained oscillation that it justifies special discussion.

There must be distinguished two principal types of traveling waves—infinite, or the effect of which is equivalent to that of infinitely long waves, and finite waves. A traveling wave created by lightning is typical of a finite wave.

For the present we will assume that the surge impedance $\sqrt{\frac{L}{C}}$ of the circuit, connected to the terminals of the network under consideration, is negligibly small compared with the surge impedance of the network.

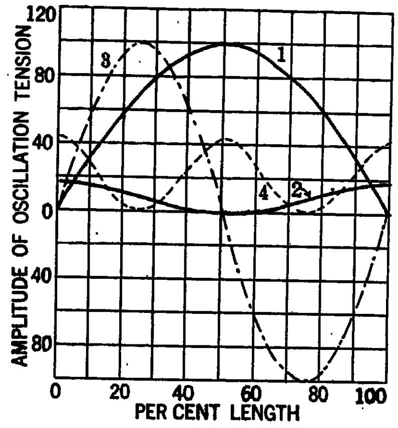

Fig. 9a—Oscillation of Elastic Rod

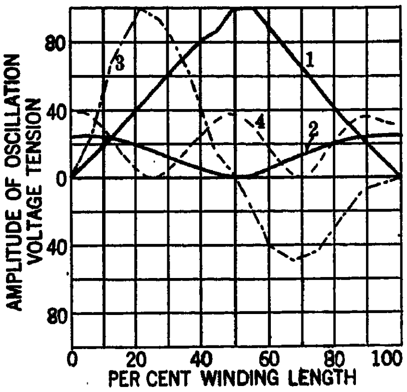

Fig. 9b—Oscillation of Transformer Winding

In such a case a traveling wave emerging from the external circuit undergoes practically complete reflection from the terminals of the network. Thus the amount of energy lost by the wave to the network, from the moment the front of the wave arrives at the terminals to the moment its tail leaves them, is negligible in comparison with the total energy of the wave.

In the case of complete reflection, the steepness of the front and of the tail of the voltage wave and its crest value at the point of reflection (the terminals of the network) is twice as great as that of the traveling wave; while the lengths of the front, crest, and tail of the voltage wave are equal to the lengths of the corresponding parts of the traveling wave (Fig. 10).

Under these conditions, the investigation of the influence of the traveling wave No. 1 of Fig. 10 on a network is reduced to finding the behavior of a network with voltage wave No. 2 maintained at its terminals.

2. *Infinite Traveling Wave.*

a. Capacitance in Series with Inductance. Analysis shows that in case of a circuit of Fig. 11 being subject to impact of an infinitely long rectangular wave of + E volts, the voltage of the condenser will oscillate with amplitude E and frequency $f = \dfrac{1}{2\,\pi\,\sqrt{L\,C}}$ about a point + E volts above zero and its maximum voltage will be + 2 E.

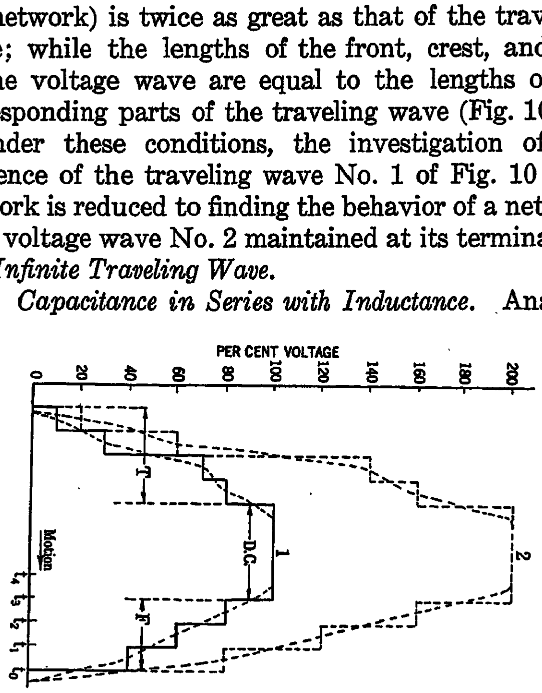

Fig. 10

No. 1 Traveling wave

No. 2 Voltage wave produced by traveling wave No. 1 at point of complete reflection

A mechanical analogy may be of assistance here.

It is well known that mass and elasticity correspond to inductance and capacitance, velocity corresponds to current, and force to voltage. It is further obvious that the mechanical system shown in Fig. 11 corresponds to an inductance L connected in series with capacitance C as shown.

Let it be assumed that E is the force necessary to hold mass L in position No. 2 (D inches above position No. 1). When a force applied to L is brought from zero to E in an extremely gradual manner so that the applied force reaches E at the same moment that L attains position No. 2, no oscillation will take place during the transient. This happens because the increase in force is controlled in such a way that the applied force exceeds the pull of the spring by an infinitesimal amount until the value of E is reached.

However, should force E be applied abruptly, at the end of the first elementary time interval, L will move a distance d, the pull of the spring will be $- E \dfrac{d}{D}$ and the total force on L will be the difference.

$$E - E\,d/D = E\,(1 - d/D)$$

Mass L, therefore, will behave for the next time interval as a free body under the influence of force $E \left(1 - \dfrac{d}{D}\right)$ and will accelerate proportionately to this force and store a certain amount of energy in kinetic form.

During the following interval, mass L will be under the influence of $E \left(1 - 2\,\dfrac{d}{D}\right)$, that is of a smaller force, and its acceleration will diminish. With every following interval, the force and hence the acceleration will grow smaller, but the velocity will be steadily increasing and therefore the total storage of kinetic energy.

At a certain time t_1, a pull of the spring will be exactly equal to E, and total force acting on L will be zero. At this moment L will occupy position No. 2, (that is, final position for a case where force E is applied so gradually that no oscillation takes place). The acceleration will become zero and the velocity will reach its maximum value. Under an abrupt force E, the kinetic energy accumulated by L during its travel from position No. 1 to No. 2 will force it to continue its motion in an upward direction. At the end of the elementary time interval, following t_1, L will experience a pull of the spring greater than force E, and therefore the total force acting on L will be

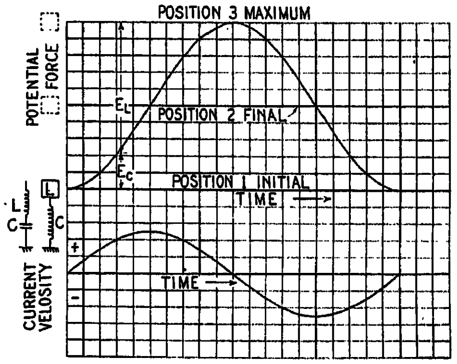

Fig. 11—Oscillation of Inductance in Series with a Condenser Due to Infinite Rectangular Wave Excitation

$$E - E\left(1 + \frac{d}{D}\right) = -\,\frac{d}{D}\,E$$

That is, the force will change its direction and become downward, which will cause a certain retardation. With every time interval following, the negative force will steadily increase, and so will the retardation. At the moment t_2, the force acting on L will become $- E$, the retardation will reach a maximum, and the velocity zero. By this time, all the kinetic energy absorbed by L on its travel from position No. 1 to No. 2 will be transferred to the spring. As this transfer followed exactly the same law as the absorption, only in reverse order, it naturally will be accomplished in the same time t_1, and the body will travel the same distance D.

Thus L will arrive at position No. 3, a distance $2\,D$ away from No. 1 after $2\,t_1$ seconds. At the same moment, the spring being expanded to $2\,D$ will be under strain $+\,2\,E$, and the mass will find itself under the action of $-\,E$. Now $-\,E$ acting on L will cause a downward motion of L which will be exactly the same function of time as the upward motion.

Thus L will oscillate between positions No. 1 and No. 3 about No. 2, subjecting spring C at regular intervals to a stress equal to $2\,E$. The oscillation about position 2 will be sinusoidal. The frequency of oscillation will be $f = \dfrac{1}{2\,\pi\,\sqrt{L\,C}}$.

In practise, oscillation is always followed by a loss of energy, and therefore it dies out with time and the system comes to rest.

In case the force or voltage is applied an indefinitely long time, this new state of rest coincides with the one about which the oscillation took place.

In the example it is position No. 2. We shall refer in the remainder of this paper to this state of distribution of force of a system as a "final" one. The state of distribution of force in which a system finds itself immediately after the application of force will be termed "initial."

We can formulate a conclusion from the above example as follows: In case a circuit is subjected to abrupt and steady voltage and its initial voltage distribution is different from the final voltage, a transient will take place. This transient will be oscillatory if losses in the circuit are relatively small. If the circuit has only one natural frequency the oscillation will be at that frequency and its amplitude equal to the difference between initial and final states.

b. Elementary Circuit. Considering again, as in case of sustained oscillation, an elementary circuit consisting of inductance in parallel with capacitance, we find that with a rectangular voltage wave applied no oscillation will take place because the initial and final states of voltage distribution are identical, as full voltage appears across condenser and inductance at the very beginning of the phenomenon and remains constant as long as the wave lasts.

c. Two Elementary Circuits in Series. Referring to Fig. 6, from a review of the behavior of this circuit under sustained oscillation, we conclude that at very high frequency the voltage will distribute between $a\,b$ and $b\,c$ as though inductance $L_1\,L_2$ were absent. This means that the voltage will distribute inversely as the capacitance. As the front of a rectangular wave acts as a quarter of a cycle of infinite frequency, it is obvious that at the first instant the voltage will distribute inversely as the capacitance, as shown below.

The initial voltage across $a\,b$, $E_{abi} = E\,\dfrac{C_2}{C_1 + C_2}$

The initial voltage across $b\,c$, $E_{bci} = E\,\dfrac{C_1}{C_1 + C_2}$

In the numerical example of Fig. 12.

The initial voltage across $a\,b = E\,\dfrac{1}{10 + 1} = 0.09\,E$

The initial voltage across $b\,c = E\,\dfrac{10}{10 + 1} = 0.91\,E$

In reviewing a sustained oscillation, we concluded that at very low frequencies or direct current, the line voltage will divide between $a\,b$ and $b\,c$, as though capacitance were absent; that is directly proportional to the inductances L_1, L_2.

As the effect of the crest of a rectangular wave of infinite length on the circuit is identical to that of direct current, it follows that the final voltage distribution will be directly proportional to the inductances as shown below:

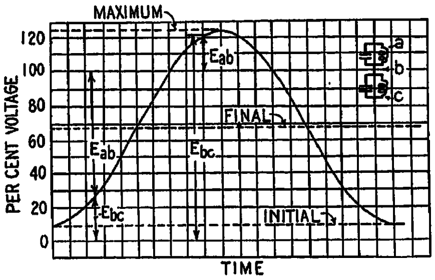

Fig. 12—Oscillation of Two Elementary Circuits Connected in Series Due to Infinite Rectangular Wave Excitation

$$E_{abf} = E\,\frac{L_1}{L_1 + L_2} \qquad E_{bcf} = E\,\frac{L_2}{L_1 + L_2}$$

in numerical example of Fig. 12.

$$E_{abf} = 0.66\,E \qquad E_{bcf} = 0.33\,E$$

Thus initial state

$$E_{abi} = E\,\frac{C_2}{C_1 + C_2} \qquad E_{bci} = E\,\frac{C_1}{C_1 + C_2}$$

and final state

$$E_{abf} = E\,\frac{L_1}{L_1 + L_2} \qquad E_{bcf} = E\,\frac{L_2}{L_1 + L_2}$$

Examination of Fig. 12 shows that the maximum voltage across C_1 is numerically equal to the final voltage of C_1 + a voltage numerically equal to the amplitude of the oscillation, which is the difference between the final and the initial state.

Thus

$$E_{abm} = \frac{L_1}{L_1 + L_2} E + \left(\frac{L_1}{L_1 + L_2} - \frac{C_2}{C_1 + C_2} \right) E$$

and across C_2

E_{bcm} = final voltage of C_2 + voltage numerically equal to amplitude of the oscillation

$$= \frac{L_2}{L_1 + L_2} E + \left(\frac{L_1}{L_1 + L_2} - \frac{C_2}{C_1 + C_2} \right) E$$

In the numerical example

$$E_{abm} = [0.66 + (0.66 - 0.09)]\, E = 1.23\, E$$

$$E_{bcm} = [0.33 + (0.66 - 0.09)]\, E = 0.90\, E$$

With L_2 and C_2 negligible in comparison with L_1 and C_1 the final voltage across $C_1 = \frac{L_1}{L_1 + L_2} E = E.$

The amplitude of oscillation

$$= \left(\frac{L_1}{L_1 + L_2} - \frac{C_2}{C_1 + C_2} \right) E = E$$

and maximum voltage $E_{abm} = 2\, E.$

The final voltage across $C_2 = \frac{L_2}{L_1 + L_2} E = 0.$

The amplitude of the oscillation is E (the same as for C_1) and maximum voltage $E_{bcm} = E$. It is obvious that if on the contrary L_1 and C_2 are negligible in comparison with L_2 and Q_2, then

$$E_{abm} = E$$

$$E_{bcm} = 2\, E$$

It is evident that $2\, E$ is the maximum possible voltage that may be created by an infinite rectangular wave across either part of the circuit.

ELIMINATION OF TRANSIENTS

As such an oscillation creates voltage across some part of the circuit in excess of the final value, it may be harmful and therefore undesirable.

As the cause of all transients is the difference between initial and final states, it follows that the elimination of this difference means elimination of the transients. In this case it can be accomplished by making

$$\frac{L_1}{L_1 + L_2} = \frac{C_2}{C_1 + C_2},$$

which is equivalent to having

$$\frac{L_1}{L_2} = \frac{C_2}{C_1} \text{ or } L_1, C_1 = L_2\, C_2.$$

To secure this relation it is sufficient to be able to choose the proper value for one of four constants. Referring to Fig. 15 it is seen that with such a relation of constants, the division of line voltage between two parts of the circuit is identical for the entire range of frequencies. The effective reactances change from inductive to capacitive at one and the same frequency f_1. Therefore their voltages are always in phase and hence at no frequency can either of them exceed line voltage.

For the same reason at no frequency can combined impedance Z_{12} become zero. Such a circuit therefore is incapable of voltage resonance.

d. Three-Element Circuit. To visualize the tendency of transient characteristics as the complexities of the network increase, we will further review the behaviour of three dissimilar elementary circuits connected in series, under the influence of an infinite rectangular voltage wave.

From considerations previously outlined, the initial and final voltages for each branch of the circuit can be written from mere inspection of Fig. 6. Taking C_0 and L_0 for capacitance and inductance from a to d

Initial voltages

$$E_{abi} = \frac{C_0}{C_1} E; \quad E_{bci} = \frac{C_0}{C_2} E; \quad E_{cdi} = \frac{C_0}{C_3} E$$

Final voltages

$$E_{abf} = \frac{L_1}{L_0} E; \quad E_{bcf} = \frac{L_2}{L_0} E; \quad E_{cdf} = \frac{L_3}{L_0} E$$

In case some or all initial values are different from final values, transient phenomena will occur in the form of oscillations of points b and c.

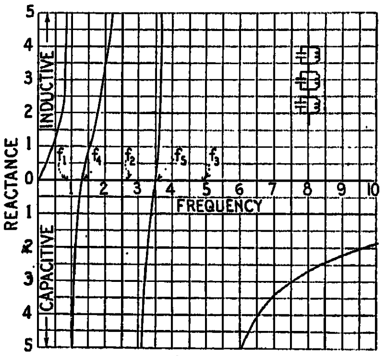

FIG. 13—VARIATION OF REACTANCE WITH FREQUENCY. THREE ELEMENTARY CIRCUITS CONNECTED IN SERIES

Fig. 13 indicates that this circuit is capable of voltage resonance at points b and c at frequencies f_2 and f_4 respectively.

With point b oscillating at its natural frequency f_2, the voltage between b and d will be pulsating with that frequency, and therefore will cause the voltage between c and d to pulsate at the same frequency with amplitude depending upon the ratio of reactances of $b\,c$ and $c\,d$. On the other hand, point c occupies in the circuit a position similar to b and therefore is equally capable of oscillating at its own natural frequency f_4; in which case the voltage between a and c will pulsate at that frequency and so will point b.

Thus point b is in a state of free oscillation at its own natural frequency f_2, and of forced oscillation at frequency f_4; similarly point c is in a state of free oscillation at frequency f_4 and of forced oscillation at frequency f_2.

The amplitude of oscillation depends upon the difference between initial and final voltage distribution, and also upon the ratio of reactance between points a and b, d and c, c and d, at frequencies f_2 and f_4.

In case $\frac{C_1}{C_2} = \frac{L_2}{L_1}$, point b has no natural frequency and the circuit oscillates at the natural frequency of point c. In case $\frac{C_2}{C_3} = \frac{L_3}{L_1}$, the natural frequency of point c disappears and the circuit oscillates at the natural frequency of b.

ELIMINATION OF TRANSIENTS

It is obvious that should constants be adjusted so that

$$\frac{C_1}{C_2} = \frac{L_2}{L_1} \quad \frac{C_2}{C_3} = \frac{L_3}{L_2}$$

the initial and final voltage distribution will be identical, no oscillation will take place. Such a circuit will be incapable of voltage resonance.

3. Infinitely Long Traveling Wave with Slanting Front.

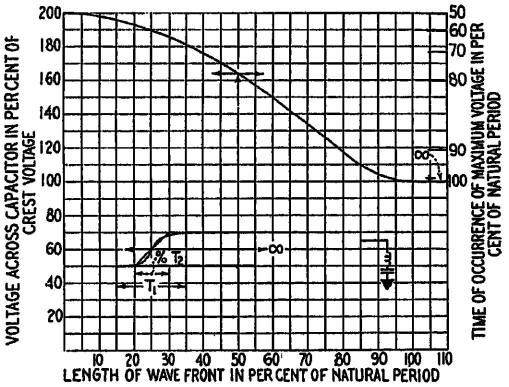

FIG. 14—INDUCTANCE IN SERIES WITH CAPACITANCE

Voltage across capacitor as the function of the slope of the front of an infinite wave

In the case of a system consisting of a spring and a mass, it was found that a given steady force E subjects the spring to a stress of either 2 E or E depending on whether it is applied suddenly or gradually.

This suggests that the maximum voltage stress created in a given network by a traveling wave depends on the steepness of the wave front.

The actual front of the wave may start to rise very slowly (say up to 10 per cent of the crest value), then fast (say up to 90 per cent of the crest value), and then slowly again. If the length of a given front T, in Fig. 14, is measured from the very beginning of the front up to the very crest of it, a wrong impression may be gained of its effect on the circuit. It was decided therefore to consider as the slope of the front of the wave the slope of the portion of the front lying between 10 per cent and 90 per cent of the crest value. The effective length of the front becomes T_2 as shown.

To illustrate the relation between the steepness of the front of the infinite wave and the maximum internal voltages created thereby across the capacitance part of the circuit, the curve of Fig. 14 was prepared for the case of an inductance in series with a capacitance.

As in such a case, the rate of change of internal voltage depends only on the front of the wave and the natural frequency of the network, the length of the front is expressed in per cent of the natural period of the network.

The lapse of time from the moment of impact of the wave to the moment the maximum voltage is reached is also given and expressed in per cent of the same period.

Fig. 14 shows that in case of inductance in series with a capacitance, there is a critical or minimum length of the front which does not produce oscillations or voltage across condenser in excess of its final value, which is the line voltage E. The same rule holds true for any other circuit. The critical length of the front depends only on natural frequency of the circuit and in case of the circuit of Fig. 14 is equal to its natural period.

On account of the above characteristics, a given front may be qualified as a very steep one for a circuit of low natural frequency, and as a very slanting one for a circuit of high natural frequency.

4. Finite Rectangular Traveling Wave.

a. General. When a transient phenomenon is the result of several causes, it may be solved by the method of superposition.

By this method, the effect on the circuit of each cause is found under the assumption that the circuit is not subjected to the effect of other causes. Then to find the actual values of currents or voltages at a given point, simultaneous values produced by all the causes are added algebraically. This is mathematically correct only where circuit constants are independent of absolute values of current and voltage.

A finite rectangular traveling wave of L miles in length (or T seconds in duration) and of E volts can be looked upon therefore as a result of two infinite rectangular waves; one of $+ E$ and the other of $- E$, traveling in the same direction with the same velocity as the finite, but with positive wave L miles (or T seconds) ahead of the negative one and with its front coinciding with the front of the finite wave, Fig. 16.

The effect of such a finite wave on a circuit is identical therefore, with the effects of the two infinite waves, determined independently and superimposed one on the other.

Transients due to waves of the same magnitude but of opposite signs are, of course, identical in form, but 180 deg. out of phase.

b. Inductance in Series with Condenser. It was found that the voltage across a condenser, due to a positive rectangular wave, will change with time, as shown by curve No. 1 on Fig. 16, as the negative wave arrives at the terminal of the circuit t seconds later, the condenser voltage created by it will be as shown by curve No. 2.

By superimposing curves No. 1 and No. 2, we derive curve No. 3 as the condenser voltage wave due to a finite wave of L miles (or t seconds). In a similar way, the voltage wave across an inductance may be determined.

To determine the effect of traveling waves of different lengths on condenser voltage, wave No. 2 should be slid along the abscissa to corresponding positions with respect to wave No. 1, and then combined with the latter.

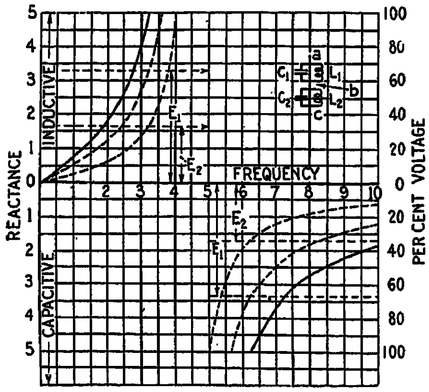

FIG. 15—VARIATION OF REACTANCE WITH FREQUENCY AND DISTRIBUTION OF LINE VOLTAGE BETWEEN $a\,b$ AND $b\,c$, WHERE $\frac{L_1}{L_2} = \frac{C_2}{C_1}$

Very often in practise it is not the shape of the voltage across a given part of a circuit that is important, but rather the crest value of the transient. Inspection of Fig. 16 indicates that the abrupt ending of the wave cannot increase the voltage across the condenser beyond 2 E (as wave No. 2 is always subtracted from No. 1). It should be remembered, that 2 E is the maximum condenser voltage that can be produced by an infinite wave. It also shows that the crest value will reach 2 E unless the wave is shorter than half of the period of the natural frequency of the circuit, that is unless

$$t < \frac{T}{2}$$

or as

$$T = \frac{1}{f} = 2\,\pi\,\sqrt{LC}$$

$$t < \pi\,\sqrt{LC}$$

The curves of Fig. 17 show the relation between the length of a traveling wave (expressed in per cent of natural period) and the value of the crest voltage produced thereby across the condenser expressed in per cent of voltage produced by the traveling wave across the terminals of the circuit. (It should be remembered that due to assumed complete reflection of the traveling wave from the terminals of the circuit, the voltage of the traveling wave is half of the terminal voltage E.) The maximum voltage across the inductance occurs at the monent of collision of the traveling wave with the circuit, and therefore is independent of the length of the wave. Its value equals terminal voltage E.

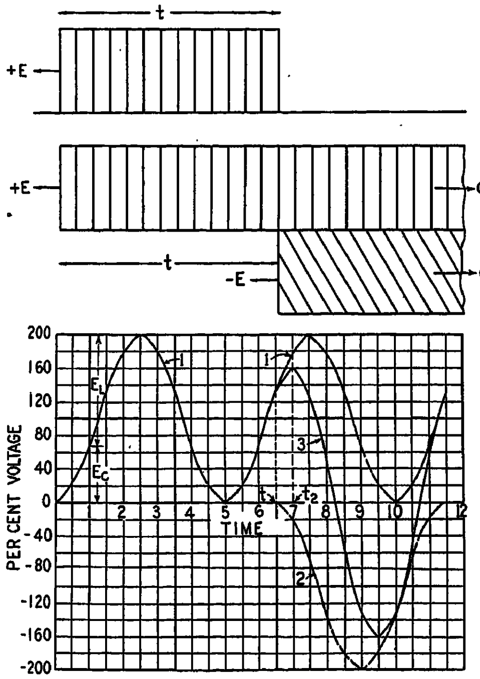

FIG. 16—INDUCTANCE IN SERIES WITH CAPACITANCE

Oscillation due to finite rectangular wave

Examination of this curve shows that the crest voltage produced by waves, shorter than half of the natural period occurs after the traveling wave leaves the circuit terminals. Thus, after the terminal voltage becomes zero again, the internal voltage may be still rising. (See Fig. 16, part of the actual condenser wave between t_1 and t_2.)

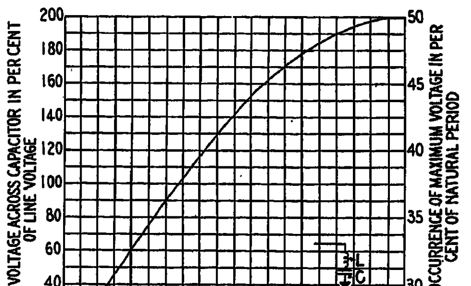

FIG. 17—EFFECT OF LENGTH OF RECTANGULAR WAVE ON MAXIMUM VOLTAGE ACROSS CAPACITOR

Examination of curve No. 3, Fig. 16, shows that abrupt ending of the finite wave does not produce any sudden change in this circuit as the oscillation that follows is still sinusoidal but the axis of oscillation changes from a level + E volts above zero potential to zero potential. Depending upon the length of the wave, this may result in increase of the amplitude of oscillation, and it may become equal to twice line voltage in case the length of the wave is an odd multiple of $\frac{1}{2}T$ as can be seen from Fig. 16 if wave No. 2 is allowed to be shifted to the right, so that its beginning coincides with crest of wave No. 1.

c. Two Elementary Circuits in Series. Analyzing in a similar manner a network consisting of two dissimilar elementary circuits in series, we obtain the voltage waves shown on Fig. 18. The wave differs in character from the wave of Fig. 16. In the case of pure inductance in series with pure capacitance, it was found that the shape of a condenser voltage wave can never have a rectangular component, as it is controlled by the natural frequency of the circuit. In the case of the circuit under consideration, however, a voltage wave across either part of the circuit contains a rectangular component equal to the initial voltage of the respective part, which coincides with the front and tail of a finite wave. This occurs because of the presence of paths of pure capacitance from point *b* to both terminals.

The oscillation, after the rectangular wave is reflected and leaves the terminals, is similar to that of the previous circuit. It also takes place about the line of zero potential, and its maximum amplitude may be twice the amplitude of the preceding oscillation.

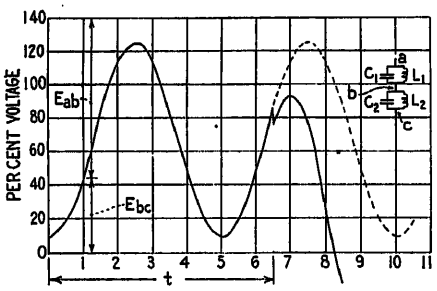

FIG. 18—TWO ELEMENTARY CIRCUITS IN SERIES

Oscillation due to finite rectangular wave of *t* seconds

It may be $2E$ (as in previous circuits) only if $\dfrac{C_1}{C_2}$ and $\dfrac{L_1}{L_2}$ is so large or so small that C_1 and L_2 or C_2 and L_1 can be neglected, but then the circuit becomes identical with the preceding one. It should be remembered, of course, that in this circuit, capacitance must be very small to be neglected while inductance must be very large.

d. Three Elementary Circuits in Series. A three-element circuit acts similarly to a two-element circuit. The presence of two natural frequencies produce transients (following the end of the traveling wave) of considerable complexity. The crest voltage across any one part may be equal to practically double line voltage if the "initial" and "final" voltage distributions are very different.

5. Traveling Wave of any Shape.

In practise, rectangular waves do not exist, and therefore the effect of deviation of wave shape from the rectangular must be considered.

A traveling wave of any shape, as well as a terminal voltage wave produced by the traveling wave, can be assumed to consist of a certain number of short rectangular waves, all of the same sign as the "main" wave, following one another in unbroken sequence, with their amplitudes and lengths corresponding to that part of the main wave with which they coincide in time. (See Fig. 10.) The voltage produced by such a wave at any point of the circuit is obtained by superimposing the simultaneous values of voltages produced by all the elementary waves at that point.

Part II.

ORDINARY TRANSFORMER

A. EQUIVALENT NETWORK

At operating frequencies, transformers act as inductances in series with some resistances. But, as was shown above, no apparatus or any part thereof can be free of capacitance,—therefore the capacitance must be present in the transformer.

In Fig. 2, a cross-section of a transformer is shown schematically.

In case of a potential difference existing between turns, turn insulation is under voltage stress, but any dielectric acts as a condenser if it is subjected to voltage stress; therefore insulation between two adjacent turns, combined with parts of conductor surfaces facing each other, comprises a condenser. Therefore, each turn has capacitance to turns on its four sides.

As there is a continuous metallic path from any point of one turn to any point of the next, in case of potential difference existing between turns, some current will flow along the conductor and produce magnetic flux.

Thus adjacent turns are equivalent to parallel conductors with distributed constants. (Fig. 3.) If the phenomenon under consideration permits the distributed constants to be lumped, the turn can be represented by a concentrated inductance in parallel with concentrated capacitance.

Between core and turns, located on the inside surface of the winding, the capacitance effect is present on account of the potential difference between the parts. The same is true of turns on the outside surface of the winding and the tank.

Instead of taking a turn as an element of an equivalent network, some problems permit the use of much larger portions of the winding for this purpose.

Taking each separate coil (disk or pancake) as an element, a transformer equivalent network becomes as shown in Fig. 3.

The network shown in this figure lacks negative inductance links representing mutual inductance between various parts of the winding. They were omitted so as not to obscure the more essential members of the network. Therefore, actually this circuit is not equivalent to a transformer at all, but their behaviors under similar conditions, while quantitatively different, are sufficiently alike qualitatively to justify the use of such a network for getting a rough idea of the action of a transformer under transients. Whenever calculated results of a transformer's behavior are given in this paper, the effect of mutual inductance is properly taken into account.

B. SUSTAINED AND DAMPED OSCILLATIONS

1. Transformer Winding.

There is a similarity between circuit No. 3 of Fig. 6 and C of Fig. 3, and therefore it can be expected that circuit No 4 has a certain number of natural frequencies of current resonance, and "sandwiched" between them, natural frequencies of voltage resonance.

It should be expected, therefore, that a curve of variation of the reactance of circuit No. 4 of Fig. 3 with frequency should be similar to that of circuit No. 3.

The result of a sustained high-frequency impedance test on a large power transformer is shown on Fig. 8. It substantiates the above expectation. As the resistance component cannot be eliminated in test, the impedance at voltage resonance frequencies is not zero but is equal to the effective resistance of the circuit at that frequency.

Similar to circuit No. 3, at different voltage resonance frequencies infinite voltages, if resistance were absent, could be created between different points of the winding and its terminals, as well as between some of the points. The location of these points in the winding is different for different natural frequencies.

At frequencies of current resonance, no infinite voltages are possible, but the entire line voltage may appear across a fraction of the winding, this fraction growing smaller the higher the frequency.

In case the winding is uniform in its construction, the constants of the equivalent circuit will be uniform, and a definitite relation between the magnitude of succeeding voltage resonance frequencies can be expected.

Examination of the test results of Fig. 8 show that, if the lowest voltage resonance frequency is taken as the first, and the next for the second, etc., the frequencies increase very nearly as the second power of their order. The lowest natural frequency is considered as the fundamental, the higher frequencies as its harmonics; therefore the frequency of a given harmonic is equal to the fundamental times the second power of the harmonic order. This law holds for harmonics of a lower order. The frequency of harmonics of very high order is $f = \frac{n}{2\sqrt{LC}}$ where C is the capacitance of one end of the winding to another through the body of the winding as if there were no shunt capacity to ground. L is the inductance of the transformer and n is the order of the harmonic.*

2. Mechanical Analogy.

It is unfortunate that a mechanical analogy to even such a simple circuit as C is so complicated that it is far easier to visualize the behavior of the circuit than to see the analogy between the two systems.

A simple mechanical system, used below, to illustrate the phenomenon of transient voltages taking place in a transformer, is not therefore analogous to a transformer, but it is hoped that apparent qualitative similarity justifies its use.

The equivalent transformer circuit of uniformly distributed inductances and capacitances is comparable to a long, very elastic rod with considerable distributed mass.

If one end of the horizontal rod is fixed, and the other is made to oscillate in a vertical direction with a frequency natural to the rod, the latter will bend into a single arc, first in one direction and then in another. With very small amplitude of oscillation at the free end, the amplitude of the middle will increase with each half cycle, and soon will exceed many times the amplitude of the free end.

As a rod, in its bent position, forms an arc, the greatest tension will be at the two ends. The tension will decrease with the approach of the middle where it will be zero, as the elongation of the middle portion of the rod is zero.

Curve No. 1 of Fig. 9A shows the rod in its position of maximum deflection of an arbitrary amplitude, and curve No. 2 the distribution of tension along it at that moment. Curves Nos. 1 and 2 of Fig. 9B show respectively the voltage to ground of various points of the winding, and voltage between adjacent points along the winding of a power transformer. These values were obtained by test, under excitation of damped oscillations of the fundamental natural frequency of the transformer.

By doubling the frequency of oscillation (2nd harmonic) of the rod, the latter will be made to bend during its oscillation in S form, Curve No. 3 of Fig. 9A, and the tension along the rod will be as shown by Curve No. 4 on Fig. 9A.

Curves Nos. 3 and 4 of Fig. 9B show the voltage distribution obtained by test on the same transformer under excitation of damped oscillations of its second harmonic (frequency of which is about four times the fundamental).

The rod oscillating with a frequency of its third harmonic will bend in three halves of a wave, and the maximum tension of individual elements will again be shifted to the new parts.

A transformer winding excited with third harmonic

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*This is in agreement with formulas given in the A. I. E. E. paper by L. F. Blume and A. Boyajian, referred to above.

(the frequency of which is approximately nine times fundamental) will give a curve of voltage distribution which also forms three halves of a wave.

The nth harmonic therefore will "bend" the voltage distribution curve in nth halves of a wave, producing high stresses between adjacent elements of the winding at each node and high voltage to ground and line terminals at each anti-node.

Considering a transformer with isolated neutral, we may compare it with the same rod having one end oscillated and the other one free. It is obvious that at its fundamental natural frequency, it would bend into a half arc, into an arc and a half at its 2nd harmonic, and into $(2n + 1)$ halves of an arc at its nth harmonic. The waves of simultaneous voltages along the winding of an isolated neutral transformer follow the same law.

3. *Natural Frequencies Experienced in Service.*

Examination of voltage distribution test data, for various frequencies of damped oscillations given in the paper, reveal that no part of the winding is free of voltage concentration, as long as the natural frequency of oscillation of the circuit, external to the transformer, cannot be kept outside the range of natural frequencies of the transformer.

It so happens that the natural frequency of circuits connected to the transformer in service range between the same limits as do the natural fundamental and harmonic frequencies of transformers.

For instance, in case of an arc-over of an insulator on a transmission line, as the frequency of oscillation of line depends only upon distance between the transformer terminal and the arc it follows:

<table>
<tr><th colspan="13">Distance between the transformer and the arc in miles</th></tr>
<tr><td>0.10</td><td>0.20</td><td>0.4</td><td>0.8</td><td>1.0</td><td>2.0</td><td>4.0</td><td>8.0</td><td>10.0</td><td>20.0</td><td>40.0</td><td>80.0</td><td>100.0</td></tr>
<tr><th colspan="13">Natural frequency of oscillation in kilocycles</th></tr>
<tr><td>470</td><td>235</td><td>117.5</td><td>58.7</td><td>47.0</td><td>23.5</td><td>11.75</td><td>5.87</td><td>4.7</td><td>2.35</td><td>1.175</td><td>0.587</td><td>0.470</td></tr>
</table>

The fundamental natural frequency of transformers range from about 1000 to 60,000 cycles, and their harmonics of practical importance reach 750,000 cycles.

4. *The Cause of the Existence of Natural Frequency of Voltage Resonance in a Transformer.*

In studying the behavior of circuits Nos. 1, 2, and 3, we found that the voltage resonance occurs due to dissimilarity of the circuit elements. We also found that, if the reactances of all adjacent parts are alike at all frequencies, no voltage resonance can take place, and the voltage will distribute uniformly between all the elements (like $a\ b$, $b\ c$, $c\ d$ of Fig. 6).

Referring to network No. 4 of Fig. 3 of a uniform winding, we find that while inductances L, as well as series or internal winding capacitances C_w, are all alike, the capacitances C_s, from the surface of the winding to ground (the "shunt" capacitances) in spite of their being alike among themselves, make reactances of elements $a\ b$, $b\ c$, $c\ d$ differ from one another.

Should capacitances C_s therefore be removed, or their effect on the winding neutralized in some manner, the reactance of all the elements of network No. 4 will become one and the same; and as such a circuit is incapable of voltage resonance, voltage of all frequencies will distribute along it uniformly.

This conclusion is of fundamental importance.

From the charcteristics of circuits 1, 2, and 3, it also follows that once the effect of shunt capacitance C is neutralized, the voltage resonance can be eliminated not only by making all corresponding constants alike, but also by making them satisfy the relation:

$\frac{L_1}{L_2} = \frac{C_2}{C_1}$; $\frac{L_2}{L_3} = \frac{C_3}{C_2}$; etc.; then voltage at all frequencies will divide in direct proportion to inductances or inversely to capacitances.

5. *Effective Reactance of a Transformer.*

Similarly to circuits 1, 2, and 3, transformer effective reactance, as it appears to the external circuits, varies in magnitude and sign with frequency as curve of Fig. 8 shows.

The important fact is that the ranges of frequencies at which a power transformer acts as a capacitance increase with the frequency, while the inductance range grows narrow; therefore the sum of all capacitance ranges is much greater than that of inductance.

C. TRAVELING WAVES

1. *Infinite Rectangular Traveling Wave.*

a. Initial Voltage Distribution. It was shown above that a transformer, similar to circuits No. 2 and No. 3 of Fig. 6, has two parallel paths from one terminal to another, one a pure inductance and the other a pure capacitance. This means that at very high frequency the current will follow the capacitance path and at low frequency the inductive path.

A sufficiently steep front of a traveling wave corresponds to a quarter of a cycle of a very high frequency, and therefore its current will follow the capacitance path alone. This means that during the rise of terminal voltage from zero to crest of the wave no current will flow along the conductor, and the transformer will act as though every turn is disconnected from the adjacent one, that is, as a pure condenser. This state is called here the initial or electrostatic.

Referring to Fig. 19, it is seen that under such a circumstance, a transformer winding represents a mass of dielectric surrounded with metal surfaces of core and tank, and with conductors distributed throughout its cross-section. The cross-section of the line turn being very small in comparison with its distance from tank and core, will cause the lines of dielectric field to crowd in its neighborhood in a way similar to that shown. The rest of the conductors "floating" in this dielectric field will assume potential corresponding to the equipotential surface with which they coincide. This means that by far the greater part of line voltage will appear across the insulation in the neighborhood of line turn.

Instead of picturing the electrostatic field distribution during the impact of a steep front wave, the voltage concentration at the line end may be explained in the following manner.

The capacitance path of the transformer network contains a number of equal condensers connected in series and a number of condensers connected in shunt with different portions of the winding as shown in the equivalent circuit of Fig. 3. This sort of a condenser system is similar to that of a string of suspension insulators, where the capacity from one hardware to the next corresponds to the series capacity C_w between coils of the transformer and capacitance from hardware to ground corresponds to the shunt capacity of the transformer coils C_s to ground.

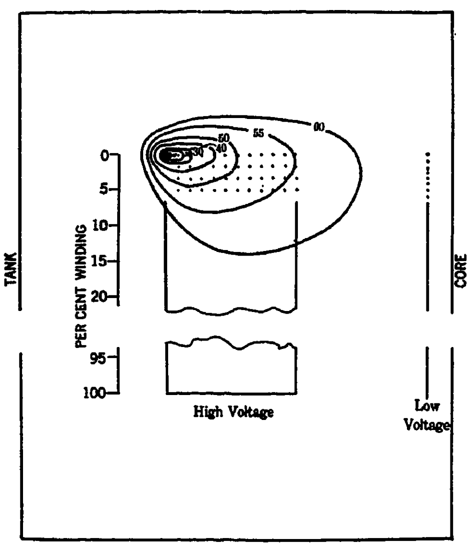


FIG. 19—INITIAL DIELECTRIC FIELD IN TRANSFORMER WINDING WITHOUT STATIC PLATE DUE TO STEEP WAVE FRONT

(Equipotential lines in per cent of applied voltage)

The fact of voltage concentration at the line end of a string of insulators is well known. It is caused by the presence of the shunt condensers, because the current of the shunt condenser must flow through the series condenser as shown on Fig. 3. Therefore, starting at the ground end each succeeding series condenser C_w carries more current than the preceding one. As all series condensers are alike it is obvious that the magnitude of their voltages will correspond to magnitudes of their currents.

The concentration of voltage will be the greater, the greater the shunt capacitance in comparison with series capacitance.

Tests, as well as calculations, show that in practise the voltage difference between adjacent turns at the line end, caused by the front of the wave, ranges between 10 and 30 per cent of the crest value of the line voltage and between adjacent coils (disks or pancakes), at the line end, may exceed 60 per cent. In case only a few coils comprise the entire winding these values become particularly high. If the length of the winding is taken for abscissa and voltage between the line terminal and various parts of the winding for ordinate, the curve of electristatic distribution will have a shape similar to those of Fig. 20, which were obtained by test on three power transformers.

b. Voltage Distribution during Oscillation. Assume for the time that the traveling wave is infinitely long and therefore its crest acts on the transformer network (Fig. 3) as d-c. voltage and thus chooses the inductance path for its current. The permanent or "final" voltage distribution will depend therefore on the ratios of the elementary inductances of the network (as with circuits 2 and 3). Under the assumption of uniformity of the winding all elementary inductances are alike, and therefore the final voltage distribution will be perfectly uniform and appear as the straight line *L G*, Fig. 21, while the initial distribution was as shown by curve 1 of Fig. 21.

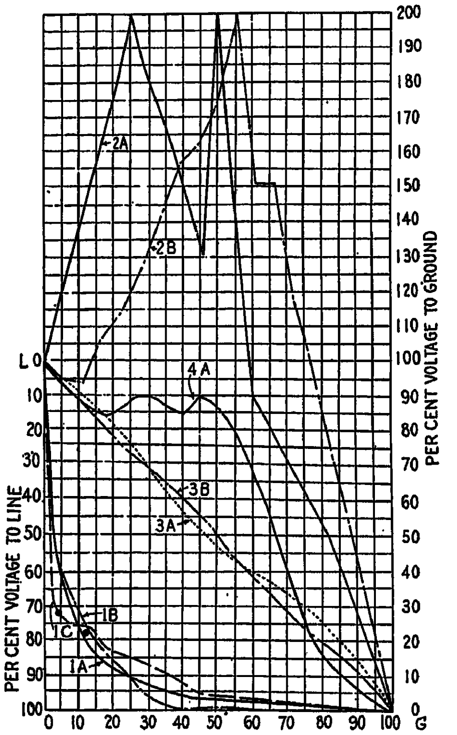


FIG. 20—VOLTAGE STRESSES MEASURED IN TRANSFORMERS UNDER TEST

Transformers tested: (A), core type; (B), core type; (C) shell type.

Non-Shielded

Curves 1A, 1B, 1C: Initial or electrostatic voltage distribution with traveling wave on transformers *A*, *B*, and *C* respectively (without entrance bushing). (Dots indicate measured voltages on *C* with entrance bushing).

Curves 2A and 2B: Envelope of voltages in transformers *A* and *B* produced by damped oscillations.

Curve 4A: Envelope of oscillations caused by traveling wave on transformer *A*.

……

Shielded (Non-resonating)

Curves 3A and 3B represent envelopes of all voltages caused by damped oscillations from 3 to 1000 kc. as well as steep front travoling waves on transformers *A* and *B*, respectively.

The difference in "initial" and "final" states is apparent and will cause a transient state which will consist of a number of sinusoidal oscillations of various frequencies and amplitudes superimposed on one another. As long as voltages at the terminals remain constant or change abruptly, these frequencies and amplitudes depend only on transformer constants, as it were with the simpler circuits.

It was found above with simple networks, that when subjected to infinite wave of steep front, they oscillate at all their natural frequencies of voltage resonance. Transformers follow the same law.

In the chapter on sustained oscillation it was pointed out that the transformer possesses a fundamental natural frequency and a number of its harmonics. At these frequencies, standing voltage waves are created along the winding composed of a number of halves of a wave. Their number corresponds to the harmonic order for transformers with grounded neutral.

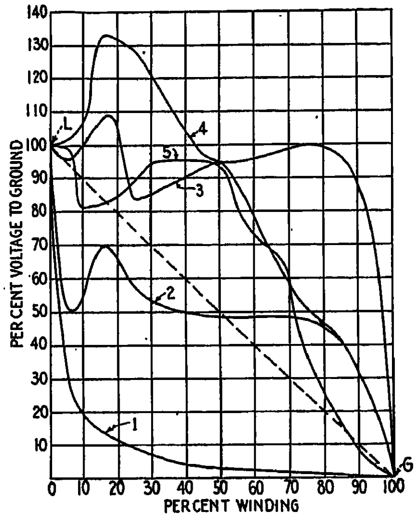


Fig. 21—Maximum Voltage to Ground Due to Rectangular Wave. Transf. A.

No. 1 initial distribution (tested)
No. 2, 3 due to waves of 1.5 and 4.6 long (calculated)
No. 4, due to waves infinitely long (calculated)
No. 5, due to waves 5 miles long (tested)

It was also shown with simple circuits that the axis of oscillation is the final state of a given point in the winding, while its amplitude is the sum of the amplitudes of all natural frequencies of the circuit.

The initial state is shown by curve No. 1 of Fig. 21. The final is a straight line *L G*. Therefore, a point of the winding 20 per cent from the line terminal will oscillate about a point of intersection of line *L G* with ordinate corresponding to the 20 per cent point. This part of the winding, therefore, starts to oscillate with an amplitude of 68 per cent of line voltage. As the oscillation contains various frequencies of different amplitudes, the rise above the point of oscillation may not be equal to the original amplitude, but may be somewhat higher or lower than that, depending upon the remoteness of the given point of the winding from the line terminal as well as the high frequency characteristics of the transformer.

The curves of Fig. 22 show results of calculation of the potential above ground of the middle point of the winding caused by a traveling wave. Curves 2, 3, 4, and 5 of Fig. 21 show the maximum voltage above ground attained by different points of the winding due to various traveling waves (Curve No. 5 is obtained by test with a wave approximately five miles long, all others are calculated).

It may be easier to visualize the behavior of a transformer under traveling wave excitation if it is again compared with the elastic rod (or here an elastic ribbon uniformly loaded with weights may better appeal to the imagination). If the ribbon were at first in a horizontal position with one end fixed, and the other end pulled upward perpendicularly to its length to a new position, with abruptness corresponding to duration of the front of the traveling wave, the ribbon would be bent in a shape very similar to the initial voltage distribution (Curve No. 1, Fig. 21).

Should the point of application of force be held at rest at its new position (which corresponds to the crest of the terminal voltage wave being held constant), the final state of the ribbon would be a straight line *L G* (similar to the final voltage distribution along the winding).

Thus the initial and final states of the ribbon are different, and therefore it must oscillate about its final state. During its oscillation different points will rise above their final position.

The distances of various points during their oscillation, from their original positions preceding the application of force, correspond to the voltage from winding to ground, while their distances to a line drawn from the new position of the "line" end parallel to the original position of the ribbon corresponds to the voltage between winding and line terminal.

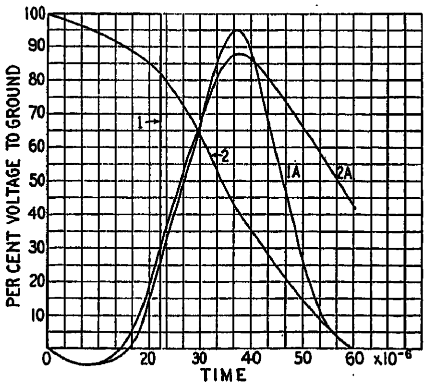


Fig. 22—Effect of Tail of Wave on Oscillation at Middle Point of Winding of Power Transformer

No. 1, and 2 voltage waves at transformer terminals
1 A, and 2 A voltage to ground at the middle of winding

2. Traveling Wave with Slanting Front.

a. Initial Voltage Distribution. From the discussion of initial voltage distribution due to rectangular front, an impression may be gained that a very steep front is necessary to create a purely electrostatic field in the winding, and therefore the concentration of voltage at the line end of the winding cannot be experienced in practise where rectangular waves do not exist.

It is clear that the higher the natural frequencies of the transformer are, the steeper must be the necessary front to produce a purely electrostatic field, since the transformer with higher natural frequencies responds more quickly to transient voltages.

In a given transformer, therefore, the concentration of voltage at the line end, due to the wave-front, is smaller for a longer wave-front, generally speaking.

This can be seen by considering an extremely long front which causes a correspondingly slow rise of terminal voltage. The current flowing into the transformer, being forced by this very slowly rising voltage, is of course of correspondingly low frequency and hence will choose the inductance path of the transformer equivalent network producing no "electrostatic" or "initial" voltage distribution, and therefore no voltage concentration.

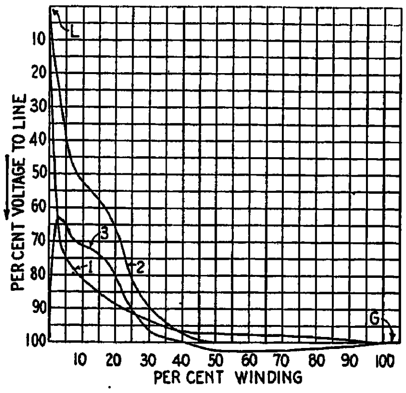


FIG. 23—EFFECT OF SLANTED FRONT ON INITIAL VOLTAGE DISTRIBUTION. (TRANSF. A.)

No. 1 for one microsecond front (test)
No. 2 for five microseconds front (calculated)
No. 3 initial voltage stress after bushing arcs-over at the crest of five microseconds front (calculated)

Here again, the analogy between the transformer and an elastic ribbon is obvious. Should the ribbon be pulled upward very slowly, it will remain straight during the entire transient.

As some traveling waves, recorded in service by the cathode ray oscillograph, had a front of five microseconds, the voltage produced by such a wave-front was calculated for the power transformer previously referred to and is shown as curve No. 2, Fig. 23. For comparison curve No. 1 obtained by test with a wave of one microsecond front is also shown. It shows that a 5-microsecond wave-front creates extremely high voltage concentration on the line end; therefore the wave front must be very long indeed, to cause no excessive voltage stress between coils and turns at the line end of the winding.

b. Voltage Distribution during Oscillation. Fig. 24 illustrates the effect of the length of the wave-front on voltages created by the oscillation following the initial state.

Curves 1, 2, 3, and 4 show the rise of voltage above ground at the middle of the transformer winding caused by voltage waves at the transformer terminals, as shown by curves 1a, 2a, 3a, and 4a respectively.

There is practically no difference between the maximums of curves 1 and 4, in spite of the great difference between the exciting waves 1a and 4a. It should be noted that 4a wave has a front of 30 microseconds.

3. Finite Traveling Wave.

The effect of a finite traveling wave on a transformer is again analogous to its effect on the ribbon.

Let it be assumed that the ribbon was suddenly pulled from its original position No. 1 to its new position No. 2, as before, but instead of holding the "line" end in position No. 2 indefinitely, after a certain lapse of time t it was suddenly pulled back. This would correspond to the application of a finite rectangular wave of t seconds long.

Let t be the moment when the entire ribbon is moving in an upward direction, the downward pull will bring the "line" end to the original position immediately, but some other points of the ribbon, remote from the line end, will still be moving in an upward direction, on account of their inertia, until the downward pull will be transmitted to them along the ribbon. The velocity of the propagation of this downward pull along this ribbon depends upon the mass and elasticity of the ribbon, and is smaller the greater either of these constants.

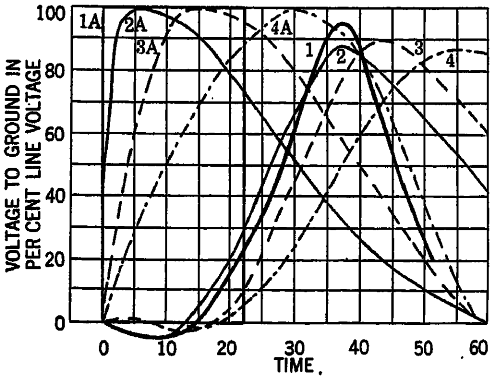


FIG. 24—EFFECT OF SLANTED FRONT ON OSCILLATION OF MIDDLE OF WINDING OF TRANSFORMER B

1 A, 2 A, 3 A, and 4 A—voltage at transformer terminals
1, 2, 3, and 4—voltage at the middle of winding

It is clear that if t (the length of the wave) is longer than the time t_1 that is necessary for a given point of the ribbon to reach its highest position, then it will reach it; but if it is shorter than t_1, then it may or may not reach it, depending on the remoteness of the point from the line end and the ratio of t_1/t.

As the downward pull was assumed to be very sudden, the method of super-position suggests looking at it as an application of an infinite negative wave of steep front at moment t. Such a wave will cause its own initial voltage distribution, which being negative must be subtracted from the potential that the respective points possessed just before t. This reveals that a traveling wave with steep tail, such as may be created by the arc-over of a bushing or a string of insulators, may set up "initial" stresses near the line end of the winding identical to those produced by the steep front, and even greater if the end of the wave should coincide in time with high potential to ground of the part of the winding near line end.

Curve No. 3 of Fig. 23 shows voltage distribution along transformer winding at the moment of arc-over of the transformer bushing. The arc-over is assumed to occur at the crest of five microseconds front.

Fig. 22 shows the change of voltage with time at the middle point of the winding of the same transformer and how it is affected by the shape of the tail of the wave. It shows that while the maximum voltage produced by a rectangular wave and a wave of the same crest, but with long tail, may be practically the same, the long tail delays the fall of the voltage to ground potential. These curves also show that internal voltage may be rising for a very long time after the potential across the transformer becomes zero. For instance, with a rectangular wave 22 microseconds long, at the time the transformer terminal voltage becomes zero, the voltage to ground of the middle point of the winding is 25 per cent of the crest of the wave and is steadily rising for the next 15 microseconds, until it reaches 98 per cent of the crest.

Part III

TRANSIENT VOLTAGES IN PRACTISE

The data on voltage transients experienced in service accumulated by various investigators can be summarized as follows:

1. *Lightning*

Taking the crest value of normal operating voltage from line to neutral as reference voltage E, the order of magnitude of transient voltages was found to be:

Lightning traveling waves:

Rarely.........	15 E
Often..........	10 E

Minimum steepness of the front, actually recorded by oscillograph five microseconds; estimated from Lichtenberg figures from 1 to more than 100 microseconds. The tail of the waves, recorded by oscillograph, exceeds 40 microseconds.

2. *Oscillation*

By switching, and other sudden changes of the network, moderately and highly damped oscillations are produced. They rarely reach 8 E, quite often 5 E, and very often 3 E.

3. *Damping*

The damping increases with magnitude of transient voltages, evidently due to corona loss.

4. *Upper Limit of Transient Voltages*

Laboratory experiments, completely substantiated by service records, show that on most of the existing transmission systems transient voltages reach values beyond the dielectric strength of the line insulation. Except near the center of long spans, therefore, the dielectric strength of the line insulation becomes a practical limit to the magnitude of transient voltages possible on the line. The effect of ground resistance of towers or poles, and of time lag in the discharge of lines to ground, must be taken into account and may raise the voltage limit.

5. *Conclusions*

In view of the behavior of transformers under transient voltage excitation, and the high magnitude of these voltages, the construction and the distribution of the insulation throughout the transformer winding is dictated not by stresses set up in the transformer, under the standard A. I. E. E. potential test, of low frequency, but by the law of distribution of voltages in the winding caused by various transients actually observed in service.

The following table shows the order of magnitude of voltage stresses produced between different elements of a transformer winding having one end permanently grounded. Column I gives stresses produced by the standard "Induced Voltage Test." Column II gives the dielectric strength necessary to withstand service transients produced in a uniform winding.

In the numerical example, Column III, the minimum values (in kv.) are given for a typical transformer to be operated on a 220-kv. system.

As a voltage of the same magnitude, but of a different order of frequency, has a materially different effect on a dielectric, all stresses and strength are expressed in values of equivalent 60-cycle one-minute test.

The voltages in Columns I and II are given in terms of effective value E, which is the normal operating line to neutral voltage of the transformer.

	I	II	III
High-voltage line end to low voltage and ground	3.46 E	3.46 E	440 Kv
Any other point between line and ground end p % away from ground end	p E	3.46 E Unless p is less than 10 %. Some reduction is permissible for p less than 10 %.	
Turn ins. of line coil . .	100 to 600 V.	0.63 E	80
Turn ins. near line coil	100 to 600 V	Gradually reduced from 0.63 E to 0.20 E	
Turn ins. in the main part of the winding . .	100 to 600 V.	0.20 E	25
at ground end	100 to 600 V.	0.35 E	44
Coil to coil at line end . .	0.04 E	1.3 E	165
Coil to coil near line end	0.04 E	Gradually reduced from 1.3 E to 0.65 E	
Coil to coil in the main part of the winding .	0.04 E	0.65 E	82
near ground end . . .	0.04 E	0.88 E	110

The difference between values of the first and the second columns is responsible for the difference in cost and volume of 20,000-kv-a. 220-kv. transformers referred to in the Introduction.

Part IV

PROTECTION AGAINST TRANSIENT VOLTAGES

1. *Non-Resonating Transformers.*

a. Theory. It was shown above that the cause of non-uniform voltage distribution along transformer windings was due to the presence of shunt capacitance (C_s in network 3 of Fig. 3), because due to this shunt capacitance, damped (or sustained) oscillations of a series of frequencies, applied to the terminals of such a network, cause different parts of it to get in voltage resonance and produce over-voltages shown by the curves (2 *A*, 2 *B* of Fig. 20).

If the shunt capacitance were to be removed, the circuit would consist of a chain of identical elements ($L\ C_w$). As was shown, such a circuit has no series natural frequency. It will distribute the voltage of all frequencies uniformly.

In the case of traveling waves, the same shunt capacitances were found to be responsible for internal over-voltages, as the "initial" and "final" voltage distributions were different because the charging current of all shunt capacitances was supplied through the series capacitances, causing concentration of voltage at the line end at the time of impact of the wave.

With shunt capacitances removed, there would be no shunt current to be supplied by any series capacitances, and as they are all alike the initial voltage would be uniform. The final voltage distribution also will be uniform, as all elementary inductances are alike, and therefore there will be no transient.

Thus with shunt capacitances removed, the voltage will be distributed between all elements uniformly, no matter what the shape of the terminal voltage wave is, whether it is ocillatory or unidirectional, of infinite frequency or direct current. But the removal of shunt capacitances is physically impossible. There must always be ground potential surfaces and surfaces of the winding facing them. If the entire winding should be screened from the ground by metal sheets interposed between the winding and the grounded surfaces, it would do no good as long as the potential of the screens is not fixed, and if they are connected to any part of the winding, including the line end, they would create a new system of shunt capacitance and the matter would be worse than before.

Fortunately, as J. M. Weed[2] pointed out in his A. I. E. E. paper on *Prevention of Transient Voltage in Windings* in 1922, the removal of shunt capacitances is not necessary; what is necessary, is the neutralization of their effect on series capacitances. The neutralization of their effect on series capacitances simply means prevention of the flow of their charging current through the latter.

Referring to *A* of Fig. 25 on the left side of the transformer equivalent network a new system of shunt condensers C_P is shown. This system, however, is not connected to ground as C_s are, but to the line terminal.

It is obvious that values of each capacitance (C_P) can be selected so that, with all inductances open circuited, and with the voltage uniformly distributed throughout the winding, the current through respective C_P to a given element will be exactly equal to the current through the shunt capacitance C_S of that element to ground. In such a case the current through all series capacitances will be equal and the voltage across them alike. This would hold true, of course, at all frequencies.

If all inductances now are connected as before, and the circuit subjected to impact of a rectangular traveling wave, the voltage distribution, as was shown, will be purely electrostatic.

As the protecting condensers are so proportioned that the voltage divides equally between the series capacitances, the initial voltage would be a straight line (as shown by *L G* of Fig. 21), the final distribution is also a straight line (as all *L*'s are alike),—therefore there can be no transient.

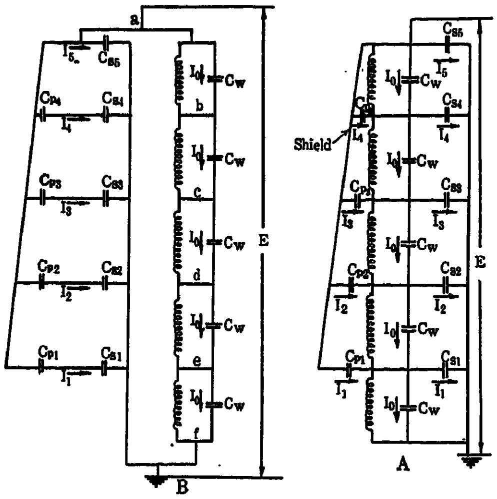


FIG. 25—EQUIVALENT NETWORK OF NON-RESONATING TRANSFORMER

As there is no exchange of current at any frequency between the two circuits at the point of the contacts *a, b, c, d, e*, we have a perfect right to think of them as two independent circuits connected in parallel—one consisting of all the shunt capacitances, and the other of series capacitances and inductances. (*B* of Fig. 25.)

The first circuit is a pure capacitance and therefore cannot have any voltage resonance frequency. The second circuit, consisting of like links $C_w\ L$, also can have no voltage resonance frequency. Therefore there can be no voltage resonance frequency for the two circuits connected in parallel. That is, the reactance-frequency curve of this circuit cannot cross zero line at any frequency, and must be similar to curve of elementary circuit No. 1 of Fig. 6 (see *Z* of Fig. 4).

b. Test Data. On Fig. 26 impedance *versus* frequency curves are given. They were obtained by test with sustained oscillation on a large power transformer which was mentioned before. Compare this with Fig. 8 obtained on the same transformer before the effect of its C_w's was neutralized by means of C_p's.

Extensive tests were made on some power transformers with damped oscillations of frequency ranging from 3 to 1000 kilocycles, as well as with impulse waves of various shapes.

The results as summarized on Fig. 20, curves 2*A* and 2*B* show the maximum voltage to ground produced by damped oscillation of various frequencies on transformers *A* and *B* respectively before they were equipped with shields, and curves 3*A* and 3*B* after they were so equipped.

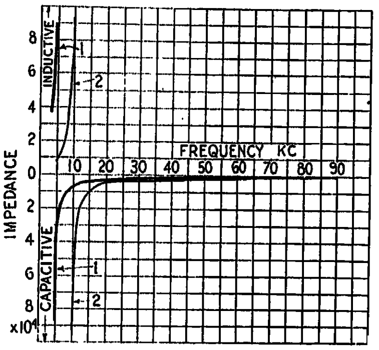


Fig. 26—Impedance-Frequency Curve. Results of Test on Non-Resonating Power Transformer B. 1, L. V. Open. 2, L. V. Shorted

Curves 1*A* and 1*B* show the initial voltage distribution of the same transformers at steep wave-front. Initial voltage distribution, after they were equipped with shields, is shown by the same curves 3*A* and 3*B*. Curve 4*A* shows the maximum voltage to ground due to a traveling wave obtained on transformer *A* before it was shielded, which became curve 3*A* after it was shielded.

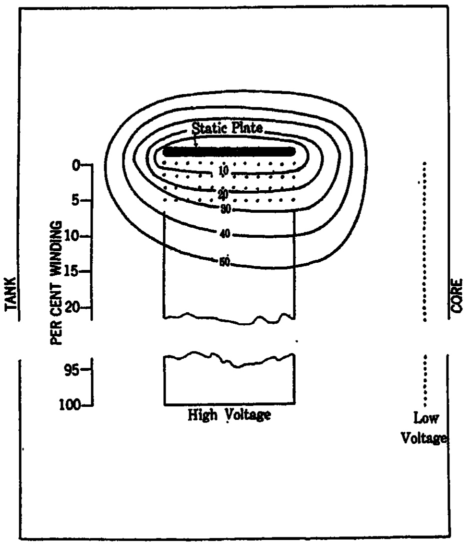


Fig. 27—Dielectric Field Due to Steep Wave Front in Transformer Winding Equipped with Static Plate (Calculated)

The difference in stress, as shown by these curves, in the new and the conventional type of transformer, is so great that it seems unnecessary to make any additional comments in its regard.

If the behavior of an ordinary type of transformer can be likened to that of a very flexible rod, or even a ribbon, the behavior of the "non-resonating" transformer is analogous to that of a perfectly rigid rod, which with one end fixed and the other oscillating does not bend at all, no matter what character of oscillation is applied.

In practise, system of condenser C_p is simply a smooth metal surface or a small number of such surfaces properly spaced and insulated from ground and the windings, and connected to the line terminal of the protected winding. These surfaces are called "shields."

They are located on the outside of the high-voltage windings, so as not to interfere with its usual construction. Fig. 29 shows a power transformer of the non-resonating type. The non-resonating type so far has been adopted for more than half a million kv-a. of transformers.

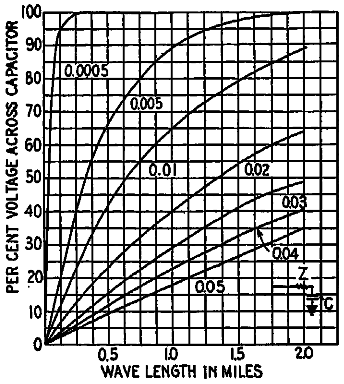


Fig. 28—Effect of Capacitor in Shunt with Transformer on Maximum Voltage of Traveling Waves of Different Lengths (Capacitances are Given in Microfarads)

2. Static Plate.

A device, known as the static plate, which has been in use for many years, has a shape usually similar to a coil of the winding that it protects. It is a continuous conducting surface with smooth edges placed adjacent to the line coil of the winding and connected to the line. It is insulated with varnish cambric cloth wrapped around it.

The object of this plate is to reduce the concentration of the initial voltage due to steep wave-front on the line end of the winding. It does this by virtue of its relatively large surface, as contrasted with that of one turn, thus changing the electrostatic field set up by the front of the wave from the one shown on Fig. 19 to that shown on Fig. 27.

From examination of Figs. 19 and 27 it will be clearly seen that the voltage concentration at the line end, as well as the beneficial effect of the static plate, depends entirely upon the geometric proportions of the winding.

It is clear that while the static plate reduces stresses materially near the line end, it cannot prevent oscillations and voltage resonances of the winding.

3. Effect of the Transformer Bushing.

It has sometimes been stated that the capacitance of the transformer entrance bushing has an appreciable effect in slanting the wave-front and decreasing the voltage concentration within the winding.

The capacitance of the bushing is in shunt with the transformer, and is of the order of 0.0002 microfarads, which corresponds to the capacity of 0.15 of a mile of transmission line. This value is so negligibly small that it can have no effect on the transformer behaviour.

Curve No. 1*C* of Fig. 20 was obtained by tests on a power transformer which, in this case, happened to be of the shell type without any bushing. It was subjected to the impact of a traveling wave transmitted to it over a circuit having a surge impedance of 500 ohms. The tests were then repeated with the bushing in place and connected, and the points marked by dots were obtained. These tests show that the bushing had no effect on the voltage distribution. It did not slant the wave-front sufficiently to cause any reduction in initial voltage distribution.

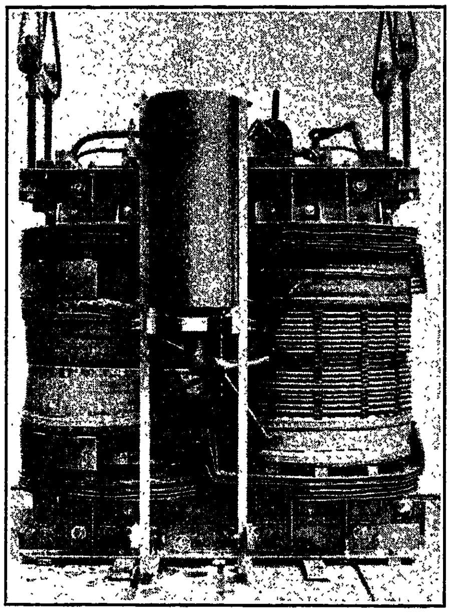


FIG. 29—NON-RESONATING 20,000-KV-A. 132,000-VOLT TRANSFORMER

A condenser used for protection of a transformer against transient voltages, must be so large that it slants the wave-front to an amount sufficient to reduce the initial voltage concentration appreciably, which, as has previously been shown, requires a wave-front greater than some 40-50 microseconds, and it must absorb the entire energy of the wave without requiring a voltage in excess of the breakdown of the apparatus.

Fig. 28 gives the relation between the size of the condenser and the maximum voltage that waves of various lengths will produce across it. The curves were obtained with surge impedance of transmission line assumed to be 500.

Higher voltages will be obtained with smaller surge impedance.

4. Inductance Coil.

a. Choke Coil. In the light of the relation of various circuits discussed in the paper, it appears that a choke coil alone, in series with a transformer, may do more damage than good.

The choke coil, while its reactance also varies with frequency on account of its small capacitance, acts as an inductance up to relatively very high frequencies (several hundred kilocycles).

Consider a transformer, having a high-frequency reactance similar to the one shown in Fig. 8, connected to a transmission line through a choke coil.

At a frequency of 45,000 cycles, the transformer acts as a condenser, while the choke coil still acts as an inductance. Should the line arc to ground about a mile (0.96 of a mile) away from the transformer, the line will oscillate at 45,000 cycles (see table above), and the voltage across the transformer will rise above the high-frequency voltage applied across the transformer and the choke coil. Should 45,000 cycles happen to be the natural frequency of capacitance of the transformer and of the inductance of the choke coil combined, the voltage across transformer may become many times the line voltage. And if this frequency, in addition, is near one of the natural frequencies of the transformer (in the case of transformer of Fig. 8, 47,000 cycles is its second harmonic), very high internal voltage may be created with the amplitude of the line disturbance of comparatively small magnitude.

The danger of resonance is often underestimated due to a faulty impression that for the creation of overvoltages of dangerous magnitude the frequency of applied voltage must be exactly equal to the natural frequency of the circuit under consideration.

Examination of reactance *versus* frequency curves of transformers and other circuits given in this paper shows that in many cases the reactance curve approaches and leaves the zero line very gradually, which means that a correspondingly wide band of frequencies produce a very substantial rise of internal voltage. In our example the choke coil still would cause a substantial increase of transformer transient voltage over line transient voltage, with variation of the distance between the transformer and the point of line fault, or variation of the natural frequency of choke coil plus transformer of some 40 per cent.

In case a traveling wave strikes a choke coil, in series with a grounded transformer, initially a considerable part of its voltage will appear across the choke coil, while finally practically the entire voltage will appear across the transformer. Therefore, an oscillation will take place.

As the natural frequency of the point of the contact of the choke coil and the transformer is generally very high, it means that transformer voltage will rise above the line voltage unless the traveling wave is very short (see Fig. 17). For instance, with the natural frequency being 45,000 cycles, the traveling wave must be shorter than four microseconds (or 4000 ft.). Most of the waves, recorded in service, were found to be considerably longer.

b. Current Transformer. A current transformer acts very much like a choke coil on account of its inductance, and therefore should always be bridged on the line side with a "by-pass" in the form of a gap with resistance in series or by other suitable arrangement.

c. Current Limiting Reactor. The current limiting reactor, being similar to a choke coil, also may cause rise of transient voltage across the transformer terminals. Very interesting tests, substantiating the above, were described by F. W. Peek[4] in an A. I. E. E. paper on *Effect of Transient Voltages on Dielectrics—III* in June 1923.

It was recommended there to shunt reactors with some voltage limiting device, as for instance carborundum resistors.

For simplification of the discussion, the effect of the surge impedance of a transmission line, resistance, and dielectric loss of a transformer on transient phenomena within the transformer, was neglected in this paper. As the surge impedance of transmission lines is measured in hundreds of ohms, and that of transformers in thousands of ohms, neglecting the former does not affect the phenomena within limits of practical interest. The theoretical, as well as experimental data so far in possession of the writer, indicate that very little help in reducing the transient voltage stress can be obtained from the damping characteristics of the transformers. This is particularly true for transformers of large sizes. On account of the limitation of space, the damping effect of losses could not be discussed in this paper at length.

I wish to acknowledge here that in the preparation of this paper the interest and assistance of Mr. F. F. Brand proved to be invaluable.

The valuable assistance of Mr. J. H. Hagenguth in the preparation of data is also recognized.

Discussion

J. F. Peters: This paper is a very interesting review of the subject of voltage distribution within transformers. When it comes to applying the fundamentals to an actual transformer it is necessary either to use a rather rough approximation or to introduce other conditions that may be a serious hazard. The windings of all high-voltage power transformers consist of a stack or stacks of pancake coils in some form or other. These coils are connected in series either as indicated by Fig. 1 or by Fig. 2 herewith. If connected according to Fig. 1 it is obvious that Mr. Palueff's Fig. 3 does not give a correct representation of the capacitance from windings to core and tank, and a shielding on the outside of the coils cannot offset the capacitance to core. If the coils are connected according to Fig. 2, his representation in Fig. 3 is correct, but it is then necessary to carry the series connections through the oil ducts between coils where the space is necessarily limited and therefore produce somewhat of a hazard.

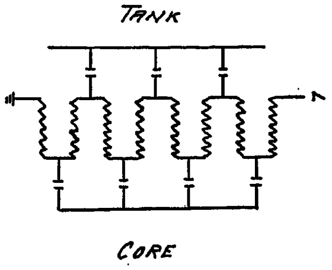


Fig. 1

In any structure that has a number of electrostatic capacitances connected in series with parallel capacitances connected in at various points along the structure, the voltage distribution will not be uniform throughout the structure. This principle has been recognized and applied to line insulators. The principle applies equally well to stacks of coils. The distribution of voltage along a string of insulators is a function of its dimensional ratio, length to diameter. Long strings have poor distribution. The same conditions exist in transformer windings. Where the stack is long and narrow the initial voltage distribution is relatively poor while where wide coils are used, thus giving a short wide stack of coils, the distribution is relatively good without the addition of shields. Where the initial distribution is relatively good the amplitude of oscillation between initial distribution and final distribution is relatively small.

Since transformers of different construction differ tremendously in physical proportions one may find it highly desirable to provide shields to improve the distribution in some transformers where in other transformers it may be quite inadvisable to provide shields.

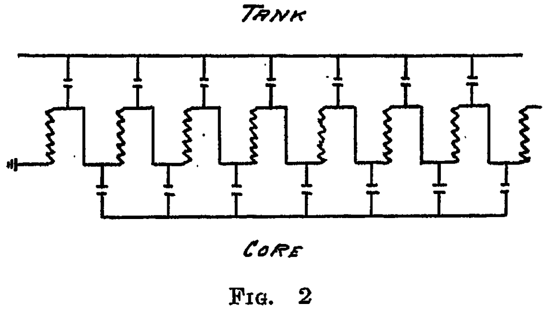


Fig. 2

F. F. Brand: I think everyone will admit that the transformer is in general a highly reliable device, but a small percentage of failures still takes place. With the growth of high-voltage networks, and the increasing power concentration in extra high-voltage systems, a reliability practically equivalent to that for metropolitan service is now required in even the highest-voltage systems so that transformers of even greater reliability are now necessary.

It is probably safe to say that the majority of transformer failures are caused by the transformer being subjected to transient over-voltages.

The author in his paper has given a fairly complete analysis of what happens in the ordinary transformer winding when it is subjected to transient over-voltages such as are caused by lightning or switching surges.

A practical transformer designer realizes that voltage failures are caused by oscillations within the transformer winding. These cause high local voltages to appear, by the concentration of voltage on the end turns when lightning strikes the transformer. In a few rare cases actual breakdown of the insulation to ground occurs on the end of the winding caused by excessive voltage applied to the transformer or breakdown to ground in the main body of the winding caused by the oscillation building up a high voltage to ground.

The difficulty with which the transformer designer is confronted in the ordinary transformer is to know the value and position of the local voltages within the transformer winding, because these depend on two things, first,, the constants of the transformer itself, and secondly, the frequencies as well as the voltage of the transient at which the transformer is subjected. Over these latter, the designer has no control. Any means, therefore, which would prevent oscillations from building up within the transformer winding at once eliminates uncertainty as to the amount and distribution of insulation throughout the winding which is necessary to withstand service conditions. All that is necessary is to know the maximum voltage and to some extent the time during which the transformer will be subjected to this voltage.

The object of the paper is not only to describe and analyze the voltage distribution throughout the transformer winding under widely varied conditions of wave form and frequency of transients, but to describe a new type of transformer winding which is non-oscillating. It is perhaps to be regretted that the author did not explain the design of this new and much improved type of transformer more fully.

Any practical design of transformer which can reduce local stresses within the winding in the order of 10-to-1 to 100-to-1, is a tremendous step forward in the design of such apparatus, and must result in greatly improved reliability. At once, it places the operation of the transformer under transient conditions beyond the realm of conjecture and makes the customary factory low-frequency test a real measure of the ability of the transformer to withstand lightning and other transients.

There is a relation between the 60-cycle strength of insulation and its strength under very short-time transients, but it is somewhat dangerous to consider this ratio as applied to completed apparatus since the voltage distribution throughout the winding of such apparatus under transient conditions may differ widely from that under low-frequency conditions, and the stress which various parts of the apparatus winding may be subjected to under transients will bear no definite relation to the stresses under low-frequency operation.

For a transformer operating with a neutral grounded, it has been customary in some cases to use graded insulation. It is obviously dangerous to grade the insulation appreciably towards the neutral if the design of the transformer permits oscillations to occur, which might raise the voltage to ground or to the low-voltage winding, at some intermediate point in the winding, very much above the proportion of the terminal voltage stress which the intermediate point has been designed to stand. Certainly if the voltage at intermediate points of the winding can rise higher than the terminal voltage, graded insulation in the ordinary transformer is unsafe if the transformer is to be subjected to high transient overvoltages such as are common on overhead lines.

Graded insulation has been successfully used in testing transformers, but in these the surges which are set up are initiated at the voltage at which the testing transformer is operated, and probably do not give voltages much above the normal rating of the transformer.

With power transformers, however, subject to lightning, the transient overvoltages may be many times normal voltage to ground that the transformer is designed to withstand at low frequency. It is true that the time lag of the insulation in resisting short-time transient voltage comes into play. This may be sufficient to prevent failure of a transformer having uniform insulation to ground through the winding unless the terminal voltage is really excessive. It may not be sufficient to cause failure if the insulation is graded.

The ability to produce a transformer which has a straight-line voltage distribution from line terminal to ground no matter what transient it is subjected to, thus places graded insulation in a class of apparatus which can be safely used, since the value of the stress to ground at any point in the winding is the same proportion of terminal voltage during factory test and during service.

That the means of rendering the transformer non-oscillating can be applied to a standard type of winding which has proved highly reliable in service is indeed fortunate, since it eliminates the possibility of the new type of transformer being considered experimental. As a matter of fact, the shielded arrangement can be added to a standard disk-coil transformer without any change in the winding at all.

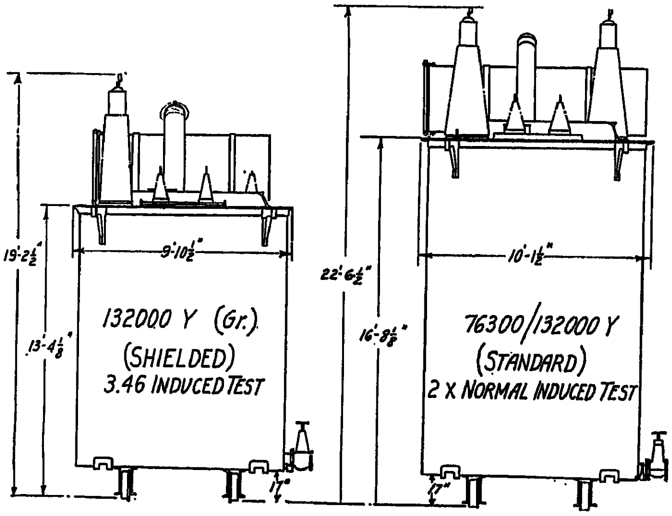


Fig. 3

With the new-type transformer, it is only necessary to set up some protective means to limit the value of the voltage applied so that the insulation of the terminal and of the winding to ground is safe under these conditions. Other than this, lightning or switching surges have no effect on the transformer.

Philip Sporn: In connection with the practical results of the problems discussed by Mr. Palueff, it may be of interest to see just how some transformers designed in accordance with the principles brought out in the paper appear.

Fig. 3 herewith shows the outlines of two transformers installed in the same station having exactly the same rating and being used for exactly the same purpose. The one on the left shows a transformer rated at 21,000 kv-a. single-phase, of the shielded design. The one on the right shows a transformer of the same rating non-shielded. The first is designed for grounded-neutral operation, and the second for either grounded or isolated operation. It is interesting to know, however, that the induced voltage tests on the first transformer were 3.46 normal, and on the second, only twice normal. Based on that, the estimated impulse strength of the two transformers is the same, but both the designers of the transformers and we with them believe that actually the shielded transformer will stand a considerably higher impulse than the non-shielded transformer. Fig. 3

shows the great difference in size between the two units. These transformers have been installed at the Philo plant of the Ohio Power Company.

In a paper which I presented before the Institute last year on the question of rationalization of transmission system insulation strength, I pointed out what Mr. Palueff has brought out forcibly in his conclusions, that the present standards of the Institute are not being followed by the manufacturers in the construction of apparatus involving high voltages, particularly in specifications with regard to high-voltage insulation, and that before a definite and rational solution of the problem can be obtained, it is essential that the 60-cycle strength and test be divorced from and made independent of, the impulse strength and test.

The investigations and work carried out by Mr. Palueff make such a step much more practicable and feasible than seemed possible at that time.

F. W. Peek, Jr.: Mr. Palueff's paper presents the results of a transformer research that has been carried on for a number of years. During this research it was discovered that lightning impulses or transients applied to the terminals of a transformer may cause very high voltages at any point in the winding. For example, the full lightning voltage applied to the transformer terminals may appear across a very small portion of the winding on the ground end of the transformer. This, during the early work, seemed remarkable because the transformer was generally thought of as a very high inductance and high inductance was supposed to prevent the penetration of high-frequency or steep waves. As a matter of fact the transformer does consist of a high inductance and the current flowing through this inductance causes even distribution of voltage along the winding for operating frequencies and for very low-frequency transients.

However, there is also in combination with this inductance, capacitance. This capacitance plays a negligible part in the distribution of voltage at low frequency. At high frequency, however, the capacitance becomes the controlling factor in voltage distribution. In all conventional transformers, of either shell-type or core-type design, the arrangement of the capacitance is such as to cause a very uneven distribution of voltage,—generally a very high concentration of voltage on the line end. This uneven capacitance distribution is caused principally by the capacitance of the windings to the grounded core and tank.

It follows that when a steep impulse is applied the inductance at the first instant acts exactly as an open circuit. The capacitance is the controlling factor and causes the voltage to pile up on the line end. Eventually as time goes on this uneven distribution of voltage must even up and reach the steady-state condition as determined by the inductance. However, the transition from the initial distribution to the steady-state distribution cannot occur instantly but must do so through an oscillation. A few microseconds after the impulse is applied the voltage swings past the steady-state line and reaches a maximum above the line in excess of the distance it was below the line at the first instant. This causes a very high voltage concentration in some other part of the winding which makes the insulation problem extremely different in the conventional transformer.

An entirely new transformer, a *non-resonating transformer*, has solved this problem. The prevention of these transient localized stresses has been accomplished in a remarkably simple manner. The electrostatic voltage distribution has been made to correspond to the inductance voltage distribution. When an impulse is applied it instantly takes the steady-state distribution because it instantly takes the capacitance distribution and this corresponds to the inductance or steady-state distribution. Therefore there can be no oscillation but each section of the winding takes its share of the applied transients. The voltage distribution is the same for all frequencies, all transients, and all impulses and corresponds to the 60-cycle distribution. To accomplish this it has not been found necessary to go to a freak design. In fact, all the features of the well tried standard designs have been retained.

It may well be asked if these extremely high and dangerous voltage concentrations can be caused by transients that are likely to occur in practise. The very beautiful oscillograms that Mr. Palueff has secured on large power transformers connected to transmission lines answer this quite decidedly. These Dufour cathode ray oscillograms show dangerous voltage distributions not only for lightning impulses of very steep wave front but also for lightning voltages of slanting front and extremely long duration. They also show that the shield reduces local stresses in many cases as much as 100 to 1.

One very important point in favor of the non-resonating transformer is that the 60-cycle test is a true measure of the insulation value of the transformer under the lightning conditions incidental to operation. This follows only in this type of transformer because the lightning distribution and the 60-cycle distribution correspond. This, of course, is not the case in the conventional transformer.

Mr. Palueff's paper and cathode ray oscillograms show that a high series inductance coil is more likely to be a hazard than a help during lightning disturbances unless the inductance is shunted by the proper value of resistance. This information, which is of great importance to operating engineers, while not new, is very definitely confirmed by the oscillograms.

V. M. Montsinger: I wish to discuss this paper from a somewhat broader standpoint than that of the distribution of voltage within a transformer when subjected to lightning. This phase of the question is covered very well in the paper itself. I will discuss the various steps in our recent progress towards the mastery of lightning as relating to protection against it under service conditions and at the same time point out how the non-resonating transformer fits into the trend of the times and helps to complete the picture.

Our somewhat more than usual progress made within the past two or three years can be attributed to the attack on three problems, namely, (1) the study of lightning voltages in the field by means of the surge-voltage recorder and the cathode ray oscillograph; (2) the proper correlation of transformer insulation with the factors that govern or limit the lightning voltage received by the transformer under service conditions; and (3) the building of a practical transformer that eliminates unpredictable and dangerous internal high-frequency voltages.

It is not necessary to comment on the progress made in the first problem as the status of this work is well known to Institute members through the series of recent papers.

As to the second problem, let us go back a few years and see what our practise has been with reference to the design and application of transformers from the standpoint of lightning.

Previous to 1925 the A. I. E. E. Rules' only requirement of the amount of insulation to be used in a transformer was that it withstand one-minute 60-cycle test of twice the rated circuit voltage plus 1000 volts. This was supposed to be entirely satisfactory under service conditions and I will add it was generally satisfactory for the line insulation used up to this time. A few operating companies used lightning arresters, but certainly not all.

The above refers, of course, to transformers used on isolated or ungrounded systems. When it became the general practise of large operating companies to ground solidly the neutral of the system, the demand grew that transformers operating on this kind of a system need not be insulated for twice the rated circuit voltage but for something less. At this stage there was serious argument as to whether the transformer should be tested for twice line to neutral voltage or some voltage between this value and twice line-to-line voltage. This argument must have been based on the premises that there was no danger from lightning voltage but that twice normal operating voltage was the sole criterion. It was finally agreed to take a value half way between twice line-to-neutral and twice line-to-line, which is equal to 3.46 × line-to-neutral voltage. Taking the average of 2 and 3.46 gives us 2.73,—hence, the derivation of the well known 2.73 transformer.

The A. I. E. E. Rules in 1925 adopted the 2.73 transformer as standard for solidly grounded systems. No reference was made to the amount of adjacent line insulation used although by this time it had become quite general practise to over-insulate some transmission lines to cut down outages.

The other point which shows that the section adopted by the Institute in 1925 on reduced and graded insulation was not quite correct is that, according to these rules, a transformer with solidly grounded neutral and where the only dielectric test is the induced voltage test. can be built with the insulation between the high-voltage windings and low-voltage winding graded strictly proportional to the 60-cycle or induced voltage stress along the coil stack. It is well known and has again been shown in Mr. Palueff's paper that this is not a safe practise to follow where lightning is encountered.

It can be said in defense of the adoption of the 2.73 design, however, that the first grounded high-voltage systems of the 220-kv. class in this country were in localities where lightning voltage is very infrequent, namely, in the Pacific Coast States. The 2.73 transformers which have been in service on these systems for several years have given a very good account of themselves, largely, no doubt, because of the absence of severe lightning rather than because the neutral was grounded.

Experience has shown that while this practise is satisfactory when applied to sections where lightning is not severe, it is not always safe when applied to sections where lightning is very severe. As a matter of fact there is no reason why it should be safe unless the transformer insulation is properly coordinated with the line insulation, because it is an entirely wrong application and one that was not intended when the 2.73 transformer was first introduced into practise.

Standard Rules for coordinating the transformer insulation with the line insulation used has been under preparation by the A. I. E. E. Subcommittee of the Electrical Machinery Committee during the past two or three years, and at the present time this problem is pretty well in hand. At least, we are on the road to a better understanding of the proper relationship between expensive high-voltage apparatus and the factors that limit lightning voltages received by the apparatus.

So far then we have made two important steps, namely, (1) obtained a better knowledge of what lightning is, and (2) learned how to apply the apparatus more intelligently to conditions as existing in service. This, of course, has no bearing on what the lightning will do after it is received into the apparatus. It has been well known for years that dangerous internal voltages far in excess of the 60-cycle stresses can be built up within the windings. The big problem, however, was how to prevent it in a practical way. In other words, the next problem was to make the lightning behave in a well ordered manner. Once this was done the design of transformers could be placed on a better basis.

The development of the non-resonating transformer, therefore, will not only make it possible to arrange the insulation in a more scientific manner but will enable us to demonstrate for the first time by low-frequency tests the ability of a transformer to resist high-frequency voltages found in service, something which is not possible with the ordinary transformer. It can readily be appreciated that the non-resonating transformer fulfills a long felt want. It marks one more important step in our progress.

L. F. Hickernell: The Consumers Power Company, Michigan, has in operation approximately 70 transformer units connected to 500 mi. of metallically connected 140-kv., 60-cycle steel-tower transmission line and 18 units connected to 250 mi. of 140-kv., 30-cycle line. The line insulation for the most part consists of 10 units spaced 4¾ in. These transformers have all been purchased according to A. I. E. E. Standards in so far as dielectric tests are concerned.

On the basis that the impulse strength of the transformers is 10 times their rating, the theoretical impulse flashover strength, based on the Pittsfield wave, of the transformers, bushings, and line insulators is as follows:

	Impulse flashover kv.
Transformers	1400
Bushings	1390
Insulators	1200

To the best of my knowledge, there has not been a single case of failure of the insulation from line to ground due to lightning. There have been, however, several failures of the turn insulation.

These experience data are offered for the consideration of the Transformer Subcommittee of the Electrical Machinery Committee who have recently submitted a proposal on the "Relation of Transformer Insulation to Line Insulation" for inclusion in the A. I. E. E. Standards.

Judging from our experience, I am inclined to believe that line-to-ground insulation is only a small part of the story and that further attention and study should be directed toward the turn insulation, revising, if necessary, the present induced test value now contained in the Standards.

W. M. Dann: (communicated after adjournment) It is well known that in a device like a power transformer, having in effect a number of electrostatic capacities in series and a number of parallel capacities connected in at various points, the voltage distribution following a surge will not be uniformly distributed throughout the windings. However, the actual distribution will naturally be different in different designs of transformers because of different arrangements of condensive elements.

The phenomenon of uneven voltage distribution has been recognized in the case of high-voltage line insulators made up of a number of disks and it seems to me that the practise in shielding long strings of line insulators constitutes an analogy which is of interest in studying the shielding of the windings of a high-voltage transformer.

The voltage distribution over a long string of insulators without grading rings is not uniform, the end units taking more than their proportionate share of the voltage. This is the reason that many people advocate the use of grading rings on long strings of insulators because they actually do help to improve the voltage distribution. However, no one proposes to use grading rings on short strings of insulators where the voltage distribution is inherently better, because the distribution is satisfactory and the addition of rings would be not only unnecessary but also objectionable.

Mr. Palueff makes the point that for very high frequencies the voltage is distributed over a circuit made up of inductances and condensers (for example, a transformer winding) just as if the inductances were not present. The analogy between strings of insulators of differing structure and stacks of coils in transformers of differing structure is. therefore, a true one and stacks of coils in transformers of different structures will be subject to the same differences in voltage distribution that are brought about by structural differences in strings of insulators.

The longer and narrower the stack of coils the poorer will be the voltage distribution from one end of the stack to the other. When the stack is quite long compared to the width of the coils, as shown in Fig. 2 of Mr. Palueff's paper, it may become desirable to introduce the otherwise objectionable grading rings just as in the case of long strings of insulators, in order to improve the voltage distribution.

In structures of the shell type where the coils are much wider and where the length of the group of coils is very much less in proportion to the width, just as in the case of the short string of insulators, a shielding structure would be not only unnecessary but objectionable. This has been fully demonstrated in tests within the past few years.

Transformers in general are already much more complicated than they used to be and it would seem that voltage distributors or shields ought to be used only when they are necessary to bring about a reasonable voltage distribution.

K. K. Paluef: As Mr. Peters pointed out, the connection of the inside turn of a coil to the outside turn of the next, as shown on Fig. 2 of his discussion, is indeed a hazard. This was recognized by us about twenty years ago, and we have not used it since that time. It was replaced by the so-called twin-section coil with connections between coils, as shown on Fig. 1 of Mr. Peters' discussion.

Mr. Peters suggested that, while shields may bring the potential of the outside turns of the stack to proper values, they cannot do that to the inside turns, which are removed too far from the shields to be affected by them. This is correct only if the coils are very wide, say 10 or 14 in., such as are used in the transformers with which Mr. Peters is most familiar. This width is one of the reasons that make it extremely difficult to produce a non-resonating transformer of shell type construction.

With concentric windings, the coils are narrow and the shield consists not only of a metal surface on the outside of the winding, but also of a static plate at the line end of the stack (see Fig. 1 of the paper), where the effect of which Mr. Peters was speaking is greatest. These features are responsible for the test results shown by curves 3*A* and 3*B* of Fig. 20.

I fear that Mr. Peters is attaching too much importance to the influence of the interconnection of coils on transient voltage concentration. While the interconnection of coils has a decided effect on the voltage distribution produced by the oscillation in the winding ensuing after the impact of a traveling wave, it has very little effect on the "initial" voltage distribution, during which the transformer acts practically as if the turns were disconnected from each other. In such a case the windings, shown by Figs. 1 and 2 of Mr. Peters' discussion, act very similarly.

I am glad to find that Mr. Peters' impulse tests of voltage distribution in the shell-type transformer agree with the results of my tests on transformers of the same type, as shown by Curve 1*C* of Fig. 20. It is unfortunate that Mr. Peters did not show us the voltage to ground that he found for various points of the winding during these tests, as these values are of vital importance to a transformer designer. However, his results are sufficient to enable one to state (in accordance with the principles described in the paper) that the voltage to ground, of at least half of the winding near the ground end, did rise far above its normal share of the applied voltage, (*i. e.*, above the straight line of uniform distribution).

This statement is based on the following considerations:

a. Mr. Peters' tests show a severe voltage concentration at the line end;

b. This means that the initial voltage distribution is of a character shown by Curve 1 of Fig. 21;

c. The final voltage distribution, of course, is a straight line *L G* of the same figure;

d. The difference between the initial and final voltage distribution requires a transient state;

e. The damping factor of power transformers, being small, allows this transient state to be oscillatory. Therefore, various points during the oscillation will rise above the respective points of their final state (which lie on line *L G*).

f. The rise above the final state depends on the shape of the wave as explained in the paper. (See Fig. 21.)

Voltages produced in the winding by damped oscillations of the transmission line are cumulating. They may reach, therefore, as high a magnitude as that of lightning voltages, in spite of the fact that the amplitude of the damped oscillation of the line is roughly half of that of unidirectional lightning wave. For this reason, I believe Mr. Peters' underestimates the hazard of this class of transients. (See Fig. 20.)

Transient voltage stresses between turns on the line end of the winding increase with an increase of turn insulation, at a rate depending upon the size and shape of the conductor and of the winding. The strength of turn insulation increases with its thickness, usually at a different rate. Increasing the turn insulation at the line end, therefore, may not increase the reliability of the winding at all. This is particularly true for wide coils. Therefore, the strengthening of the winding against lightning stresses, by merely increasing the insulation, is not as simple a matter as may appear from Mr. Peters' remarks.

In his discussion of my paper, Mr. Dann drew an analogy between transformer windings and insulator strings.

Realizing that "an analogy is a good servant, but a poor master," I caution the reader that in several places in my paper, the analogies used were only approximate.

While the capacitance circuit of a transformer winding may be compared with that of an insulator string for an elementary discussion of its behavior under high-frequency excitation, such a comparison is entirely inadequate for a quantitative analysis. Where it is desired to determine the effect of the configuration of the winding on the phenomenon, the separate coils (disk or pancake) cannot be looked upon as solid metal plates comprising condensers C_w of Fig. 3 of the paper, but must be considered as rather a complicated network shown on *A* of Fig. 3 where *a b* is the first turn, *b c* is the second turn, *c d* is the third, etc.

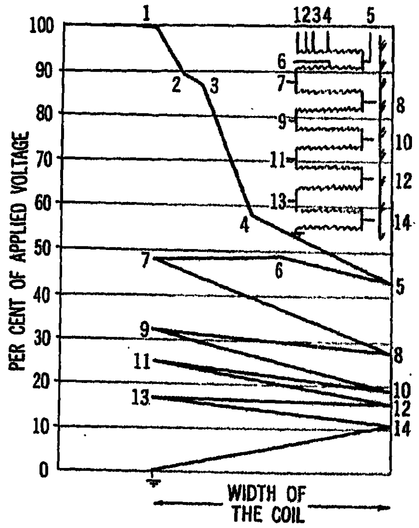


Fig. 4—Initial Voltage Distribution in Stack. 10 In. Long with Wide Coils (10 in.)

For this reason, the voltage concentration across the line coils increases with increase of its width. If each coil acted as a solid plate, the voltage would decrease.

For the same reason, the relative shortness of the shell type transformers does not exert the beneficial effect expected by Mr. Dann.

Fig. 4 herewith gives a result of lightning tests on a very short stack, (10 in.) with wide coils (10 in.).

Fig. 19 of the paper shows that, with wide coils, during initial voltage distribution, the voltage drop across the line coil is approximately 55 per cent of the total applied voltage. Curve 1*C* of Fig. 20 shows that 65 per cent of the applied voltage appeared across 2.5 per cent of the shell type winding stack while curves 1*A* and 1*B* of the same figure, show that same voltage appeared across 7½ per cent of the stack of the concentric type of transformer. Thus, in this particular case, the voltage concentration in the shell type transformer happened to be three times as great as in the other. If it is remembered, as Mr. Dann points out, that a shell type stack is substantially shorter than a concentric type stack. it means that the voltage stress per unit length in shell-type transformer is even more than three times as great as in concentric type transformers.

That the severe concentration of voltage in shell type transformer does take place is also substantiated by tests by Mr. Peters. That the severe oscillations are produced by a single lightning wave is illustrated by the cathode ray oscillogram, Fig. 5, herewith.

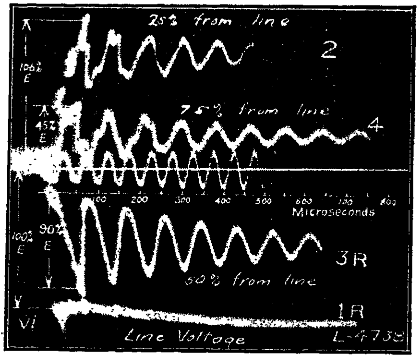


FIG. 5—OSCILLATION PRODUCED IN SHELL TYPE TRANSFORMER BY SINGLE UNIDIRECTIONAL WAVE

I should like to call to Mr. Hickernell's attention that, in the paper, only a transformer with a solidly grounded neutral is discussed. In such a case, the A. I. E. E. standard insulation test, consisting of a so-called induced voltage test with the neutral end of the winding grounded, allows grading of major insulation in the order of the voltage to ground existing during this test. It is this grading of the major insulation that is criticized by the author.

In case of a transformer operating with neutral isolated or with the high-voltage winding connected in delta, the insulation test consists of not only an induced voltage test, but also of a high-potential test between the high-voltage winding and ground. The latter test, subjecting the entire high-voltage winding to full test voltage to ground, allows no grading of the major insulation. No trouble should be expected with major insulation of such a transformer.

As far as I know, the Consumer's Power Company of Michigan, referred to by Mr. Hickernell, operates with isolated neutral and with most of the transformers connected in delta on the high-voltage side. The absence of trouble with the major insulation on that system, therefore, has no bearing on the conclusions suggested in the paper.

I am glad to find that Mr. Hickernell's operating experience emphasizes the importance of coil and turn insulation. On the seventeenth page of the paper, the table of insulation allowances and the test voltage stresses is given. These allowances were used for many years, and show that where the transient stresses are particularly high, the strength of the turn insulation may be more than 800 times the turn-to-turn stress produced by the A. I. E. E. test.

In a non-resonating transformer the insulation between coils and turns is practically the same as shown in the above table, but the stresses due to the elimination of internal oscillation are reduced in the order of from 10-to-1 up to 100-to-1, and thereby very much greater reliability is secured.

I am glad indeed to find that Mr. Sporn, an operating engineer of very wide experience, shares my confidence in the reliability of non-resonating transformers.

It is hoped that the series of cathode ray oscillograms obtained on a power transformer with a transmission line will be published in the near future. Fig. 5 of this discussion is one of these oscillograms.

Transient Analysis of A-C. Machinery

BY YU H. KU*
Associate, A. I. E. E.

Synopsis.—This paper shows how the Heaviside operational methods may be applied to determine the transient currents that are produced in synchronous and induction machines by some sudden alteration of their electric circuits. The observed and computed values of the transient currents have been plotted for a number of cases. There is also a table giving the names of the principal investigators who have written on this subject and the methods of analysis that they have employed.

DURING the last fifteen or twenty years the problem of calculating the transient currents in a-c. machinery has become increasingly important. Many notable papers on this subject have appeared in technical literature both in this country and abroad.

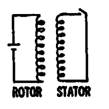
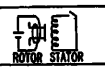
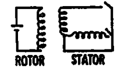
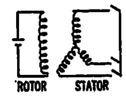
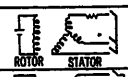
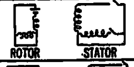
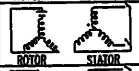
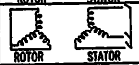
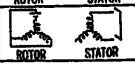
In the analysis of these transient problems it is necessary to make some assumptions, either in regard to the nature of the phenomena or in regard to the characteristics of the fundamental physical quantities involved. The earlier authors, such as Steinmetz and Boucherot, followed the first method, while the later authors, with few exceptions, have followed the second method. The important physical quantities involved are the resistances and inductances of the stator and rotor windings, and the characteristics of the magnetic circuit that enter into the determination of the eddy-current and hysteresis losses. It has been customary to assume that the resistances and self-inductances of the stator and rotor windings are constant, and the mutual inductance between any two windings varies with the cosine of their relative angular displacement. These assumptions in regard to the inductances are practically equivalent to assuming that the air gap is uniform and the flux in it, sinusoidally distributed. Whenever eddy-currents have been considered they have been assumed to exist in an electric circuit of constant resistance and self-inductance.

It seems appropriate to present at this time a brief summary of the technical literature by the principal investigators of these transient problems, indicating the type of solution and the essential features of the methods they have employed. Those authors who adopted the foregoing or other reasonable assumptions in regard to the characteristics of the fundamental physical quantities, and have carried out the mathematical analysis with rigor, are said to have given an exact solution. All other solutions are classed as approximate. Thus all cases where the resistances have been neglected or have been taken only partially into account, as, for example, in determining values for the damping coefficients, have been classed as approximate, even though such solutions may have constituted very important and valuable contributions to this subject.

These transient problems are readily grouped in three general classes. In the first the circuits on each side of the air gap are symmetrical, in the second the circuits on but one side of the gap are symmetrical, and in the third the circuits on neither side of the gap are symmetrical. Exact solutions for problems in the first two classes can be obtained by operational methods, but those in the third class do not readily lend themselves to mathematical treatment, except in one special case.

In the accompanying table is given the summary of the methods employed by the principal investigators.

TABLE I

CASE	DIAGRAM OF CONNECTIONS	AUTHOR OF SOLUTION	TYPE OF SOLUTION	ESSENTIAL FEATURES
III		Steinmetz 1909 Boucherot 1911 Berg 1915 Biermanns 1915 Doherty 1918 Shimidzu and Ito 1922 Otake 1924	Approximate Approximate Approximate Exact Approximate Approximate Exact	"Flux Theory" "E.M.F. Theory" For Special Case $r_a/L_a = r_f/L_f$ "Constant Flux Linkages Theorem" Resistances First Neglected. Mathematical
III[1]		Doherty and Nickle 1927	Approximate	Double—Reaction Method and Constant Linkages Theorem
II		Boucherot 1911 Laffoon 1924 Franklin 1925	Approximate Approximate	"Constant Flux Linkages" Doherty's Method
II		Steinmetz 1909 Arnold 1913 Diamant 1915 Dreyfus 1916 Lyon 1923 Laffoon 1924 Franklin 1925	Approximate Approximate Approximate Exact Exact Approximate Approximate	Boucherot's Method 'Vector' Method 'Vector' Method or Method of 'Shrinking Vectors'.
III		Karapetoff 1925	Approximate	Neglect Resistances
I[2]		Dreyfus 1912	Exact	Mathematical
I[2]		Rudenberg 1925 Bekku 1927	Exact Exact	Dreyfus' Vector Method Operational
I		Doherty and Williamson 1921 Lyon 1923 Dreyfus 1911	Approximate Exact Exact	"M.M.F's. Considered and Const. Flux Linkages" "Shrinking Vectors" Vector Method
II[2]		Bekku 1927	Exact	Operational

1 Salient Pole Rotor 2 Symmetrical Field Excitation.

One of the principal advantages of the Heaviside operational method is that the magnitudes of the transient currents are directly determined by means of the expansion theorem without recourse to the solution of simultaneous equations which is a feature of other methods of analysis. This advantage alone justifies the use of the method.

After the present investigation had been completed the Heaviside operational solution by Bekku[1] was brought to the writer's attention. Bekku considered only the case of an alternator with symmetrical excitation and so in this slight respect his solution differs from that presented by the author. In addition the author believes that such a powerful method deserves to be more widely known than would be possible from Bekku's publication in a foreign journal.

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

The following assumptions that have been made conform with the general practise. First, it will be assumed that the stator windings are symmetrical and that the rotor is turning at constant speed. Second, it will be assumed that the resistances and self-inductances of the windings are constant and that the mutual inductance between any two windings is proportional to the cosine of their relative angular displacement. Thus the effect of frequency upon resistance, the effect of saturation upon inductance, and the effect of harmonics in the distribution of the air-gap flux density are neglected. It should be observed that these assumptions in regard to the inductances cannot be realized except when the machine has a cylindrical rotor. It must be noted particularly that the solution as here presented does not apply unless the circuits on at least one side of the air gap are symmetrical. For example, the method does not apply to the sudden single-phase short-circuit of an alternator with the usual type of field excitation.

Briefly this solution consists in applying first the Stokvis-Fortescue method of symmetrical-phase components to reduce the polyphase relations to equivalent single-phase relations; second, an operational method by which the equations that apply to the stator and rotor circuits and that recognize their relative angular velocity are transformed so that they become similar to the equations that apply to circuits which have no relative rotation; third, the Heaviside operational method, including the principle of superposition as applied to static circuits, is employed to determine the integration constants in the solution.

For the sake of brevity it is only shown how this method of analysis can be applied to the problem of the simultaneous polyphase short circuit of an alternator. Several other cases have been calculated and the results are compared with the measured current wave forms. The agreement between the observed and computed values is quite satisfactory.

FUNDAMENTAL EQUATIONS

With the foregoing assumptions relative to the character of the resistances and inductances, superposition of currents is possible and the principle underlying Thévenin's[2] theorem can be used in determining the currents which flow when the terminals are suddenly short circuited. Thus the current in any phase after the short circuit occurs can be considered as the sum of two components: first, that current which would exist if no disturbance occurred, and second, the additional current that would be produced by suddenly applying to the terminals an alternating e. m. f. equal and opposite to the potential existing there before the short circuit.

If the short circuit takes place under steady conditions, the first component is the normal steady-state current of the phase, and can be readily obtained by the conventional schemes of steady-state analysis.

A short circuit suddenly reduces the terminal potentials to zero. The net effect of this on the circuit currents is equivalent to suddenly applying e. m. fs. to the terminals which will nullify the effect of the potentials existing there. The current in any phase resulting from these suddenly applied e. m. fs.—all other potentials being considered zero—is the additional component mentioned above. This principle is convenient to use because it reduces the short-circuit case to the conventional operational calculus problem—that of finding the response of a circuit to a suddenly applied e. m. f.

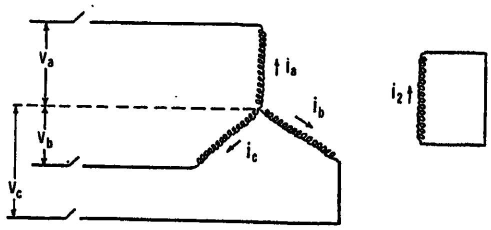

FIG. 1—CONNECTION FOR SUDDEN APPLICATION OF E. M. Fs. TO ALTERNATOR TERMINALS

The operational equations for the alternator's response to the suddenly applied terminal e. m. fs. can be written as follows, reference being made to Fig. 1:

$$(R_1 + L_1\,p)\,i_a + M\,p\,[i_2 \cos n\,t] = v_a\,1 \tag{1}$$

$$(R_1 + L_1\,p)\,i_b + M\,p\left[i_2 \cos\left(n\,t - \frac{2\,\pi}{3}\right)\right] = v_b\,1 \tag{2}$$

$$(R_1 + L_1\,p)\,i_c + M\,p\left[i_2 \cos\left(n\,t - \frac{4\,\pi}{3}\right)\right] = v_c\,1 \tag{3}$$

$$(R_2 + L_2\,p)\,i_2 + M\,p\left[i_a \cos n\,t + i_b \cos\left(n\,t - \frac{2\,\pi}{3}\right) + i_c \cos\left(n\,t - \frac{4\,\pi}{3}\right)\right] = 0 \tag{4}$$

Where

R_1 = resistance of stator, per phase.

R_2 = resistance of field.

L_1 = synchronous self-inductance of stator, per phase. It is the self-inductance of one stator phase, plus the mutual effect of each of the other stator phases.

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[1] S. Bekku, "Sudden Short-Circuit of an Alternator," *Researches of the Electrotechnical Laboratory*, No. 203, June 1927, Tokyo, Japan.

[2] L. Thévenin, "Sur un Nouveau Théorème D'Electricité Dynamique," *Comptes Rendus*, Vol. 97, 1883, pp. 159-161. While this theorem was written for the steady state, its extension to the transient case can be readily made.

L_2 = self-inductance of field.

M = maximum value of the mutual inductance between one phase of the stator and the field winding.

i_a, i_b, i_c = instantaneous values of stator currents in phases a, b, and c.

i_2 = instantaneous value of field current.

v_a, v_b, v_c = instantaneous values of the stator terminal potentials in phases a, b, and c.

n = relative angular velocity between windings of stator and field.

$p = \dfrac{d}{d\,t}$, the time differential operator.

1 = unit function. In operational calculus notation this means a time function which is discontinuous at $t = 0$, being zero before and unity thereafter.

Although the e. m. fs. which are applied are equal and opposite to the terminal potentials, when considered with respect to the assumed positive direction of current flow in the armature winding, they have the same direction as the original e. m. fs. in the phase.

SIMPLIFYING THE EQUATIONS

If the stator potentials are balanced, the e. m. fs. which are applied will also be balanced, so

$$v_a + v_b + v_c = 0 \tag{5}$$

With no connection to the neutral, the currents in the stator must satisfy the relation

$$i_a + i_b + i_c = 0 \tag{6}$$

It is seen from these relations that Equation (3) is obtainable from (1) and (2), and hence need not be considered further.

Also by means of (6) it is seen that Equation (4) can be reduced to

$$(R_2 + L_2\,p)\,i_2 + \sqrt{3}\,M\,p\left[i_a \cos\left(n\,t - \frac{\pi}{6} \right) + i_b \cos\left(n\,t - \frac{\pi}{2} \right)\right] = 0 \tag{7}$$

The equations for solution now consist of (1), (2), and (7).

INTRODUCTION OF SYMMETRICAL COMPONENTS

By introducing the Stokvis-Fortescue scheme[3] of symmetrical component analysis, the phase currents can be resolved into their positive- and negative-sequence components. Writing these relations for the currents in phases a and b,

$$\left.\begin{aligned} i_a &= i_+ + i_- \\ i_b &= i_+\,a^2 + i_-\,a \end{aligned}\right\} \tag{8}$$

where a and a^2 are respectively $\epsilon^{j\frac{2\pi}{3}}$ and $\epsilon^{j\frac{4\pi}{3}}$.

The e. m. fs. applied to phases a and b can be similarly resolved, so

$$\left.\begin{aligned} v_a &= v_+ + v_- \\ v_b &= v_+\,a^2 + v_-\,a \end{aligned}\right\} \tag{9}$$

and the induced e. m. fs. due to mutual induction can be written

$$\left.\begin{aligned} M\,p\,[i_2 \cos n\,t] &= e_+ + e_- \\ M\,p\left[i_2 \cos\left(n\,t - \frac{2\,\pi}{3} \right)\right] &= e_+\,a^2 + e_-\,a \end{aligned}\right\} \tag{10}$$

If the cosine terms in (10) are written in their exponential form, it is seen that

$$\left.\begin{aligned} e_+ &= \frac{M}{2}\,p\,i_2\,\epsilon^{jnt} \\ e_- &= \frac{M}{2}\,p\,i_2\,\epsilon^{-jnt} \end{aligned}\right\} \tag{11}$$

Equations (1) and (2) can now be rewritten so as to include the relations of Equations (8), (9), and (10). They become

$$(R_1 + L_1\,p)\,(i_+ + i_-) + (e_+ + e_-) = (v_+ + v_-)\,1 \tag{12}$$

$$(R_1 + L_1\,p)\,(i_+\,a^2 + i_-\,a) + (e_+\,a^2 + e_-\,a) = (v_+\,a^2 + v_-\,a)\,1 \tag{13}$$

These can be solved to obtain

$$(R_1 + L_1\,p)\,i_+ + e_+ = v_+\,1 \tag{14}$$

$$(R_1 + L_1\,p)\,i_- + e_- = v_-\,1 \tag{15}$$

and upon insertion of the expressions for e_+ and e_- there results

$$(R_1 + L_1\,p)\,i_+ + \frac{M}{2}\,p\,i_2\,\epsilon^{jnt} = v_+\,1 \tag{16}$$

$$(R_1 + L_1\,p)\,i_- + \frac{M}{2}\,p\,i_2\,\epsilon^{-jnt} = v_-\,1 \tag{17}$$

This transformation of (1) and (2) into (16) and (17) has resulted in considerable simplification for now one equation has been obtained which contains only positive-sequence terms, and another which contains only negative-sequence terms.

When the relations of (8) are substituted into (7), the latter becomes

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[3] L. G. Stokvis, "Sur la Création des Harmoniques 3 dans les Alternateurs par Suite du Déséquilibrage des Phases," *Comptes Rendus*, 159, 1914, pp. 46-49. L. G. Stokvis "Analysis of Unbalanced Three-Phase Systems," *Electrical World*, 65, 1915, pp. 1111-15. R. E. Gilman and C. L. Fortescue, "*Single-phase Power Service from Central Stations*," A. I. E. E. TRANS., Vol. XXXV, 1916, pp. 1329-1347. C. L. Fortescue, "*Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks*," A. I. E. E. TRANS., Vol. XXXVII, 1918, pp. 1027-1115.

$$(R_2 + L_2 p)\, i_2 + \frac{3}{2} Mp\, (i_+ \epsilon^{-jnt} + i_- \epsilon^{jnt}) = 0 \quad (18)$$

Equations (16), (17), and (18) are now in form to be readily handled by operational methods.

REDUCTION TO STATIONARY COUPLED-CIRCUIT PROBLEM

There is an operational transformation formula by means of which an exponential time function ϵ^{-kt} forming part of the operand of an operational expression can be shifted around its operator to the position of a multiplying factor. The compensating change in the operator to show that this has taken place is made by replacing p throughout with $p - k$. This process can be expressed operationally as follows:[4]

$$\frac{1}{Z\,(p)}\, \epsilon^{-kt} f\,(t)\, 1 = \epsilon^{-kt} \frac{1}{Z\,(p-k)} f\,(t)\, 1,$$

where $1/Z\,(p)$ is the operator, and $\epsilon^{-kt} f\,(t)\, 1$ is the operand, or time function, before the shift is performed.

The converse of this process is possible also, and an exponential ϵ^{-kt} can be shifted into the operand if the operator is changed by replacing p with $p + k$. This transformation affords a convenient means of simplifying the alternator equations still further before attempting to evaluate them. If Equation (17) is multiplied by ϵ^{jnt}, and Equation (16) by ϵ^{-jnt}, and then these exponentials shifted to the operands, the two equations become

$$[R_1 + L_1 (p + j\,n)]\, i_+ \epsilon^{-jnt} + \frac{M}{2} (p + j\,n)\, i_2 = v_+ \epsilon^{-jnt}\, 1 \quad (20)$$

$$[R_1 + L_1 (p - j\,n)]\, i_- \epsilon^{jnt} + \frac{M}{2} (p - j\,n)\, i_2 = v_- \epsilon^{jnt}\, 1 \quad (21)$$

By this transformation two linear equations with constant coefficients have been secured which can be combined with Equation (18) to solve for the three variables i_+, i_-, and i_2. Thus the alternator equations, written with a variable mutual inductance parameter to care for the relative motion between circuits, have been reduced to those of the equivalent coupled circuits having windings which are stationary.

Solving equations (18), (20), and (21) simultaneously, the following operational expressions for i_+ and i_- are obtained:

$$i_+ = \epsilon^{jnt} \frac{1}{\sigma\, L_1\, D\,(p)} \Big[(G_+ + H_+)\, v_+ \epsilon^{-jnt} - H_-\, v_-\, \epsilon^{jnt} \Big] 1 \quad (22)$$

$$i_- = \epsilon^{-jnt} \frac{1}{\sigma\, L_1\, D\,(p)} \Big[(G_- + H_-)\, v_-\, \epsilon^{jnt} - H_+ v_+ \epsilon^{-jnt} \Big] 1 \quad (23)$$

where $\sigma = 1 - \dfrac{3}{2}\, \dfrac{M^2}{L_1 L_2}$,

$$D\,(p) = p^3 + \Big[h_1 + \frac{h_1 + h_2}{\sigma} \Big] p^2 + \Big[n^2 + \frac{h_1}{\sigma} (h_1 + 2\,h_2) \Big] p + \frac{h_2}{\sigma} [n^2 + h_1^2]$$

$$= (p - p_1)\,(p - p_2)\,(p - p_3)$$

$$h_1 = \frac{R_1}{L_1}, \text{ and } h_2 = \frac{R_2}{L_2}$$

$$G_+ = (p + h_2)\,(p + h_2 - j\,n)$$

$$G_- = (p + h_2)\,(p + h_2 + j\,n)$$

$$H_+ = \frac{1}{2} (\sigma - 1)\, p\, (p - j\,n)$$

$$H_- = \frac{1}{2} (\sigma - 1)\, p\, (p + j\,n)$$

The e. m. f. applied to phase a is of the form

$$v_a = V_m \sin n\, t = \frac{V_m}{2j} \epsilon^{jnt} - \frac{V_m}{2j} \epsilon^{-jnt} = v_+ + v_-$$

so

$$\left. \begin{aligned} v_+ &= \frac{V_m}{2j} \epsilon^{jnt} \\ v_- &= -\frac{V_m}{2j} \epsilon^{-jnt} \end{aligned} \right\} \quad (24)$$

If these values are inserted in Equations (22) and (23), and all constant factors moved to a position before the operator,

$$i_+ = \frac{V_m}{2j} \epsilon^{jnt} \frac{1}{\sigma\, L_1\, D\,(p)} [G_+ + H_+ + H_-]\, 1 \quad (25)$$

$$i_- = -\frac{V_m}{2j} \epsilon^{-jnt} \frac{1}{\sigma\, L_1\, D\,(p)} [G_- + H_- + H_+]\, 1 \quad (26)$$

It is apparent from these results that the effect of the shifting step has been to remove the rotational factors from the operand to the position of multiplying coefficients. At this point it becomes the conventional operational problem of evaluating the result of an operator operating on the unit time function.

Equations (25) and (26) have been derived on the assumption that the short circuit occurs at the instant of zero potential in phase a. As the disturbance can occur at any time in the cycle, a more general form of the equations can be written with the angle $n\,t$ replaced by $n\,t + \theta$. The equations then become

$$i_+ = \frac{V_m}{2} \epsilon^{j(nt+\theta)} \epsilon^{-j\frac{\pi}{2}} \frac{1}{\sigma\, L_1\, D\,(p)} [G_+ + H_+ + H_-]\, 1 \quad (27)$$

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[4] O. Heaviside, "Electromagnetic Theory," Vol. II, p. 294. V. Bush, "Operational Circuit Analysis," (Wiley), 1929, Ch. VIII, Sec. 10.

$$i_- = \frac{V_m}{2} \epsilon^{-j(nt+\theta)} \epsilon^{j\frac{\pi}{2}} \frac{1}{\sigma L_1 D(p)} [G_- + H_- + H_+] 1 \tag{28}$$

The factors $1/j$ and $-1/j$ have here been written in their exponential form $\epsilon^{-j\frac{\pi}{2}}$ and $\epsilon^{j\frac{\pi}{2}}$.

EVALUATION BY HEAVISIDE EXPANSION FORMULA

Equations (27) and (28) have been reduced to the ordinary form of operational equations, and there remains the evaluation of operational expressions such as

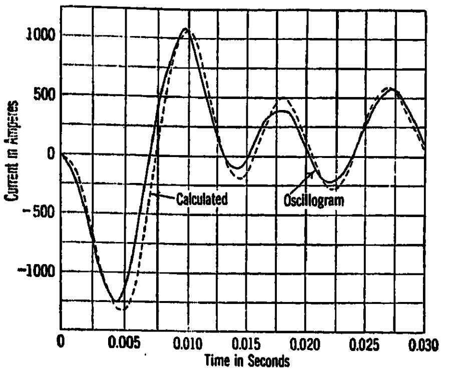

FIG. 2—POLYPHASE SHORT CIRCUIT OF ALTERNATOR AT NO-LOAD CURRENT TRANSIENT IN PHASE A

The full lines represent oscillograph records and the dotted lines the calculated values

$$\frac{A(p)}{B(p)} 1 = \frac{G_+(p) + H_+(p) + H_-(p)}{D(p)} 1. \tag{29}$$

A convenient way of doing this is by means of the Heaviside expansion formula in the form[5]

$$i = \sum_{k=0}^{k=n} \frac{A(p_k)}{F(p_k)} \epsilon^{p_k t} \tag{30}$$

where $F(p_k) = [(p - p_0)(p - p_1) \ldots (p - p_{k-1}) (p - p_{k+1}) \ldots (p - p_n)]_{p = p_k}$ in which, for our case, $p_1, p_2 \ldots\ldots p_n$ are the roots of $D(p) = 0$, and $p_0 = 0$ because the operand in (29) is the unit function.

The result of using this on Equation (27) is the series of terms

$$i_+ = \frac{V_m}{2} \epsilon^{j\left(nt+\theta-\frac{\pi}{2}\right)} \frac{1}{\sigma L_1} \Bigg[\left\{ \frac{G_+ + H_+ + H_-}{(-p_1)(-p_2)(-p_3)} \right\}_{p=0}$$

$$+ \left\{ \frac{G_+ + H_+ + H_-}{p_1 (p_1 - p_2)(p_1 - p_3)} \right\}_{p=p_1} \epsilon^{p_1 t}$$

$$+ \left\{ \frac{G_+ + H_+ + H_-}{p_2 (p_2 - p_1)(p_2 - p_3)} \right\}_{p=p_2} \epsilon^{p_2 t}$$

$$+ \left\{ \frac{G_+ + H_+ + H_-}{p_3 (p_3 - p_1)(p_3 - p_2)} \right\}_{p=p_3} \epsilon^{p_3 t} \Bigg] \tag{31}$$

and a similar solution can be written for (28).

When Equations (27) and (28) have been finally interpreted into functions of time it is found that i_+ and i_- are conjugate quantities, so that from Equation (8),

$$\left.\begin{aligned} i_a &= i_+ + i_- = 2\,(i_+)_{real} \\ i_b &= i_+ a^2 + i_- a = 2\,(i_+ a^2)_{real} \end{aligned}\right\} \tag{32}$$

Consequently the actual numerical work is reduced to the interpretation of i_+ from (27) by means of the expansion formula given in (30).

APPLICATION TO POLYPHASE SHORT CIRCUIT UNDER LOAD

When the short circuit occurs under load, the angle θ does not correspond to the rotor position angle as in the no-load case. This can be seen by consideration of the excitation e. m. f. E and the terminal potential V in Fig. 3. The terminal potential lags behind E by an angle

$$\alpha = \beta - \theta.$$

Under these conditions

$$\left.\begin{aligned} v_+ &= \frac{V_m}{2j} \epsilon^{j(nt+\theta)} \\ v_- &= -\frac{V_m}{2j} \epsilon^{-j(nt+\theta)} \cdot \end{aligned}\right\} \tag{34}$$

the same as before, but the exponentials in Equations (22) and (23) are now $\epsilon^{j(nt+\theta+\alpha)}$ and $\epsilon^{-j(nt+\theta+\alpha)}$. This results from the fact that the e. m. fs. due to mutual induction in (11) depend upon the position of the rotor, so under load conditions they take the form

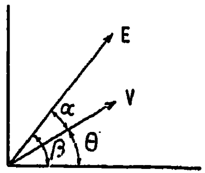

FIG. 3—RELATIVE POSITION OF EXCITATION E. M. F. AND TERMINAL POTENTIAL AT INSTANT OF SHORT CIRCUIT UNDER LOAD

$$e_{\pm} = \frac{M}{2} p\, i_2\, \epsilon^{\pm j(nt+\theta+\alpha)},$$

and the exponentials involved in the shifting process correspond. When these are combined with the values for v_+ and v_- given in (34), there results

$$i_+ = \frac{V_m}{2j} \epsilon^{j(nt+\theta+\alpha)} \frac{1}{\sigma . L_1 D(p)} \left[G_+ + H_+ \epsilon^{-j\alpha} + H_- \epsilon^{j\alpha} \right] 1$$

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[5] L. Cohen, "The Heaviside Expansion Theorem," *Jour. Franklin Inst.*, 194, 1922, pp. 765-770. V. Bush, *loc. cit.*, Chap. VII, Sec. 4.

$$= \frac{V_m}{2} \epsilon^{j\left(nt+\theta-\frac{\pi}{2}\right)} \frac{1}{\sigma L_1 D(p)} \left[G_+ + H_+ + H_- \epsilon^{2j\alpha}\right] 1 \tag{35}$$

$$i_- = \frac{V_m}{2} \epsilon^{-j\left(nt+\theta-\frac{\pi}{2}\right)} \frac{1}{\sigma L_1 D(p)} \left[G_- + H_- + H_+ \epsilon^{-2j\alpha}\right] 1 \tag{36}$$

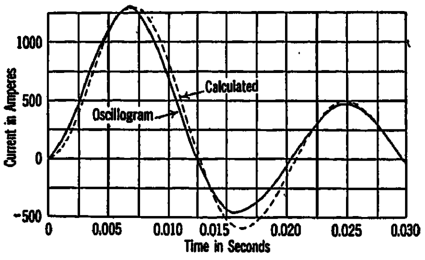


Fig. 4—Field Current Transient Corresponding to Case of Fig. 2

The full lines represent oscillograph records and the dotted lines the calculated values

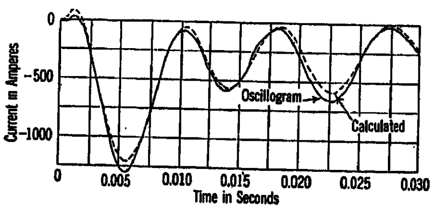


Fig. 5—Polyphase Short Circuit of Alternator Having Unity Power Factor Load. Current Transient in Phase *a*

The full lines represent oscillograph records and the dotted lines the calculated values

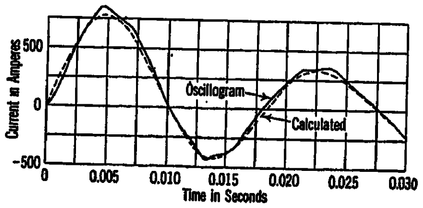


Fig. 6—Field Current Transient Corresponding to Case of Fig. 4

The full lines represent oscillograph records and the dotted lines the calculated values

For the particular case of $\alpha = 0$, these two equations reduce to those of the no-load case given in (27) and (28). The expressions for i_+ and i_- are again conjugate, so it is necessary to evaluate only i_+, and the phase currents are given again by Equations (32).

Curves showing comparisons between experimental and calculated phase currents for short circuits occurring under loads having power factors which are unity, lagging, and leading, are given in Figs. 5, 7, and 9. Calculations for these are given in the Appendix.

FIELD CURRENT TRANSIENTS

The expression for the field current transient can be obtained by solving Equations (18), (20), and (21) simultaneously for i_2. For the general case of short circuit under load

$$i_2 = \frac{3}{2} \frac{M}{L_1 L_2} \frac{V_m}{\sigma} \frac{p}{D(p)} [n \cos\alpha + (p + h_1) \sin\alpha] 1 \tag{37}$$

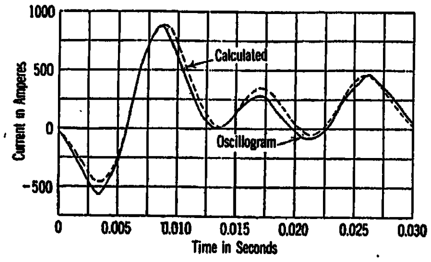


Fig. 7—Polyphase Short Circuit of Alternator Having Lagging Power-factor Load. Current Transient in Phase *a*

The full lines represent oscillograph records and the dotted lines the calculated values

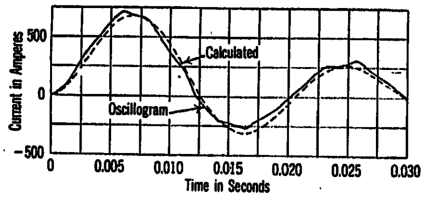


Fig. 8—Field Current Transient Corresponding to Case of Fig. 6

The full lines represent oscillograph records and the dotted lines the calculated values

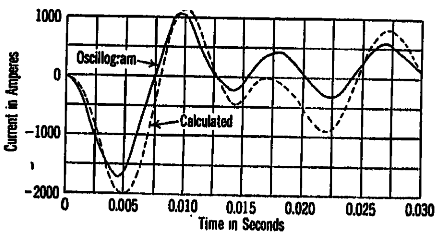


Fig. 9—Polyphase Short Circuit of Alternator Having Leading Power-factor Load. Current Transient in Phase *a*

The full lines represent oscillograph records and the dotted lines the calculated values

When the short circuit takes place at no load, $\alpha = 0$, and the expression simplifies to

$$i_2 = \frac{3}{2} \frac{M}{L_1 L_2} \frac{n}{\sigma} V_m \frac{p}{D(p)} 1 \tag{38}$$

Since the evaluation of (38) is not difficult, in the no-load case it is often convenient to solve for the field transient first. Since the field transient consists of a sum of terms, there will be a term in the stator transient corresponding to each; so it can be expressed by

$$i_a = \sum_{k=1}^{k=3} \epsilon^{j(nt+\theta)} \left\{ \frac{-M(p_k + jn)}{L_1(p_k + h_1 + jn)} \, i_2(p_k) \right\}_{real} \qquad (39)$$

The field current transients to correspond to the above cases of short circuit at zero load, and at loads of unity, lagging, and leading power factors are given in Figs. 4, 6, 8, and 10. Calculations for these are given in the Appendix.

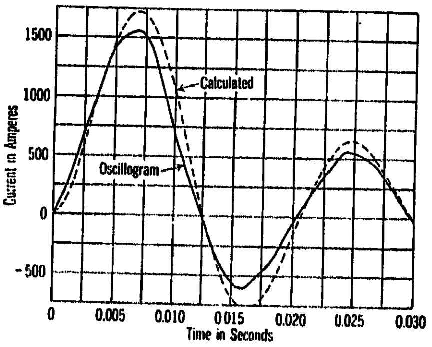
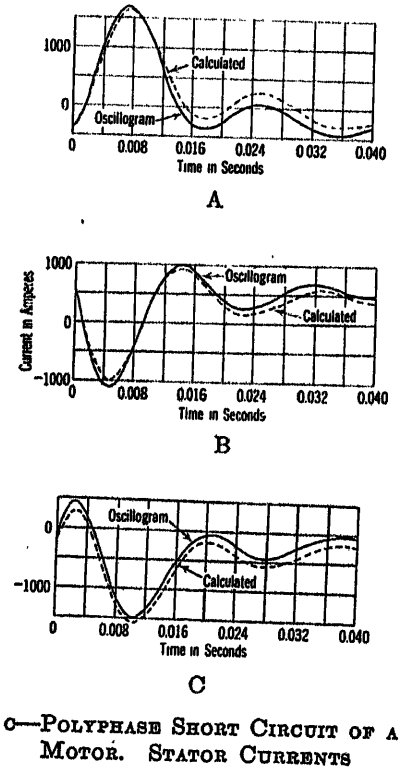


Fig. 10—Field Current Transient Corresponding to Case of Fig. 9

The full lines represent oscillograph records and the dotted lines the calculated values



A

B

C

Fig. 11-a, b, c—Polyphase Short Circuit of an Induction Motor. Stator Currents

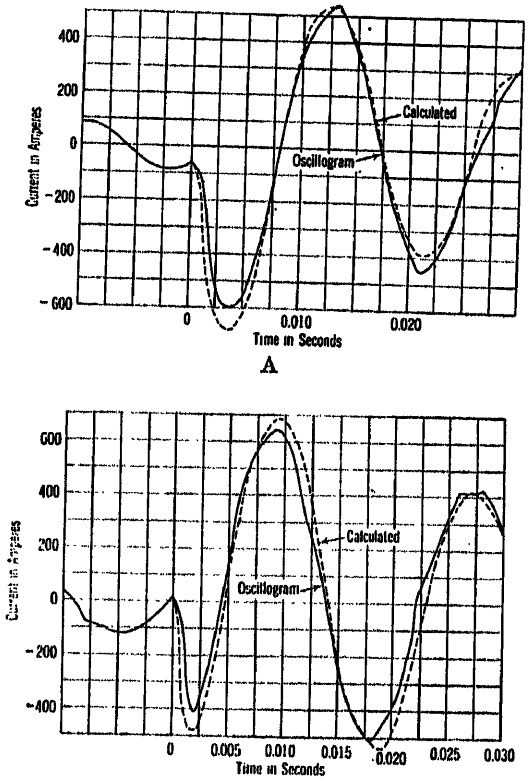
The full lines represent oscillograph records and the dotted lines the calculated values

Additional Applications

In addition to the above applications to the alternator, the author has applied this scheme of analysis to the short circuit of the induction motor, both the balanced and unbalanced cases being considered. The case of sudden voltage reduction at the induction-motor terminals was also studied. Curves giving a comparison between calculated and experimental results for two of these cases are added in Figs. 11 and 12.

Acknowledgment

The author wishes to acknowledge his indebtedness to Professor V. Bush and Dr. B. A. Behrend for their suggestions and encouragement, and particularly to Professor W. V. Lyon and Mr. M. F. Gardner, who have revised the paper for publication. Credit is also due to those former graduate students of the Massachusetts Institute of Technology whose theses were the source of helpful experimental results.



A

B

Fig. 12-a, b—Sudden Voltage Reduction at Induction-Motor Terminals. Stator Currents

The full lines represent oscillograph records and the dotted lines the calculated values

Appendix

Numerical Data and Calculations

Machine Constants

$R_1 = 0.0232$ $R_2 = 0.138$ $M = 8.34 \times 10^{-3}$
$L_1 = 5.66 \times 10^{-3}$ $L_2 = 19.3 \times 10^{-3}$ $n = 377$
$h_1 = 4.10$ $h_2 = 7.13$ $\sigma = 0.046$

$$\frac{1}{\sigma L_1} = 3840$$

General Calculations

$$D(p) = p^3 + 248\,p^2 + 144 \times 10^3\,p + 220 \times 10^5 = 0$$

gives the roots: $p_1 = -169$

$$p_2 = -39.6 + j\,359 = -361\ \underline{/-83.7}$$

$$p_3 = -39.6 - j\,359 = -361\ \underline{/83.7}$$

$G_{\pm}(0) = 2690\ \underline{/\mp 89.4}$ $H_{\pm}(0) = 0$

$G_{\pm}(p_1) = 66{,}500\ \underline{/\pm 66.4}$ $H_{\pm}(p_1) = 33{,}300\ \underline{/\pm 66.9}$

$G_{+}(p_2) = 14{,}200\ \underline{/-58}$ $H_{+}(p_2) = -7{,}500\ \underline{/-59.3}$

$G_{-}(p_2) = -265{,}000\ \underline{/7.93}$ $H_{-}(p_2) = 127{,}000\ \underline{/9.4}$

$G_{+}(p_3) = [G_{-}(p_2)]_c$ $H_{+}(p_3) = [H_{-}(p_2)]_c$

$G_{-}(p_3) = [G_{+}(p_2)]_c$ $H_{-}(p_3) = [H_{+}(p_2)]_c$

$F(0) = 22.1 \times 10^6$ $F(p_2) = -98.8 \times 10^6 \underline{/76.5}$

$F(p_1) = -24.6 \times 10^6$ $F(p_3) = -98.8 \times 10^6 \underline{/-76.5}$

Short Circuit with no load

$V_m = 188$ $\theta = -71.3$

Initial steady-state current = 0

$$(G_{\pm} + H_{\pm} + H_{\mp})_{p_1} = -61{,}000\ \underline{/\mp 89.5}$$

By Equation (27):

$i_{+}(p_1)$

$$= \frac{188}{2}\,\epsilon^{j(377t-71\cdot 3-90)}\,\frac{3840 \times (-61{,}000\ \underline{/-89.5})}{-24.6 \times 10^6}\,\epsilon^{-169t}$$

$$= 894\,\epsilon^{-169t}\,\epsilon^{j(377t+109)}$$

By Equation (32):

$$i_a(p_1) = 1790\,\epsilon^{-169t}\cos(377t + 109)$$

and similarly for roots p_2, p_3, and p_0:

$$i_a(p_2) = 949\,\epsilon^{-39\cdot 6t}\cos(736t - 51.1)$$

$$i_a(p_3) = 1038\,\epsilon^{-39\cdot 6t}\cos(18t - 88.7)$$

$$i_0(0) = 88\cos(377t + 109)$$

For plotting in Fig. 2:

$$i_a = i_0(0) + i_a(p_1) + i_a(p_2) + i_a(p_3)$$

By Equation (38):

$$\frac{3}{2}\,\frac{M}{L_1 L_2}\,\frac{n}{\sigma} = 93.9 \times 10^4$$

$$i_2(0) = 0$$

$$i_2(p_1) = 1210\,\epsilon^{-169t}$$

$$i_2(p_2) + i_2(p_3) = 1290\,\epsilon^{-39\cdot 6t}\cos(359t - 160)$$

For plotting in Fig. 4:

$$i_2 = i_2(p_1) + i_2(p_2) + i_2(p_3)$$

Short Circuit with Unity Power Factor Load

$V_m = 125$ $\theta = 180$ $\alpha = 46.7$ $\beta = 227$

By Equations (35) and (32):

$$i_a(p_1) = 437\,\epsilon^{-169t}\cos(377t + 47.7)$$

$$i_a(p_2) = -585\,\epsilon^{-39\cdot 6t}\cos(736t + 115)$$

$$i_a(p_3) = 637\,\epsilon^{-39\cdot 6t}\cos(18t + 161)$$

i_{as} (total steady-state current) $= 84.3\cos(377t + 47.3)$

For plotting in Fig. 5:

$$i_a = i_{as} + i_a(p_1) + i_a(p_2) + i_a(p_3)$$

By Equation (37):

$$i_2(p_1) = 296\,\epsilon^{-169t}$$

$$i_2(p_2) + i_2(p_3) = 796\,\epsilon^{-39\cdot 6t}\cos(359t - 112)$$

For plotting in Fig. 6:

$$i_2 = i_2(p_1) + i_2(p_2) + i_2(p_3)$$

Short Circuit with Lagging Power Factor Load

$V_m = 99$ $Z_L = 2.52\ \underline{/84.3}$ $\alpha = 2$ $\beta = -35$

By Equations (35) and (32):

$$i_a(p_1) = -930\,\epsilon^{-169t}\cos(377t - 34.5)$$

$$i_a(p_2) = 499\,\epsilon^{-39\cdot 6t}\cos(736t - 13)$$

$$i_a(p_3) = -544\,\epsilon^{-39\cdot 6t}\cos(18t + 126)$$

$$i_{as} = -81\cos(377t - 34.4)$$

For plotting in Fig. 7:

$$i_a = i_{as} + i_a(p_1) + i_a(p_2) + i_a(p_3)$$

By Equation (37):

$$i_2(p_1) = 628\,\epsilon^{-169t}$$

$$i_2(p_2) + i_2(p_3) = 676\,\epsilon^{-39\cdot 6t}\cos(359t - 158)$$

For plotting in Fig. 8:

$$i_2 = i_2(p_1) + i_2(p_2) + i_2(p_3)$$

Discussion

Louis Cohen: Mr. Ku made effective use of the Expansion Formula in his study of a very interesting problem, and he has also brought in the use of the "Operand Shift" principle. He has of course used the formula correctly, but in shifting the operand the impression is created that in applying the formula to a-c. problems it is always necessary to shift the operand and operate on unity function. This of course is not at all necessary. In my paper on the derivation of the Expansion Formula referred to by Mr. Ku, and also in my recent book "Heaviside's Electrical Circuit Theory," I have given a modification of the Expansion Formula suitable for the case of applied alternating voltages by which the solution can be obtained directly without resorting to the shift principle, and then operate on unity function. It may be pointed out, however, that the successful use of the Expansion Formula depends on the possibility of evaluating the roots of the determinantal equation $Z(p) = 0$, and this imposes a limitation on its applicability for the solution of engineering problems.

M. F. Gardner: There is a number of points of mathematical interest in Dr. Ku's paper, one of which seems to me to warrant particular attention. It is the use of the principle of superposition in the solution of a system which contains a parameter which is not a constant.

The self parameters of the stator and field circuits are assumed to be constants, but the mutual inductance between these circuits is assumed to vary sinusoidally as a function of time. The principle of superposition is considered applicable to the system, so the effects of the various e. m. fs. are solved for separately and an algebraic addition made of these results to obtain the actual response. Since discussions of the superposition principle are generally presented from the standpoint of circuits with constant coefficients, its application in such a case as this where there is a parameter variable with the time needs some investigation. While the use of this principle under such conditions is frequently made in engineering analysis, there does not seem to be available in engineering literature any mathematical demonstration that such applications are legitimate.

When the parameters of the circuit are fixed, that is when they are independent of time or circuit performance, and the case of impulsive response is barred, the principle of superposition can be expressed by the definite integral.

$$i(t) = \frac{d}{dt}\int_0^t A(t-\lambda)\,e(\lambda)\,d\lambda. \tag{1}$$

In this $A(t-\lambda)$ is formed from $A(t)$ which is the characteristic of the circuit as embodied in its response to a suddenly applied unit continuous e. m. f. when the circuit is initially in equilibrium. The integral then expresses the response $i(t)$ to an applied e. m. f. $e(t)$ of arbitrary form in terms of the same circuit's response to the unit e. m. f.

When the parameters of the circuit are not all fixed, but include some which are varying with time, this integral no longer has any significance because there is now no characteristic $A(t)$ for the entire circuit which is independent of the time at which it is determined. Such circuits can still be treated, however, for in general they can be divided so as to separate the variable portion from the invariable portion.[1] The differential equation for this case can then be written in the operational form

$$i(t) = \frac{e(t) - f(p, t)\, i(t)}{Z(p)} \tag{2}$$

where $f(p, t)$ represents the operator formed from the variable portion, and $Z(p)$ from the invariable portion. The first is expressed in general form to show that it may be a function of derivatives and integrals as well as time. The differential equation written in this modified form shows that the effect of the variable part of the circuit has been accounted for by the substitution of a fictitious e. m. f. equivalent to the impedance drop in that portion. This combines with the applied e. m. f. to produce the current that flows in the fixed part of the network. Since the superposition theorem holds for the fixed portion, it can be written

$$i(t) = \frac{d}{dt}\int_0^t A(t-\lambda)\,[e(\lambda) - f(p, \lambda)\, i(\lambda)]\, d\lambda \tag{3}$$

and we now have an integral equation. It should be noted that $A(t)$ is the characteristic of the invariable part of the circuit only, and the problem has been expressed as the application to this part of two related e. m. fs.

Now let us extend this idea to the case where there are several electromotive forces impressed in the circuit. Consider that a voltage $e_1(t)$ is applied. The response is given by

$$i_1(t) = \frac{d}{dt}\int_0^t A(t-\lambda)\,[e_1(\lambda) - f(p, \lambda)\, i_1(\lambda)]\, d\lambda \tag{4}$$

Had a voltage $e_2(t)$ been applied instead, an expression of similar form would have been obtained for its response $i_2(t)$. Now consider that $e_1(t)$ and $e_2(t)$ are applied in the network simultaneously, and write the resulting current as $[i_1(t) + i_2(t)]$. There then is obtained

$$[i_1(t) + i_2(t)] = \frac{d}{dt}\int_0^t A(t-\lambda)\,[(e_1(\lambda) + e_2(\lambda)) - f(p, \lambda)\,(i_1(\lambda) + i_2(\lambda))]\, d\lambda. \tag{5}$$

On the other hand if the above results, $i_1(t)$ and $i_2(t)$, of applying the two voltages separately are added and combined by transformation into one integral, the resultant expression is identical with (5).

The transformation necessary in making this step is

$$f(p, t)\, i_1(t) + f(p, t)\, i_2(t) = f(p, t)\,[i_1(t) + i_2(t)]. \tag{6}$$

This is legitimate because $f(p, t)$ is a linear operator, and obeys the distributive law although not the laws of algebra in general. It is interesting to note at this point that it is the lack of this property in functional operators, such as $f(p, i)$, that prevents a similar treatment of networks containing parameters which are dependent upon the currents and voltages present,—cases in which the superposition principle is known to be invalid.

Since it is seen that the result of adding the effects of two voltages applied separately to a circuit having time-varying parameters is the same as that obtained when these voltages are applied simultaneously, it justifies the splitting of a voltage into its parts and solving for the effects of these parts separately when such parameters are present. Since this is precisely the principle of superposition, it is seen that this principle can be legitimately applied when there are parameters present which are variable provided their change is one with time only.

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[1] J. R. Carson, "Theory and Calculation of Variable Electrical Systems," *Phys. Rev.*, Vol. 17, Feb. 1921, pp. 116-134.

Two-Reaction Theory of Synchronous Machines

Generalized Method of Analysis—Part I

BY R. H. PARK*
Associate, A. I. E. E.

Synopsis.—Starting with the basic assumption of no saturation or hysteresis, and with distribution of armature phase m. m. f. effectively sinusoidal as far as regards phenomena dependent upon rotor position, general formulas are developed for current, voltage, power, and torque under steady and transient load conditions. Special detailed formulas are also developed which permit the determination of current and torque on three-phase short circuit, during starting, and when only small deviations from an average operating angle are involved.

In addition, new and more accurate equivalent circuits are developed for synchronous and asynchronous machines operating in parallel, and the domain of validity of such circuits is established.

Throughout, the treatment has been generalized to include salient poles and an arbitrary number of rotor circuits. The analysis is thus adapted to machines equipped with field pole collars, or with amortisseur windings of any arbitrary construction.

It is proposed to continue the analysis in a subsequent paper.

* * * * *

THIS paper presents a generalization and extension of the work of Blondel, Dreyfus, and Doherty and Nickle, and establishes new and general methods of calculating current power and torque in salient and non-salient pole synchronous machines, under both transient and steady load conditions.

Attention is restricted to symmetrical three-phase† machines with field structure symmetrical about the axes of the field winding and interpolar space, but salient poles and an arbitrary number of rotor‡ circuits is considered.

Idealization is resorted to, to the extent that saturation and hysteresis in every magnetic circuit and eddy currents in the armature iron are neglected, and in the assumption that, as far as concerns effects depending on the position of the rotor, each armature winding may be regarded as, in effect, sinusoidally distributed.[3]

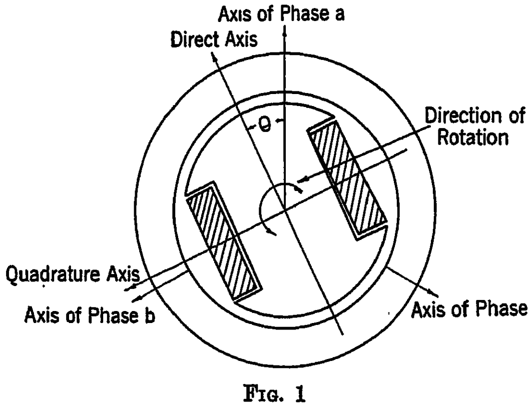

FIG. 1

A. Fundamental Circuit Equations

Consider the ideal synchronous machine of Fig. 1, and let

i_a, i_b, i_c = per unit instantaneous phase currents
e_a, e_b, e_c = per unit instantaneous phase voltages
ψ_a, ψ_b, ψ_c = per unit instantaneous phase linkages
t = time in electrical radians

$$p = \frac{d}{d\,t}$$

Then there is

$$\begin{aligned} e_a &= p\,\psi_a - r\,i_a \\ e_b &= p\,\psi_b - r\,i_b \\ e_c &= p\,\psi_c - r\,i_c \end{aligned} \qquad (1)$$

It has been shown previously[3] that

$$\begin{aligned} \psi_a = {} & I_d \cos\theta - I_q \sin\theta \\ & - \frac{x_0}{3}\,[i_a + i_b + i_c] - \frac{x_d + x_q}{3}\left[i_a - \frac{i_b + i_c}{2}\right] \\ & - \frac{x_d - x_q}{3}\,[i_a \cos 2\,\theta + i_b \cos(2\,\theta - 120) \\ & \qquad + i_c \cos(2\,\theta + 120)] \\ \psi_b = {} & I_d \cos(\theta - 120) - I_q \sin(\theta - 120) \\ & - x_0\,\frac{i_a + i_b + i_c}{3} - \frac{x_d + x_q}{3}\left[i_b - \frac{i_c + i_a}{2}\right] \\ & - \frac{x_d - x_q}{3}\,[i_a \cos(2\,\theta - 120) + i_b \cos(2\,\theta + 120) \\ & \qquad + i_c \cos 2\,\theta] \qquad (2) \\ \psi_c = {} & I_d \cos(\theta + 120) \\ & - I_q \sin(\theta + 120) - x_0\,\frac{i_a + i_b + i_c}{3} \\ & - \frac{x_d + x_q}{3}\left[i_c - \frac{i_a + i_b}{2}\right] \\ & - \frac{x_d - x_q}{3}\,[i_a \cos(2\,\theta + 120) + i_b \cos 2\,\theta \\ & \qquad + i_c \cos(2\,\theta - 120)] \end{aligned}$$

where,

---

*General Engg. Dept., General Electric Company, Schenectady, N. Y.

†Single-phase machines may be regarded as three-phase machines with one phase open circuited.

‡Stator for a machine with stationary field structure.

[3] For numbered references see Bibliography.

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Synopsis.—Starting with the basic assumption of no saturation or hysteresis, and with distribution of armature phase m. m. f. effectively sinusoidal as far as regards phenomena dependent upon rotor position, general formulas are developed for current, voltage, power, and torque under steady and transient load conditions. Special detailed formulas are also developed which permit the determination of current and torque on three-phase short circuit, during starting, and when only small deviations from an average operating angle are involved.

In addition, new and more accurate equivalent circuits are developed for synchronous and asynchronous machines operating in parallel, and the domain of validity of such circuits is established.

Throughout, the treatment has been generalized to include salient poles and an arbitrary number of rotor circuits. The analysis is thus adapted to machines equipped with field pole collars, or with amortisseur windings of any arbitrary construction.

It is proposed to continue the analysis in a subsequent paper.

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THIS paper presents a generalization and extension of the work of Blondel, Dreyfus, and Doherty and Nickle, and establishes new and general methods of calculating current power and torque in salient and non-salient pole synchronous machines, under both transient and steady load conditions.

Attention is restricted to symmetrical three-phase† machines with field structure symmetrical about the axes of the field winding and interpolar space, but salient poles and an arbitrary number of rotor‡ circuits is considered.

Idealization is resorted to, to the extent that saturation and hysteresis in every magnetic circuit and eddy currents in the armature iron are neglected, and in the assumption that, as far as concerns effects depending on the position of the rotor, each armature winding may be regarded as, in effect, sinusoidally distributed.[3]

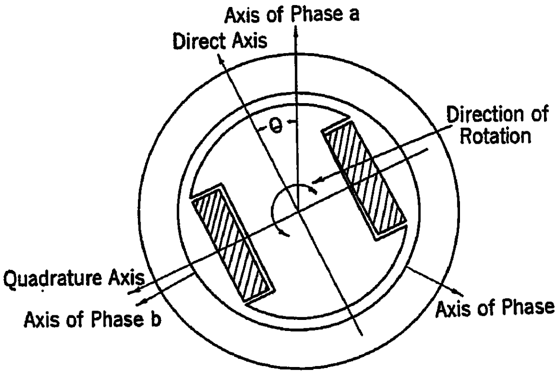

FIG. 1

A. Fundamental Circuit Equations

Consider the ideal synchronous machine of Fig. 1, and let

i_a, i_b, i_c = per unit instantaneous phase currents
e_a, e_b, e_c = per unit instantaneous phase voltages
ψ_a, ψ_b, ψ_c = per unit instantaneous phase linkages
t = time in electrical radians

$$p = \frac{d}{dt}$$

Then there is

$$\begin{aligned} e_a &= p\,\psi_a - r\,i_a \\ e_b &= p\,\psi_b - r\,i_b \\ e_c &= p\,\psi_c - r\,i_c \end{aligned} \qquad (1)$$

It has been shown previously[3] that

$$\begin{aligned} \psi_a = {} & I_d \cos\theta - I_q \sin\theta \\ & - \frac{x_0}{3}[i_a + i_b + i_c] - \frac{x_d + x_q}{3}\left[i_a - \frac{i_b + i_c}{2}\right] \\ & - \frac{x_d - x_q}{3}[i_a \cos 2\theta + i_b \cos(2\theta - 120) \\ & \qquad + i_c \cos(2\theta + 120)] \\ \psi_b = {} & I_d \cos(\theta - 120) - I_q \sin(\theta - 120) \\ & - x_0 \frac{i_a + i_b + i_c}{3} - \frac{x_d + x_q}{3}\left[i_b - \frac{i_c + i_a}{2}\right] \\ & - \frac{x_d - x_q}{3}[i_a \cos(2\theta - 120) + i_b \cos(2\theta + 120) \\ & \qquad + i_c \cos 2\theta] \\ \psi_c = {} & I_d \cos(\theta + 120) \\ & - I_q \sin(\theta + 120) - x_0 \frac{i_a + i_b + i_c}{3} \\ & - \frac{x_d + x_q}{3}\left[i_c - \frac{i_a + i_b}{2}\right] \\ & - \frac{x_d - x_q}{3}[i_a \cos(2\theta + 120) + i_b \cos 2\theta \\ & \qquad + i_c \cos(2\theta - 120)] \end{aligned} \qquad (2)$$

where,

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†Single-phase machines may be regarded as three-phase machines with one phase open circuited.

‡Stator for a machine with stationary field structure.

[3]For numbered references see Bibliography.

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I_d = per-unit excitation in direct axis
I_q = per-unit excitation in quadrature axis
x_d = direct synchronous reactance
x_q = quadrature synchronous reactance
x_0 = zero phase-sequence reactance

As shown in the Appendix, if normal linkages in the field circuit are defined as those obtaining at no load* there is in the case of no rotor circuits in the direct axis in addition to the field,

Φ = per-unit instantaneous field linkages
$= I - (x_d - x_d')\, i_d$

where,

I = per-unit instantaneous field current

$$i_d = \frac{2}{3}\,\{i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)\} \tag{3}$$

On the other hand, if n additional rotor circuits exist in the direct axis there is,

$$\Phi = I + X_{f1d}\, I_{1d} + X_{f2d}\, I_{2d} + \ldots + X_{fnd}\, I_{nd} - (x_d - x_d')\, i_d$$

where,

$I_{1d}, I_{2d}, \ldots$ etc., are the per-unit instantaneous currents in circuits 1, 2, etc., of the direct axis, $X_{f1d}, X_{f2d}, \ldots$ etc., are per-unit mutual coefficients between the field and circuits 1, 2, etc., of the direct axis.

Similar relations exist for the linkages in each of the additional rotor circuits except $x_d - x_d'$ is to be replaced by a term x_m. However, since all of these additional circuits are closed, it follows that there is an operational result

$$\begin{aligned} I_d &= I + I_{1d} + I_{2d} + \ldots + I_{nd} \\ &= G(p)\, E + H(p)\, i_d \end{aligned} \tag{4}$$

where E is the per-unit value of the instantaneous field voltage, and $G(p)$ and $H(p)$ are operators such that

$$G(o) = 1 \qquad G(\infty) = 0$$
$$H(o) = 0 \qquad H(\infty) = x_d - x_d''$$

x_d'' = the subtransient reactance[2]

It will be convenient to write $H(p) = x_d - x_d(p)$ and to rewrite (4) in the form,

$$I_d = G(p)\, E + [x_d - x_d(p)]\, i_d \tag{4a}$$

If there are no additional rotor circuits, there is, as shown in Appendix I,

$$\Psi = I - (x_d - x_d')\, i_d$$
$$E = T_0\, p\, \Psi + I$$

where T_0 is the open circuit time constant of the field in radians.

There is then,

$$G(p) = \frac{1}{T_0\, p + 1}$$

$$x_d(p) = \frac{x_d'\, T_0\, p + x_d}{T_0\, p + 1}$$

If there is one additional rotor circuit in the direct axis there is,

$$\Psi = I + X_{f1d}\, I_{1d} - (x_d - x_d')\, i_d = \frac{E - I}{T_0\, p}$$

$$\Psi_{1d} = X_{11d}\, I_{1d} + X_{f1d}\, I - x_{m1d}\, i_d = \frac{-I_{1d}}{T_{01d}\, p}$$

which gives,

$$G(p) = \frac{[X_{11d} - X_{f1d}]\, T_{01d}\, p + 1}{A(p)}$$

$$x_d(p) = x_d - \frac{T_0\, T_{01d}\, [X_{11d}\,(x_d - x_d') - X_{f1d}\, x_{m1d}]\, p^2 + [(x_d - x'_d)\, T_{01d} + x_{m1d}\, T_0]\, p}{A(p)}$$

where,

$$A(p) = [X_{11d} - X_{f1d}^2]\, T_0\, T_{01d}\, p^2 + [X_{11d}\, T_0 + T_{01d}]\, p + 1$$

If there is more than one additional rotor circuit the operators $G(p)$ and $x_d(p)$ will be more complicated but may be found in the same way. The effects of external field resistance may be found by changing the term I in the field voltage equation to $R\, I$. Open circuited field corresponds to R equal to infinity.

Similarly, there will be

$$I_q = [x_q - x_q(p)]\, i_q \tag{5}$$

where,

$$i_q = -\frac{2}{3}\{i_a \sin\theta + i_b \sin(\theta - 120) + i_c \sin(\theta + 120)\} \tag{3a}$$

$$x_q(o) = x_q,\ x_q(\infty) = x_q''.$$

So far, 10 equations have been established relating the 15 quantities e_a, e_b, e_c, i_a, i_b, i_c, ψ_a, ψ_b, ψ_c, i_d, i_q, I_d, I_q, E, θ in a general way. It follows that when any five of the quantities are known the remaining 10 may be determined. Their determination is very much facilitated, however, by the introduction of certain auxiliary quantities e_d, e_q, e_0, i_0, ψ_d, ψ_q, ψ_0.

Thus, let

$$i_0 = \frac{1}{3}\{i_a + i_b + i_c\} \tag{3b}$$

$$e_d = \frac{2}{3}\{e_a \cos\theta + e_b \cos(\theta - 120) + e_c \cos(\theta + 120)\}$$

$$e_q = -\frac{2}{3}\{e_a \sin\theta + e_b \sin(\theta - 120) + e_c \sin(\theta + 120)\} \tag{6}$$

$$e_0 = \frac{1}{3}\{e_a + e_b + e_c\}$$

$$\psi_d = \frac{2}{3}\{\psi_a \cos\theta + \psi_b \cos(\theta - 120) + \psi_c \cos(\theta + 120)\}$$

$$\psi_q = -\frac{2}{3}\{\psi_a \sin\theta + \psi_b \sin(\theta - 120) + \psi_c \sin(\theta + 120)\} \tag{7}$$

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*This definition is somewhat different from that given in reference 2.

$$\psi_0 = \frac{1}{3}\ \{\psi_a + \psi_b + \psi_c\}$$

then from Equation (1) there is

$$e_d = \frac{2}{3}\{\cos\theta\, p\,\psi_a + \cos(\theta-120)\, p\,\psi_b + \cos(\theta+120)\, p\,\psi_c\} - r\,i_d$$

$$e_q = -\frac{2}{3}\ \{\sin\theta\, p\,\psi_a + \sin(\theta-120)\, p\,\psi_b + \sin(\theta+120)\, p\,\psi_c\} - r\,i_q$$

$$e_0 = p\,\psi_0 - r\,i_0$$

but,

$$p\,\psi_d = \frac{2}{3}\ \{\cos\theta\, p\,\psi_a + \cos(\theta-120)\, p\,\psi_b + \cos(\theta+120)\, p\,\psi_c\}$$

$$-\frac{2}{3}\{\sin\theta\ \psi_a + \sin(\theta-120)\, p\,\psi_b + \sin(\theta+120)\, p\,\psi_c\}\ p\,\theta$$

$$= e_d + r\,i_d + \psi_q\, p\,\theta$$

$$p\,\psi_q = -\ \frac{2}{3}\ \{\sin\theta\, p\,\psi_a + \sin(\theta-120)\, p\,\psi_b + \sin(\theta+120)\, p\,\psi_c\}$$

$$-\frac{2}{3}\{\cos\theta\,\psi_a + \cos(\theta-120)\,\psi_b + \cos(\theta+120)\,\psi_c\}\,p\,\theta$$

$$= e_q + r\,i_q - \psi_d\, p\,\theta$$

hence there is

$$e_d = p\,\psi_d - r\,i_d - \psi_q\, p\,\theta \tag{8}$$

$$e_q = p\,\psi_q - r\,i_q + \psi_d\, p\,\theta \tag{9}$$

$$e_0 = p\,\psi_0 - r\,i_0 \tag{10}$$

Also it may be readily verified that

$$\psi_d = I_d - x_d\, i_d = G\,(p)\,E - x_d\,(p)\,i_d \tag{11}$$

$$\psi_q = I_q - x_q\, i_q = -\,x_q\,(p)\,i_q \tag{12}$$

$$\psi_0 = -\,x_0\, i_0 \tag{13}$$

Equations (8) to (13) establish six relatively simple relations between the 11 quantities e_d, e_q, e_0, i_d, i_q, i_0, ψ_d, ψ_q, ψ_0, E, θ. In practise it is usually possible to determine five of these quantities directly from the terminal conditions, after which the remaining six may be calculated with relative simplicity. After the direct, quadrature, and zero quantities are known the phase quantities may be determined from the identical relations

$$\begin{aligned} i_a &= i_d \cos\theta - i_q \sin\theta + i_0 \\ i_b &= i_d \cos(\theta-120) - i_q \sin(\theta-120) + i_0 \\ i_c &= i_d \cos(\theta+120) - i_q \sin(\theta+120) + i_0 \end{aligned} \tag{14}$$

$$\begin{aligned} \psi_a &= \psi_d \cos\theta - \psi_q \sin\theta + \psi_0 \\ \psi_b &= \psi_d \cos(\theta-120) - \psi_q \sin(\theta-120) + \psi_0 \\ \psi_c &= \psi_d \cos(\theta+120) - \psi_q \sin(\theta+120) + \psi_0 \end{aligned} \tag{15}$$

$$\begin{aligned} e_a &= e_d \cos\theta - e_q \sin\theta + e_0 \\ e_b &= e_d \cos(\theta-120) - e_q \sin(\theta-120) + e_0 \\ e_c &= e_d \cos(\theta+120) - e_q \sin(\theta+120) + e_0 \end{aligned} \tag{16}$$

Referring to Fig. 2, it may be seen that when there are no zero quantities, that is, when $e_0 = \psi_0 = i_0 = 0$, the phase quantities may be regarded as the projection of vectors $\bar{e}$, $\bar{\psi}$, and $\bar{i}$ on axes lagging the direct axis by angles θ, $\theta - 120$ and $\theta + 120$, where taking the direct axis as the axis of reals,

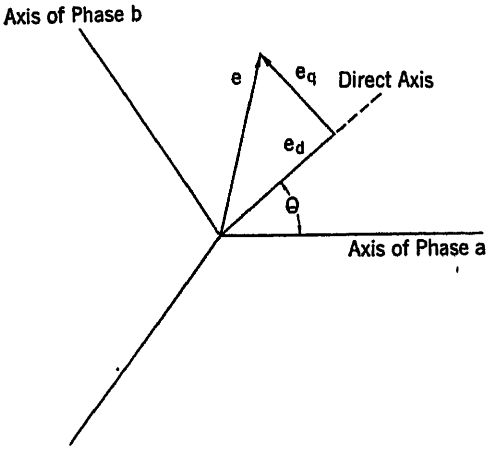

FIG. 2

$$\begin{aligned} \bar{e} &= e_d + j\,e_q \\ \bar{\psi} &= \psi_d + j\,\psi_q \\ \bar{i} &= i_d + j\,i_q \end{aligned}$$

If we introduce in addition the vector quantity,

$$\bar{I} = I_d + j\,I_q$$

the circuit equations previously obtained may be transferred into the corresponding vector forms,

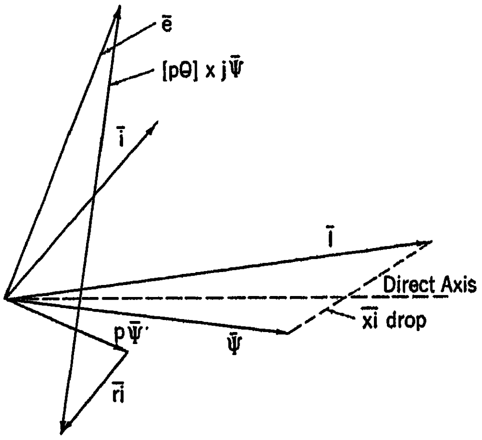

FIG. 3

$$\begin{aligned} \bar{e} &= p\,\bar{\psi} - \overline{ri} + [p\,\theta]\,j\,\bar{\psi} \\ \bar{\psi} &= \bar{I} - \overline{xi} \end{aligned}$$

where, $\overline{xi} = x_d\, i_d + j\, x_q\, i_q$

Fig. 3 shows these relations graphically.

B. Armature Power Output

The per-unit instantaneous power output from the armature is necessarily proportional to the sum

$e_a i_a + e_b i_b + e_c i_c$. By consideration of any instant during normal operation at unity power factor it may be seen that the factor of proportionality must be 2/3. That is,

$$P = \text{per-unit instantaneous power output}$$
$$= 2/3 \; \{ e_a i_a + e_b i_b + e_c i_c \}$$

Substituting from Equations (14) and (16) there results the useful relation,

$$P = e_d i_d + e_q i_q + e_0 i_0 \qquad (17)$$

C. Electrical Torque on Rotor

It is possible to determine the electrical torque on the rotor directly from the general relation,

{Total power output} =

{mechanical power transferred across gap}

+ {rate of decrease of total stored magnetic energy}

− {total ohmic losses} (18)

However, since this torque depends uniquely only on the magnitudes of the currents in every circuit of the machine, it follows that a general formula for torque may be derived by considering any special case in which arbitrary conditions are imposed as to the way in which these currents are changing as the rotor moves.

The simplest conditions to impose are that I_d, I_q, i_d, i_q, and i_0 remain constant as the rotor moves. In this case there will be no change in the stored magnetic energy of the machine as the rotor moves, and the power output of the rotor will be just equal in magnitude and opposite in sign to the rotor losses. It follows that under the special conditions assumed, Equation (18) becomes simply,

{armature power output} =

{mechanical power across gap} − {armature losses}

$$\text{or, } P = T\, p\, \theta - \frac{2\, r}{3} \{ i_a^2 + i_b^2 + i_c^2 \}$$
$$= T\, p\, \theta - r \{ i_d^2 + i_q^2 + i_0^2 \}$$

Then,

$$T = \text{per-unit instantaneous electrical torque}$$
$$= \frac{e_d i_d + e_q i_q + e_0 i_0 + r\, \{ i_d^2 + i_q^2 + i_0^2 \}}{p\, \theta}$$

but subject to the conditions imposed,

$$e_d = -\psi_q\, p\, \theta - r\, i_d$$
$$e_q = \psi_d\, p\, \theta - r\, i_q$$
$$e_0 = -r\, i_0$$

It therefore follows that,

$$T = i_q \psi_d - i_d \psi_q \qquad (19)$$
$$= \text{vector product of } \bar{\psi} \text{ and } \bar{i}$$
$$= \bar{\psi} \times \bar{i} \qquad (19a)$$

a result which could have been established directly by physical reasoning. Formula (19) is employed by Dreyfus in his treatment of self-excited oscillations of synchronous machines.[14]

D. Constant Rotor Speed

Suppose that the constant slip of the rotor is s. Then there is,

$$e_d = p\, \psi_d - r\, i_d - (1 - s)\, \psi_q$$
$$e_q = p\, \psi_q - r\, i_q + (1 - s)\, \psi_d$$

but,

$$\psi_d = G(p)\, E - x_d(p)\, i_d$$
$$\psi_q = -x_q(p)\, i_q$$

Putting

$$p\, x_d(p) + r = z_d(p)$$
$$p\, x_q(p) + r = z_q(p)$$

there is

$$e_d = p\, G(p)\, E - z_d(p)\, i_d + (1 - s)\, x_q(p)\, i_q \qquad (20)$$
$$e_q = (1 - s)\, [G(p)\, E - x_d(p)\, i_d] - z_q(p)\, i_q \qquad (21)$$

Solving gives,

$$i_d = \{ [p\, z_q(p) + (1 - s)^2 x_q(p)]\, G(p)\, E - z_q(p)\, e_d - (1 - s)\, x_q(p)\, e_q \} \div D(p) \qquad (22)$$

$$i_q = \frac{(1 - s)\, r\, G(p)\, E - z_d(p)\, e_q + (1 - s)\, x_d(p)\, e_d}{D(p)} \qquad (23)$$

where, $D(p) = z_d(p)\, z_q(p) + (1 - s)^2 x_d(p)\, x_q(p)$

E. Two Machines Connected Together

Suppose that two machines which we will designate respectively by the subscripts g and h, are connected together, but not to any other machines or circuits, and assume in addition that there are no zero quantities. In this case the voltages of each machine will be equal phase for phase, and it therefore follows that the voltage vectors of each machine must coincide, as shown in Fig. 4.

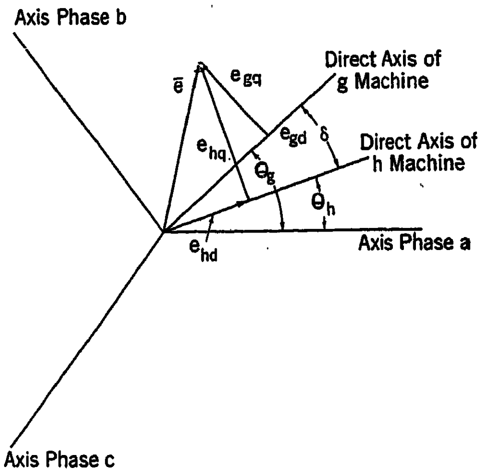

FIG. 4

Referring to the figure it will be seen that the direct and quadrature components of voltage of the two machines are subject to the mutual relations,

$$e_{hd} = e_{gd} \cos\delta - e_{gq} \sin\delta$$
$$e_{hq} = e_{gd} \sin\delta + e_{gq} \cos\delta \qquad (24)$$
$$e_{gd} = e_{hd} \cos\delta + e_{hq} \sin\delta$$
$$e_{gq} = -e_{hd} \sin\delta + e_{hq} \cos\delta \qquad (25)$$

On the other hand, for currents there will be

$$i_{hd} = -\,\{i_{gd}\cos\delta - i_{gq}\sin\delta\}$$
$$i_{hq} = -\,\{i_{gd}\sin\delta + i_{gq}\cos\delta\} \qquad (26)$$
$$i_{gd} = -\,\{i_{hd}\cos\delta + i_{hq}\sin\delta\}$$
$$i_{gq} = -\,\{-\,i_{hd}\sin\delta + i_{hq}\cos\delta\} \qquad (27)$$

F. *One Machine on an Infinite Bus*

In (*E*), if machine h has zero impedance, it follows from (20) and (21) that $e_{hd} = 0$, $e_{hq} =$ bus voltage say $= e$.

Then for machine g there is,

$$e_d = e\sin\delta$$
$$e_q = e\cos\delta \qquad (28)$$

G. *Torque Angle Relations*

From Equations (11), (12), and (19), there is,

$$T = \frac{I_q\,\psi_d}{x_q} - \frac{I_d\,\psi_q}{x_d} - \frac{x_d - x_q}{x_d\,x_q}\,\psi_d\,\psi_q$$

Then if the rotor leads the vector $\bar{\psi}$ by an angle $\bar{\delta}$ there is

$$\psi_q = -\,\psi\sin\delta$$
$$\psi_d = \psi\cos\delta$$

$$T = \frac{I_q\,\psi}{x_q}\cos\delta + \frac{I_d\,\psi\sin\delta}{x_d} + \frac{x_d - x_q}{2\,x_d\,x_q}\,\psi^2\sin 2\,\delta \qquad (29)$$

A derivation of this formula for steady load conditions has been previously given by Doherty and Nickle.[9]

H. *Three-Phase Short Circuit with Constant Rotor Speed Maintained*

Since a three-phase short circuit causes e_d and e_q to vanish suddenly, its effect with constant rotor speed maintained may be found by impressing $e_d = -\,e_{d0}$, $e_q = -\,e_{q0}$ in (22) and (23) where e_{d0} and e_{q0} are the values of e_d and e_q before the short circuit. The initial currents existing before the short circuit must be added to the currents found in this way in order to obtain the resultant current after the short circuit.

With $s = 0$ and E constant there is in detail,

$$i_d = \frac{z_q\,(p)\;e_{d0} + x_q\,(p)\;e_{q0}}{D\,(p)}\,.\,1 + \frac{x_q\,E - r\,e_{d0} - x_q\,e_{q0}}{r^2 + x_d\,x_q}$$

$$i_q = \frac{z_d\,(p)\;e_{q0} - x_d\,(p)\;e_{d0}}{D\,(p)}\,.\,1 + \frac{r\,E - r\,e_{q0} + x_d\,e_{d0}}{r^2 + x_d\,x_q}$$

$$(30)$$

The working out of the formulas may be illustrated by consideration of the simple case of a machine with no rotor circuits in addition to the field. In this case there is

$$x_q\,(p) = x_q$$

$$x_d\,(p) = \frac{x_d'\,T_0\,p + x_d}{T_0\,p + 1}$$

$$D\,(p) = \left\langle \frac{x_d'\,T_0\,p + x_d}{T_0\,p + 1}\,p + r \right\rangle \langle x_q\,p + r\rangle$$

$$+ \frac{x_d'\,T_0\,p + x_d}{T_0\,p + 1}\,x_q$$

$$\begin{aligned} x_d'\,x_q\,T_0\,p^3 & \\ + [x_d'\,r\,T_0 + (x_d + r\,T_0)\,x_q]\,p^2 & \\ + [r\,(x_d + x_q + r\,T_0) + x_d'\,x_q\,T_0]\,p & \\ = \frac{+\,r^2 + x_d\,x_q}{T_0\,p + 1} & \\ = \frac{d\,(p)}{T_0\,p + 1} & \qquad (31) \end{aligned}$$

By the expansion theorem there is, finally,

$$i_d = \frac{x_q\,E}{r^2 + x_d\,x_q} + \sum_1^3 \frac{(T_0\,\alpha_n + 1)\,\langle (x_q\,\alpha_n + r)\,e_{d0} + x_q\,e_{q0}\rangle\,\epsilon^{-\alpha_n t}}{\alpha_n\,d'\,(\alpha_n)}$$

$$i_q = \frac{r\,E}{r^2 + x_d\,x_q} + \sum_1^3 \frac{\langle x_d'\,T_0\,\alpha_n^2 + (x_d + r\,T_0)\,\alpha_n + r\rangle\,e_{q0} - (T_0\,\alpha_n\,x'_d + x_d)\,e_{d0}}{\alpha_n\,d'\,(\alpha_n)}\;\epsilon^{-\alpha_n t} \qquad (32)$$

where the summation is extended over the roots of

$$d\,(\alpha) = 0 \text{ and } d'\,(p) = \frac{d}{d\,p}\,d\,(p)$$

The phase currents may, of course, be found from Equations (32) by the application of Equations (14). For the particular case

$$T_0 = 2{,}000,\; x_d = 1.00,\; x_q = 0.60,\; x_d' = 0.30$$

the roots α_1, α_2, α_3 of the equation $d\,(p) = 0$, were found to be as shown in Figs. 5, 6, and 7, where

$$\alpha_2 = \alpha_a + \alpha_b$$
$$\alpha_3 = \alpha_a - \alpha_b$$

It will be noted that, as would necessarily be the

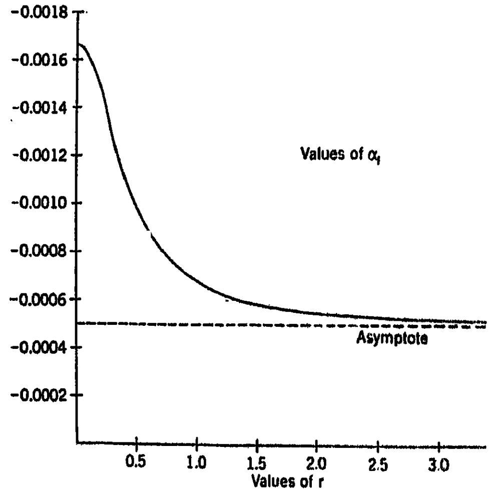

FIG. 5

case, where $r = 0$, α_1 is equal to the reciprocal of the short circuit time constant of the machine, *i. e.*, for $r = 0$,

$$\alpha_1 = - \frac{x_d}{x_d'} \frac{1}{T_0} = - 0.001667$$

while for $r = \infty$

$$\alpha_1 = - \frac{1}{T_0} = - 0.000500$$

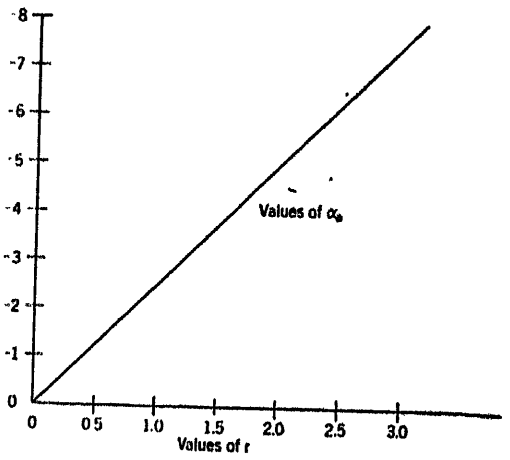

FIG. 6

The root α_a is found to be almost exactly equal to the value which it would have were $T_0 = \infty$, *i. e.*,

$$\alpha_a = \frac{r\,(x_d' + x_q)}{2\,x_d'\,x_q} \text{ approximately}$$

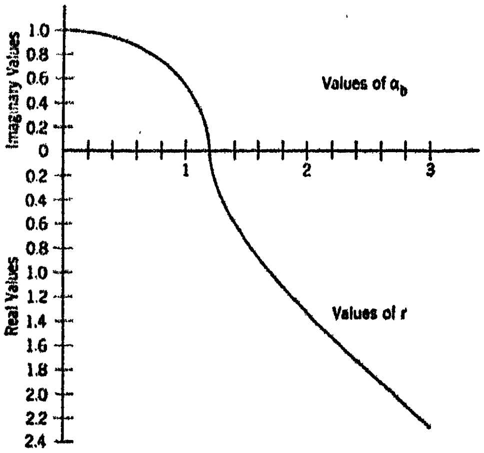

FIG. 7

Thus, in the special case considered this approximate formula gives

$$\alpha_a = \frac{(0.30 + 0.60)\,r}{2 \times 0.30 \times 0.60} = 2.50\,r$$

which checks the result found by the exact solution of the cubic.

I. Starting Torque

On infinite bus and with slip s, there will be, choosing $\delta = \frac{\pi}{2} - s\,t$, and referring to Equation (28),

$$e_d = \cos s\,t$$
$$e_q = \sin s\,t$$

If we now introduce a system of vectors rotating at s per-unit angular velocity there is

$$ed = 1.0$$
$$e_q = -j$$
$$p = j\,s \qquad (33)$$

Then from (22) and (23),

$$i_d = \{ j\,s\,x_q\,(j\,s) + r - j\,(1 - s)\,x_q\,(j\,s) \} \div \{ [j\,s\,x_d\,(j\,s) + r]\,[j\,s\,x_q\,(j\,s) + r] + (1 - s)^2\,x_d\,(j\,s)\,x_q\,(j\,s) \}$$

$$= \frac{j\,(1 - 2\,s)\,x_q\,(j\,s) - r}{r^2 + (1 - 2\,s)\,x_d'(j\,s)\,x_q'(j\,s) + j\,s\,r[x_d'(j\,s) + x_q'(j\,s)]}$$

$$= \left\{ j\,x_q\,(j\,s) - \frac{r}{1 - 2\,s} \right\} \div \left\{ x_d\,(j\,s)\,x_q\,(j\,s) + \frac{r}{1 - 2\,s}\,[r + j\,s\,\langle x_d\,(j\,s) + x_q\,(j\,s) \rangle\,] \right\} \qquad (34)$$

$$i_q = - \frac{[j\,s\,x_d\,(j\,s) + r]\,(-\,j) - (1 - s)\,x_d\,(j\,s)}{r^2 + (1 - 2\,s)\,x_d\,(j\,s) x_q\,(j\,s) + j\,s\,r[x_d\,(j\,s) + x_q\,(js)]}$$

$$= \left\{ x_d\,(j\,s) + \frac{j\,r}{1 - 2\,s} \right\} \div \left\{ x_d\,(j\,s)\,x_q\,(j\,s) + \frac{r}{1 - 2\,s}\,[r + j\,s\,\langle x_d\,(j\,s) + x_q\,(j\,s) \rangle\,] \right\} \qquad (35)$$

The expressions for average power and torque then become,

$$P_{av} = 1/2\,[e_d \,.\, i_d + e_q \,,\, i_q]$$
$$T_{av} = 1/2\,[i_q \,.\, \psi_d - i_d \,.\, \psi_q]$$

where the dot indicates the scalar product, or

$$P_{av} = 1/2\,[1 \,.\, i_d - j \,.\, i_q]$$
$$= 1/2\,[\text{Real of } i_d - \text{Imaginary of } i_q] \qquad (36)$$

There is in general,

$$e_d + r\,i_d = p\,\psi_d - (1 - s)\,\psi_q$$
$$e_q + r\,i_q = (1 - s)\,\psi_d + p\,\psi_q$$

$$\psi_d = \frac{\begin{vmatrix} e_d + r\,i_d & -(1 - s) \\ e_q + r\,i_q & p \end{vmatrix}}{\begin{vmatrix} p & -(1 - s) \\ (1 - s) & p \end{vmatrix}}$$

$$= \frac{p\,(e_d + r\,i_d) + (1 - s)\,(e_q + r\,i_q)}{p^2 + (1 - s)^2} \qquad (37)$$

$$\psi_q = \frac{p\,(e_q + r\,i_q) - (1 - s)\,(e_d + r\,i_d)}{p^2 + (1 - s)^2} \qquad (38)$$

$$\psi_d = \frac{j\,s\,(e_d + r\,i_d) + (1-s)\,(e_q + r\,i_q)}{1-2\,s}$$

$$\psi_q = \frac{j\,s\,(e_q + r\,i_q) - (1-s)\,(e_d + r\,i_d)}{1-2\,s}$$

with $e_d = 1.0,\ e_q = -\,j$

$$\psi_d = \frac{j\,s + j\,s\,r\,i_d + (1-s)\,(-\,j) + (1-s)\,r\,i_q}{1-2\,s}$$

$$= \frac{-\,(1-2s)\,j + r\,[j\,s\,i_d + (1-s)\,i_q]}{1-2\,s}$$

$$= -\,j + \frac{r}{1-2\,s}\,[j\,s\,i_d + (1-s)\,i_q] \qquad (39)$$

$$\psi_q = \frac{j\,s\,(-\,j + r\,i_q) - (1-s) - r\,(1-s)\,i_d}{1-2\,s}$$

$$= \frac{-\,(1-2\,s) + r\,[j\,s\,i_q - (1-s)\,i_d]}{1-2\,s}$$

$$= -\,1 + \frac{r}{1-2\,s}\,[j\,s\,i_q - (1-s)\,i_d] \qquad (40)$$

Thus,

$$T_{av} = 1/2\left[\begin{array}{l} i_q\,.\,(-\,j) + i_q\,.\,\dfrac{r}{1-2\,s}\,\langle j\,s\,i_d + (1-s)\,i_q\rangle \\ -\,i_d\,.\,(-\,1) - i_d\,.\,\dfrac{r}{1-2\,s}\,\langle j\,s\,i_q - (1-s)\,i_d\rangle \end{array}\right]$$

$$= P_{av} + \frac{r}{2(1-2\ s)}\left[\begin{array}{l} (1-s)\,\langle i_q^2 + i_d^2\rangle \\ +\,2\,s\,i_q\,.\,j\,i_d \end{array}\right]$$

$$= P_{av} + \frac{r}{2}\,(i_q^2 + i_d^2) + \frac{r\,s}{2(1-2\,s)}\left[\begin{array}{l} i_q^2 + i_d^2 \\ +\,2\,i_q\,.\,j\,i_d \end{array}\right]$$

$$= P_{av} + r\,\frac{i_q^2 + i_d^2}{2} + \frac{r\,s}{2(1-2\ s)}\,(i_q + j\,i_d)^2 \qquad (41)$$

Mr. Ralph Hammar, who has been engaged in the application of the general method of calculation outlined above, to the predetermination of the starting torque of practical synchronous motors, has suggested an interesting modification of formulas (36) and (41), based upon the fact that, since the total m. m. f. consists of direct and quadrature components pulsating at slip frequency, it may be resolved into two components, one moving forward at a per-unit speed $1 - s + s = 1.0$, and the other moving backward at a per-unit speed $1 - s - s = 1 - 2\,s$. Thus from this standpoint half of both the direct and quadrature components will move forward, and half backward. Since the quadrature axis is ahead of the direct it follows that as far as concerns the forward component the quadrature current i_q is equivalent to a d-c. $j\,i_q$, while as regards backward component it is equivalent to a direct component $-\,j\,i_q$. It follows that the vector amounts of forward and backward m. m. f. or current are

$$\text{forward current} = i_f = \frac{1}{2}\,(i_d + j\,i_q)$$

$$\text{backward current} = i_b = \frac{1}{2}\,(i_d - j\,i_q) \qquad (42)$$

If we define by analogy,

$$\text{forward voltage} = \frac{1}{2}\,(e_d + j\,e_q)$$

$$\text{backward voltage} = \frac{1}{2}\,(e_d - j\,e_q) \qquad (43)$$

There is,

$$i_f = \frac{1}{2}\left\{\frac{-\,2\,r}{1-2\,s} + j\ [x_d\,(j\,s) + x_q\,(j\,s)]\right\} \div \left\{x_d\,(j\,s)\,x_q\ (j\,s) + \frac{r}{1-2\,s}\,\langle r + j\,s\,[x_d\,(j\,s) + x_q\,(j\,s)]\rangle\right\}$$

$$i_b = \frac{1}{2}\left\{j\,[x_q\,(j\,s) - x_d\,(j\,s)]\right\} \div \left\{x_d\,(j\,s)\,x_q\,(j\,s) + \frac{r}{1-2\,s}\ \langle r + j\,s\,[x_d\,(j\,s) + x_q\,(j\,s)]\,\rangle\right\}$$

$$e_f = 1.0 \qquad (44)$$

$$e_b = 0 \qquad (45)$$

$$P_{av} = e_f\,.\,i_f = \text{real of } i_f \qquad (46)$$

$$T_{av} = P_{av} + r\,i_f^2 + \frac{r}{1-2\,s}\,i_b^2 \qquad (47)$$

J. Zero Armature Resistance, One Machine Connected to an Infinite Bus

Assume that a machine of negligible armature resistance is operating from an infinite bus of per-unit voltage e, at synchronous speed, with a steady excitation voltage E_0, and displacement angle δ_0. At the instant $t = 0$, let δ and E change.

There is,

$$i_d = \frac{E_0 - \psi_{d0}}{x_d} - \frac{1}{x_d\,(p)}\,\Delta\,\psi_d + \frac{G\,(p)}{x_d\,(p)}\,\Delta\,E$$

$$i_q = -\,\frac{\psi_{q0}}{x_q} - \frac{1}{x_q\,(p)}\,\Delta\,\psi_q$$

$$\psi_d = e\cos\delta$$

$$\psi_q = -\,e\sin\delta$$

From which there is, by obvious re-arrangement,

$$i_d = \frac{E - e\cos\delta}{x_d} + e\,\frac{x_d - x_d(p)}{x_d\,x_d(p)}(\cos\delta_0 - \cos\delta)$$

$$- \frac{x_d(p) - G(p)\,x_d}{x_d\,x_d(p)}\,\Delta E$$

$$i_q = \frac{e\sin\delta}{x_q} + e\,\frac{x_q - x_q(p)}{x_q\,x_q(p)}(\sin\delta - \sin\delta_0) \qquad (48)$$

Then,

$$T = \frac{E\,e\sin\delta}{x_d} + \frac{x_d - x_q}{2\,x_d\,x_q}e^2\sin 2\delta$$

$$+ e^2\cos\delta\,\frac{x_q - x_q(p)}{x_q\,x_q(p)}(\sin\delta - \sin\delta_0) \qquad (49)$$

$$+ e^2\sin\delta\,\frac{x_d - x_d(p)}{x_d\,x_d(p)}(\cos\delta_0 - \cos\delta)$$

$$- e\sin\delta\,\frac{x_d(p) - x_d\,G(p)}{x_d\,x_d(p)}\,.\,\Delta E$$

But quantities a_{dn}, a_{qn}, α_{dn}, α_{qn}, b_n, β_n may be found such that

$$\frac{x_q - x_q(p)}{x_q(p)}\,.\,1 = \frac{x_q - x_q''}{x_q''}\sum a_{dn}\,\epsilon^{-\alpha_{dn}t}$$

$$\frac{x_d - x_d(p)}{x_d(p)}\,.\,1 = \frac{x_d - x_d''}{x_d''}\sum a_{qn}\,\epsilon^{-\alpha_{qn}t} \qquad (50)$$

$$\frac{x_d(p) - x_d\,G(p)}{x_d(p)}\,.\,1 = \sum b_n\,\epsilon^{-\beta_n t}$$

$$x_q'' = x_q(\infty)$$
$$x_d'' = x_d(\infty)$$
$$\Sigma\,a_{dn} = 1.0$$
$$\Sigma\,a_{qn} = 1.0$$
$$\Sigma\,b_n = 1.0$$

It therefore follows from the operational rule that,

$$f(p)\,F(t) = F(o)\,\phi(t) + \int_0^t \phi(t - u)\,F'(u)\,d\,u \qquad (51)$$

where,

$$\phi(t) = f(p)\,.\,1$$

that if

$$\delta = \delta(t)$$
$$p\,\delta = \delta'(t)$$
$$\Delta E = \Delta E(t)$$
$$p\,\Delta E = \Delta E'(t)$$

Equations (48) and (49) may be rewritten in the form,

$$i_d = \frac{E - e\cos\delta}{x_d}$$

$$+ e\,\frac{x_d - x_d''}{x_d\,x_d''}\sum a_{dn}\,\epsilon^{-\alpha_{dn}t}\int_0^t \epsilon^{\alpha_{dn}u}\sin\delta(u)\,\delta'(u)\,d\,u$$

$$- \frac{1}{x_d}\sum b_n\,\epsilon^{-\beta_n t}\int_0^t \epsilon^{\beta_n u}\,\Delta E'(u)\,d\,u \qquad (48a)$$

$$i_q = \frac{e\sin\delta}{x_q}$$

$$+ e\,\frac{x_q - x_q''}{x_q\,x_q''}\sum a_{qn}\,\epsilon^{-\alpha_{qn}t}\int_0^t \epsilon^{\alpha_{qn}u}\cos\delta(u)\,\delta'(u)\,d\,u$$

$$T = \frac{E\,e\sin\delta}{x_d} + \frac{e^2(x_d - x_q)}{2\,x_d\,x_q}\sin 2\delta$$

$$+ e^2\frac{x_d - x_d''}{x_d\,x_d''}\sin\delta\sum a_{dn}\,\epsilon^{-\alpha_{dn}t}\int_0^t \epsilon^{\alpha_{dn}u}\sin\delta(u)\,\delta'(u)\,du$$

$$+ e^2\frac{x_q - x_q''}{x_q\,x_q''}\cos\delta\sum a_{qn}\,\epsilon^{-\alpha_{qn}t}\int_0^t \epsilon^{\alpha_{qn}u}\cos\delta(u)\,\delta'(u)\,du$$

$$- \frac{e\sin\delta}{x_d}\sum b_n\,\epsilon^{-\beta_n t}\int_0^t \epsilon^{\beta_n u}\,\Delta E'(u)\,d\,u \qquad (49a)$$

Formula (49a) may be used to determine starting torque and current with zero armature resistance, by introducing $\delta(t) = s\,t$, $\delta'(t) = s$. Thus the average component of torque is found to be,

$$T_{av} = \frac{1}{2}\,\frac{x_d - x_d''}{x_d\,x_d''}\sum a_{dn}\,\frac{\alpha_{dn}\,s}{\alpha_{dn}^2 + s^2}$$

$$+ \frac{1}{2}\,\frac{x_q - x_q''}{x_q\,x_q''}\sum a_{qn}\,\frac{\alpha_{qn}\,s}{\alpha_{qn}^2 + s^2} \qquad (52)$$

Since

$\frac{\alpha\,s}{\alpha^2 + s^2}$ is never greater than ½, and

$$\Sigma\,a_{dn} = \Sigma\,a_{qn} = 1.0$$

it follows that T_{av} is never greater than

$$\frac{1}{4}\left\{\frac{x_d - x_d''}{x_d\,x_d''} + \frac{x_q - x_q''}{x_q\,x_q''}\right\} \qquad (53)$$

Equation (53) thus provides a very simple criterion of the maximum possible starting torque of a synchronous motor of given dimensions, when armature resistance is neglected.

The same formula may also be used to obtain a simple expression for the damping and synchronizing components of pulsating torque due to a given small angular pulsation of the rotor.

Thus if the angular pulsation is

$$\Delta\delta = [\Delta\delta]\sin(s\,t)$$

and if the pulsation of torque is expressed in the form

$$\Delta T = T_s \Delta \delta + T_d \frac{d}{d\,t} \Delta \delta$$

there results,

$$T_s = T_{s0} + e^2 \sin^2 \delta_0 \frac{x_d - x_d''}{x_d\, x_d''} \sum \frac{a_{dn}\, s^2}{(\alpha_{dn})^2 + s^2}$$

$$+ e^2 \cos^2 \delta_0 \frac{x_q - x_q''}{x_q\, x_q''} \sum \frac{a_{qn}\, s^2}{(\alpha_{qn})^2 + s^2} \qquad (54)$$

$$s\, T_d = e^2 \sin^2 \delta_0 \frac{x_d - x_d''}{x_d\, x_d''} \sum \frac{a_{dn}\, s\, \alpha_{dn}}{(\alpha_{dn})^2 + s^2}$$

$$+ e^2 \cos^2 \delta_0 \frac{x_q - x_q''}{x_q\, x_q''} \sum \frac{a_{qn}\, s\, \alpha_{qn}}{(\alpha_{qn})^2 + s^2}$$

where,

$$T_{s0} = \frac{e\, I_{d0} \cos \delta_0}{x_d} + \frac{e^2\, (x_d - x_q)}{x_d\, x_q} \cos 2\, \delta_0$$

δ_0 = average angular displacement, *i. e.*, total angle = $\delta = \delta_0 + \Delta\, \delta$.

It can be shown that for the case of no additional rotor circuits, Equations (**54**) are exactly equivalent to Equations (**24**) and (**25**) in Doherty and Nickle's paper; *Synchronous Machines III.* The new formulas herein developed are, however, very much simpler in form, especially since in the case which Doherty and Nickle have treated, there is only one term in the summation; that is, $n = 1$, and α is merely the reciprocal of the short circuit time constant of the machine, expressed in radians.

K. The Equivalent Circuit of Synchronous Machines Operating in Parallel at No Load, Neglecting the Effect of Armature Resistance

Let, δ_a = angle of rotor a and bus

θ_0 = angle of rotor a in space

In general, the shaft torque of a machine depends on its acceleration and speed in space, and the magnitude and rate of change of the bus voltage as a vector. If all of the machines are operating at no load and if there is no armature resistance, a small displacement of any one machine will change the magnitude of the bus voltage only by a second order quantity; consequently for small displacements the magnitude of the bus voltage may be regarded as fixed, and only the angle of the bus and rotor need be considered. Furthermore, the electrical torque may be found in terms of (δ) by employing an infinite bus formula. But Equation (**49a**) implies the alternative general operational form,

$$T = \frac{e\, I_{d0} \sin \delta}{x_d} + \frac{e^2\, (x_d - x_q) \sin 2\, \delta}{2\, x_d\, x_q}$$

$$- \frac{x_d - x_d''}{x_d\, x_d'} e^2 \sin \delta \sum \frac{a_{nd}\, p}{p + \alpha_{nd}} . \cos \delta \qquad (\mathbf{49b})$$

$$+ \frac{x_q - x_q''}{x_q\, x_q''} e^2 \cos \delta \sum \frac{a_{nq}\, p}{p + \alpha_{nq}} . \sin \delta$$

Therefore in the case under consideration there is for machine a,

$$T_a = \left[\frac{e\, I_a}{x_{da}} + e^2 \frac{(x_{da} - x_{qa})}{x_{da}\, x_{qa}} \right] \delta_a$$

$$+ \frac{x_{qa} - x_{qa}''}{x_{qa}\, x_{qa}''} e^2 \sum a_{nqa} \frac{p}{p + \alpha_{nqa}} . \delta_a \qquad (55)$$

where: e = per-unit bus voltage

I_a = per-unit excitation of machine a, etc.

This equation can be represented by Fig. 8, in which the charge through the circuit represents (δ_a) and the voltage across the circuit represents the electrical torque of the machine (T_a).

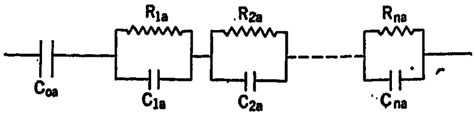

FIG. 8

The capacitances and resistances must be chosen so that

$$C_{0a} = \frac{x_{da}\, x_{qa}}{e\, I_a\, x_{qa} + e^2\, (x_{da} - x_{qa})} \qquad (56)$$

$$C_{na} = \frac{x_{qa}\, x_{qa}''}{e^2\, a_{nqa}\, (x_{qa} - x_{qa}'')}$$

$$R_{na} = \frac{1}{C_{na}\, \alpha_{nqa}}$$

The equation for the mechanical torque is

$$T_{sa} = T_a + M_a\, p\, s_a \qquad (57)$$

where:

M_a = inertia factor of machine a in radians

$$= \frac{2 \times \text{stored mech. energy at normal speed}}{\text{base power}}$$

$$= 2\, \pi\, f \frac{0.462 \;\; W R^2 \left(\dfrac{\text{rev. per min.}}{1000}\right)^2}{\text{base kw.}}$$

s_a = per-unit speed of machine a

t = time in seconds $\left(p = \dfrac{d}{d\, t} \right)$

But, $s_a = p\, \theta_a$

Thus there is

$$T_{sa} = T_a + M_a\, p^2\, \theta_a \qquad (\mathbf{57a})$$

which corresponds to the equivalent circuit of Fig. 9, in which change = θ_a

$$L_a = M_a$$

The machine operating on an infinite bus can be represented by the equivalent circuit of Fig. 10, since the condition

$$\theta_a = \delta_a = 0$$

is fulfilled.

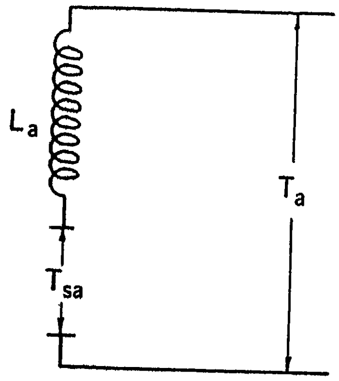

FIG. 9

Several machines in parallel on the same bus may be represented by the diagram of Fig. 11, since the conditions

$$\theta_a - \delta_a = \theta_b - \delta_b = \ldots (= \text{bus angle in space})$$

$$T_a + T_b + T_c, \text{ etc.} = \text{bus power output} = 0$$

A transmission line may be represented by a condenser.

Thus two machines connected by a line of reactance (x) would be represented by the circuit of Fig. 12, where

$$C = \frac{x}{e^2} \tag{58}$$

Shaft torques are, of course, represented by voltages.

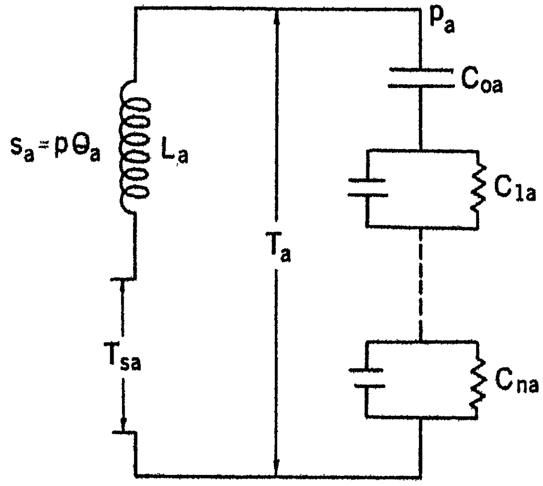

FIG. 10

Mechanical damping, such as that due to a fan on a motor shaft or that due to the prime mover, is represented by resistance in series with the inductance (L) as in Fig. 13. (R) must be chosen equal to the rate of decrease in available driving torque with increase in speed.

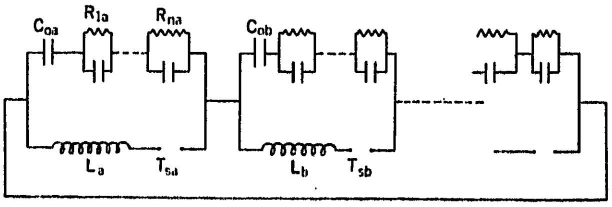

FIG. 11

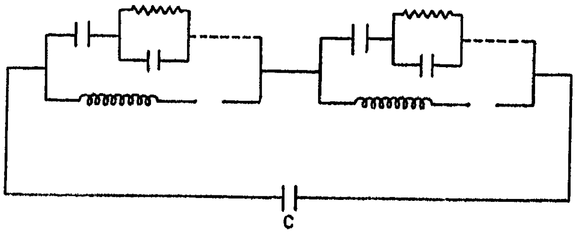

FIG. 12

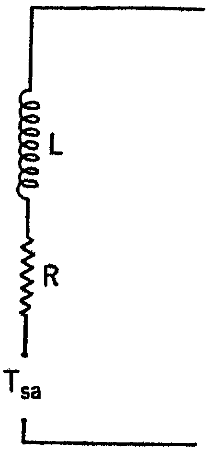

FIG. 13

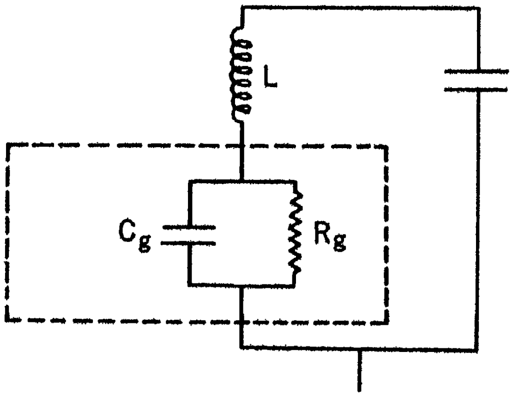

FIG. 14

Governors and other prime mover characteristics may also be represented by connecting their circuits in the inductive branch of the circuit. Thus a governor which acts through a single time constant may be represented by the circuit of Fig. 14, where

$$R_g = \frac{1}{\text{regulation}}$$

$$C_g = \frac{\text{time constant of governor in elec. radians}}{R_g} \tag{59}$$

An induction motor is represented by the simple circuit of Fig. 15 and is precisely the circuit of a synchronous machine with only one time constant and $C_0 = \infty$ on account of $I = 0$.

Results similar to these have been previously shown by Arnold, Nickle,[10] and others, but simpler and more approximate circuits were used, the branches of the several circuits were not directly evaluated in terms of machine constants, and the derivation was incomplete in that the limitation to no load and zero resistance was not appreciated.

L. *Torque Angle Relations of a Synchronous Machine Connected to an Infinite Bus, for Small Angular Deviations from an Average Operating Angle*

There is, in general,

$$T = T_0 + \Delta T = (\psi_{d0} + \Delta \psi_d)(i_{q0} + \Delta i_q) - (i_{d0} + \Delta i_d)(\psi_{q0} + \Delta \psi_q)$$

For small angular deviations,

$$\Delta T = i_{q0} \Delta \psi_d + \psi_{d0} \Delta i_q - i_{d0} \Delta \psi_q - \psi_{q0} \Delta i_d$$
$$= \{\psi_{d0} + i_{d0} x_q (p)\} \Delta i_q - \{\psi_{q0} + i_{q0} x_d (p)\} \Delta i_d \qquad (60)$$

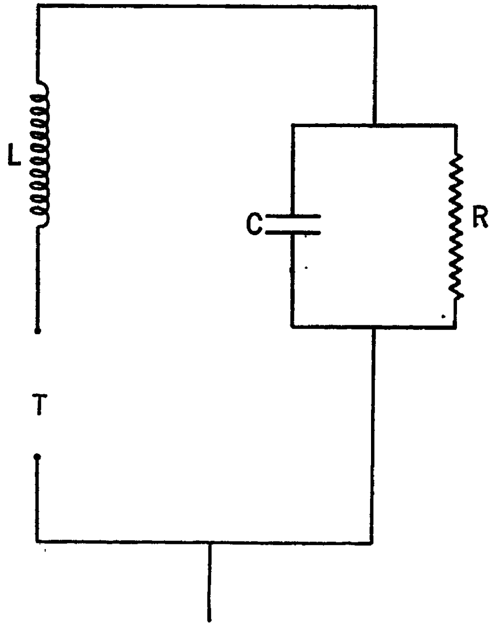

FIG. 15

$$e_{d0} + \Delta e_d = p \Delta \psi_d - r(i_{d0} + \Delta i_d) - (\psi_{q0} + \Delta \psi_q)(1 + p \Delta \delta)$$
$$e_{q0} + \Delta e_q = p \Delta \psi_q - r(i_{q0} + \Delta i_q) + (\psi_{d0} + \Delta \psi_d)(1 + p \Delta \delta)$$
$$\Delta e_d = p \Delta \psi_d - r \Delta i_d - \psi_{q0} p \Delta \delta - \Delta \psi_q$$
$$\Delta e_q = p \Delta \psi_q - r \Delta i_q + \psi_{d0} p \Delta \delta + \Delta \psi_d$$

from which there is

$$z_d (p) \Delta i_d - x_q (p) \Delta i_q = - \Delta e_d - \psi_{q0} p \Delta \delta$$
$$z_q (p) \Delta i_q + x_d (p) \Delta i_d = - \Delta e_q + \psi_{d0} p \Delta \delta$$

$$\Delta i_d = \frac{z_q (p)\langle - \Delta e_d - \psi_{q0} p \Delta \delta\rangle + x_q (p)\langle - \Delta e_q + \psi_{d0} p \Delta \delta\rangle}{D (p)}$$

$$\Delta i_q = \frac{z_d (p)\langle - \Delta e_q + \psi_{d0} p \Delta \delta\rangle - x_d (p)\langle - \Delta e_d - \psi_{q0} p \Delta \delta\rangle}{D (p)} \qquad (61)$$

where,

$$D (p) = z_d (p) z_q (p) + x_d (p) x_q (p)$$

but from Equations (28),

$$e_{d0} + \Delta e_d = e \sin (\delta_0 + \Delta \delta)$$
$$e_{q0} + \Delta e_q = e \cos (\delta_0 + \Delta \delta)$$
$$\Delta e_d = e \cos \delta_0 \Delta \delta$$
$$\Delta e_q = - e \sin \delta_0 \Delta \delta \qquad (62)$$

$$\Delta i_d = \frac{-(e \cos \delta_0 + \psi_{q0} p) z_q (p) + (e \sin \delta_0 + \psi_{d0} p) x_q (p)}{D (p)} . \Delta \delta$$

$$\Delta i_q = \frac{(e \sin \delta_0 + \psi_{d0} p) z_d (p) + (e \cos \delta_0 + \psi_{q0} p) x_d (p)}{D (p)} . \Delta \delta \qquad (63)$$

$$\Delta T = \frac{[\psi_{d0} + i_{d0} x_q (p)] \left\{ \begin{array}{l} (e \sin \delta_0 + \psi_{d0} p) z_d (p) \\ + (e \cos \delta_0 + \psi_{q0} p) x_d (p) \end{array} \right\} + [\psi_{q0} + i_{q0} x_d (p)] \left\{ \begin{array}{l} (e \cos \delta_0 + \psi_{q0} p) z_q (p) \\ - (e \sin \delta_0 + \psi_{d0} p) x_q (p) \end{array} \right\}}{D (p)} . \Delta \delta \qquad (64)$$

say,

$$\Delta T = f (p) . \Delta \delta$$

From (57a) the equation for shaft torque becomes

$$\Delta T_s = (M p^2 + f (p)) . \Delta \delta$$

Thus,

$$\Delta \delta = \frac{1}{M p^2 + f (p)} . \Delta T_s \qquad (65)$$

Appendix

Formula for Linkages and Voltage in Field Circuit with no Additional Rotor Circuits

In this case the per-unit field linkages will depend linearly on the armature and field currents. That is, in general,

$$\Psi = a I - b i_d$$

Then if normal linkages are defined as those existing at no load there must be $a = 1.0$.

The quantity b may be found by suddenly impressing terminal linkages ψ_d with no initial currents in the machines and $E = 0$.

By definition there is, initially

$$i_d = - \frac{\psi_d}{x_d'}$$

but also there must be from the definition of $x_d{}^2$

$$i_d = \frac{I - \psi_d}{x_d}$$

hence there must be an initial induced field current of amount

$$I = \psi_d \left(1 - \frac{x_d}{x_d'} \right)$$

But, initially the field linkages are zero, thus

$$\Psi = \psi_d 1 - \left[\frac{x_d}{x_d'} + \frac{b}{x_d'} \right] \psi_d = 0$$

hence $$b = x_d - x_d'$$

Similarly, there will be

$$\begin{aligned} E &= \text{per-unit field voltage} \\ &= c\,p\,\Psi + d\,I \end{aligned}$$

Normal field voltage will be here defined as those existing at no load and normal voltage. This requires that $d = 1$. The quantity c may then be recognized as the time constant of the field in radians when the armature is open circuited, since with the field shorted under these conditions there is

$$\begin{aligned} (T_0\,p + 1)\,I &= 0 \\ c\,p\,\Psi + I &= 0 \\ \psi &= I \end{aligned}$$

$c = T_0$ = time constant of field with armature open circuited.

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Discussion

H. C. Specht: I should think Mr. Park's theory could be applied just as well to the so-called synchronous induction motor, that is an induction motor in which the rotor teeth between the poles are cut out for a distance of about one-third of the pole pitch. Such a motor runs at synchronous speed. However, the pull-out torque is much less than that of an induction motor with the full number of teeth.

C. MacMillan: There was one statement in the first page of Mr. Park's paper to the effect that "Idealization is resorted to, to the extent that saturation and hysteresis in every magnetic circuit and eddy currents in the armature iron are neglected...". And with regard to Fig. 5, Mr. Park remarked that it represented a rigorous solution. Perhaps Mr. Park could give us a little more insight into the effect of taking into account saturation, and give other cases in which certain elements have been neglected with more or less effect upon the final results.

W. J. Lyon: In a paper of this description, certain premises should be chosen and, with these always in mind, the mathematical development should be rigorous. The paper may then be criticized either because of insufficient premises or because of incorrect mathematical development. I believe that the former is the kinder method; it is the one I shall employ.

The premises that Mr. Park chooses are that the field and armature windings are symmetrical, that saturation and hysteresis are neglected and that the armature windings are in effect sinusoidally distributed. I take this last to mean that the air-gap flux due to the armature currents is sinusoidally distributed, for if the armature windings themselves were sinusoidally distributed, there would be produced space harmonics in the air-gap flux distribution due to the saliency of the poles, which, as we all know, would complicate the problem tremendously. In order that the mathematical method used by Mr. Park shall be rigorous, I believe it is necessary to make one further assumption. I think I can best explain this by asking you to consider the result of supplying the field winding with a sinusoidal current while the armature rotates at some speed which may be called synchronous. Under these conditions, there will first be produced in the armature windings two sets of balanced currents each of which will produce 3 component flux distributions in the gap. The first of these is what would be produced if the air-gap were uniform, and is proportional to $1/2\,(x_d + x_q - x_a)$, where x_a equals the armature leakage reactance. The second of these components is proportional to $1/2\,(x_d - x_q)$. The third component is of the same size as the second. Using the values that Mr. Park gives under Section H of his paper, the relative magnitudes of these components would be $(0.8 - x_a)$ and 0.2. The first and second components react on the field, and produce in it a current of the impressed field frequency. These are the components that Mr. Park has recognized, but the third component produces an entirely different frequency in the field, which will then be reflected into the armature and the process will be repeated. That is, in this respect, it is similar to the condition that exists in a single-phase alternator. As far as I am aware, the Heaviside operational method cannot be used to obtain a rigorous solution for the single-phase alternator. In spite of this, I think the objection that I have raised is of no more importance than the effect of neglecting saturation or hysteresis or the space harmonics in the air-gap flux that are actually produced, but I do think that Mr. Park should have mentioned this point in order that the mathematical work which he builds up should be rigorous. I discovered the necessity of this additional premise about a year ago while working on the problem of the transient short-circuit currents in a salient-pole alternator.

There is another problem of considerable interest that Mr. Park has not mentioned. About two years ago I became aware that Mr. Fortescue's historic 1918 paper could be extended to the transient case by using an operational method. That is to say, any of the problems involving unbalanced circuits that Mr. Fortescue rigorously solved in the steady state can also be solved in the transient state by an operational method.

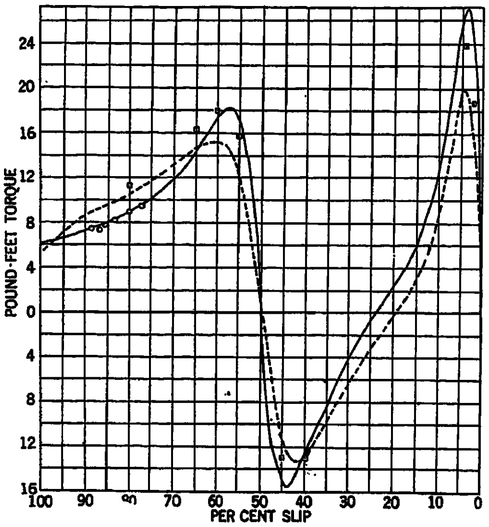

FIG. 1—ELECTRICAL TORQUE OF A H I

Solid Line—Calculated
Dotted Line—Experimental

Note: The six points plotted above were taken experimentally at a temperature of 25 deg. cent.

The balance of the dotted curve was taken at varying higher temperatures The solid curve was calculated for a temperature of 25 deg. cent., and the points in squares for 100 deg. cent.

W. B. Heinz: (communicated after adjournment) Since the completion of Mr. Park's paper as presented, experimental work has been done which illustrates the worth of the new theory developed.

A small synchronous motor rated 15 hp., 220 volts, 1800 rev. per min. was selected for study. Its squirrel-cage winding was removed in order to simplify the mechanical work of calculation.

This motor was set up with an electric dynamometer, and its torque-slip curve was taken at 110 volts over the entire range of slip from 100 per cent to zero. Power was supplied to it by a special 220-volt sine-wave generator of 375-kv-a. capacity. The constants of this machine as well as of the connecting line and switches were determined, and these constants were included in the calculation of torque in order to simulate as closely as possible infinite-bus conditions.

The torque as indicated by the dynamometer was corrected for windage and friction and for dynamometer errors, the net electrical torque exerted on the rotor thus being obtained.

By the use of measured constants of the test machine and its circuit, a mathematical evaluation of torque was made for a temperature of 25 deg. cent. These constants as determined, on a per unit base of 24.75 kv-a. at 220 volts, are as follows:

$x_d = 1.0$
$x_d' = 0.178$
$x_q = 0.442$
$x_q' = 0.442$
$r = 0.0325$
$T_0 = 159$ radians
Normal torque = 96.8 lb-ft.

The calculated curve appears as a solid line in Fig. 1 herewith and near it is the dotted curve obtained experimentally.

During the progress of the test run with the dynamometer no control or regular measurement of the synchronous motor temperature was made. It fluctuated widely with changes in slip, and the only points taken at a known temperature are the ones near the beginning of the run. For the first six, those surrounded by circles, the temperature probably had not risen far from ambient of 25 deg. cent. They check quite closely the calculated curve.

In order to evaluate the effect of temperature change eight points were calculated on the basis of 100 deg. cent. These are shown within squares. They show a better coincidence of slips at which maximums occur, but little improvement in the maximum error. Previous theories have usually neglected the effect of salient poles. The solid line in Fig. 2 shows the calculated curve in this case, that is, for $x_q = x_d$. A comparison of the solid curves in Figs. 1 and 2 shows the large differences between the two calculations.

I. H. Summers: (communicated after adjournment) One outstanding feature of this paper is that the equations are expressed in "per-unit" quantities. This feature presents the important advantage that the quantities involved are those which are known directly from test or from the design of the machine. Furthermore, machines are more readily compared with others of different rating when their constants are known in "per-unit" values.

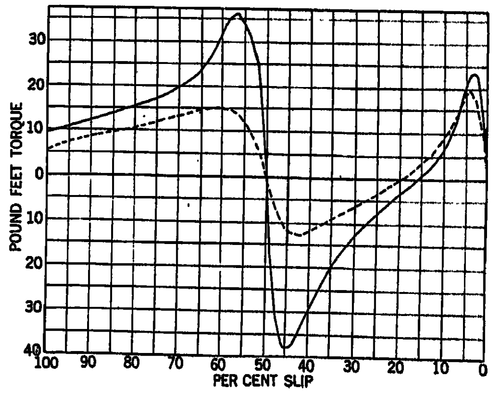

FIG. 2—TORQUE CURVES FOR A H I

Solid Line—Calculated
Dotted Line—Experimental

However, these equations may not look familiar at first sight to those who are more used to dealing in the physical quantities such as ohms, amperes, volts, henrys, and farads. It therefore seems appropriate to point out the correlation between this new method and the more usual one. Consider, for example, Equations (4a) in the paper, together with the two immediately following.

Ordinarily we should have written for a machine with no rotor circuits in addition to the field,

$$(R_f + L_f\,p)\,I_f - M\,p\,[j_a \cos\theta + j_b \cos(\theta - 120°) + j_c \cos(\theta + 120°)] = E_f \qquad (1)$$

In this equation R_f is field resistance, L_f is field inductance, M is mutual inductance between field and any one armature phase in the position of maximum mutual inductance, j_a, j_b, and j_c

are armature phase currents, and E_f is field voltage. All these quantities are in practical units.

Let

$$j_d = \frac{2}{3}\left[j_a \cos\theta + j_b \cos(\theta - 120^\circ) + j_c \cos(\theta + 120^\circ)\right] \tag{2}$$

Divide through by $I_n R_f$ where I_n is that value of field current that causes normal voltage to be generated in the armature at no load.

$$\frac{E_f}{I_n R_f} = (1 + T_0 p)\frac{I_f}{I_n} - \frac{3}{2}\frac{L_f}{R_f}\frac{M_p}{I_n L_f} j_d \tag{3}$$

where

$$T_0 = L_f / R_f \tag{4}$$

Let

$$\left.\begin{aligned} E_f / I_n R_f &= E \\ I_f / I_n &= I \\ j_d / i_n &= i_d \end{aligned}\right\} \tag{5}$$

Where i_n is normal armature current, maximum value. Then

$$E = T_0 p\left(I - \frac{3}{2}\frac{M}{L_f}\frac{i_n}{I_n} i_d\right) + I \tag{6}$$

This equation may be put into the form:

$$\left.\begin{aligned} E &= T_0 p \Psi + I \\ \Psi &= I - \left(\frac{3}{2}\frac{M i_n}{L_f I_n}\right) i_d \end{aligned}\right\} \tag{7}$$

These results may be compared with those immediately following (4a) in the paper. It is easy to show by methods similar to those used by Mr. Park that

$$\frac{3}{2}\frac{M i_n}{L_f I_n} = x_d - x_d' \tag{8}$$

where x_d and x_d' are the per-unit direct axis synchronous and transient reactance respectively. Then the equations are the same as those given by the author.

Without this last relation we should doubtless have written the equations as in (7) or even as in (1). But both of these contain cumbersome quantities involving mutual and self inductances. The expression used by Mr. Park is simpler and more expressive and contains quantities which are familiar to most engineers who have to do with synchronous machines.

Throughout this paper, although the problem is inherently complex and some of the demonstrations appear long, the results are relatively simple and in terms adapted for immediate use.

R. H. Park: Mr. Specht is entirely correct in his thought that the present theory could be applied to synchronous induction motors such as those to which he refers. In principle this type of motor is not greatly different from a salient-pole synchronous motor with an amortisseur winding.

In reply to Mr. MacMillan's question, I would point out that it is always necessary to make some assumptions; in fact, even in the simplest problems there invariably exists an enormous number of assumptions most of which are not recognized as such. Therefore a rigorous solution invariably means only "rigorous on the basis of the assumptions." The solutions presented in the paper are rigorously correct in this sense.

Professor Lyon has criticized the assumptions made in the paper. This is due to a very definite misconception as to their character. Thus, referring to the assumption "that the armature windings are in effect sinusoidally distributed," Professor Lyon states that it is necessary to interpret the statement to mean something other than it says. I believe, however, that this is not necessary. Thus he states that were the armature windings themselves sinusoidally distributed, there would be produced space harmonics in the air-gap flux distribution due to the saliency of the poles, which "would complicate the problem tremendously." The reason that it does not complicate the problem is that with a sinusoidal distribution of turns only space fundamental flux produces any armature flux linkages.

This follows from the fact that

$$\int_0^{2\pi} \cos n\,\theta \cos m\,\theta \, d\,\theta = \int_0^{2\pi} \cos n\,\theta \sin m\,\theta \, d\,\theta = 0$$

if n and m are integers and $n = m$.

Although this answers Professor Lyon's objection the impression should not be derived that the theory developed strictly presupposes an exactly sinusoidally distributed winding. The full statement of the assumption referred to is that the armature windings are in effect sinusoidally distributed "as far as concerns effects depending on rotor position." Interpreted in terms of self and mutual inductive coefficients this statement is exactly equivalent to the two assumptions,

(a) that the self inductance of the armature circuits is expressible by an equation of the form,

$$L = L_0 + L_2 \cos 2\,\theta$$

(b) that the mutual inductance between the armature and any rotor circuit is expressible by an equation of the form,

$$M = M_1 \cos\theta$$

The assumptions which have been made in previous studies, such as those which Mr. Ku in his paper "Transient Analysis of A-C. Machinery" has referred to as "exact," have been precisely the same except that the second harmonic term in the armature self inductance equation has not been considered, while amortisseur and other circuits in addition to the field have been neglected.

Actually the expressions for both armature and rotor self inductance will involve all of the even harmonics of angular position, and the expression for armature mutual inductance will involve all of the odd harmonics of angular position. However, tests on actual machines have shown that in most cases only the zero and second harmonics of armature self inductance, the first harmonic of mutual inductance, and the zero harmonic of rotor circuit self inductance are of primary importance. About the only case in which a consideration of any other harmonics would be of value appears to be in the study of locking torques. With proper design, however, the tendency to lock at other than half speed is slight.

Professor Lyon is, in my opinion, also in error when he endeavors to point out the inadequacy of the theory by means of an example in which alternating current is supplied to a rotating field in a salient-pole machine. In this connection he merely makes certain dogmatic statements without proof, which would not agree with the writer's findings. However, such mere statements or opinions do not constitute a proof, and the writer continues to disagree with them.

As I understand the example given, it corresponds to the application of alternating current to a synchronous machine field winding with a three-phase short circuit across the armature terminals.

From Equations (11) and (22) of the paper there is in this case

$$I_d = \left[1 + \frac{< x_d - x_d(p) > < p z_q(p) + (1 - s^2) x_q(p) >}{D(p)}\right] G(p) E$$

$= $ total per unit rotor excitation.

Evidently this is of the same frequency as that of the field voltage E. That is, there are no harmonics, Mr. C. A. Nickle has submitted a physical interpretation of this result. Thus he states

"Let $I \cos t$ be impressed on the field which is running at a speed S expressed as a fraction of synchronous speed for the frequency of the current impressed on the field winding. The current $I \cos t$ produces a space fundamental flux $I \cos t$ in the direct axis and space harmonics alternating in time. But since the armature winding is assumed to have sine distribution, these harmonic fluxes can induce no voltage and may be neglected. The fundamental flux, $I \cos t$, may be resolved into two rotating components $I/2$ rotating forward at normal speed with reference to the pole structure and $I/2$ rotating backward at the same speed. The first component rotates forward with respect to the armature at a speed $1 + S$ and since the armature is short circuited polyphase, a positive phase sequence current of frequency $1 + S$ will flow in the armature winding. Similarly a negative phase sequence current of frequency $1 - S$ will flow in the armature winding due to the backward rotating component of field flux. The polyphase current i_{1+s} will produce a rotating m. m. f. also i_{1+s} and this m. m. f. rotates forward relative to the pole structure at normal speed. Two fluxes are produced due to dissymmetry of the direct and quadrature axes. These are

$$\frac{i_{1+s}}{2}(x_d' + x_q')$$

rotating forward with respect to the poles at normal speed or $1 + S$ with respect to the armature, and

$$\frac{i_{1+s}}{2}(x_d' - x_q')$$

rotating backward with respect to the poles at normal speed or at $1 - S$ backward with respect to the armature. The first will induce an armature voltage of frequency $1 + S$, and the second a voltage of frequency $1 - S$.

The armature current of frequency i_{1-s} produces an m. m. f. i_{1-s} rotating backward with respect to the poles at normal speed and produces two rotating fluxes. One,

$$\frac{i_{1-s}}{2}(x_d' + x_q')$$

rotates backward with respect to the armature at a speed $1 - S$ and

$$\frac{i_{1-s}}{2}(x_d' - x_q')$$

rotates forward with respect to the armature at a speed $1 + S$. Thus, this current also produces armature voltages of frequencies $1 + S$ and $1 - S$. Therefore, no frequencies other than $(1 + S)$ and $(1 - S)$ need be assumed, in the armature winding and therefore, these are the only frequencies that exist there. Moreover, this implies that only fundamental frequency can exist in the rotor."

Magnetic Analysis†

BY RAYMOND L. SANFORD*
Non-member

Synopsis.—This paper presents a brief outline of the development and application of magnetic analysis for the nondestructive testing of iron and steel and their products. The subject is treated under the following headings: Historical development, Experimental results, Practical commercial applications, and Conclusion.

* * * * *

INTRODUCTION

THE use of electrical methods for the measurement of non-electrical quantities, such as temperature, displacement, time, etc., is quite common. In the majority of such applications, these methods provide the most convenient and accurate means for making the desired determinations. The time is approaching, if indeed it has not already arrived, when magnetic analysis should be included in the list. The term "magnetic analysis" has been adopted to distinguish magnetic tests made with a view to their interpretation in terms of structure, mechanical characteristics, or soundness of ferrous materials from those made solely for the purpose of determining magnetic properties as such. The term connotes, therefore, not only the process of testing but also the analysis of the results from the point of view of mechanical characteristics. Although the development of such magnetic testing methods has not by any means reached a satisfactory state of perfection, their practicability has been demonstrated sufficiently well at least in a few cases to warrant the expectation that they will ultimately have a much wider application and thus meet the need which is becoming more and more urgent for a reliable nondestructive testing method.

That there is a close connection between the magnetic properties of ferrous materials and their other physical properties has long been recognized, and many investigations have been made for the purpose of discovering the nature of the connection. It is only in recent years, however, that investigations along this line have been undertaken with the definite object in view of developing practical magnetic testing methods for routine inspections and for the study of the phenomena associated with the thermal and mechanical treatment of steel.

It is the purpose of the present paper to trace briefly the development of magnetic analysis, to indicate its present status, and to suggest lines along which further investigation might profitably be directed.

HISTORICAL DEVELOPMENT

Pioneers in the development of magnetic analysis were Burrows and Fahy, who, in the latter part of 1911, undertook at the Bureau of Standards under the joint sponsorship of the Bureau and the Pennsylvania Railroad, an investigation on spring steel. This investigation was continued over a period of about five years, during which time its scope was somewhat extended to cover preliminary experiments on the applicability of magnetic analysis to various forms of steel products. Three papers[1, 6, 8] were published giving the results of this investigation. Burrows also published a paper[5] in which the experimental evidence on the relationship between magnetic and mechanical properties recorded prior to 1916 was summarized. From the evidence presented he concluded that there was every reason to believe that the two sets of properties are intimately connected and that practical research and inspection methods could be developed. During this same period other papers[2, 3, 4] were published having a definite bearing on the problem.

In 1918 the American Society for Testing Materials authorized the formation of a technical committee to be known as Committee A-8 on Magnetic Analysis with Dr. Burrows as its Chairman. This committee was composed initially of investigators already working in the field, but from time to time its membership has been strengthened by the addition of members who have subsequently become actively interested in the development of magnetic analysis. The committee has functioned primarily as a clearing house for the interchange of ideas and experience among its members who have carried on individual investigations on various phases of the subject; but it has itself also carried out a number of investigations of a general and fundamental nature.[28, 32, 40, 51] A considerable proportion of the literature available on the subject of magnetic analysis has been contributed by members of this committee, either independently or through the auspices of the committee.

A second major source of technical literature is papers published by the Bureau of Standards. In general, the work of the Bureau has been directed along the more fundamental lines although some work on practical applications has been carried out.

Worthy of special mention is the work of deForest.[22, 24, 25, 31, 43] This ingenious experimenter has confined his attention in general to the development of methods employing alternating currents, and has brought out a number of applications of considerable interest and wide applicability.

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†Part of symposium on Electrical Measurements of Non-Electrical Quantities.

For numerical references see Bibliography.

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Most of the work definitely belonging in the field of magnetic analysis has been done in the United States. There has, however, been some interest in other countries. In this connection the work of Honda[7,56] in Japan, and of Fraichet[29, 30] in France, should be especially mentioned. Honda has employed magnetic methods in the study of structural transformations occurring during heating and cooling, and, with his associates, has published a great many papers on the subject. He is probably the first one to use the term "magnetic analysis" but he employed it, in a limited sense, to denote only the single phase in which his work is preeminent. The work of Fraichet has consisted mainly of the study of relations between magnetic and elastic properties with special reference to the determination of the elastic limit by magnetic means.

EXPERIMENTAL RESULTS

In discussing the results of investigations in the field of magnetic analysis, it will be found convenient to consider the following topics: Fundamental correlation, tests for general quality, detection of flaws, alternating-current methods, and thermomagnetic analysis.

Fundamental Correlation. In his paper published in 1916[5] on the correlation of magnetic and mechanical properties of steel, Burrows[5] enunciated what he believed to be the fundamental principle of magnetic analysis as follows: "There is one and only one set of mechanical characteristics corresponding to a given set of magnetic characteristics, and, conversely, there is one and only one set of magnetic characteristics corresponding to a given set of mechanical characteristics." He goes on to say: "Although there is no evidence to refute the preceding rather broad statement, the utility of this generalization is decidedly limited by the complexity of the relations due to the large number of variables and the lack of sufficient quantitative data. . . . Even with these limitations, magnetic testing, in conjunction with mechanical testing, may be expected to be of considerable value in determining mechanical properties." Unfortunately, some have made extravagant claims based on the first part of this statement while overlooking entirely the caution as to the inherent limitations due to the complexity of the relationships involved and the lack of fundamental data. As a rule, such claims are detrimental rather than advantageous, as failure to substantiate them has a tendency to discredit the whole proposition and deter some from entering the field.

In the attempt to discover fundamental correlations between the two sets of properties, it has been the general practise to make magnetic measurements by some well-known testing method on specially prepared samples. The results of these tests are then correlated either with the heat treatment or with the results of mechanical tests subsequently made on the same samples. Such correlation is not easy because the magnetic properties are in themselves complex. Attempts have been made to simplify the problem by using certain critical values, such as maximum permeability, induction corresponding to a specified value of magnetizing force, residual induction, coercive force and the like, as criteria. These attempts have not been very successful and it has been found necessary to consider the magnetic properties as a whole in drawing conclusions. Another lesson that has been learned is that it is generally not safe to draw too definite conclusions from the results of correlating magnetic properties with heat treatment alone, for we can never be assured that some accidental factor has not entered into the heat treatment and thus produced mechanical properties not normally associated with the specified heat treatment. Too much emphasis cannot be laid upon the necessity for proper metallurgical control in correlation investigations. The results of experiments in which this factor has not been adequately cared for must necessarily have but limited value in the quantitative sense.

Space does not permit a detailed discussion of the results of all the investigations recorded in the technical literature. These results, however, are in the main concordant and the following general conclusions may be drawn:

1. Any treatment which alters to a measurable extent the mechanical properties of a piece of steel at the same time appreciably alters its magnetic characteristics, though not necessarily to a corresponding degree.

2. The nature of the effect of various treatments on magnetic properties is generally similar for different types of steel but this is not a universal rule, as notable exceptions have been observed.

3. Although the magnetic and mechanical properties appear to be functions of the same variables,—that is, composition and structure,—there are certain secondary effects, notably that of mechanical strain, which exert an influence on the magnetic properties out of all proportion to their effect on the mechanical properties. This is an important point which should never be overlooked in the search for laws of correlation.

Tests for General Quality. In their final application, tests for general quality must be applied to manufactured products of various shapes and sizes, and for this reason a solution of the problem calls for a considerable amount of ingenuity in the design of apparatus for the testing of irregular shapes. In this type of application it should not be necessary to await the establishment of definite laws of correlation, for if pieces of steel having identical magnetic properties also have identical mechanical characteristics, it should only be necessary to compare the pieces under test magnetically with one known to have the requisite mechanical characteristics and which may therefore be taken as a standard of quality. A considerable number of investigations has been carried out as to the applicability of magnetic analysis for the testing of various products on this basis. Among the products investigated are cutlery, small tools, ball bearing races and balls, small case-hardened chain, and twist drills.

Encouraging results have been obtained in the majority of cases, but the obstacle in the way of practical application on a commercial basis is usually the effect of secondary influences as noted above. In other words, the test generally fails on the side of safety, but leads to the rejection of an excessive amount of good material. This should not be considered to imply that the outlook is hopeless, but only that the problem is not so simple as it might appear at first glance. The need for a satisfactory nondestructive test is so great that efforts in this direction should not be abandoned until every possibility has been exhausted. And the fact that successful application has been accomplished in even a few cases leads to the conviction that the general problem is not impossible of solution.

The Detection of Flaws. A large proportion of the failures of steel parts in service is due not to poor general quality of the materials but to the presence of hidden flaws or defects. It is for the detection and location of hidden flaws that a reliable nondestructive test is most needed. For this reason, a considerable amount of attention has been given to this phase of magnetic analysis, and at least one successful application has been recorded.[47] A large number of different products such as steel rails, rifle-barrel steel, rifle barrels, ball-bearing races, and welded chain, has been the subject of study. One of the most important problems is that of the test of welds, for the applicability of welding is seriously limited by the lack of a suitable test for quality of the weld.

The usual test for the detection of flaws consists in an exploration for the determination of the degree of magnetic uniformity of the specimen. Abrupt changes in magnetic permeability may be the result of flaws; on the other hand, they may indicate the presence of internal strain or may result from relatively unimportant surface conditions. The problem therefore, at least in part, is to discover some way of differentiating between the indications due to strain and those due to actual flaw. A partial solution has been found[39] in the fact that variations due to strain are less in magnitude under high values of magnetizing force than for low or medium values. Unfortunately, the sensitivity to the effect of real flaws is materially decreased when high magnetizing forces are used. A great deal of work remains to be done on this important problem.

Alternating-Current Methods. For routine inspection on a commercial scale, alternating-current methods appear to have some practical advantages. The quantitative interpretation of the magnetic results is generally more difficult than for the ordinary direct-current methods, due primarily to the phase relations involved. With standardized equipment and arbitrarily chosen limits, however, these difficulties are of minor importance.

During the course of an investigation on high-speed steel carried on by Committee A-8 of the A. S. T.M.,[40] several methods were studied by different members of the committee. Although the results of the investigation left much to be desired in the way of definite correlations, the alternating-current methods developed by deForest[24] seemed to offer the most encouraging field for further study. Two types of inspection have been developed. In the earlier form, the sample under test is compared with a standard by means of an inductance bridge using a separately excited a-c. galvanometer as a detector. By adjusting the phase of the current in the galvanometer field with respect to the induced voltages, it is possible to obtain readings on the galvanometer which depend mainly on the difference in permeability of the two specimens, the difference in hysteresis loss, or various combinations of the two factors. Thus it may be seen that in addition to speed in operation, the method offers great flexibility.

The second method[43] consists of the use of an oscillograph, preferably of the cathode ray type, with which the magnetic differences are shown graphically and appear as closed figures or loops of various shapes and sizes. By adjusting the phase of the time coordinate, it is possible to emphasize the difference in any particular part of the hysteresis loop. This method, first adapted to magnetic analysis by deForest, has been discussed with respect to its more theoretical aspects by Spooner.[44] From the evidence already presented, it would appear that these two types of alternating-current testing provide exceedingly valuable methods for the application of magnetic analysis both in the laboratory and on a commercial scale.

Thermomagnetic Analysis. Thermomagnetic analysis consists of the study of magnetic changes which occur during heating or cooling of a specimen. While not properly classed as a nondestructive test, thermomagnetic analysis should be considered as one phase of magnetic analysis. Its development and application have received little attention in the United States, but in the hands of the Japanese investigator Honda[7, 36] it has proved to be an exceedingly valuable tool for the study of the phenomena associated with the heat treatment of steel. Apparatus for the application of thermomagnetic analysis has been set up recently at the Bureau of Standards and it is hoped to develop further this phase of magnetic analysis.

Practical Applications. The methods of magnetic analysis have found some practical application both in the laboratory and in commercial routine inspection. In the laboratory, the applications have been of two kinds. As a research tool for the study of the constitution and structure of ferrous materials, it has found use which will undoubtedly be extended. Indeed, it may well be that in this connection magnetic analysis will find its most valuable application. Magnetic tests have been used also in the laboratory in connection with long-time mechanical tests, such as endurance tests and tests upon the effect of elevated temperatures on mechanical properties. In tests of this sort, it is of great importance to be sure that all of the specimens used are initially in the proper condition and that they contain no flaws. Under such circumstances, the rejection of some good material is of less consequence than the acceptance of unsatisfactory material, since the results on such material may be not only valueless but even misleading.

In the realm of commercial routine inspection there are three outstanding examples: the inspection of small case-hardened chain,[22] the testing of small heat-treated forgings,[43] and the examination of steam turbine bucket wheels.[47] The first two are applications of the two types of alternating-current tests developed by deForest for the American Chain Company. The third mentioned method is a d-c. exploration test developed by the General Electric Company with the collaboration of Dr. Burrows.

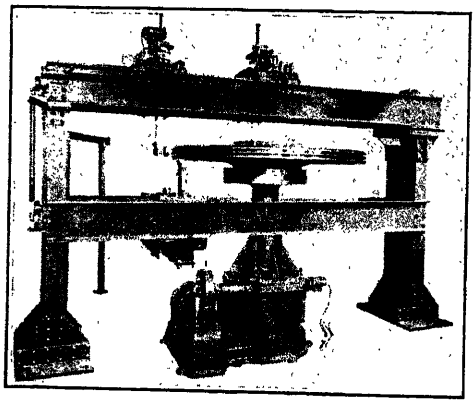

Fig. 1—Magnetic Analysis for Turbine Bucket Wheels.

In the testing of case-hardened chain, the inductance bridge method is used. The phase of the galvanometer field current is so adjusted as to give readings which experience has shown are proportional to brittleness. Readings which lie outside of certain specified limits indicate material which is either too brittle or too soft. This method for the control of the heat treatment of the manufactured product has been in use for over five years, several thousand individual specimens being inspected each day.

For the inspection of small heat-treated forgings, the oscillograph method is utilized. Experience has shown what form of loop indicates a satisfactory condition of the material. Variations from this form, greater than a certain amount, indicate defective material.

One form of apparatus for testing turbine bucket wheel forgings is shown in Fig. 1. The forging under test is rotated on its own axis between the poles of an electromagnet which can be adjusted to any given position on the radius of the circular forging. The magnetic flux passes across the air-gap between the pole tips, in which air-gap the disk is rotated. Any variation in the magnetic quality of the material in this air-gap is indicated by means of a sensitive galvanometer connected to coils on the specially designed pole tips. In most cases, the value of magnetic explorations for the detection of flaws and irregularities is limited by the effects of strain and other obscure conditions. This test, however, is not applied until the disk has been heat treated for the removal of strain, and machined to final dimensions, and hence operates under the most favorable conditions. Although rejections are very few, in view of serious damage and sometimes even loss of life resulting from failure, it is exceedingly important that no defective piece be used in the finished machine.

Conclusions

From the foregoing rather cursory outline of the history and development of magnetic analysis, it is evident that the subject is one of great interest both from the point of view of commercial application and that of scientific investigation. The problem is not a simple one, as it involves not only the complex magnetic properties of materials, but also their composition and structure which determine the mechanical properties. The more rapid development of magnetic analysis has been hindered by a number of factors, some of which may be inevitable, but others of which might be obviated,—at least to some extent.

With relation to the magnitude of the problem in all its phases, the number of workers in this field is very small, and their activities have extended over such a broad area that general progress has been necessarily slow. More workers and better coordination would do much to remedy the situation. One circumstance which may be inevitable, but which, nevertheless, operates to retard development, is the tendency for most workers to cover each discovery and application as fully as possible with patents. The potential new worker in this field, particularly in industrial laboratories, is somewhat uncertain as to his own status, and in this case the tendency is to await further development in the hope that such development as may interest him can be made on a secure basis.

Above all, the need is for more fundamental work. The possibilities in the way of practical applications are definitely limited by the lack of fundamental data. To be of maximum value, such fundamental investigations must be planned and carried out not only from the standpoint of the magnetic phenomena but also from the point of view of the metallurgical principles involved. This involves the closest cooperation between the magnetic investigator and metallurgist. One of the most important phases of the problem is the study of the so-called secondary effects, which greatly influence the magnetic properties without a correspondingly great effect on the mechanical properties.

The outlook, on the whole, is decidedly encouraging. Magnetic analysis has already proved of use in research on the treatment and structure of steel and for routine inspection on a commercial scale. There seems to be no reason why future progress should not be more rapid, and the hope that magnetic analysis may be found more and more useful as time goes on appears to be amply justified.

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Discussion

B. M. Smith: Mr. Sanford has covered the field of magnetic analysis quite thoroughly, but there is one application which he has not mentioned and which I have found quite reliable. I refer to the determination of the chemical composition of magnetic materials with special reference to the carbon content of iron and steel. In this direction there have been two methods successfully used, one utilizing magnetic saturation and the other, the unsymmetrical hysteresis loop.

Hadfield and Hopkinson presented a paper before the Institution of Electrical Engineers December 8, 1910 showing how the magnetic saturation of iron could be lowered by increasing the amounts of carbon within certain limits, the reduction being in a linear relation to the increase in carbon and sufficiently exact to suggest the determination of the carbon content by magnetic measurements.

Russel J. Eddy, in *Testing*, Jan. 1924 described a "Carbometer" devised by Messrs. Malmberg and Holmstroem which determined the carbon content of steel by utilizing the unsymmetrical hysteresis loop. For percentages of carbon between certain limits the calibration curve is practically a straight line. Experiments which we have conducted along this line have shown the method to be very reliable and quite suited for determining the carbon content of steel at the melting furnace.

During my work of the past 10 years on magnetic analysis I have collaborated with Dr. Burrows and S. L. Gokhale in developing the turbine bucket-wheel test and with others in developing the Durescope. I have, however, found no application more interesting or more promising for further investigation than the application of magnetic analysis to the chemical composition of magnetic materials.

W. B. Kouwenhoven: Last June Dr. J. D. Tebo of the Bell Telephone Laboratories and I presented a paper before the American Society for Testing Materials on a new method of magnetic testing. Briefly, this method involves the applying of a constant d-c. magnetizing field to a specimen and then superimposing a second field which may be alternating or of some other form and determining the change in magnetic characteristics due to the superimposed field. This method we have named an incremental permeability method.

We are continuing this year at Johns Hopkins. I have one man actively engaged on it. The further investigations with this method and the results at present are fairly promising. However, a great deal of work still remains to be done and we need more fundamental data. We need more investigators and cooperation would be welcome.

A. V. de Forest: I should like to draw attention to the very intimate relationship between magnetic measurements and all the unknown properties of steel. In the magnetic method we introduce the possibilities of measuring all kinds of properties that do not come out in any of the formal methods of mechanical testing, and yet we attempt to correlate our magnetic measurements with the results of these exceedingly crude mechanical tests. No wonder it requires quite a long time. Also, no wonder unpleasant experiences occur every once in a while in that a wild kind of material will come along and give the same mechanical tests but different magnetic ones, or vice versa, the same magnetic tests and wild mechanical ones.

I have found it quite desirable to consider the magnetic method as far more fundamental, and very much more subtle than the mechanical tests.

One of the reasons for this is that almost all of the magnetic methods bring in the question of internal stress and yet the internal stress conditions have very little to do with the ordinary measured static mechanical properties. It is necessary to go into fatigue and impact and probably into corrosion and perhaps corrosion fatigue phenomena in order to get reflections of the internal stress conditions which the magnetic is quite able to pick up without any trouble at all.

Perhaps another way of looking at the thing is to consider that both ordinary mechanical static tests and the magnetic tests are measurements on the average properties of large groups of crystalline grains. When we get into mechanical tests of the fatigue variety, perhaps in a long length of wire there will be only one grain, an infinitesimal thickness of material, which will determine the actual life under working conditions of the entire mass of material that we are testing. No static test and no hitherto devised magnetic test can be applied to a single grain or a single section of a large piece. We, therefore, have to study both mechanical and magnetic relationships in the light of average results and not on the single individual specimens. The difficulty is that the mechanical testing of large numbers of specimens is so far almost hopeless. There aren't fatigue-testing machines enough and there isn't time enough to accomplish the correlation.

Electrical Instruments Used in the Measurement of Flow

BY W. H. PRATT[1]

Fellow, A. I. E. E.

Synopsis.—This paper outlines some important flow-measurement problems and points out the character of electrical apparatus that may be used with other equipment for measuring flow.

IN the measurement of fluid flow by electrical means, the electrical devices, though frequently a major portion of the equipment, are essentially auxiliaries. Fundamentally, fluid measurement is a problem in hydromechanics and electrical apparatus is used as a convenient supplement to the mechanical actions which are generally utilized to make manifest the behavior of the fluid. The significance of this statement will appear as we proceed.

Measurements of the flow of water, oil, air, fuel gas, and steam are, as far as numbers of applications, the most important, but there are many other actual and possible occasions for utilizing similar ideas and apparatus. Certain instruments described in the latter part of this paper are as electrical devices, instruments for measuring the relation of two electrical quantities and as such are capable of a wide variety of applications. As here described they are for the measurement of flow.

A broad division of the quantities to be measured may be made according to the conditions of measurement; *i. e.*, 1, measurements of unconfined flow, 2, measurements of flow in confined channels or pipes. The first is illustrated by the measurement of wind velocities and by the determination of the air speed of an aeroplane, the second by the flow of steam in a steam main.

Other divisions may be made following the ideas utilized in effecting a connection between the fluid flow and the electrically responsive measuring devices; for example, we may have an anemometer wheel whose speed response to the fluid flow is used to determine a voltage or a frequency of electrical impulses, or a drop in pressure may be brought about by an orifice or by a venturi tube and the resulting pressure difference communicated to electrically responsive apparatus, or again the change by convection of the temperature of electrically heated wires when exposed to the moving fluid may be the means of establishing a connection with the electrically responsive apparatus.

In measurements of unconfined flow the quantity determined is essentially a velocity or a velocity-density function, and the first stage of measurement is to establish a pressure, a displacement, a motion, or a temperature change which is a function of that velocity or function.

I. If the first alternative is chosen, *i. e.*, if the pressure resulting from the relative motion of the fluid and the measuring apparatus is employed, it may be utilized to change a resistance as for instance that of a carbon-pile, or indeed it is conceivable that through the ministrations of the piezoelectric effect a suitable electrical measurement could be achieved if not directly at least by the use of amplifying devices.

II. Displacements may be used to alter the capacity of a capacitor, to change the inductance of a reactor, or to change the resistance or conductance of a suitable conducting circuit. There are many ways in which electrical effects may be derived from such combination.

III. If the third alternative is chosen the motion will undoubtedly be that of the wind-wheel of an anemometer. From here on to the indication or record the connection may be electrical and may take one of the many forms of electrical speed indicators among which may be mentioned,

1. A magneto, which produces a voltage proportional to speed, connected to a suitable voltmeter.

2 (a) A contact maker which releases electrical impulses at a rate proportional to the speed of the moving member. These may be condenser discharges which thus may release a quantity of electricity proportional to the speed, which may be indicated on a milliammeter acting as a balistic instrument; or the impulses may control an independent source of mechanical motion which may be indicated by the magnetic drag induced by the relative motion of permanent magnets and a body of conducting material or by purely mechanical forms of indication.[2]

(b) Instead of electrical contacts periodic variations of electrical capacity or of inductance may be used to control electrical pulses which in such cases will probably be used in connection with vacuum tubes and electrical oscillations of audio or higher frequencies. The final indications will be proportional to the rate of pulses and the actual apparatus closely paralleling that indicated under (a).

IV. The heat imparted to the surrounding medium

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

[2] These ideas have been employed for the distant indication of electrical quantities.

by a wire carrying an electric current is a function of certain properties of the medium itself, of the temperature and surface conditions of the wire, and of the relative motion of medium and wire. If now energy at a constant rate is supplied to the wire and other conditions remain unchanged the temperature of the wire will be a function of the relative motion of medium and wire. By using a wire having a suitable temperature coefficient of resistance the resistance becomes a function of the fluid flow and a measurement of undisturbed flow resolves itself into a measurement of resistance after the necessary constants have been determined.[3]

The measurement of flow of fluids in pipes has much in common with the measurement of the relative motion of an unconfined fluid medium, and a measuring device. The measurement of steam or of air under pressure is generally carried out by recourse to pressure drop through an orifice or by the pressure-velocity relations determined by pitot and static-pressure tubes. In general the quantities of interest are the mass of fluid per unit of time passing the point of observation, and the total amount of fluid which shall have passed between the epoch of start and any chosen times for observation. Generally these measurements are made at pressures fairly near chosen standards; the one is a velocity density function, the other an integration with respect to time of this function.

The electrical apparatus used generally consists of two portions, 1, a container and a body of mercury whose configuration is changed to correspond to the pressure changes communicated to it and which thereby alters, by shorting out resistance or inductively, the conductivity of an electric circuit; 2, an electrically responsive member which will translate the changes of conductance or admittance into indications or integrate an appropriate function of one of these quantities with respect to time.

In the measurement of the flow of liquids, mechanisms quite similar to those used for gaseous flow may be employed. From the hydromechanic point of view the problem is simplified by the absence of change of density.

Electrical power is thus utilized to perform the work incident to indication or integration and the hydromechanical organizations determine the magnitude or other characteristic of the electrical action.

For most of the measurements here discussed alternating current has many advantages over direct current. In any case an instrumentation whose indications are independent within moderate limits of the supply voltage and in the case of alternating current, independent of the frequency changes that may be expected, is much to be preferred as compared with one which must rely on a fixed or automatically controlled source.

When a body of mercury is caused to change its level by being subjected to a differential pressure it can be caused successively to make contacts which thus change the resistance or impedance of an electric circuit, and in so doing no pressure reaction should result. On the other hand a finite and practically a rather limited number of steps is imposed.

If the body of mercury is so disposed that it acts as the secondary of a transformer, a change of level in the mercury surface may be made to change correspondingly the amount of mercury in the secondary path and consequently the characteristics of the primary circuit. Measurements made in the primary circuit will correspond to conditions in the secondary and these in turn correspond to the equilibrium conditions established by the hydromechanical portion of the equipment. For an arrangement of this kind the electro-magnetic reaction between primary and secondary circuit must be limited to an amount that will not importantly change the mercury level. Small changes can be provided for in the calibration and voltage compensation of the instruments.

In the remaining portion of this paper two mechanisms are described both actuated by alternating current. The one is intended for indication on a scale or for recording on a chart flow as determined by a flow meter in which a body of mercury acts as a secondary and thus influences the characteristics of the circuit in which the instrument is connected; the other integrates with respect to time a quantity similarly determined.

1. An instrument that indicates the conductance of an electric circuit.

This instrument indicates the ratio between the power expended in a standard circuit and that of another whose admittance is controlled by a flow meter or other device. It consists of two elements, similar to the electromagnetic system of the ordinary induction watthour meter, arranged to oppose their torques on a common disk. The disk is so modified that the torque developed by one of the elements, or if conditions should require it, of each of the elements, is inter alia dependent on the position, with regard to deflection, of the disk. Thus for a steady indication, the torques acting on the disk are balanced and

$$k_1 E I_1 \cos \phi_1 f_1(\alpha) = k_2 E I_2 \cos \phi_2 f_2(\alpha)$$

$$k_1 \frac{E^2}{Z_1} \cdot \frac{R_1}{Z_1} f_1(\alpha) = k_2 \frac{E^2}{Z_2} \cdot \frac{R_2}{Z_2} f_2(\alpha)$$

$$k_1 G_1 f_1(\alpha) = k_2 G_2 f_2(\alpha)$$

$$\frac{G_1}{G_2} = \frac{k_2}{k_1} \frac{f_2(\alpha)}{f_1(\alpha)} = F(\alpha)$$

in which

α represents the angular position of the moving system

k_1, k_2 are constants

E is the e. m. f. of the circuit

I_1 is the current in the first circuit

I_2 is the current in the second circuit

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[3] This idea in a modified form is used in the Thomas Gas Meter.

G_1 is the conductance of the first circuit
G_2 is the conductance of the second circuit
ϕ_1 is the lag angle of the first circuit
ϕ_2 is the lag angle of the second circuit

If the constants of the second circuit are fixed and this circuit is used as a standard

$$G_1 = G_2 F(\alpha)$$

If now each value of G is uniquely determined by the quantity we seek to measure, the angular position of the instrument pointer gives an indication of that quantity and a scale may be appropriately marked in terms of it.

2. To integrate a quantity $f(G)$ with respect to time we may utilize an induction watthour meter structure and by changing the proportioning of its part cause the damping effect of the magnetic field from the voltage coil to be large enough to supply all the damping necessary to maintain the speed at a suitable value. As then constituted the torque of the meter will be proportional to the square of the voltage since this torque is determined by the reaction of currents in two circuits in each of which the flow is proportional to the voltage. Also the damping effect is proportional to the square of the voltage, for the magnetic field and hence the Foucault current induced thereby are both proportional to the voltage. The interaction of this field and current produces the damping.

With a simple structure as outlined above a change of frequency would produce an inconsequential change of torque but the current in the voltage coil would vary inversely as the frequency and thus the damping would change inversely as the square of the frequency.

If, however, a capacity is introduced in the voltage circuit, as shown in the diagram, and if its value is so chosen that the resonant frequency of that circuit shall be about double the actual frequency, the current in the potential circuit instead of decreasing in inverse ratio as the frequency will increase with increasing frequency and the damping which increases as the square of the current in the potential circuit can be made to change at the same rate as the resulting change of torque, thus making the speed of the meter independent of the frequency over moderate ranges of frequency. The conditions that make for independence of voltage remain effective. The constants of the circuit are such that resonance from the fundamental frequency of the circuit or its harmonics is precluded.

The first of the two instruments described is clearly one that indicates the ratio between two similar electrical quantities and so can be utilized to measure the ratio between any two quantities that can be represented by suitable electrical quantities, as for instance, temperature. In this case, standard temperature may be represented by a standard resistance having a small or zero temperature coefficient, while the temperature to be measured may be represented by a resistance that varies suitably with that temperature.

Instead of measuring flow as such, two pressure-sensitive devices may be used, one to influence the one, the other the second element of the instrument whereby a ratio of flows may be measured. The ratio of steam delivered to air used might be one application or the measurement of the ratio of steam extracted to total steam used, another.

An instrument could be constructed with two moving systems and associated elements, the two indicators arranged to move in closely parallel surfaces over a single scale in such a way that the reading point should be the intersection between the two indicators. In this way indications of such functions of four quantities as may be expressed in the form $F\left(\frac{X}{Y}, \frac{U}{V}\right)$ could be obtained as a single reading. Non-electrical quantities that can be represented by suitable electrical quantities can be so measured. The only application (of which the author is aware) of such a structure was for the measurement of the product, or ratio of electrical quantities for the measurement of another electrical quantity, namely power factor, $\frac{P}{E\ I}$. The actual relation made use of in the instrument was $\frac{P}{E^2} \cdot \frac{P}{I^2}$ but the occasion may easily arise in connection with other measurements, and a not impossible application may be in the determination of a compound ratio of the flow of several fluids.[4]

Induction instruments of the ratio type are nearly free from one of the drawbacks that characterize certain other forms, notably induction voltmeters and wattmeters, in that the indications result from the equilibrium of two torques that have the same characteristics with respect to temperature. Small residual errors are cared for by a small amount of temperature-sensitive magnetic material in the air gaps.

In the integrating meter, temperature errors are self-eliminating.

It is not expected by the author that this paper more than outlines the nature of some important fluid measurement problems and points out the general character of the electrical apparatus that may be employed. The essence of fluid measurement is contained in the subject of hydromechanics. The electrical apparatus which generally may be advantageously used has characteristics that suggest its usefulness in connection with other types of measurement that may have nothing in common with fluid measurement aside from the electrical apparatus used for giving final indication.

Numerous bibliographical references may be found in the bibliographies relating to the Measurement of Non-Electrical Quantities by Electrical Means which have appeared in the A. I. E. E. TRANSACTIONS.[5]

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[4] Pratt patent 719,609, Feb. 3, 1903. Induction elements could replace the electro-dynamometer elements illustrated therein.

Appendix

By way of explanation of the diagrams:

In Fig. 1 the disk is represented as divided by a curved line into two parts. The main portion is of high conductivity material and the irregular boundary of this part is so chosen that the torque of the back element is a function of the angular position of the disk as well as of the current, voltage, etc., to which that element is subjected. The smaller part of this disk is of high resistance material and its purpose is solely to give mechanical balance to the element.

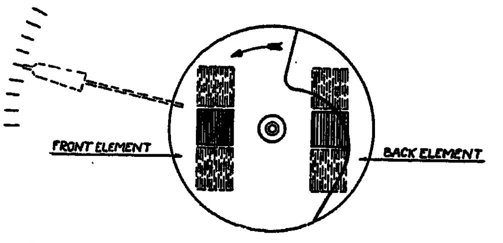

FIG. 1—DISK OF CONDUCTANCE METER

Fig. 2 gives the connections of the electrical elements associated with the moving system shown in Fig. 1. In Fig. 2, M represents a flow meter element, a transformer with a mercury secondary.

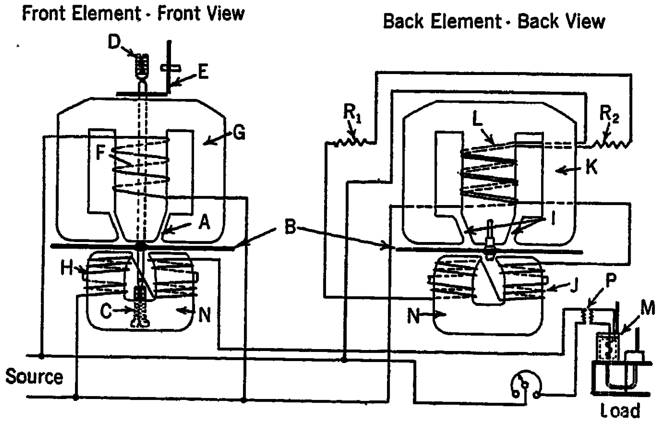

FIG. 2—SCHEMATIC DIAGRAM OF CONDUCTANCE METER

G, K, N, N........ laminated iron cores

B........the disk shown in Fig. 1.

L........the exciting coil of the back element which also serves as a primary to which the circuit including R_1, R_2, and J is a secondary. The resistance $R_1 + R_2$ constitutes the standard electrical quantity with reference to which the indications of the instrument are determined.

In Fig. 3, C and K are laminated iron cores. The magnetic flux which crosses the gap between them in which disk E rotates not only produces the torque but also supplies all the damping.

The circuit including B and I is highly reactive, leading, thus with increasing frequency, the current in this circuit increases and the damping, which may be taken as proportional to the square of the current in B, increase at a rate sufficient to compensate for the increases of torque which also increases with the frequency. In the absence of the capacitor I the torque would remain almost unchanged with change of frequency while the damping would vary inversely as the square of the frequency, since the predominant part of the damping is due to the flux from coil B. The circuit including coil F and resistors R_1 and R_2 is of high resistance. Its purpose is to supply a small amount of torque to balance out the excitation losses of the transformers O and M. H is the main current winding and L is an adjustable reactive shunt to effect calibra-

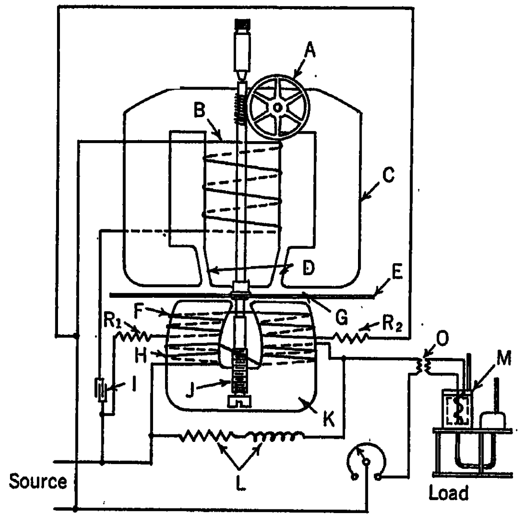

FIG. 3—SCHEMATIC DIAGRAM OF INTEGRATING METER

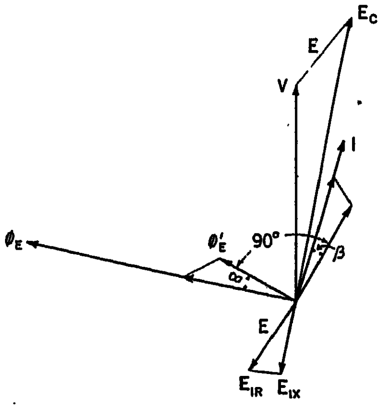

FIG. 4—VECTOR DIAGRAM OF INTEGRATING METER

tion and provide a small adjustment for the phase angle of the current in coil *H*.

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[5] A. I. E. E. TRANS., XLIV, 1925, p. 258.
A. I. E. E. TRANS., XLVI, 1927, p. 710.
A. I. E. E. Quarterly TRANS., Vol. 47, October 1928, p. 1168.

Fig. 4 shows the vector relation of the quantities involved in the action of the meter shown in Fig. 3.

V represents the voltage impressed on the voltage circuit and is the sum of the two components

E_c the voltage across the condenser

E the voltage across the voltage coil

the latter is the resultant of reactive and energy components as indicated by E_{IX} and $E_{IX} - E_{IR}$.

ϕ_E represents the flux in phase with the current in the voltage coil

ϕ_E' is that portion of the flux ϕ_E that crosses the disk, the phase shifted through the angle α by the light load adjusting plate (not shown)

I represents the current in the current coils of the meter

ϕ_I' the resultant flux due to the current I and current in short-circuited bands around the current poles (not shown in Fig. 3).

Discussion

I. F. Kinnard: Mr. Pratt shows (Fig. 3) a very interesting application of the induction watthour meter to measure conductance. It will be observed that there are no damping magnets in this arrangement, but the damping on the disk is supplied entirely by the potential flux. In other words, the same flux drives the disk and produces damping. It should also be noted that a condenser is used in series with the potential coil in this device in order to correct for frequent variations.

There has recently been developed a modified scheme also utilizing the watthour meter which may be of interest to the Institute and ought to be on record.

The watthour meter, of course, has a moving element, usually an aluminum disk. It has a driving element consisting of a potential winding and a current winding, and the control torque is supplied by a permanent magnet. If we dispense with the permanent damping magnet and substitute instead another electromagnet which is connected directly across the line through a fixed resistor, we have a conductance meter which is very closely allied to the standard watthour meter. As a matter of fact, it has been found possible to get very excellent results using standard watthour meter disks, bearings, and driving elements in this way. It is obvious just to get a physical picture of the way this device works that the only variable that can cause a change in speed of the meter is a change of the conductance of the load. That is, if the potential should vary, both the driving torque and the damping torque varies according to the square of the potential and there is no resultant speed of the meter. There may be many applications in the future whereby this very interesting modification of the watthour meter could be profitably used.

Use of the Oscillograph for Measuring Non-Electrical Quantities

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Member, A. I. E. E.

and

W. B. BATTEN*
Associate, A. I. E. E.

Synopsis.—The electromagnetic oscillograph has proved to be a useful instrument for obtaining recorded measurements of non-electrical quantities. The problems of application involve the translation of mechanical phenomena into electrical changes which are proportional. Several typical applications are described showing how motion, time, stress, pressure, etc., are measured.

Although the electromagnetic oscillograph was developed primarily for, and has found its principal application in, recording electrical quantities in terms of current, voltage, and watts, its inherent characteristics have made it useful for non-electrical applications. This oscillograph offers a means of recording photographically phenomena which are of too short a duration for the eye to watch or for other types of instruments to record. Furthermore, it is frequently desirable to record simultaneously both electrical quantities and mechanical actions of some apparatus. For this, the oscillograph is admirably adapted. However, these quantities must be of such a nature that they can be translated into electric potentials or currents which have a definite relation to the quantities to be recorded.

The element of the instrument is a low inertia galvanometer, the two ribbons of which move with respect to each other in a magnetic field. The motion tilts a mirror and through an optical lever causes a beam of light to be deflected on a screen or film. The motion is proportional to current in the ribbons so that equal increments of current through the galvanometer or equal increments of potential applied to the galvanometer with series resistance are faithfully recorded.

The problem in adapting the oscillograph for other than electrical uses lies in the translation of the phenomena to be recorded, into proportionate electrical values. It is the aim of this paper to show a number of typical applications illustrating how this object was obtained. Many phenomena, such as changes in pressure, vibrations, stresses, and time intervals, give rise to mechanical motion, either linear or angular, resulting in a variation of speed or an acceleration of the body as a whole or in part. If this motion can be made to vary an impedance in an electrical circuit as a function of this motion, the current through the circuit, or the potential across a portion of this impedance, may be utilized to actuate the galvanometer for a record.

Travel Indicator

Where the motion of a part of a machine is to be studied for uniformity, acceleration, rebound, etc., a resistance wire can be placed along a stationary part, and a moving contact attached to the moving part. The contact slides along the wire and changes the resistance included in the galvanometer circuit as in Fig. 1, in which $A\,B$, the resistance, may be either of continuous length or steps in a wound resistor. C is the sliding contact for indicating relative motion between the moving member to which it is attached and the fixed member to which $A\,B$ is fixed. In such a circuit, a constant current flows from a battery through $A\,B$. The resistance of the galvanometer circuit $A\,G\,C$ is made sufficiently high so that changes in the position of C do not alter the potential distribution along the resistor $A\,B$ appreciably. The proportions of current in the two circuits (resistor and galvanometer) can be adjusted to secure any reasonable degree of accuracy.

A device of this kind has proved particularly useful in studying the travel of circuit breaker parts. As shown in Fig. 2, the resistor wire is wound on the edge of an insulating disk which can be rotated against the action of a special spring. The device is clamped to the frame at a convenient point and a cord, connected to the disk and wrapped around a drum on its shaft, is attached to the moving part to be studied.

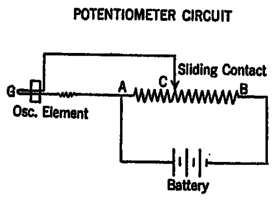

Fig. 1—Potentiometer for Travel Indicator

Speed Recorder

When a rotational speed record is desired in a form, such as an oscillogram, particularly adjacent to some other record for the purpose of timing, rotation must be translated into an electrical current in the oscillograph element proportional to the speed. A magneto or other constant field generator has a voltage characteristic proportional to the speed, but the usual electric tachometer has a wave form that is very bad, being full of high-frequency harmonics. At various times homopolar generators have been used but they generally develop current collection troubles and generate a very low potential. A remedy to the magneto trouble can be obtained by using a special oscillograph element, which is equipped with a damping vane and will not respond to high frequencies.

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*Both of Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

Fig. 3 is a section of an oscillogram recording the operation of two reversing mill motors. The top record shows the speed of one motor as measured with a magneto and oscillograph element having a damping vane. The fifth record from the top shows the speed of a second motor as measured with a magneto and standard oscillograph element. These two records show the effect of a damping vane on an oscillograph element when making this type of record.

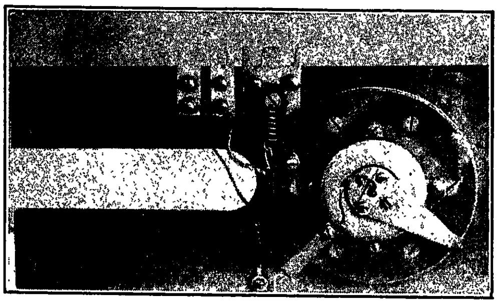

Fig. 2—Circuit-Breaker Travel Indicator

It is sometimes desirable to record minute speed variations of rotating parts as when investigating governor action. This may be accomplished by coupling or belting a small magneto or constant field generator to the machine under test. The magneto or generator is connected in series with a battery, the potential of which opposes and equals the magneto voltage during steady state condition. The oscillograph records the differential potential and can be adjusted to respond to a fraction of one per cent change in speed.

Vibration

If vibratory motion of small amplitude can be made to change the reluctance of a magnetic circuit of proper design an electrical equivalent of the motion can be obtained. This was used in exploration of turbine blade vibration by placing a vibration converter on the end of the turbine shaft. This device is really a small generator with a permanent magnet field. It is so designed that the normal position of a field pole is midway between two armature teeth. As the field moves with respect to the armature, the flux shifts from one tooth to the other and induces a potential in the coils on the armature teeth. In practise, the armature is clamped rigidly to the end of the turbine shaft and hence follows the motion of the turbine rotor. The field of the vibration converter is free to move with respect to its armature and its inertia is such that it does not follow the high-frequency vibrations which may be present in the turbine shaft, though it rotates at the same average speed as its armature.

Timing

Sequence of operation without reference to magnitude of motion or resulting phenomena can be shown nicely by the oscillograph. Two factors are necessary, one an indication of operation, and the other a time scale. If, for example, the relation between the opening or closing of several switches is desired, each unit can be so arranged as to short-circuit a resistor in the galvanometer circuit or insert resistance. At each operation the recording line makes a jump up or down. The time scale may be made by an a-c. wave of known frequency, simultaneously recorded a timing record from a tuning fork or the tick of a watch. In the last case a small mirror mounted on the hair spring deflects a beam of light giving impulses on the film. This is accomplished by removing a small section from the edge of the watch case opposite the hair spring. The mirror, which is about 0.017 by 0.070 in. in size, is fastened directly to the hair spring at a point near the place where the hair spring is secured to the frame. Hence the mirror moves only slightly, this motion causing a small rotational movement about its transverse axis. The light spot reflected from the mirror moves up and down as the hair spring moves. As the light spot passes the slit in the oscillograph film holder, a spot on the film is exposed. Hence the record consists of a series of dots or dashes, such as that shown at the bottom of Fig. 3 for slow speed work. This device is very dependable. It has the advantage of not requiring a regular oscillograph element, thus leaving an extra element for some other measurement. The mirror affects the speed of the watch only slightly.

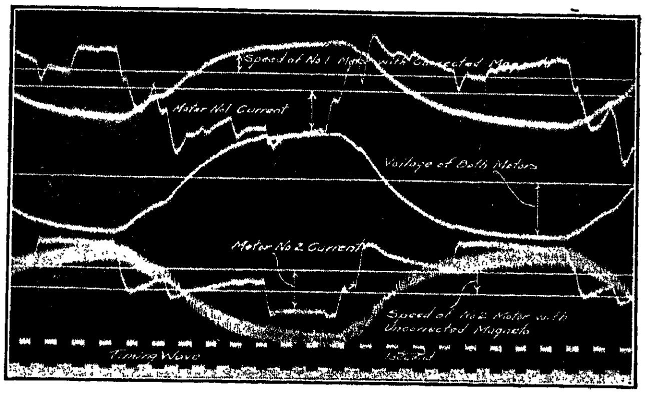

Fig. 3—Oscillogram Showing Speed Record with and without Corrected Magneto Wave

Stresses

For most materials the strain from load on a mechanical member is proportionate to stress within the elastic limit, so that a record of stretch or compression can be interpreted in terms of stress. Usually the amount of motion is small, so that slide wire scheme is not applicable. However, other methods such as change in resistance of a pile of carbon disks with small motion or change in reluctance in a magnetic circuit in combination with suitable coils, are available.

The strain gage is an interesting form for studying railroad track and locomotive side rod stresses. This can be applied wherever it is desired to obtain oscillographic records of mechanical movements of very small amplitude (a few thousandths of an inch). The gage consists of two stacks of U-shaped sheet-steel punchings with a coil on each and an armature of sheet-steel punchings as shown in Fig. 4. The open ends of the two stacks of punchings are placed facing each other, with the armature between them, and the gap between the armature and U-punchings is sufficient to permit the armature to move with respect to the punchings. In other words, the strain gage records the movement of the point to which the armature is fixed with respect to the part to which the U-punchings are attached. The two coils are connected in series and a fixed current at relatively high frequency (400 to 800 cycles) is sent through them. The oscillograph records the voltage across one of the coils. This potential varies with the distance between the armature and U-punchings. Experience has proved that practically a straight line relation exists between potential across one coil and armature position as long as the armature does not actually touch either stack of U-punchings. The record shows a modulated high-frequency current; hence vibrations having a frequency higher than about 15 per cent of the frequency of the current through the coils cannot be recorded accurately. Inasmuch as the modulated high-frequency wave occupies a large space on the film, it is difficult to record six stress measurements simultaneously without overlapping, giving a record confusing to analyze. To eliminate this condition, the voltage to be recorded is sent through a full-wave copper-oxide disk rectifier, and then through a choke coil before reaching the oscillograph galvanometer. The choke coil is of such value that the rectified high frequency is practically ironed out, but is low enough so that it does not affect the envelope curve (40 cycles or less). This arrangement gives a straight line record, the position of the line being varied only by a change in the position of the strain indicator armature or in other words by the stress in the member to which the strain gage is attached.

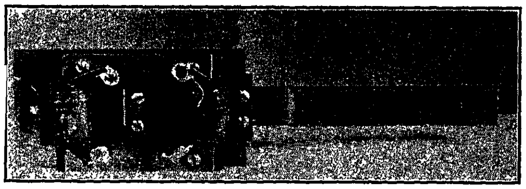

FIG. 4—THE STRAIN GAGE

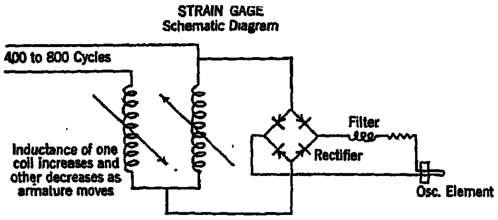

FIG. 5—STRAIN GAGE CIRCUIT

FIG. 6—PRESSURE RECORDER

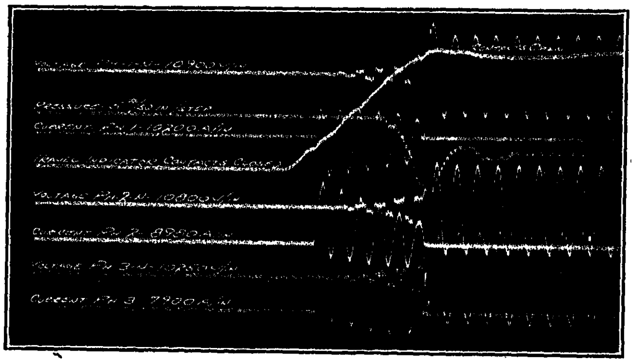

FIG. 7—OSCILLOGRAM SHOWING CIRCUIT-BREAKER TEST WITH TRAVEL INDICATOR AND PRESSURE RECORDS

PRESSURE RECORDER

A great deal of effort has been expended upon the subject of recording transient steep wave front pressures. Strictly speaking, any device in which motion occurs due to the pressure cannot give a true record, for the motion has changed the volume and consequently the pressure. Furthermore, the motion of a piston or diaphram involves time lag. But for ordinary purposes we can be satisfied with much less than the idea. Probably the piezo crystal comes as near being perfect as anything. Certain crystals exhibit electrical changes when subjected to pressure which, however, alter their shape very little. For many commercial uses, however, such a crystal is far too delicate a device, and the electrical change much too feeble.

The carbon pile is often used but even though a bridge arrangement is employed it is difficult to maintain a constant calibration and it is too delicate for rough handling.

A much better instrument for circuit-breaker pressure records was developed later using a very light-weight gas-engine indicator modified to translate the piston motion into change in resistance. The pointer which is ordinarily used for tracing an indicator card was made part of an electrical circuit and travels over a drum made up of alternate copper and insulating disks, the copper being connected to suitable external resistance units. An inch of travel on the pointer tip covers 20 steps. Thus each step represents a pressure of 1/20 of the piston spring calibration. This makes the record self-calibrating. The pressure record is shown in steps but if these are made of the proper magnitude there is no objectionable lack of smoothness in the curve. The device is very convenient and withstands test floor handling without damage. For protection during tests the recorder is placed in a fireproof metal box and a short length of metal hose is used for the connection to the circuit-breaker tank. It is interesting to relate that at first a two ft. length of heavy rubber hose was used. Records were obtained of explosion pressures long after the phenomenon was over. There was a time delay in propagation of the wave of pressure through the rubber sufficient to cause this. Substituting a rigid metal connection eliminated this.

We have described above a few interesting uses of the oscillograph outside of the purely electrical field. Many more could be cited but these will suffice to show the inherent usefulness and versatility of this instrument.

Bibliography

J. W. Legg, "The Multi-Element Oscillograph," *Elec. Journal*, Nov. 1927.

J. W. Legg, "Six-Element Portable Oscillograph," *Elec. Journal*, Mar. 1925, p. 108.

J. W. Legg, *Portable Oscillograph*, A. I. E. E., July 1920, p. 674.

The following bibliographies previously published contain many references to the application of the oscillograph to non-electrical measurements.

TRANSACTIONS A. I. E. E., Vol. XLIV, 1925, p. 257.

TRANSACTIONS A. I. E. E., Vol. XLVI, 1927, p. 709.

TRANSACTIONS A. I. E. E. Quarterly TRANS., Vol. 47, October, 1928, p. 1168.

Discussion

M. S. Rusher: One who is constantly using the oscillograph probably has found that there are more applications of this instrument to the measurement of non-electrical quantities than to electrical ones. The methods used for adapting it to these measurements will naturally differ greatly and no particular method can be called the best for all cases.

The authors state that the "Travel Indicator" described in their paper, gives any reasonable degree of accuracy with proper adjustment, but it will be found that accuracy by this method is at the expense of considerable power loss in the resistor circuit. The step-by-step method of measuring displacement which Dr. H. L. Curtis called a "displaceometer" in his paper on *Use of an Oscillograph in Mechanical Measurement*, (A. I. E. E. TRANS., Vol. 44, p. 264, 1925) has been used by us for many years with the added feature that the resistance in series with the oscillograph vibrator on different steps varies over a certain range and then repeats itself so that direction of travel and rebound are apparent at a glance. Displacements of greater magnitudes are quickly indicated by noting the number of peaks of a given height on the film the same as when using the long marks only across an ordinary scale. For measuring smaller variations it is necessary to examine the deflections of various heights. Time and distance can be measured with great accuracy by this method.

For measuring rotational speed of motors or generators, a small constant-field generator has been found very satisfactory. The wave shape being comparatively free from high harmonics, it therefore gives a curve sufficiently smooth for the required accuracy.

Vibrations of any parts made of magnetic material or to which a piece of magnetic material can be attached without changing the vibrations appreciably, have been successfully recorded by the oscillograph using the variations in the reluctance of the magnetic circuit through an exploring coil placed near the vibrating part. In order to measure this vibration accurately, the *Vibration Recorder*, described by Mr. A. V. Mershon (A. I. E. E. TRANS., Vol. 45, p. 1007, 1926) was found very useful. This same apparatus has been adapted to the measurement of pressure variations by means of a steel diaphram. This has been used for measuring pressure variations in the draft tube under a large water wheel as well as for obtaining indicator diagrams of the pressure inside an electric household refrigerator unit (*General Elec. Rev.*, Vol. 31, p. 526, 1928).

The applications of the oscillograph for timing purposes have been too numerous and varied to describe here. A portable timing device controlling a vibrating reed or timing vibrator is best adapted for very slow records. The voltage wave of known frequency is satisfactory for the great majority of tests while in some cases where great accuracy is required, the use of a tuning fork such as described by Dr. Curtis in the paper mentioned above is very satisfactory. Measurements have been made of the time required for the explosion of dynamite as easily as that of the time of operation of an oil circuit breaker.

In addition to the above applications to the measurement of non-electrical quantities are those in which the emphasis is placed on recording the variations in such quantities. Sound was recorded with the oscillograph by Mr. L. T. Robinson in 1905 (A. I. E. E. TRANS., Vol. 24, p. 212). This has developed into the present talking-movie recording apparatus.

The oscillograph was also used with a special galvanometer for recording heart beats. This too has developed into a special instrument, the "electro-cardiograph."

When new apparatus of any kind is being developed, it is usually found that the oscillograph can be used to advantage in solving some of the problems which arise.

Bela Gati: Applying a microphone and an oscillograph, the curves show quite distinctly the difference between a new and a used gasoline engine. In the same manner, the microphone and oscillograph can be used for studying various noises and for determining means for abatement of the noises, for instance in the subway. The oscillograph and its accessories are too expensive as yet for a private engineer. Perhaps the proposed Navy reserve institution could supply our Institute with an oscillograph equipment closet where each engineer could work on his special problem.

The noise of a cruiser can be detected in 12 km. distance, and by the noise a sound-governed torpedo can be released. It is to the interest of the Navy to support these and similar investigations and researches.

A. V. de Forest: In connection with the oscillograph on non-electrical measurements, I should like to mention further a matter which has been already spoken of, and that is the carbon pile, devised by Peters at the Bureau of Standards. It seems to be working out into an exceedingly flexible mechanism for pressures and motions of all kinds and along with it there is a twelve-element oscillograph that Peters has worked up which is a very compact and pleasant piece of apparatus to use compared with the ordinary oscillograph of some years ago.

That particular field seems to be opening up commercially into all kinds of directions. I have recently seen some figures on the stresses in track bolts made by that method that looked extremely suggestive of the possibilities of redesigning all kinds of members subjected to unknown stresses.

W. B. Kouwenhoven: One of the previous discussors has mentioned the electrocardiograph, which is used by medical men for studying the heart. Doctors are using electricity more than ever in their work. At Hopkins I often receive the query from our Medical School, "Can we do this electrically?" I think that it behooves the electrical profession to help them whenever possible, and to aid them in properly applying electricity to their work.

One of the latest non-engineering electrical measurement problems that I have worked on is the determination of the effect of diet on teeth. A poor tooth is more porous than a sound one and is more quickly permeated by a solution. The teeth of animals that have been fed special diets are extracted, and tested as follows: The nerve space of the tooth to be tested is filled with mercury; which serves to make electrical connection to the inside, and the tooth is then partly immersed in a conducting solution. It is connected in one arm of an a-c. bridge and its resistance is measured at certain intervals of time. The decrease of resistance with time of immersion is more rapid with poor teeth than with good ones.

G. D. Floyd: The measurement of non-electrical quantities affords an opportunity for combining more than one quantity on the same element of an oscillograph, the reason being that in most of these measurements of non-electrical quantities, the unidirectional electrical quantity is recorded on the oscillograph. In this case it is possible to take an alternating quantity and place on the same element the unidirectional quantity, giving what one might call a bias to the combined measurement and detracting nothing from either quantity. I think this point is important and could be made use of in a large number of cases.

Perry A. Borden: The use of the oscillograph for work other than the actual determination of current and voltage wave-forms has received a great stimulus in the advent of an instrument as flexible as that used in the paper under discussion. Those not so fortunate as to have at their disposal an instrument giving six or more simultaneous records are frequently handicapped, and are sometimes forced to resort to peculiar expedients to make best use of the equipment at hand. Such an instance is recalled in the case of tests performed some years ago in the investigation of performance of a remotely controlled hydroelectric station. Here it was required to obtain as many simultaneous records as possible with an instrument having only three vibrators; and it became necessary to "crowd" one vibrator, to the extent of having it perform three distinct duties, the record on the film afterwards being "unscrambled."

The vibrator in question was energized by a battery current upon which were superimposed oscillations set up by a tuning fork, thus establishing a standard of time. The main current was periodically interrupted by a contactor attached to the shaft of the machine under test, thus giving a count of the revolutions of the unit, while the magnitude of the whole current was regulated through a rheostat attached to the gate mechanism of the governor. There was thus obtained a record in which the height from the zero line represented gate opening, while the periodic breaks in the line showed revolutions of the generator shaft; and the ripple introduced by the tuning fork completed the data from which could be plotted the desired curves of velocity against gate opening, leaving the remaining vibrators free for the recording of electrical phenomena.

C. A. Mead: In a recent meeting of the Iron and Steel Institute, W. B. White of the American Steel and Wire Company gave a very interesting discussion of his research work in connection with steel wire, piano wire principally. That has caused considerable interest in connection with piano tuning and the loading of steel wire, etc. He has taken some very interesting oscillograms showing that the touches of various pianists differ, the same note being struck by different artists not being reproduced identically.

F. J. Schlink: One very simple method of obtaining the necessary variation of resistance for use in recording by the oscillograph, has not been mentioned: that is, a rheostat, made by winding a fine enameled wire on a cylinder with adjacent turns touching, and then removing the enamel insulation along one element of the cylinder by the use of a fine abrasive like crocus paper. In this way steps of resistance as small as two or three thousandths of an inch can easily be obtained without the necessity of piling up thin conducting and insulating plates which then have to be laboriously calibrated as to the distance from contact to contact. In the method described, the enamel wire can very easily be wound so that successive turns are in physical but not electrical contact, and no calibration is necessary beyond determining the diameter of the wire over the insulation. Such a device was described by the speaker in the *Journal of the Optical Society of America* and *Review of Scientific Instruments*, June 1925, p. 661. Very little shop work is necessary, and a rheostat of this type can be prepared ready for connection to the oscillograph with perhaps an hour's work.

Moreover, the contact surface being very perfect, depending upon the finish of the cylinder on which the wire is wound, motions can be recorded at extremely high speed and in some oscillograms taken in this way contacts were made at rates as high as 5000 per second with perfect legibility in the oscillograph record. It would be interesting to see how small a contact interval could be obtained by the use of wire especially drawn for the purpose and winding facilities devised for the particular problem. It may be that steps of one thousandth of an inch could be made available by this method with special wire, a simple device to assure a light uniform winding tension, and some manipulation skill.

F. G. Graf: I might mention a very interesting application of the osiso to the oil industry. The osiso elements are situated throughout the area in which a pool has been located and by setting off charges of dynamite and making records of the variations in the vibrations in the ground, the extent of the pool has been very accurately plotted. I think that as this application becomes better known and is studied further, considerable money will be saved not only to the oil industry but probably to other mining industries.

W. B. Batten: The travel indicator described in our paper gives a true position—time curve on the oscillogram. This, in our opinion, is preferable to any device which varies in steps, and we use the step method only when the motion is small as in the case of the pressure recorder. A power consumption of five or ten watts in the resistance wire has never offered us any difficulty.

There is no constant-field generator suitable for measuring rotational speed which, with the exception of homopolar machines, gives a good wave form. It is not a question of accuracy but of appearance.

On the pressure recorder described in the paper the use of disks was adopted because of the difficulty in obtaining satisfactory contact conditions when the pointer moved directly over the turns of a rheostat.

Study of Noises in Electrical Apparatus

T. SPOONER*
Member, A. I. E. E.

and

J. P. FOLTZ*
Associate, A. I. E. E.

Synopsis.—The objections to unnecessary noises in electrical apparatus are becoming so insistent that the manufacturers are making every effort to reduce them. In order to attack the problem intelligently it is first necessary to determine the amplitude and frequency of the components of the complex sounds. The first requirement then is a simple portable sound analyzer. Descriptions are given of two resonance types which have been developed. There is included also a short discussion of the laws of sound variation and sound units. Finally there are discussed various applications of the sound analyzer to the study of noises in gears, motors, generators, induction regulators, loud speakers, vacuum sweepers, and the like.

* * * * *

STUDY OF NOISES IN ELECTRICAL APPARATUS

Introduction. Psychologists have found that noise reduces human efficiency. This effect may be very considerable even for the most phlegmatic individuals. Moreover, most of us object to unnecessary noises. In recent years there has arisen a very insistent demand for quieter electrical apparatus. This applies particularly to household apparatus such as refrigerator motors, vacuum sweepers and the like. In order intelligently to attack the problem of reducing noises, it is first necessary to study them in detail and resolve them into their components.

That the human ear is very inadequate for such purposes is well known. Two examples from our own experience will serve to illustrate this. A few years ago nine loud speakers, practically alike, were mounted at one end of a long hall with an observer stationed at the other end. This observer was unable to tell with certainty whether or not one of these loud speakers or all nine were operating. Ordinarily the human ear can detect much smaller differences in sound volume than this but the illustration serves to show that under certain circumstances the ear is a very poor judge of sound intensity. The second illustration has to do with the estimate of frequency. A certain member of the Laboratory Staff has a very good sense of absolute pitch. If a note is struck on the piano this individual can usually tell what note it is and give the corresponding frequency in cycles per second. There was under test in a noise proof room a compressor motor which was very noisy, the frequency of this sound corresponding to the number of stator teeth. The above mentioned observer was asked to listen to this motor and make a guess concerning the frequency of the noise. He stated that the motor had two frequencies, one at about 350 and another at 450 cycles per second. As a matter of fact there was only one frequency of any magnitude and this was 900 cycles per second. This shows that one's judgment of frequency may be very poor for unaccustomed sounds. Many have known of cases where an inspector has rejected a machine one day because of noise and the next day passed the same machine without any change in the apparatus having been made in the mean time. Such a thing may happen because the room noises were greater on the second day than on the first or perhaps simply because the inspector had had a good breakfast or for some other reason was in an amiable frame of mind.

Theory. The sensation of loudness is a function of the sound energy but is not directly proportional to it. For a given frequency the loudness is approximately directly proportional to the logarithm of the r. m. s. sound pressure. This is a form of Weber's law; namely,

Loudness (sensation) = $K \log_{10}$ pressure (stimulus) where K is a function of the frequency. The r. m. s. pressure is a function of the frequency (f) and amplitude (a) thus:

$$p = 41.2\ (2\ \pi\ f\ a)\ \text{dynes/cm.}^2$$

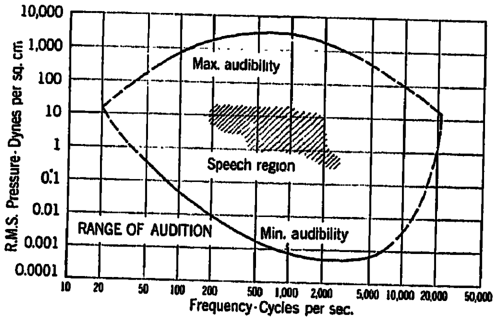

Fig. 1—Maximum and Minimum Limits of Audibility.

Fig. 1, due to the Bell Telephone Engineers,[1] shows the maximum and minimum limits of audibility as a function of sound pressure and frequency. The ordinary speech range is given by the cross hatched portion. Of course the minimum limit of audibility curve is an average for many observers and corresponds to perfectly quiet conditions. With ordinary room noises present the limit would be raised very considerably.

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Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

[1] Fletcher, "Useful Numerical Constants of Speech and Hearing," *Bell System Technical Journal,* Vol. IV, July, 1925.

Above the maximum audibility limit the sensation is that of feeling or even pain.

In order to obtain arbitrary loudness units which may be used for comparing various sounds, the audibility range has been divided into ten equally spaced sections (double log paper), each section corresponding to ten units. Such curves are shown by Fig. 2 as a function of r. m. s. pressure and frequency. These curves apply to single frequencies only. For a composite sound the loudness is proportional to the logarithm of the sum of the components weighted with respect to frequency, as shown by Steinberg[2] and Kingsbury[3] in the Bell Telephone Laboratories.

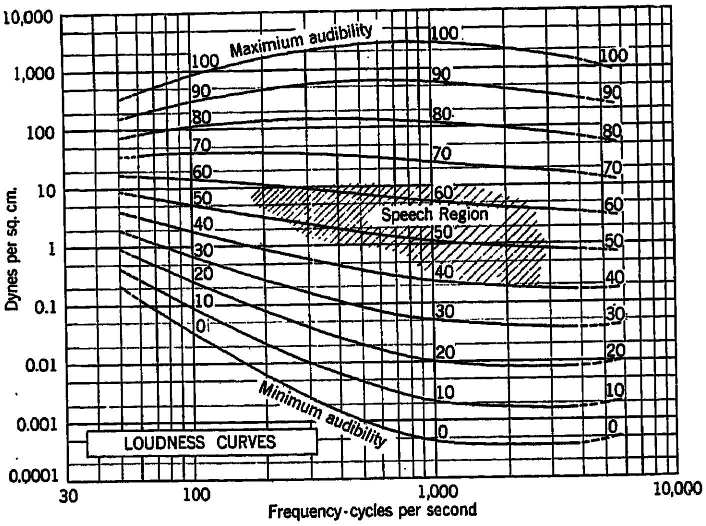

FIG. 2—LOUDNESS UNITS FOR COMPARING VARIOUS SOUNDS

SOUND ANALYZERS

Various devices for measuring the sound components of recurrent phenomena have been developed. The analyzers used for the investigations about to be described were of the simple resonance type. Other analyzers using different methods and usually much more complicated have been built and used where it was not essential that the analyzers be portable.

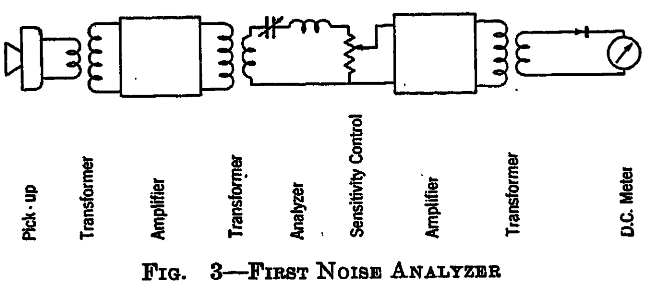

FIG. 3—FIRST NOISE ANALYZER

Figs. 3 and 4 show diagramatically two types of analyzers which have been constructed. Referring to Fig. 3, the pick-up consists of an electro-dynamic microphone feeding into a step-up audio-frequency transformer, vacuum-tube amplifier, and step-down transformer. The resonant circuit of the analyzer consists of a fixed inductance and variable condenser. The voltage across the non-inductive resistance with a variable tap is fed into a second amplifier, a step-down transformer, and a d-c. milliammeter and copper oxide rectifier. Obviously if a sound having a frequency component corresponding to the setting of the resonant circuit impinges on the microphone a reading will be obtained on the d-c. meter. If the analyzer circuit is not in resonance it will offer such a high impedance that the particular frequency in question will not produce sufficient current in the analyzer circuit to produce an appreciable drop in the non-inductive resistance. The second type of analyzer (Fig. 4) is similar to the first except that the sensitivity control is transferred to the microphone circuit and the analyzer circuit is arranged differently. In this case the resonant arrangement provides practically a short circuit for the secondary of the second transformer except when tuned to the incoming frequency. When the tuning corresponds to a certain component of the sound frequency, the analyzer circuit offers a high impedance to this frequency and an appreciable voltage is applied to the input of the second amplifier. The rectifier for this second analyzer makes use of a vacuum tube rather than a copper oxide rectifier. The second analyzer gives a meter reading which varies very considerably with frequency for a given sound pressure. This is no serious disadvantage, however, since the analyzers are calibrated over the whole range of frequency.

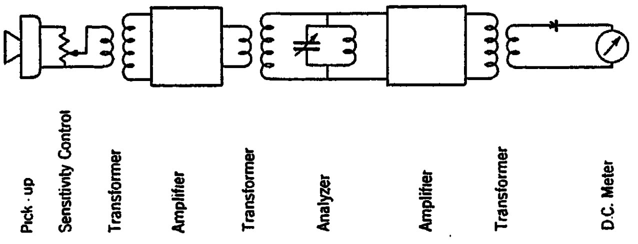

FIG. 4—SECOND NOISE ANALYZER

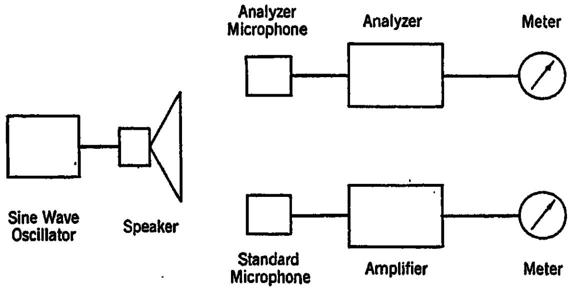

FIG. 5—METHOD OF CALIBRATING ANALYZERS

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[2] J. C. Steinberg, "The Relation between the Loudness of a Sound and Its Physical Stimulus," *Physical Review*, Second Series, Vol. 26, pp. 507-23, October, 1925.

[3] B. A. Kingsbury, "A Direct Comparison of the Loudness of Pure Tones," *Physical Review*, Second Series, Vol. 29, pp. 588-600, April, 1927.

The method of calibrating is as follows: (see Fig. 5) An electro-dynamic loud speaker capable of going to low frequencies is excited from a variable frequency sine wave source of voltage. In front of the loud speaker is placed the electro-dynamic pick-up of the analyzer to be calibrated. Beside the analyzer pick-up is placed another very carefully built electro-dynamic pick-up, the constants of which are known; namely, for a given sound pressure the generated voltage can be calculated. This standard microphone is connected through an amplifier to a milliammeter. This amplifier combined with the milliammeter is calibrated between input volts and milliammeter readings. Since the r. m. s. sound pressure is directly proportional to the generated voltage the pressure can be estimated directly from the reading of the milliammeter. Next a sine wave a-c. voltage of a known frequency is applied to the loud speaker element, the analyzer adjusted for a maximum reading by moving the variable condenser and the condenser setting and the meter reading noted. This gives an amplitude and frequency point on the calibration curve of the analyzer. This is repeated for various frequencies and amplitudes until the complete calibration curves have been obtained.

In connection with noise studies it is often necessary to measure mechanical vibrations as well. For this purpose an electrical phonograph pickup specially adapted for the purpose, together with an amplifier and osiso, has been used. Such apparatus has been developed by Mr. J. W. Legg.[4] An apparatus of this kind can be readily calibrated for amplitude. If a two-element osiso is used, a study of vibration phase angles as well as frequency and amplitude may be made. This frequently makes it possible to trace the noise further to its source by finding the particular surface from which the disturbance is radiated. The mode of vibration of the surface can also be determined and then it is possible in a large number of cases to arrive at the direct cause as well as to prescribe a remedy.

APPLICATIONS

We shall now proceed to describe certain typical applications for the analyzers which have been made.

I. *Gears.* One of the first applications made of the noise analyzer was for the purpose of studying electric railway motor gear noises. These noises are of three general types:

a. Ringing noises which are the same as those which appear when a gear is struck with a hammer.

b. Tooth frequency noises.

c. Hobbing noises which are a result of imperfections in the gear cutting machine.

The ringing noise occurs only at no-load. This can readily be noted, frequently, when the power is shut off and a trolley car is running down hill. The frequency is usually of the order of from 1000 to 2000 cycles per second. If the gear is struck with a hammer when stationary its ringing frequency may be determined by means of a noise analyzer. The frequency of pinions is usually so high as to be above the audio range.

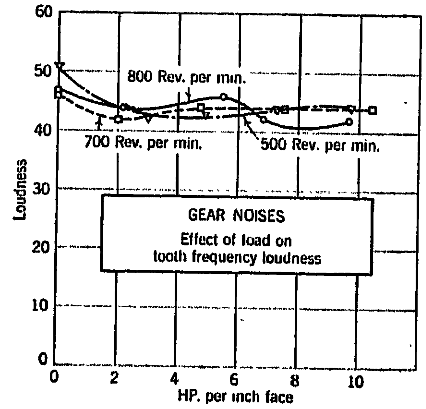

Fig. 6—Gear Noise Tests

This ringing noise can easily be eliminated by a method devised by Mr. R. E. Peterson which will be described in a paper soon to be published by him.

When the gears are running there is given forth a noise having a frequency which is proportional to the gear speed and number of teeth. The pitch is usually much lower than the ringing frequency.

If the master gear on the gear cutting machine has imperfections in the teeth, which sometimes occurs, the gears which are cut by this machine will have a frequency when running corresponding to the rev. per min. and number of teeth on the gear cutting machine. This is known as the hobbing noise.

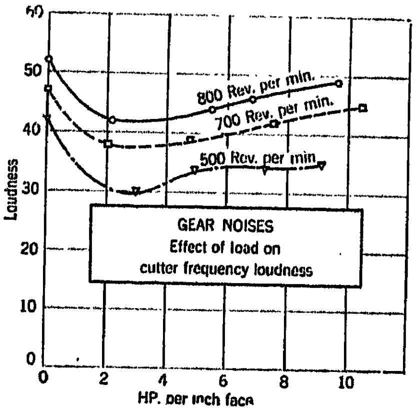

Fig. 7—Gear Noise Tests

If a set of gears is put in operation and the various frequencies are determined by the analyzer, it is very easy to determine the gear, or gears, responsible for the noises because the ringing and tooth frequencies are already known or can easily be determined. Any other frequency must be due to imperfections in the cutting of the teeth. Figs. 6, 7, 8, and 9 show some typical gear noise tests made on commercial somewhat imperfect gears as a function of load and frequency.

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[4] J. W. Legg, *Expansion of Oscillography by the Portable Instrument*, A. I. E. E. Jour., Vol. 42, pp. 106-11, February, 1923.

J. W. Legg, "Oscillography," *Elec. Jour.*, Vol. XXIV, No. 6, pp. 268-72; No. 7, pp. 341-46; No. 8 pp., 398-405; June-August, 1927.

Electric Motors and Generators. There are several causes of noise from rotating electrical machines as follows:

a. Tooth frequency noises (stator and rotor).
b. Windage (fans).
c. Natural period of frames.
d. For single-phase motors, variations in torque and forced frame vibrations.

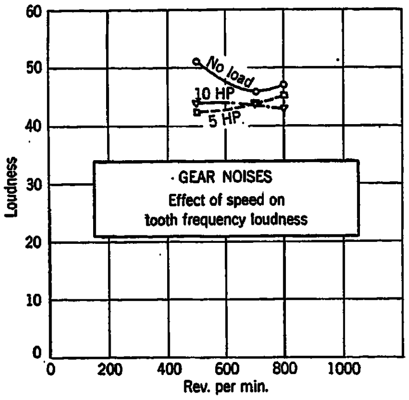

Fig. 8—Gear Noise Tests

In induction motors having rotor slots there is nearly always an appreciable amount of vibration which is proportional to the number of stator and rotor teeth and the rotor speed. These noises are the result of reluctance pulsations which produce vibrations of the teeth and sometimes the frame. Often these noises are very considerably increased as the load becomes larger. An interesting case of magnetic noise recently came to our attention in connection with a salient pole synchronous machine. This machine produced a very considerable noise having a frequency corresponding to the number of stator teeth and the rotor speed. An analysis showed that the pole width and stator slot pitch were such that at one instant practically three teeth were under the pole and an instant later when the pole had advanced one-half a stator slot pitch practically four teeth were under the pole. This gave rise to very appreciable reluctance pulsations between the rotor and stator. This condition probably would not have produced any appreciable noise if it had not been for one further unfortunate circumstance. The pole pitch was such that at a certain instant several adjacent poles were subjected to approximately minimum reluctance conditions and then an instant later to maximum reluctance. This resulted in a very considerable frame vibration having the frequency noted above.

When motors are supplied with ventilating fans it is quite usual to obtain appreciable noise which is a function of the motor speed and the number of blades on the fan. This is known as the siren frequency and often consists not only of the fundamental but also of harmonics. Other portions of the rotor also may produce siren frequencies.

It occasionally happens that a machine may be operated at such a speed that some frequency component may be very close to the natural period of the frame or some other portion of apparatus. In this case noises may be produced which are considerably in excess of those which occur at some slightly different speed. A slight change in speed will reveal such a condition.

Single-phase, 60-cycle motors almost always have appreciable 120-cycle noises. These are due to torque variations or to frame vibrations. Also not infrequently harmonics of 120-cycles may appear.

In studying noises from small motors where the sounds are of no great magnitude and the microphone is placed close to the motor, care must be taken to shield the microphone from stray fields or to keep it sufficiently far removed from the apparatus under test. Otherwise, odd harmonics of the voltage will appear in the analyzer which are not caused by the sound.

Induction Regulators. For 60-cycle regulators sound analyses so far as we have observed will always show that the frequency consists mainly of a 120-cycle component. In the past it has usually been thought that when a regulator was particularly noisy the causes were dissymmetry in the magnetic fields and loose bearings. Recently some three-phase regulators were studied to determine the source of the noise by means of the vibration apparatus mentioned above. The phonograph pick-up and amplifier were connected to one element of an osiso, a second element being operated from the exciting current of one of the phases in order to supply a phase reference point. Vibration measurements on the rotor and various parts of the stator disclosed the fact that the rotor movements were much less than the stator vibrations and there were no appreciable torque vibrations. It was disclosed that the stator punchings and frame assumed an elliptical form, the ellipse revolving with the rotating field.

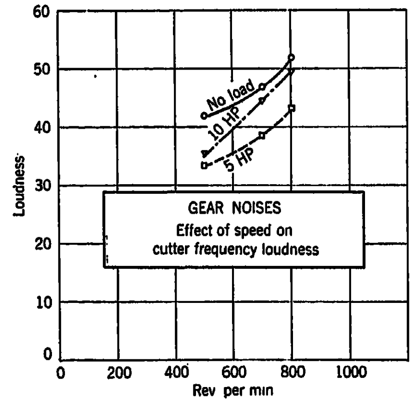

Fig. 9—Gear Noise Tests

Obviously then the method of minimizing the noise from this particular type of regulator was to stiffen up the stator punchings and frame.

Vacuum Sweepers. These household devices operate at a very high speed, usually of the order of 10,000 rev. per min. The various principal causes of noise are as follows:

a. Sweeper fan noises.
b. Motor and ventilating fan noises.
c. Unbalance.

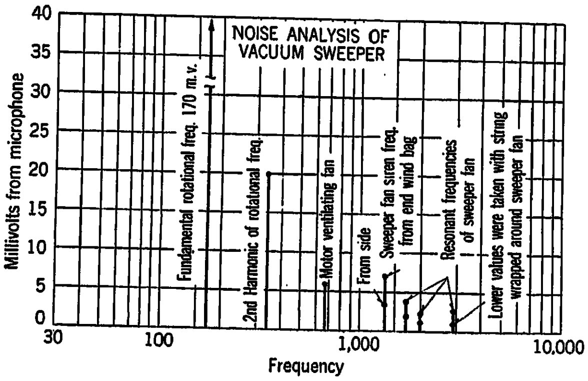

FIG. 10—NOISE ANALYSIS OF VACUUM SWEEPER

Figs. 10 and 11 show noise analysis results on a well-known vacuum sweeper. Fig. 10 gives the noise components as a function of microphone millivolts which are proportional to sound pressures and Fig. 11 gives the same data as a function of loudness units. (See Fig. 2.) It will be seen that in terms of loudness units the higher frequency components are much more important than would be indicated from the sound pressure spectrum, due to the greater sensitivity of the ear at the higher frequencies. It was found by striking the fan, thus causing it to vibrate and measuring the frequencies with the noise analyzer, that the sweeper fan had three natural frequencies corresponding to three different modes of vibration. It will be seen from the spectrum that under operating conditions all of these frequencies appeared to be giving comparatively large noise components. Of course the siren frequency of the sweeper fan was a rather large percentage of the total noise. The motor ventilating fan gave a surprisingly loud noise considering the fact that it was much smaller than the main sweeper fan. The particular sweeper under test was not very accurately balanced; therefore, a very large frequency component of approximately 170 cycles resulted. This corresponds to approximately 10,000 rev. per min. Also there appeared a very appreciable second harmonic of the rotational frequency.

Obviously if an attempt were made to alter this sweeper so that it would be quiet it would be of very little use to reduce any one component to a small value because the total noise could thereby be decreased only slightly. The production of a really quiet vacuum sweeper is a problem of no little difficulty.

Loud Speakers. Since its development in connection with noise problems, the sound analyzer has found other applications. As an example, it may be used for obtaining response curves for loudspeakers. While other methods now in use may have the advantage of greater speed, the use of the analyzer eliminates the necessity of a pure voltage wave for the speaker supply. This is true since the analyzer can measure the pressure of the fundamental of the sound wave produced by the speaker and reject any harmonics whether they are due to harmonics in the supply or are generated in the speaker.

Conclusions. We might enumerate other examples of noise analyses but those given are sufficient for illustration. The quantitative determination of noise is very difficult even with an analyzer due to nodes and sound reflections. Noise often varies very greatly in different locations around a piece of apparatus and it varies with the position of the objects in the room. For the accurate study of noise, the apparatus should be taken to a sound proof room with deadened walls so that there will be little reflection. However, in a large number of instances where only one or two frequency components are important qualitative results are quite sufficient. In other cases when tests must be run under shop conditions only qualitative results can be obtained due to a large amount of reflection as well as interference from the shop noises. In these cases results can be obtained very quickly by means of the analyzer by using head phones in place of the meter ordinarily used to measure amplitude. Then even though the surrounding noise level is very high it is possible to obtain accurate frequency determinations of the noise being studied. Usually if the frequencies of the principal components can be determined the cause or causes of the noise can be assigned and means taken to eliminate or reduce it. Therefore, a qualitative test is very often sufficient to enable the designer to avoid similar conditions in future apparatus.

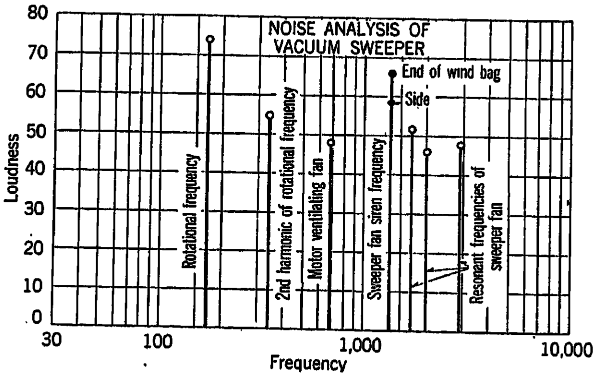

FIG. 11—NOISE ANALYSIS OF VACUUM SWEEPER

We are greatly indebted to Mr. R. E. Peterson who has given considerable assistance and many valuable suggestions in the work on gear noise. We also wish to acknowledge the assistance rendered by Mr. C. F. Royce in certain phases of this work.

Discussion

H. C. Specht: The method described in this paper is for measuring the frequency of noise; however, as manufacturers of electrical machines we are also interested in the amount of noise. To measure the degree of loudness is very difficult as there is no accepted unit for comparison and because of different responses of different ears. It is very difficult to agree on a definite maximum level of noise which may be considered unobjectionable. Also, the loudness depends on the kind of body which is in contact with the noise producer. If the natural frequency of the body is the same as that of the noise, there may result very great amplification.

Unfortunately, the test of noise in regular production is conducted by means of the human ear, which cannot be considered as very satisfactory, as the ears of different persons are not the same and even the sensitiveness of the same person at different times is not always alike. Another difficulty is to build inside a noisy factory building, a noise-proof room. From time to time the room must be opened, at which time the factory noise will enter and it will then take some time before the ears of the tester are again adjusted for testing. It would be far better to have the noise-proof room entirely separated from the factory building, but this, of course, would be undesirable on account of the cost of transportation.

J. P. Foltz: I believe the points mentioned by Mr. Specht were very well taken. We have been confronted from time to time with problems of commercial noise testing and it has been quite a problem to us inasmuch as it is rather complicated. We have to deal with unequal ear sensitivity as regards different frequencies as well as surrounding shop noises. In the case of induction regulators, however, we have used an entirely different principle. We measure the mechanical vibration of the outer surface. That eliminates entirely the interference of shop noises and inasmuch as in that case only one frequency exists, the variation of sensitivity of the ear also is eliminated. It was very easy to devise a commercial test for regulators in that way. We have since applied the same principle to gear cases with some success and we may be able to extend it to other apparatus. The fact is that in enclosed apparatus we do not get any noise unless we have a corresponding mechanical vibration to radiate that noise, so this principle may in most cases offer better possibilities as a commercial test than measurement of the actual noise produced.

Electrical Aids to Navigation*

BY ROBERT H. MARRIOTT†

Fellow, A. I. E. E.

Synopsis.—Electrical and magnetic aids to navigation by water and air are outlined in this paper. The principal subjects discussed are the earth inductor compass, the radio compass, the radio beacon, submarine signals, depth finders, distance finders, channel-marking signals, fog-penetrating lights, height indicators for air-craft, etc. A bibliography is included.

THIS paper points out that there are numerous ways in which electricity and magnetism aid air and water navigation, partially describes and illustrates some of the less familiar aids which are in use or are coming into use, and mentions some aids that may come into use.

Magnetism as an aid to navigation goes back to the more romantic probabilities, history, and tradition of the past. By tradition natural magnetic iron is said to have been known as far back as 2634 B. C., by history, as far back as 800 B. C. The now poetic name "Lode Stone" was the name of the natural permanent magnet found in Magnesia. The Lode Stone was the first magnetic compass used to aid navigation. The attracting power of the Lode Stone and the name "Magnesia" gave rise to such names as "magnet" and "magnetism." Speaking broadly, experiments with the Lode Stone resulted in the production of electric current, as the steel magnet was first made by contact with the Lode Stone and later used to produce electric current. In other words, the Lode Stone was one of the first instruments used as an aid to navigation and was an ancestor of dynamo-made electricity.

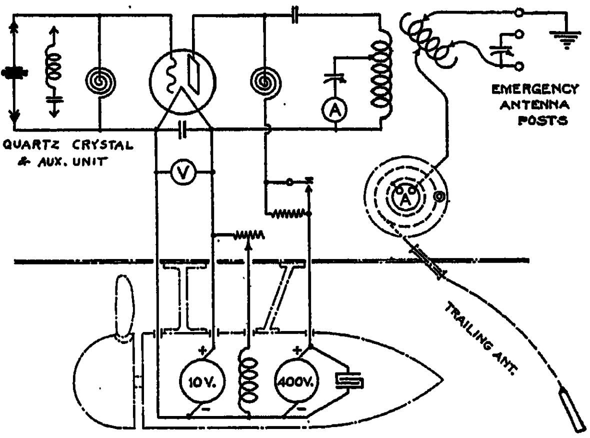

FIG. 1—DIAGRAM OF CRYSTAL-CONTROLLED AIRCRAFT RADIO TRANSMITTER

The word "navigation," judging from the Latin words from which it was made, originally related to the moving or propelling of a ship. Now we usually consider that word in connection with the steering of a ship. However, electricity aids under both meanings. Battleships are driven by a-c. motors and submarines are driven from storage battery power by d-c. motors. Electric winches are used to pull vessels into dock. Electrically driven gyroscopes are used to make ice breaking ships roll and to keep yachts from rolling. Electric ignition is used in the propulsion of both air and marine craft.

Electric navigation lights and docking or landing lights are used for both air and marine craft. Almost every form of electrical communication, signaling, and remote control device is used as an aid to navigation. Even loud speakers with audio amplifiers are used to carry the words of the officer on the bridge to men on shore when navigating a vessel into dry dock.

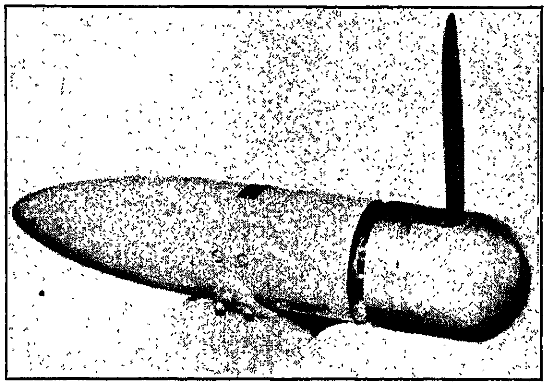

FIG. 2—WIND-DRIVEN GENERATOR WITH DESLAURIERS SELF REGULATING PROPELLER

The gyro-compass is a gyroscope but it depends on electricity for its motive power and for the operation of the control devices that go with it.

Practically all of the electrical comforts used on land contribute to the comforts in the navigation of the modern steamships.

The coming of air navigation is expanding the use of electrical devices for aiding that rapidly growing navigation. The success of air navigation apparently depends largely on the ability of air craft to dodge dangerous storms. To do this the air craft must be fast and the pilots must know where storms are, how violent they are, how much area they cover, how fast they are moving, and what direction they are following.

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*Part of Symposium on Electrical Measurement of Non-Electrical Quantities.

†Consulting Engineer, Brooklyn, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

More and more information regarding storms is being supplied for air craft by wire line and radio communication. The fundamental principle is to collect weather information from a number of widely distributed observers rapidly, translate all of the information into definite statements of storm conditions and deliver those statements to the pilots, promptly.

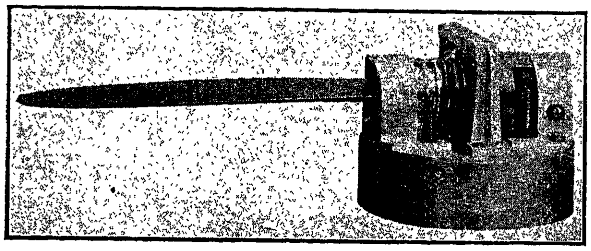


FIG. 3—MECHANISM OF SELF-REGULATING PROPELLER

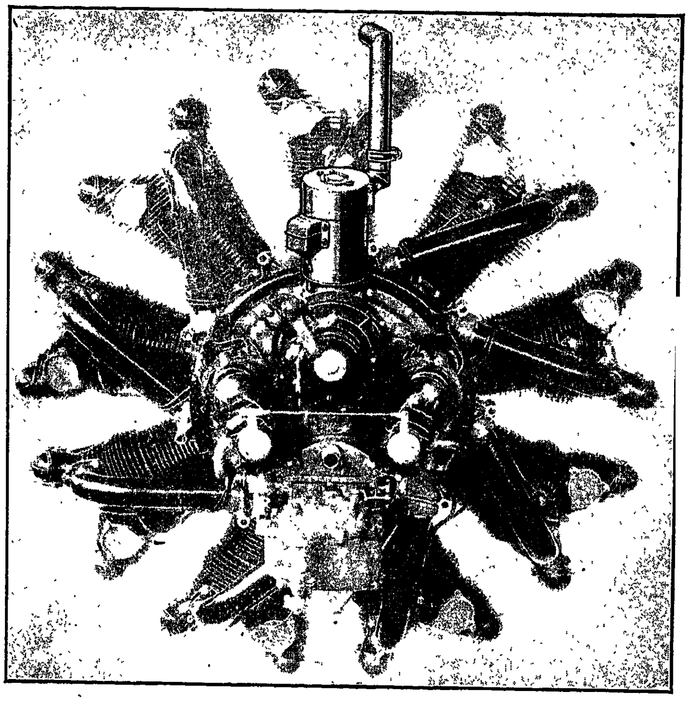


FIG. 4—GENERATOR GEARED TO AN AIR-CRAFT ENGINE

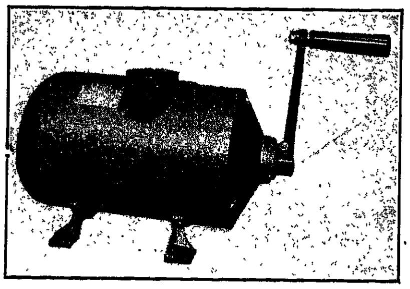


FIG. 5—HAND-DRIVEN GENERATOR FOR EMERGENCY AIRCRAFT RADIO USE

We have repeatedly seen that great feats have been carried out by airplanes and dirigible balloons when severe storms were not met and we have seen great catastrophies when those craft have met storms. The electrical means of communication by wire and by radio applied to the rapid supplying of weather information to the pilots evidently must be one of the greatest aids for successful air navigation.

The devices to be used for that purpose are the radio apparatus on ships, radio and wire apparatus on land, and to some extent the cables under the seas.

The life and property saving ability of the radio S. O. S. call in connection with water navigation forced the use of radio on practically all large ships and for some years the position of icebergs and other menaces to navigation have been regularly reported to ships by the same radio communication that is used for other purposes.

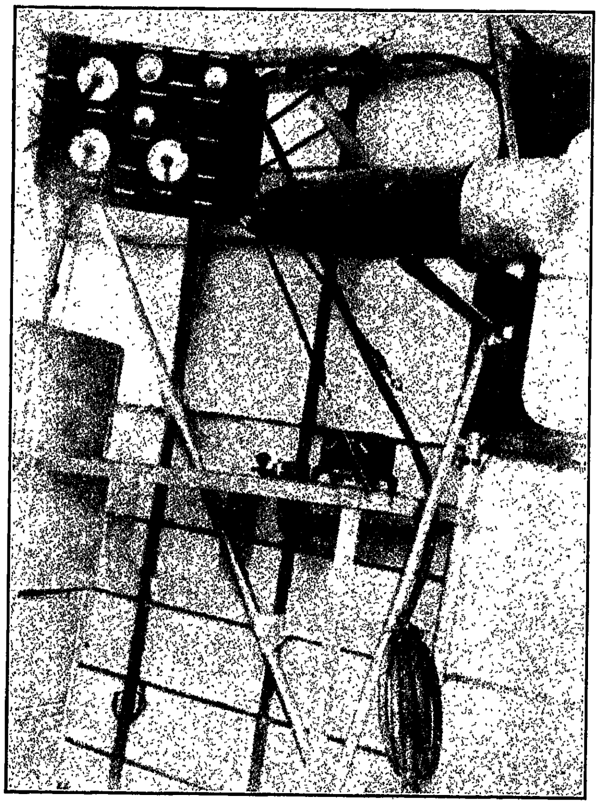


FIG. 6—RADIO CABIN OF AIRPLANE *Roma*. RETRACTABLE WIND-DRIVEN GENERATOR SWUNG IN

The foregoing electrical aids to navigation need no description. The main point about them is that they are numerous examples of the electrical engineer's contributions to navigation. There are some special electrical aids that will bear some description, such as for example, the earth inductor compass, the radio compass, and the depth finder.

There is some relationship between the old magnetic compass and some of our more modern aids. The magnetic compass, the earth inductor compass, the radio compass, and the compass used to follow an energized cable in a channel, are all magnetic flux indicators. The first two indicate the south-to-north

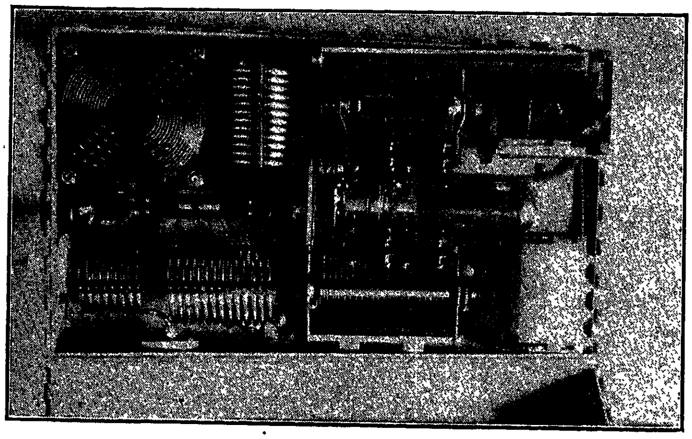


Fig. 7—Interior of an Airplane Transmitter

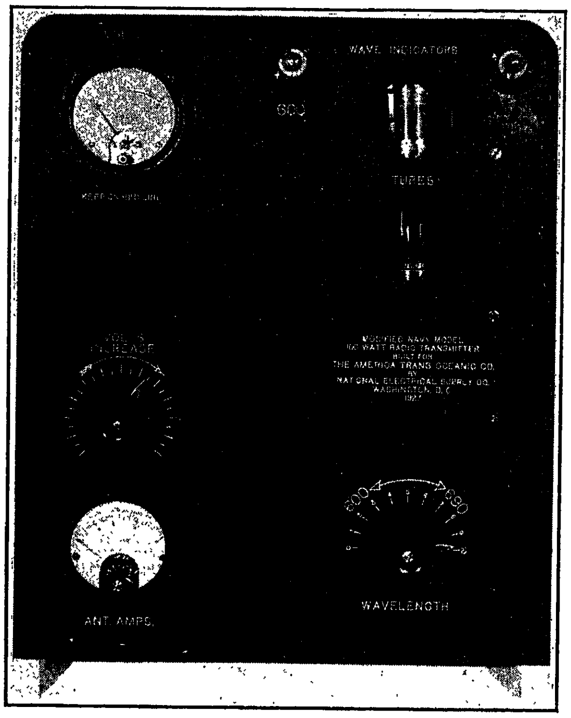


Fig. 8—150-Watt Radio Transmitter of Airplane *America*

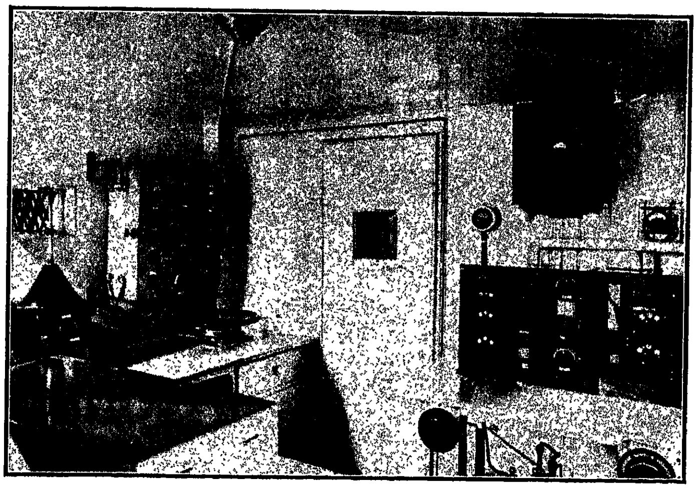


Fig. 9—Radio Room of the *Graf Zepplin*, 1928

Including a 70- and a 140-watt telephone and telegraph transmitter, a short wave transmitter, radio compass, and four receivers

pole direction of the flux lines of the earth. The other two indicate the direction of the side movement of lines of flux set up at some center such as an antenna or other conductor of alternating current and thereby are used to locate that center. The first two indicate a steady field while the second two indicate an alternating field of flux that is accompanied by an alternating electric field.

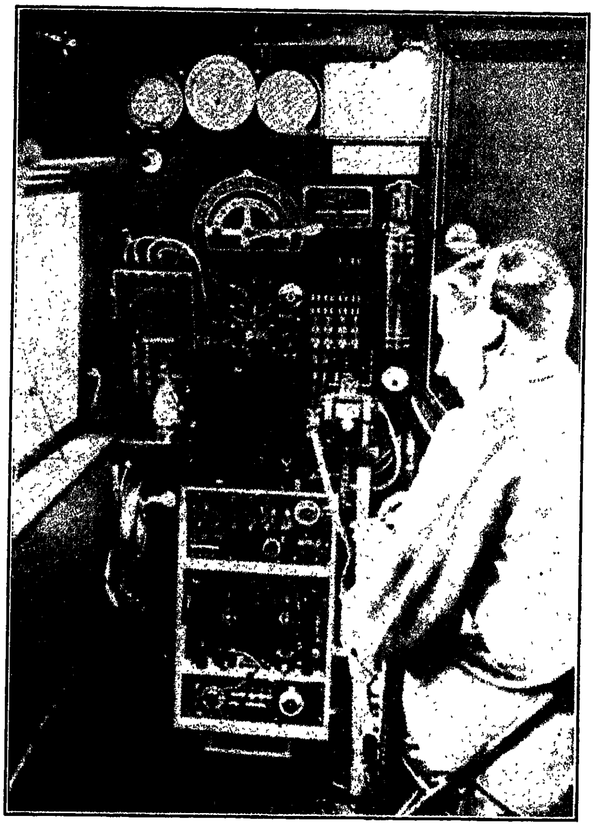
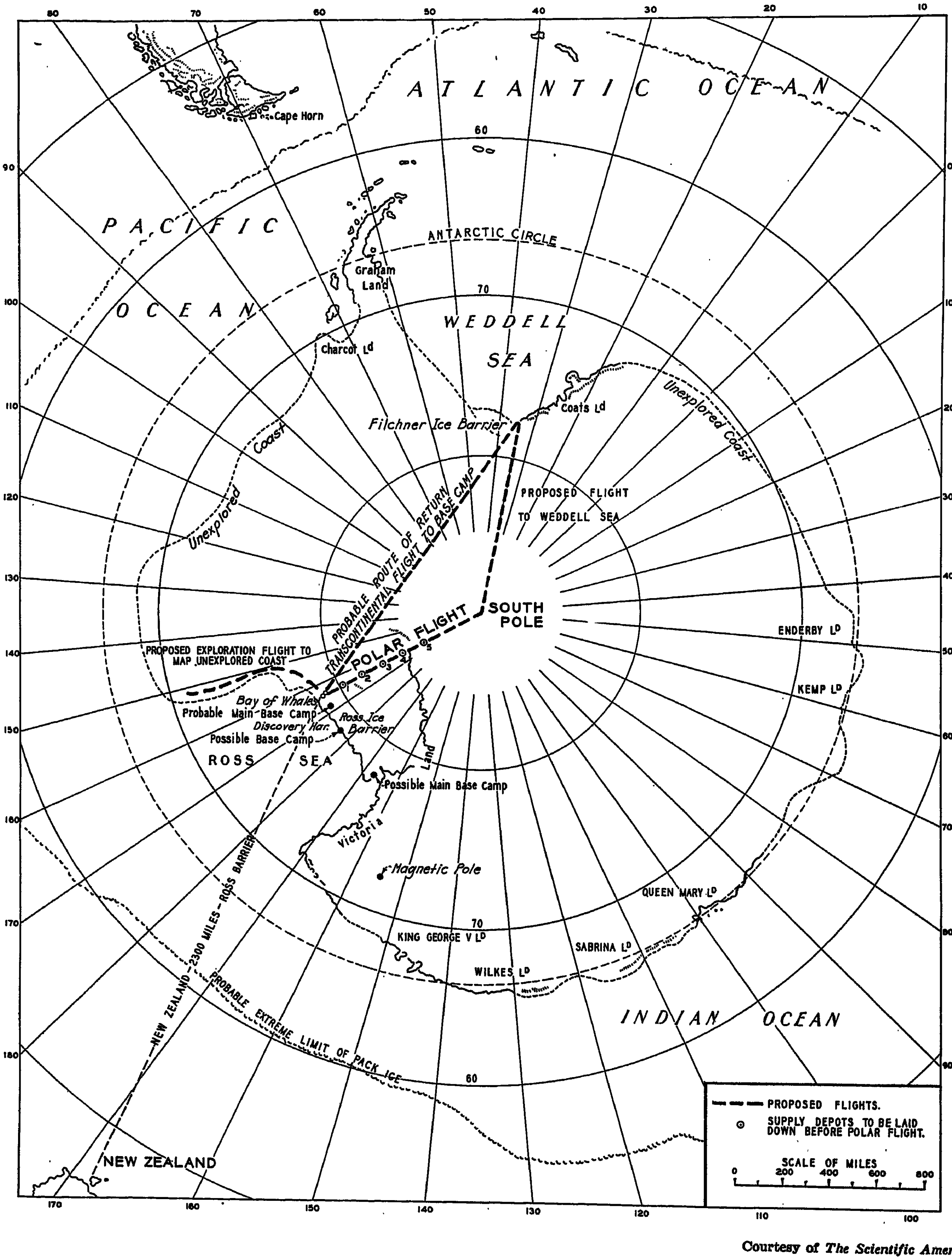


Fig. 10—Radio Installation in a British Airline

Fig. 11—Waterproof Self-Contained Emergency Spark Coil Radio Transmitter

The magnetic compass is a polarized iron armature that is attracted into a position which is parallel to



Courtesy of *The Scientific American*

FIG. 12

The Antartic regions showing main bases and routes of exploratory marches and flights of the Byrd expedition. Here, radio and other electrical aids are expected to be of great service in navigating ice, air, and water.

the earth's flux. The earth inductor compass is a dynamo, its poles are the north and south poles of the earth, and its rotating armature consists of loops of wire connected to commutator segments.

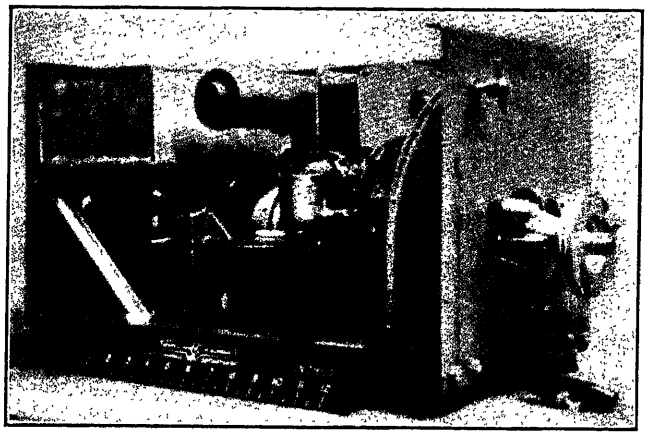


Fig. 13—Generator Unit Driven by a Gasoline Engine, for the Advance-Parties of the Byrd Antartic Expedition

This expedition includes in its equipment 33 radio transmitters and receivers for use in navigating the air, water, and ice. Two radio compasses will be used at base locations suitable for triangulation to locate all planes and ice navigating parties. Dry batteries and gasoline engine-driven generators are provided for locations where ship power is not available

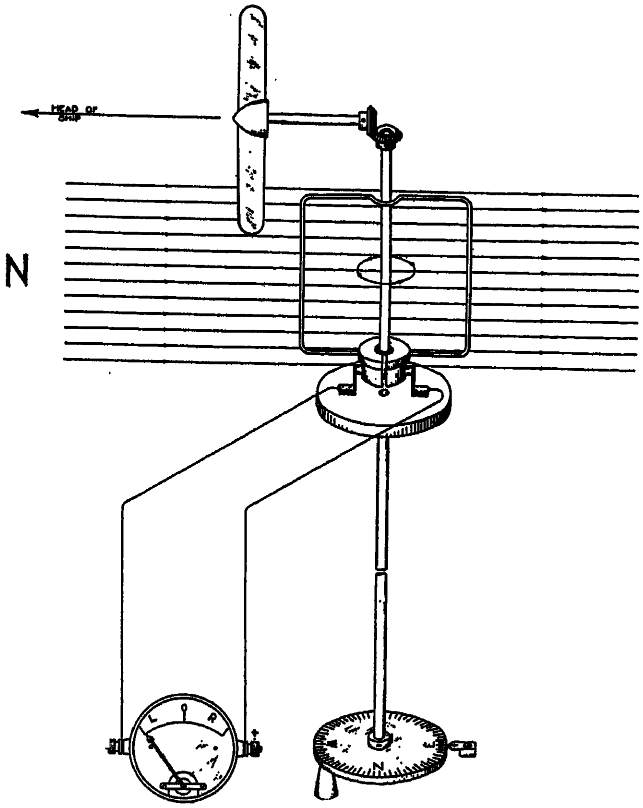


Fig. 14—Schematic Representation of the Earth Inductor Compass

The other two compasses may be said to be of the stationary armature type that are cut by alternating magnetic fields. They consist fundamentally of single loops that may be revolved to positions for cutting from a maximum to a minimum phase difference in the alternating fluxes. When the planes of these loops are at right angles to the direction of the source of the alternating magnetic flux the minimum current is set up, and that indicates the direction of the source.

Earth Inductor Compass

The earth inductor compass is the device that is most closely related to the old magnetic compass. In an airplane the compass must be close to the pilot or navigator where he can see it. But there may be iron nearby which makes the magnetic compass inaccurate. The revolving coil of the earth inductor compass can be located at some distance from interfering iron and the current from it can be conducted to a galvanometer located where the pilot or navigator can see it.

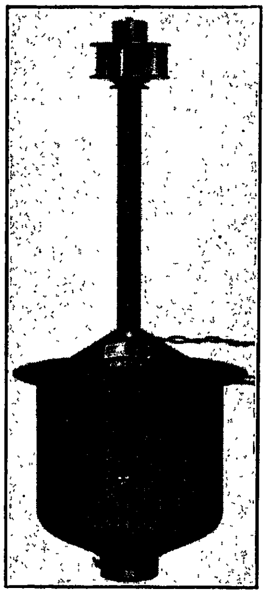


Fig. 15—Earth Inductor Compass Wind Mill and Generator

The earth inductor compass is a d-c. dynamo in which the magnetic field is supplied by the earth. It will be seen by reference to Fig. 14 that a windmill at the top revolves a one-turn armature winding connected to a commutator, to cut the earth's north-to-south field, and that the brushes connect to a meter. Also it will be noted that the position of the brushes can be shifted. The meter is a d-c. galvanometer; therefore if the brushes are set in one position a maximum deflection will be obtained when the windmill is turning the armature rapidly. With the brushes moved to a position at right angles no deflection will result. By revolving the brushes an additional ninety degrees a reverse deflection will be obtained. Since the field is fixed in space, turning the whole instrument produces the same result as shifting the brushes the same amount.

If the device is mounted on an airplane and the plane is headed in the direction it is to follow and the brush controller is turned to produce no deflection on the galvanometer, there will be no deflection so long as the plane continues in that course. If the airplane turns off course to the left the galvanometer pointer will move to the left and vice versa if the plane heads to the right. Should the airplane be turned completely around, the galvanometer pointer will move to the right when the plane moves to the left. The device is used in the simplest way by starting the plane in the direction it is to go, turning the brush controller to put the galvanometer pointer on zero, and then steering the plane so the pointer will remain on zero.

Fig. 15 shows the horizontal type of self adjusting windmill at the top and the case below which contains the revolving armature, in gimbals that permit the revolving armature to remain vertical by its gyroscopic action when the plane heads up or down. Fig. 16 shows the brush controller and Fig. 17 shows the galvanometer.

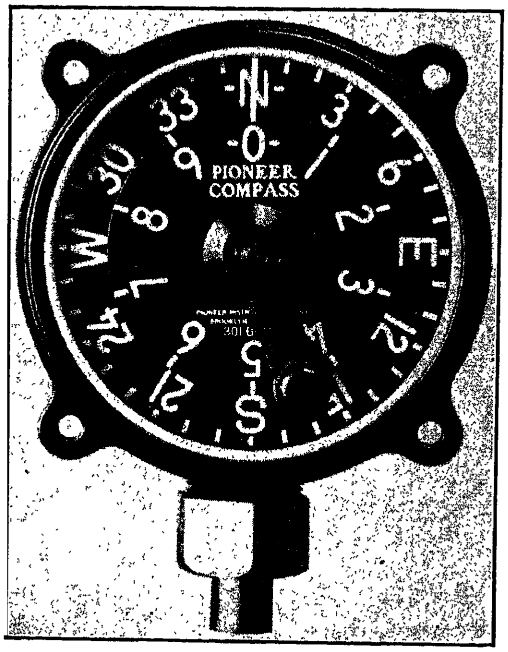


FIG. 16—CONTROLLER OF THE EARTH INDUCTOR COMPASS

While the principle is simple, fine workmanship is required to produce an accurate and strong instrument. For example, flexible shafts are used and the copper commutator with its silver brushes revolves in filtered oil with a wiper on the commutator to remove tiny particles of metal. The armature winding includes 24 sections of many turns each. The armature develops about 1.5 millivolts at 1800 rev. per min.

RADIO COMPASS

A radio compass, direction finder, or goniometer is a radio receiving device which permits determination of the line of travel of waves as received from a transmitting station.

According to the usual explanation, radio waves are composed of two parts,—an electric part and a magnetic part. The electric part moves up and down from the earth and may be imagined to be like the ripples in water that spread out when a stone is dropped into the water. That is, the electric portion of the electromagnetic waves move out about the same way from the transmitting antenna. The magnetic part also moves out from the transmitting antenna not as an up and down movement but as expanding rings of

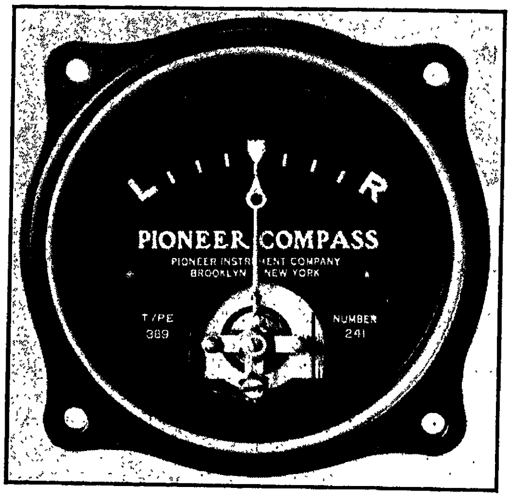


FIG. 17—GALVANOMETER OR INDICATOR OF THE EARTH INDUCTOR COMPASS

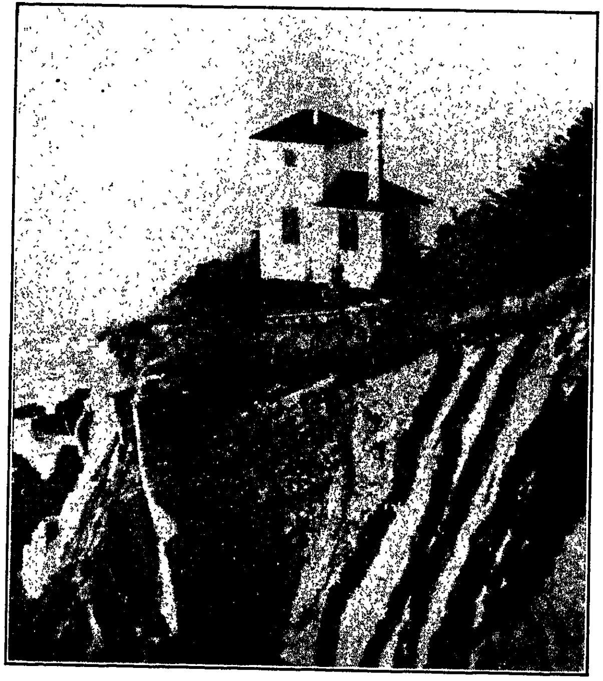


FIG. 18—U. S. NAVAL RADIO COMPASS AT CAPE HINCHINBROOK, ALASKA

The second story contains the coil. The platform with railing, above the coil, is for the calibrator and his transit while following the course of the calibrating ship. An underground control cable leads to the transmitter which is located at a distance sufficient to avoid affecting the accuracy of the compass magnetic flux which are revolving successively in opposite directions parallel to the earth's surface about the transmitting antenna. The change from a maximum in one direction to a maximum in the other occurs in a sinusoidal manner; therefore if two conductors vertical to the earth's surface are cut by these flux lines and one of the conductors is one-quarter of a wavelength farther from the transmitter than the other, then the currents set up in them will be 90 deg. out of phase and if those conductors are connected together at the top and bottom, forming a loop, these currents will not neutralize each other completely but will leave some current due to the phase difference. If now the loop is revolved so that both of the vertical conductors of the loop are the same distance from the station there will be no phase difference and there will be no current in the loop. Therefore under ideal conditions when a loop antenna is turned broadside to the radio station no signal is heard and that is the fundamental feature of a radio compass. If the radio waves arriving at that point are always tilted forward or backward the broadside position will not show a zero but a minimum which is usually satisfactory. Also if there are certain fixed conductors in certain directions, then the flux lines arriving from those directions will not be exactly at right angles to the direction of the transmitter and the scale of the compass must be corrected accordingly.

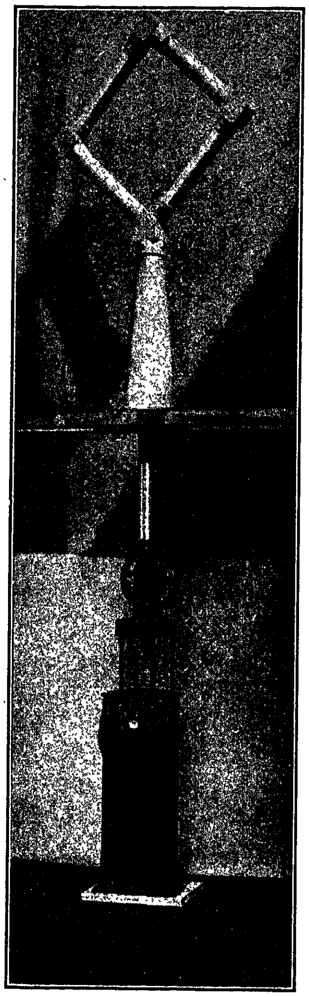


Fig. 19—Steamship Type of Radio Compass

The radio compass must be carefully built, properly located, calibrated under service conditions, and corrected to suit the peculiarities of its location. When these things are done a good degree and sometimes a remarkably fine degree of accuracy is obtainable. Otherwise the radio compass is much like the loop type of broadcast receiver with which many are familiar and with which good or poor directional effects may have been obtained depending on the receiver and the location.

The loop antenna or radio compass is not new. It is sometimes called the Hertz loop. But as the energy in it depends on phase difference, the energy is small compared to the energy in an open antenna. It was the amplifying ability of the vacuum tube that brought the loop back into use long after the days of Heinrich Hertz.

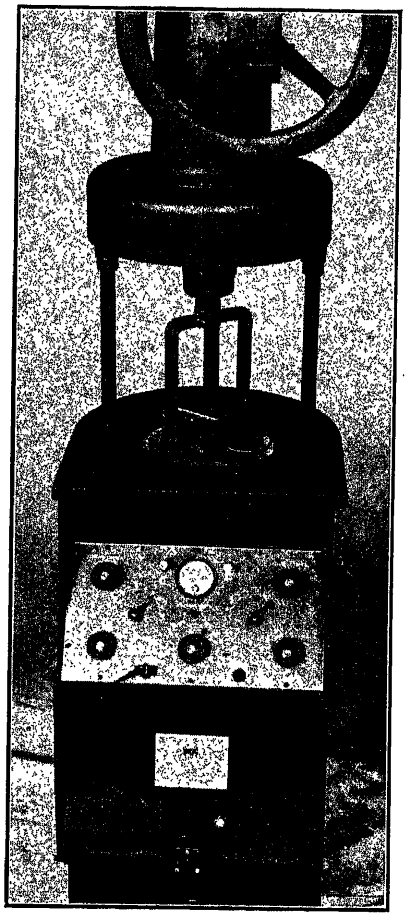


Fig. 20—The Radio Receiver, Indicators, and Hand Wheel of the Steamship Type of Radio Compass

In the United States the Navy Department maintains radio compass stations at suitable points along the coast lines, Fig. 18. All navigators have the exact location of these compass stations on their charts. When a ship wants a bearing during thick weather it calls for a navy compass station. A compass station replies and asks the ship to transmit for a sufficient interval for the compass station to get the ship's bearing relative to the compass station. This information is then transmitted to the ship. If two compass stations can get the ship's bearing at the same time, the exact location of the ship can be calculated from the triangle. Otherwise the ship moves by magnetic or gyroscopic compass for a certain distance and then gets another bearing from the one compass station, thus completing the triangle.

These compass stations provide free aid to navigation in peace time and provide the Navy with stations and trained personnel for use in war time. In Europe similar compass stations are located along the lines of air travel. They keep following the craft in the air and tell them by radio telephone where they are.

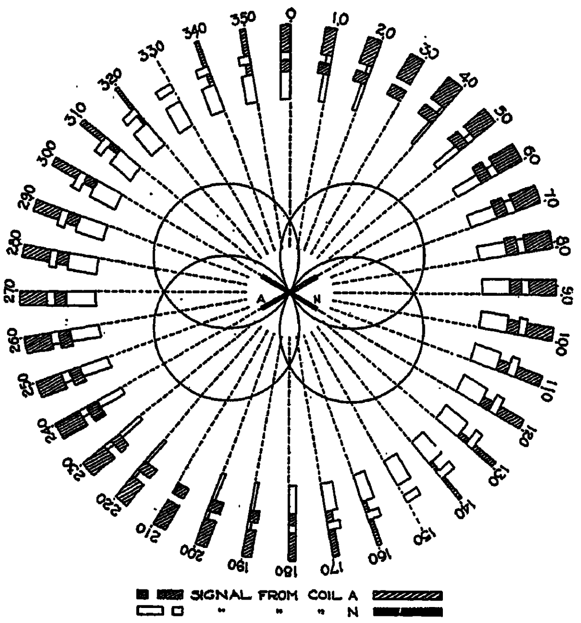


FIG. 21—TWO-LOOP ANTENNA TRANSMITTING THE SIGNALS *A* AND *N*

If the aircraft approaches the transmitter along the lines 0, 90, 180, or 270 equal volume *A* and *B* signals will be received. On either side of those lines one will be louder than the other

Fig. 19 shows a type of radio compass that is used on steamships. The hollow square at the top is a tube containing the turns of the compass loop. That square portion is above the top of the pilot or chart house and is sturdy to withstand weather. The loop is turned by the hand wheel shown below. Fig. 20 shows more plainly the hand wheel, compass scale, and radio receiver. Similar radio compasses of lighter construction are used on large aircraft.

RADIO BEACON

A radio beacon is a radio transmitting station in a fixed geographic location which emits a distinctive or characteristic signal for enabling mobile receiving stations to determine bearings.

Such a beacon may transmit in all directions, or chiefly in one, two, or four directions or it may be directive and revolve, giving different indicating signals for different directions. When the beacon transmits equally in all directions a radio compass is necessary for the receiver to determine the direction of the beacon from the receiver. The direction of a revolving or fixed directive beacon may be determined by the receiver without the use of the radio compass. With a revolving directive beacon the ship's radio operator can tell when he is at a certain angle from the charted beacon he is receiving from because at that angle the signals come to him the loudest and the character of the signals or revolution timing from north or east signal tell him what the angle is. With the fixed directive beacon the receiver gets the maximum signal or characteristic signal when following the path along which the beacon sends the maximum signal or some characteristic signal.

For aviation, low power marker beacons which radiate in all directions but give a characteristic signal are located along air routes. If the flyer sticks to his course he will hear them one after another as he flies near them. Directly over them he will not get a signal from a vertical antenna. The combination of marker beacons, directive beacons, and weather reports by radio telephone can be of great assistance to the flyer, who has no space for long range receiving apparatus and a radio compass.

Fig. 21 represents a directive beacon that sends two signals A and N in Morse code at different angles. When the flyer is on his course he will hear A and N equally well which will make them sound like a dash to him. If he gets off to one side of his course the dot-dash of the A will be noticeable, if he gets off to the other side the dash-dot of the N will be noticeable.

Fig. 22 shows a circuit diagram of one of the types of beacons used. The triangular shaped directive loop antenna are shown at the right. It will be seen that this apparatus depends largely on vacuum tubes for use as radio and audio frequency generators and amplifiers. Such use of vacuum tubes is characteristic of present day direction finding apparatus. It is the vacuum tube that has made possible many of our aids to navigation.

Fig. 23 shows the general appearance of an air-beacon station. Fig. 24 shows the small receiver in shock absorbers, that is used on small aircraft for receiving radio beacon signals and radio telephone messages.

Fig. 25 shows a visual device that may be mounted on the board in the front of the pilot. It operates something like a reed type frequency meter. A beacon station sending two signals at two angles and modulated to the frequencies of the reeds can cause one reed to vibrate farthest when the aircraft is off the course on one side, the other to vibrate farthest when off the other side, and both to vibrate the same amount when he is on the course.

SUBMARINE SIGNAL

The electrical submarine signal transmitter is a large tuned steel diaphram driven by alternating current. The submarine signal receiver is, in its simplest form, a small diaphram and microphone button, like a telephone transmitter, connected through a battery to a telephone receiver.

The submarine transmitter is, for example, suspended below a light-ship and actuated by groups of short and long signals that are characteristic of that light-ship. All vessels equipped with submarine receivers can pick up those characteristic signals through the water when the light-ship cannot be seen through fog or when a sound could not be heard over the same distance through the air.

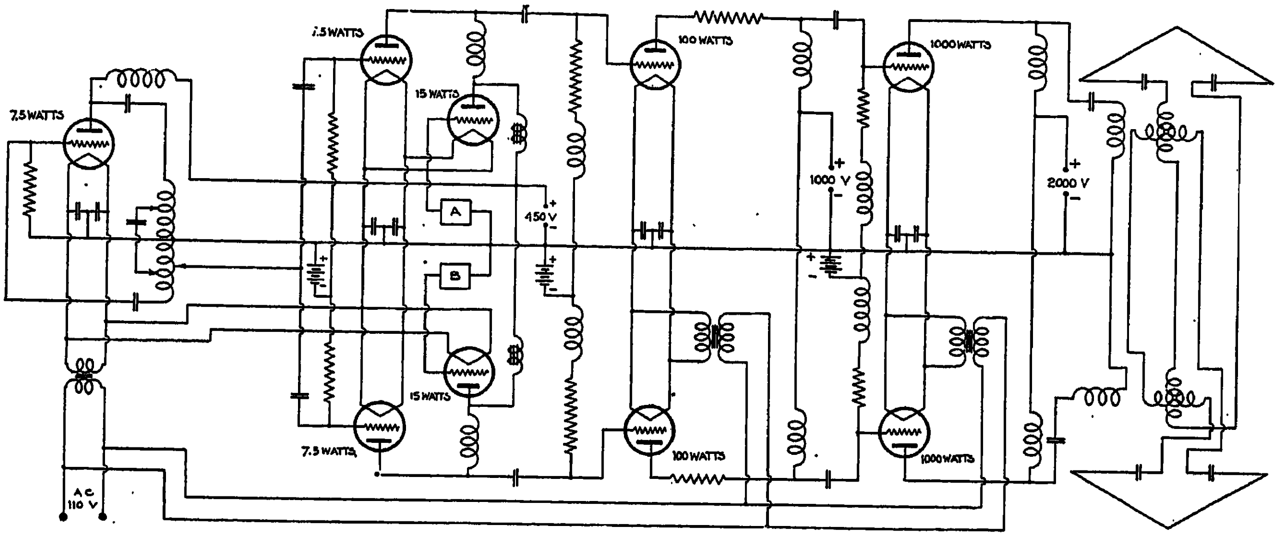


FIG. 22—CIRCUIT DIAGRAM OF MODULATION TYPE OF BEACON

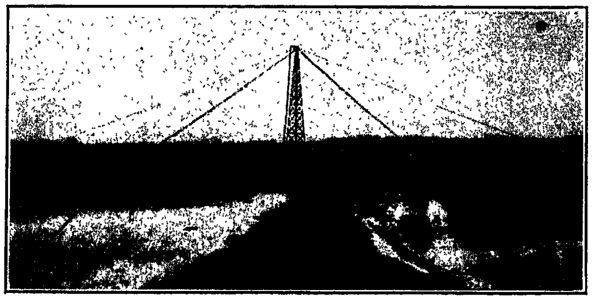


FIG. 23—BUREAU OF STANDARDS DIRECTIVE RADIO BEACON AT COLLEGE PARK, MARYLAND

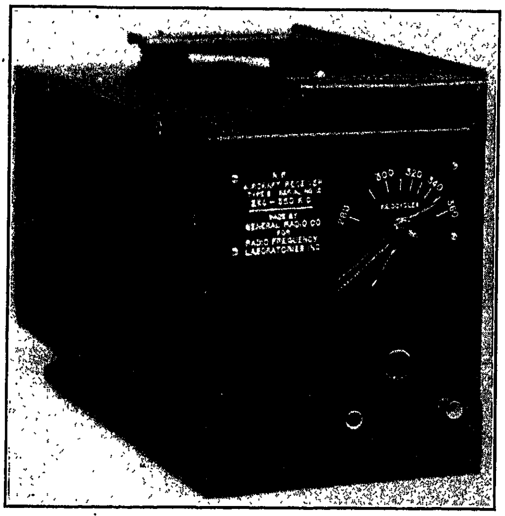


FIG. 24—AIRPLANE RECEIVER FOR RADIO BEACON AND TELEPHONE

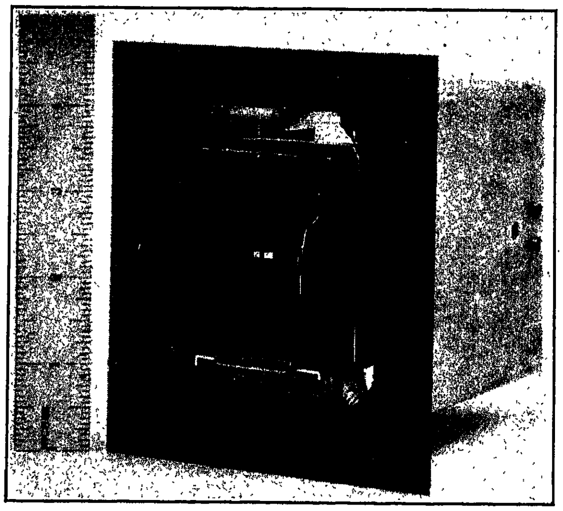


FIG. 25—REED TYPE VISUAL INDICATOR OF THE COURSE

Such signals have been heard in this way as far as 80 miles. There are numerous submarine signal transmitters of the pneumatic bell and electrical types and probably over two hundred steamship lines have equipped their vessels with submarine signal receivers.

DEPTH FINDING

Echo depth sounding, the sonic depth finder, and the fathometer all refer to the same principle, namely, sending a submarine signal from a vessel and receiving on the same vessel the sound or echo that is reflected from the bottom.

The submarine transmitter is attached to the vessel's hull and consists of a large tuned steel diaphram driven by alternating current. The signal sent is of short duration to prevent its interference with the reflected sound or echo.

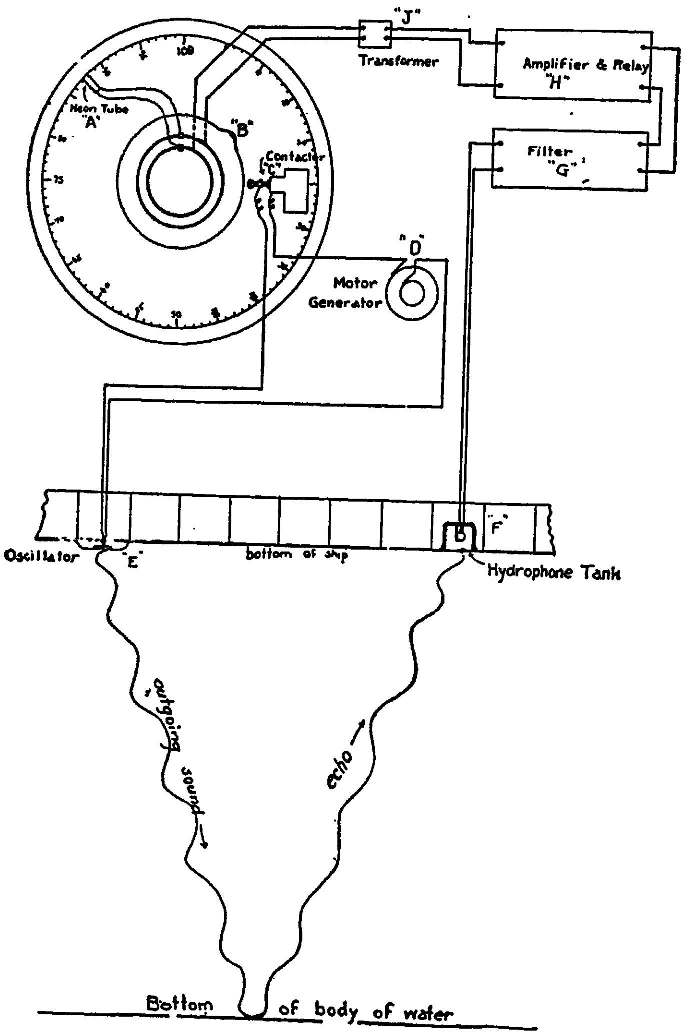


FIG. 26—SCHEMATIC REPRESENTATION OF THE MECHANICS OF DEPTH FINDING

The receiver sometimes called a hydrophone is a telephone transmitter immersed in a small enclosed metal tank of fresh water on the ship's hull at some distance from the transmitter, Fig. 26. The telephone transmitter is connected through its source of direct current and a filter for reducing other sounds, to a vacuum tube amplifier which operates a relay. The relay through an electrically energized transformer causes a neon Geisler tube *A* to glow when an echo is received. The motor driven Geisler tube revolves like a pointer back of a scale that is transparent at its edge. The scale is calibrated in fathoms. The Geisler or neon tube pointer makes a revolution in one-quarter of a second and every time it passes zero the transmitter is operated by *B*. Sound travels about 4800 feet or 800 fathoms per second; therefore the neon pointer will travel for one-eighth of a second or half way around the scale while the transmitter sound is traveling 100 fathoms or 50 fathoms to the bottom and 50 fathoms back as an echo and the neon tube will glow as it passes the 50-fathom mark on the scale. As the ship passes over the hills and holes in the bottom the illuminated spot will move forward and backward, giving a continuous reading of the depth as it is at the moment. By shifting to other connections, greater depths may be measured. A dial arrangement is shown in Fig. 27, and an example of the profile of sea bottom that may be obtained in this way in Fig. 28.

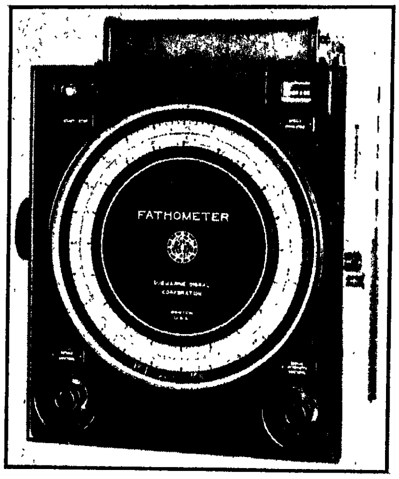


FIG. 27—THE DEPTH-INDICATING DIAL

COMBINED SUBMARINE SOUND BEACON AND RADIO BEACON

In this, the submarine signal transmitters and receivers as described above are used in cooperation with a radio beacon transmitter and receiver. For example, one key on a light-ship is used to operate both a submarine transmitter and a radio transmitter at the same time, sending a short signal. The receiving ship's officer listens with head phones that are connected to both a submarine receiver and a radio receiver. He hears the radio signal almost instantly and presses a button that starts a clock. Later he hears the submarine signal and presses the button again, stopping the clock at a figure which corresponds to the number of nautical miles between his ship and the light-ship.

POSSIBLE AND PROBABLE ELECTRICAL AIDS TO NAVIGATION

There are several electrical schemes which may prove to be valuable for use as aids to navigation,—

schemes that range all the way from those which have been tried and seem to be practical to schemes that may be only visionary.

For example, a device that will enable vessels to navigate a narrow, crooked channel, in fog, has been demonstrated on large transatlantic liners and naval craft by the U. S. Navy Department in New York Harbor. By the device a blindfolded helmsman could follow a channel. The device is comparatively simple. The transmitter includes a single conductor cable laid in the bottom of the channel, through which alternating current of say one thousand cycles is conducted to its outer end which is grounded. The receiver on a steel vessel includes two flat coils of wire hung on the port and starboard sides of the ship, parallel to the ship's sides and connected to two head phones or other indicators. When the keel of the ship is steered directly over the cable the responses from the two coils will be equal. When the ship gets off on the starboard side the greatest response will come from the port-side coil and vice versa, until the ship is brought back to where it should be. A single coil in gimbals can be used on wooden ships, Fig. 29.

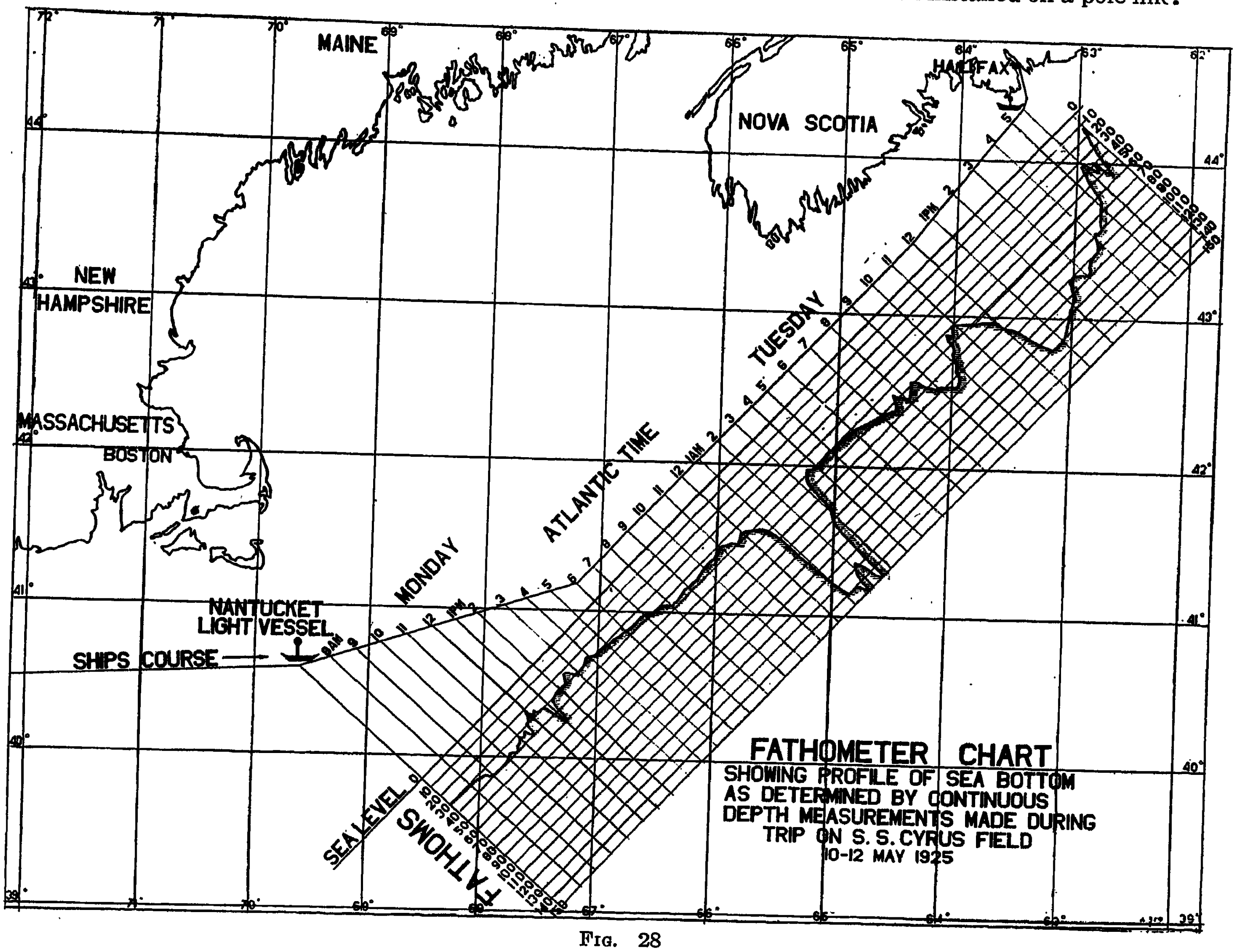


Fig. 28

One cable carrying a distinctive signal, even speech, can be used for incoming vessels and another for outgoing vessels.

Such a device has been proposed time and again and our Navy spent a great deal of money in demonstrating its practicability. However, the device is not used even in such an important channel as that leading to New York. Its lack of use seems to stand as a criticism of the traditions and principles of water navigation.

It has also beeen proposed to use that same scheme for guiding aircraft across the country. In France such a conductor has been installed on a pole line.

It is proposed to use several high-power Geisler types of tubes containing neon or other gases producing light which will penentrate farther through fog than ordinary light will penetrate.

Also it is proposed to use the photoelectric cell which is more sensitive than the eye to the light rays that penetrate fog. The effect on the photoelectric cell is to be amplified to operate an indicator such as a galvanometer.

Aviators are provided with barometers to indicate their height above sea level but that is not an indication of their height above the ground unless they know where they are and know the height of the ground below them. Therefore several devices have been proposed to measure height above the ground.

As aircraft make a great deal of noise it is proposed to produce sound signals on those craft which are higher in frequency than any of the sounds produced by the aircraft and pick up those signals after reflection from the earth, by means of an indicating device tuned to their frequency. That scheme is similar to the depth finder used in water navigation except that higher sound frequencies are proposed.

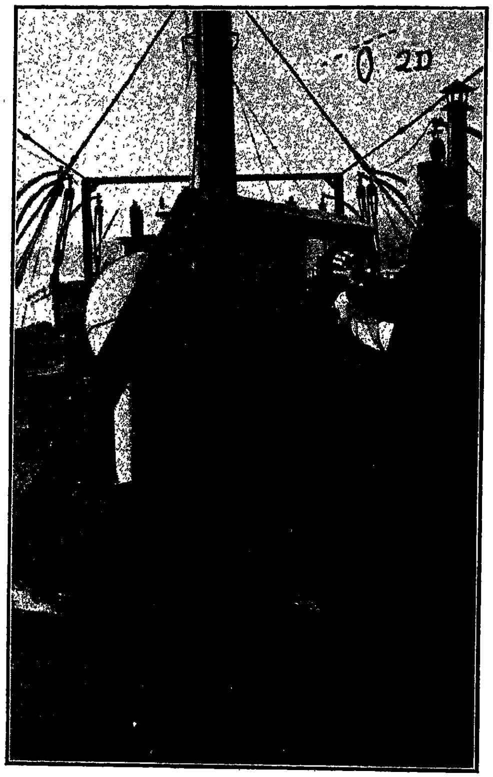


Fig. 29—Locating the Position of a Channel Cable

In which a 1000-cycle current is flowing, by turning and tilting a single coil on a wooden vessel

Another scheme involves the transmitting of radio waves to ground and picking up their reflection on the air craft. The strength of the reflected signal may work out to be the factor for indicating height or the phase relation of the outgoing frequency to the reflected frequency may give the altitude in terms of the radio wavelength.

Over a radio beacon using a vertical antenna the aviator probably could estimate his height by the diameter of the dead area directly above the station.

The variations of the air dielectric of a condenser with altitude has been proposed as a substitute for a barometer.

The future electrical aids to air navigation are particularly speculative both as to kinds and quantities, but are being taken up very seriously, as will be seen from Fig. 30. It is expected that radio will be used a great deal for aircraft and even though the air navigation industry is not sufficiently developed to ask for what it needs, the Federal Radio Commission has anticipated that it will need numerous wavelengths and therefore has tried to hold wavelengths for aids to aviation.

For illustrations and other assistance in the preparation of this paper I wish to thank Dr. J. H. Dellinger of the U. S. Bureau of Standards, Dr. Louis M. Hull of the Radio Frequency Laboratories, Mr. Haraden Pratt of Mackay Radio & Telegraph Co., Mr. Malcolm P. Hanson of the Naval Research Laboratory, Mr. G. W. Pickard of the Wireless Specialty Apparatus Company, Mr. Victor E. Carbonara of the Pioneer Instrument Company, Mr. J. E. Colloton of the Submarine Signal Corporation, and the Institute of Radio Engineers.

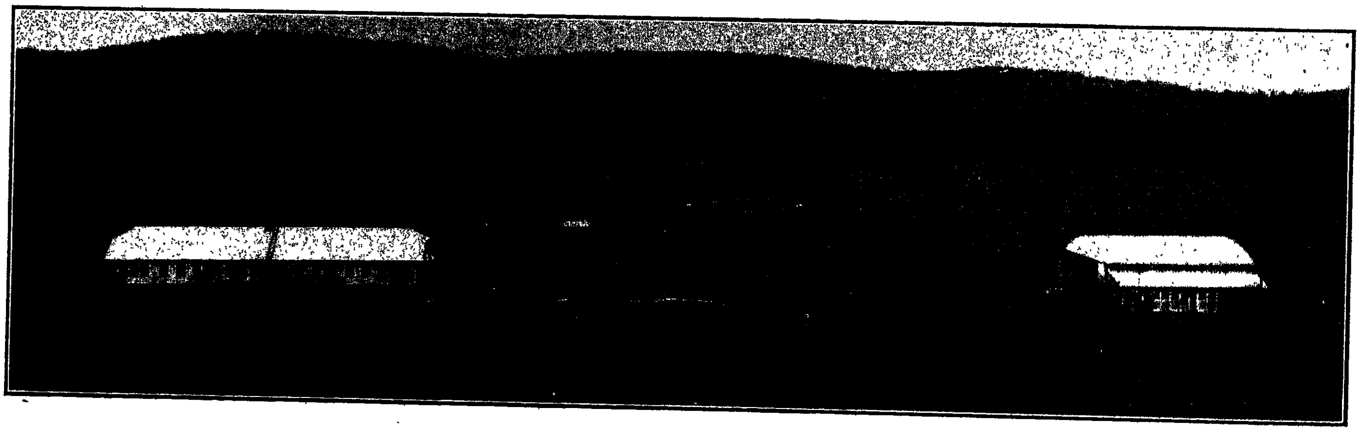


Fig. 30—Laboratory, Hangar, and Landing Field near Completion at Boonton, New Jersey

Exclusively for research in and development of radio and other aids to air navigation

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Discussion

P. A. Borden: In the depth sounding apparatus which Mr. Marriott mentioned, the telephone receiver attached to the hull of the ship is immersed in a tank of fresh water. I was a little puzzled as to why that should be necessary.

G. D. Robinson: I might venture partially to answer that question. A fluid of the same density as water is used within the tank to provide transmission of the sound to the microphone; with only air in the tank, most of the sound arriving is reflected and almost none would reach the microphone.

E. S. Lee: Of interest on the general subject of the measurement of non-electrical quantities by electrical means is the recent measurement of the shaft horsepower output of the electrically driven airplane carriers U. S. S. *Lexington* and *Saratoga*. Measurements of shaft horsepower output of large ships are usually made mechanically by torsion meters as part of the shaft. Electrical measurement is possible in the electrically driven ships by measuring the input to the motors and multiplying this by the efficiency of the motors. The means are old, but the application is new.

The method was proved several years ago on the U. S. S. *Maryland* where measurements were made with torsion meters and electrical measuring instruments at the same time.

Measurements on the U. S. S. *Lexington* and U. S. S. *Saratoga* were made by measuring the power input into each motor phase with single-phase indicating wattmeters. Ammeters and voltmeters were used for measuring current and voltage values. There being 8 three-phase motors, 24 sets of measuring equipment were necessary complete with current and potential transformers. Calibrations were required to cover a current range of five to one, a voltage range of three to one, frequency and power factor range of two to one.

The extent of the measurement can be ascertained from the fact that the weight of the measuring equipment boxed, ready to ship from Schenectady to the Pacific Coast, was three tons. Fifty men were required to observe all electrical instruments, to make calibrations, and to supervise the test, in order to get all completed results onto the trial board within one-half hour after completion of any test. The highest value of shaft horsepower output measured was 215,581 shaft hp. As far as is known, this is the largest amount of power measured at one time in a single test.

The satisfactory performance of the measurement was a distinct credit to the application of electrical measuring equipment to do this work.

Telemetering

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Synopsis.—The paper, in addition to surveying the field of application of "telemetering" and relating it to "supervisory control," presents several innovations in the types of "telemeters" available. Operating requirements and channels available for the transmission of readings are discussed, and an installation of the varying frequency type of telemeter on the Montaup Electric Company's System is described.

* * * * *

I. INTRODUCTION

AUTOMATIC performance has been applied extensively during recent years to the generation, transmission, and distribution of electrical energy. Automatic devices and equipments are very satisfactorily performing duties that were performed only a few years ago by operators, so that a large number of men heretofore performing important but minor duties have been released for more active and responsible positions. As a result, today we find stations under lock and key, visited only for inspection and maintenance. However, it has been found advisable to place these stations under the supervision of a power dispatcher, to provide for occurrences such as require prompt operation of station equipment but do not occur in the ordinary automatic cycle. Automatic supervisory equipments provide the operator with visual indications of apparatus positions, and means for controlling power apparatus located at remote points. One example is that of switching in case of fire in a section fed by a power feeder. Another example is that of switching so as to improve the economy of distribution over an entire power network. To obtain the most efficient operation, the dispatcher must be able to read electrical quantities at various outlying stations. A suggested way of doing this is to give the dispatcher television pictures of the substation instruments when a visual indication is required. While this method falls within the scope of modern engineering solution, it is not at present economically justified.

This paper deals with the telemetering problem of transmitting instrument readings or their equivalents from one point to another, which is one of transmitting approximate indications and records of quantities from remote points and is not one of transmitting quantities for metering and billing purposes.

When readings were required over relatively short distances, the permissible length of the leads of instrument transformers and shunts was sufficient for the transmission of information. However, the distances between the point of metering and the point of indication soon became greater than could be reached by instrument transformer leads, and other methods were sought for transmitting the information. Current readings at a distance were obtained by reducing the transmitted current, as, under this condition, a greater voltage drop could be tolerated without increasing the burden on the instrument transformer. As a result, the interconnecting leads could be considerably increased in length. These improvised methods, however, were not suitable for transmitting all types of readings. The result was the development of special apparatus which could be utilized for transmitting the necessary readings. This class of apparatus is described as "telemetering" equipment.

II. QUANTITIES OR EQUIVALENTS TO BE TRANSMITTED

The readings to be transmitted by telemetric means may be classified according to their use as follows:

1. Indicating
2. Recording
3. Demand
4. Integrating

The term "indicating" refers not only to readings transmitted for an instantaneous indication, but also to readings that are employed to actuate control devices such as the tap-changing device on a transformer, or the operating mechanism on an induction voltage regulator. Many such devices are now being operated manually, but probably a great number of them can be operated automatically by suitable telemetering equipment. The term "recording" applies only to readings used to actuate instruments that give a detailed permanent record of operating conditions. The term "integrating" refers to readings used directly to actuate an integrating device totalizing readings over a period of time. The term "demand" refers to readings transmitted to record the short time power demand of a network.

Experience indicates that the following list includes the quantities that are needed most commonly, arranged in order of the frequency of their probable application.

1. A-c. amperes
2. A-c. watts
3. D-c. volts
4. A-c. volts
5. Totalized watt-hours
6. Reactive volt-amperes
7. D-c. amperes
8. D-c. watts
9. Volt-amperes
10. Frequency
11. Power factor
12. Various quantities used in industrial applications

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[1] All of the General Electric Co.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

III. Operating Requirements

The following enumeration is intended to include all possible variations in operating conditions under which readings may be transmitted.

A. One reading transmitted continuously.

B. One reading transmitted continuously for the control of a device.

C. A plurality of readings transmitted continuously.

D. A plurality of readings distributed to integrating, indicating, and recording instruments, and control devices.

E. A plurality of readings, one at a time at the call of the dispatcher through a supervisory equipment.

F. One reading from each outlying station or a totalized reading from each group of outlying stations.

IV. Channels[2] Available for the Transmission of Readings

In the design of a telemetering equipment, it is necessary to study the channel limitations and to design the terminal equipments accordingly. For economic reasons, it is desirable to have as many readings as possible transmitted through a given channel. The optimum number of channels depends on, among other factors, the distance of a proposed transmission. Some equipments can be adapted to long distance transmission and others only to the shortest distance. The choice depends very largely on economic factors, although there are engineering features that must also be considered. Some methods require very expensive and complicated terminal apparatus. It is out of the question to apply such types of equipment to transmission over very short distances. A terminal equipment that is simple, but requires the use of more than one channel, may be economically practical where the cost of additional channels is not excessive.

There are three important types of channels; multiconductor cables, wire construction, and transmission lines. The first two, cables and wire construction, are essentially metallic connections, though in some instances it is the practise to install insulating transformers to protect terminal equipments from high electrostatically or electromagnetically induced voltages. The third type of channel, the transmission line, requires the use of additional terminal equipment and coupling capacitors. For this reason, the transmission line as a telemetering channel is expensive for short-distances, and is suitable only for longer-distance transmission or for short-distance transmission where it is impracticable to obtain metallic channels. Specific cases will determine the economies of the application.

Important limitations in connection with the use of telephone lines for telemetering indications relate to the allowable current, voltage, and energy levels. If the channel is owned by the telephone company, the restrictions as to energy level are definite because the telephone company must protect its communication channels against excessive interference. Some types of telemetering equipments can be used to transmit readings when a conversation is taking place over the same channel, whereas other types cannot be used in this way because the operating frequency may cause interference.

Telemetering systems employing direct current impulses or high frequency alternating current may in general make use of these channels without seriously interfering with telephonic communication.

V. Terminal Equipments

Several successful types of terminal equipments are described below. The equipments were designed to satisfy the demands discussed in the previous sections of the paper.

A. The Inverse-Current Type. A successful telemetering system, described in previous papers, and in use since 1917 on the Chicago, Milwaukee and St. Paul Railroad, indicates at a dispatcher's office the total power taken from the high tension transmission lines at several points. This system uses the inverse current principle—that is, the current flow in the metering system is inversely proportional to the power taken from the high-tension transmission lines. This scheme of remote indication has proved very satisfactory and reliable.

B. Selsyn Type. One of the simplest systems for remote indication consists essentially of two Selsyn motors. A Selsyn motor is an alternating-current motor with a three-phase stator winding and a single-phase rotor winding. When two or more such motors have their stators connected together and their rotors excited from the same single-phase alternating-current source, the rotors take the same relative angular positions. When one rotor is deflected, the rotors of all the other Selsyns are deflected by the same amount. Thus one Selsyn operating mechanically from an instrument serves as a transmitter, and another Selsyn located perhaps miles away, taking the same position as the transmitter, serves as an indicator. Satisfactory remote indication may therefore be accomplished by having the receiving Selsyn actuate either the pointer of an indicating device or the pen of a recording device.

When it is desired to operate large remote indicators that require more torque than the transmitting instrument is capable of furnishing, a device is used which is essentially an amplifier and is known as an indication transmitter.

Selsyns are well adapted for use as remote indicators at relatively short distances. They are accurate and require little attention. However, the rotors of all the Selsyns in the system must be excited from the same source of alternating current, and three wires are required to connect the stators of the Selsyns. It is essential that the matter of rotor supply voltage be considered carefully, because in some power systems the electrical angle between substations a short distance apart may be as much as 20 or 30 degrees.

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[2] The word "channel" as used in this paper refers to the path over which the readings are transmitted.

It has been found that a transmitting Selsyn will actuate a remote indicator through lines that are equivalent to two miles of standard No. 19 B. and S. telephone cable. A remote recorder will operate satisfactorily over half of this distance. If an "indication transmitter" is used, the indication may be transmitted through an equivalent of from 12 to 30 miles of cable, depending upon the type of receiver employed.

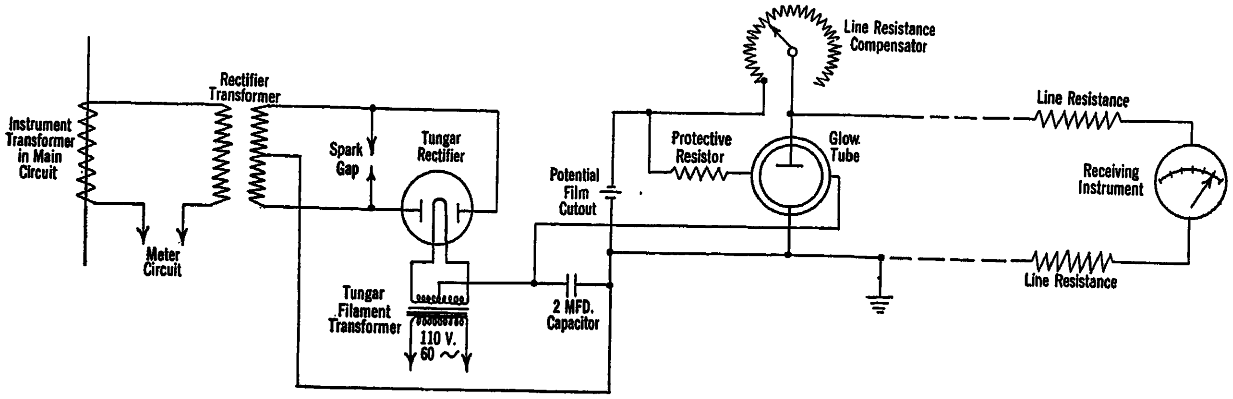

FIG. 1—RECTIFIED CURRENT TELEMETER

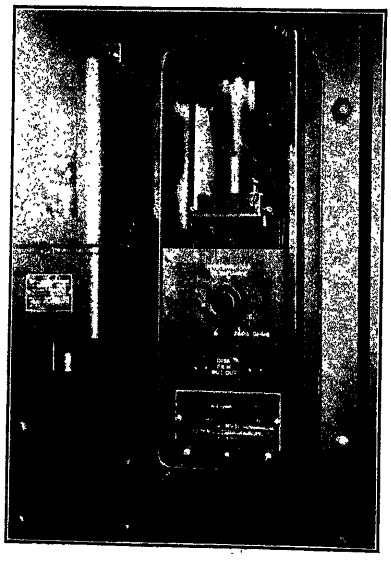

FIG. 1A—RECTIFIED CURRENT TRANSMITTER

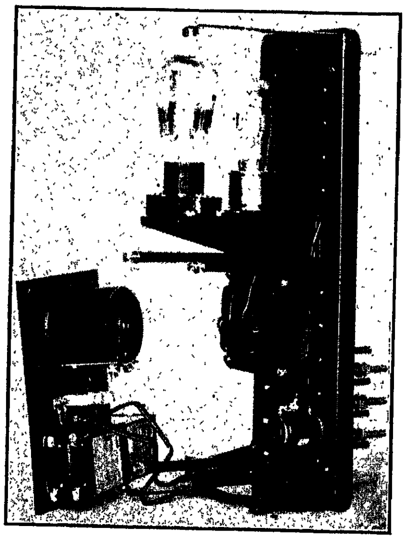

FIG. 1B—RECTIFIED CURRENT TRANSMITTER

C. Rectified Current Type. When it is desired to transmit readings of a-c. amperes and a-c. volts a very simple device, shown in Figs. 1, 1A, and 1B, is available which consists only of an instrument transformer, a rectifier, a transmission channel, and a milliammeter. Transmission by direct current instead of alternating current eliminates all trouble due to pilot-wire inductance and capacitance.

The rectifier transformer used in obtaining the voltage readings is a transformer operated from the secondary side of the main current transformer. The secondary winding is tapped at the middle; this is done in order to obtain full wave rectification of the power taken from the rectifier transformer.

In case it is desired to obtain a measurement of potential instead of current, a rectifier transformer is operated from the potential transformer in a manner similar to that used for the current indication. The voltage across the secondary winding of this rectifier transformer is dependent on the resistance of the load circuit.

The burden that each rectifier transformer imposes on the instrument transformer is approximately 10 volt-amperes, except that for the current transformer the burden is nearly doubled when the protective spark gap operates. Thus, there is very little chance for overloading the instrument transformers with this equipment.

The full-wave rectifier in the secondary of the rectifier transformer is of the hot-cathode type. The rectifier and metering circuit is as follows: The two terminals of the rectifier transformer secondary are connected to the anodes of the rectifier. The rectifier alternating current passes from the cathode of the rectifier through a pilot wire, a milliammeter at the receiving station, and another pilot wire back to the midpoint of the secondary winding of the rectifier transformer. The filament of the rectifier tube is excited from the secondary of a small transformer connected to the 110-volt a-c. source. In order to obtain long life the filament is allowed to operate continuously. This is because with intermittent operation the life would be shortened by the repeated heating and cooling of the filament.

The combined resistance of the line, receiver, compensating resistance, and the protective resistor of 2500 ohms, is 5000 ohms. Adjacent to the rectifier are located a condenser for smoothing out the rectified current, the current-limiting resistors, and the resistors for the purpose of compensating for line resistance.

The receiver is a standard direct-current indicating or recording instrument of the d'Arsonval type. It is calibrated in terms of the unit being measured.

A protective spark gap integral with the rectifier transformer is connected across the secondary to prevent the voltage across that winding from rising excessively in case of an open circuit.

The principal protection against high voltage on the transmission channel is the neon glow tube; if the voltage across the transmission channel rises excessively as it does on open circuit, the neon glow tube takes the current and causes the voltage to drop. Finally, the circuit from the rectifier to the remainder of the terminal equipment is so arranged that no current can pass unless the neon glow tube is in place. A secondary protection against high voltage on the channel wires or on the transmitter is a disk film cutout. This is connected between the negative and positive wires as shown in Fig. 1. In case the voltage between these two points rises to several hundred volts, the cutout will break down and will short-circuit the transformer. This is what would happen if the neon tube became inoperative, or if the line-resistance compensator became open circuited.

It is evident that when the rectifier type of telemeter is used to determine a-c. voltage, the resistance of the terminal apparatus and channel wires affects the reading at the receiver. Therefore, it is necessary to adjust the line-resistance compensator so that the receiver operates correctly. In the case where a current reading is desired, the line resistance has little effect on the reading, because the current transformer must maintain ampere turns in the secondary to balance the ampere turns in the primary; thus the variation of resistance in the line wires does not seriously affect the calibration of the equipment when used for current readings.

The rectified-current method is subject to a small error if the wave form of the supply is not sinusoidal. This error can be compensated for by a transformer ratio adjustment.

D. The Varying-Frequency Impulse Type. The three types of telemeter discussed above have depended primarily upon current or voltage magnitudes. Another type of telemeter is available which has a different principle of operation in that the readings are transmitted by means of impulses, the magnitude of the quantity being a function of the rate at which impulses are sent.

The transmitter used in this scheme is a meter of the same general type as the watt-hour meter. Since the operation of this system is according to the frequency of the impulses, the transmitter is usually a rotating device which sets up these impulses by means of a contact-making device—usually a small commutator. This commutator is geared to the motor shaft by a pair of gears of such a ratio that with full load (full scale reading) the meter originates approximately two impulses per second. Usually, this gear ratio is 1:1. The frictional load imposed by the commutator and brush is very small and is compensated for in the light load adjustment of the meter. (Note Appendix A.)

If the impulses from the watt-hour meter are used to actuate a speedometer, the speedometer mechanism may be calibrated so that its deflection indicates kilowatts. The actual receiver is not a simple speedometer, but is an analogous device, compensated for the temperature error usually found in speedometers. It is adapted to telemetering as follows: the impulses from the transmitter are received by a sensitive relay. This relay operates a portion of the telemeter known as the "rotary relay," whose function is to translate impulses into intermittent motion. By this means the output shaft of the rotary relay turns through one revolution for each impulse received.

This rotary relay consists of a motor controlled by an electrically operated latch that allows the motor to turn through one revolution each time the latch winding is energized by an incoming impulse. The shaft of the rotary relay is connected mechanically through a helical spring to a system of magnets comprising a flywheel. The flywheel possesses considerable inertia and the spring takes up the intermittent motion, so that the flywheel turns at a practically uniform speed proportional to the average frequency of the impulses received. The magnets induce eddy currents in a copper drum and thereby tend to produce rotation. The rotation of the drum is restricted by a spring, so that the deflection is proportional to the dragging force, and therefore to the speed of the magnets. The deflecting system has some inertia and is elastic, and consequently it must be damped to prevent mechanical oscillations. The deflecting element actuates a pointer or pen to serve as an indicating or recording receiver. This instrument when operating on a frequency of two impulses per second has a torque of about 2000 gram-millimeters. (Note Appendix B.)

It is very easy to check the operation of either the transmitter or the receiver at will by counting impulses. If the transmitter is operating correctly, the impulses come through with fair uniformity. If the transmitter is not operating properly, either the impulses do not come through, or they come through intermittently. The relation between the frequency of impulses and the receiver deflection is linear.

The same impulses that operate the indicator may also be used to determine both the total kilowatt-hours and the demand over a given time interval. The total kilowatt-hours are obtained by operating a counter known as the "distant dial." The demand is determined by operating any of the conventional demand meters by the use of the impulses. Thus this type of telemeter gives an indication of power, an integration of power, and the power demand, from the impulses sent out by a single watt-hour meter. Provision can be made to take account of the direction of power flow.

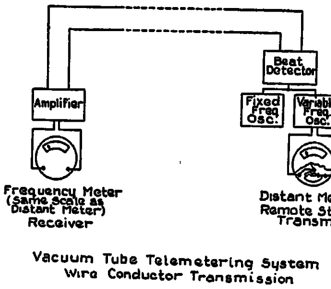
Beat Detector
Amplifier
Fixed Freq Osc.
Variable Freq. Osc.
Frequency Meter (Same Scale as Distant Meter) Receiver
Distant Meter Remote Station Transmitter
Vacuum Tube Telemetering System
Wire Conductor Transmission

FIG. 2—VACUUM TUBE TELEMETERING SYSTEM WIRE CONDUCTOR TRANSMISSION

The summation of two or more readings can be accomplished by adding the impulses from the several transmitters. This is done by the use of a device known as a differential relay. This relay sends out a new series of impulses to operate the receiver according to the sum of the original impulses. The differential relay or impulse-totalizing device consists only of the following apparatus: impulse meters, differential gearing, ratio gearing, a damping device, and a contact device to produce impulses. A differential relay can be designed to supply both additions and subtractions. Such a relay is usually designed for a given number of circuits with various impulse constants.

E. Vacuum Tube Type. Fig. 2 shows the apparatus which is employed. A small condenser is attached to the movement of the instrument whose reading is to be telemetered. This condenser is connected to an oscillator so that the frequency of the latter varies according to the meter position.

A second oscillator, operating at a frequency close to that of the first oscillator, has a fixed condenser, and therefore furnishes a constant frequency. A beat frequency is set up by these two oscillators and is detected by means of a third vacuum tube. This beat frequency is relatively low, and varies in accordance with the instrument deflection.

The beat, or telemetering frequency, is transmitted over the connecting line to the receiving station, where it operates a direct reading frequency meter. The frequency meter may be directly energized by the incoming signal or an amplifier may be provided, the method depending upon the nature of the line, its impedance, and on the energy level permissible.

The frequency meter is furnished with a scale which corresponds exactly to that of the distant instrument. It indicates at all times the position of the remote instrument. The receiving instrument indication depends upon frequency only, and, provided it receives more than the minimum power necessary to operate it, is not affected by changes in signal strength. Thus any changes in the impedance or leakage resistance of the line do not affect the accuracy.

If it is desired to transmit the instrument readings by carrier waves, over special conductors, telephone lines, or high voltage power lines, transmitting and receiving apparatus very similar to telephone communication equipment, which is already familiar in the Central Station field, is employed.

Fig. 3 is a schematic diagram of the general arrangement for carrier current telemetering. The heat frequency output of the oscillator circuit, instead of the output of a microphone speaking circuit, is applied in the regular way to the modulator. The carrier current generator consists of the usual arrangement of master oscillator and power amplifier. There is thus sent out over the channel a carrier wave of constant frequency modulated by an audio frequency which varies with the magnitude of the quantity being telemetered.

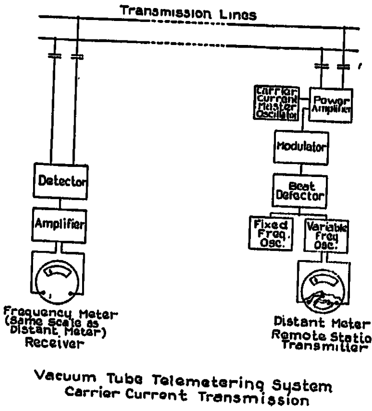
Transmission Lines
Carrier Current Master Oscillator
Power Amplifier
Modulator
Beat Defector
Fixed Freq. Osc.
Variable Freq Osc.
Detector
Amplifier
Frequency Meter (Same Scale as Distant Meter) Receiver
Distant Meter Remote Station Transmitter
Vacuum Tube Telemetering System
Carrier Current Transmission

FIG. 3—VACUUM TUBE TELEMETERING SYSTEM CARRIER CURRENT TRANSMISSION

At the receiving end, the carrier is demodulated and the audio signal amplified. The amplified signal is then taken direct to the frequency meter which reproduces the position of the pointer of the distant instrument. It will be specially noted that again the signal strength—provided it be sufficient to energize the apparatus—has no effect upon the accuracy of the indication.

In cases where it is desired to telemeter more than one reading from a remote station, a continuously operating selector[3] at the transmitting station automatically connects each instrument, one after the other, to the telemetering transmitter for a brief period. Twenty or more instruments may be taken care of in this way.

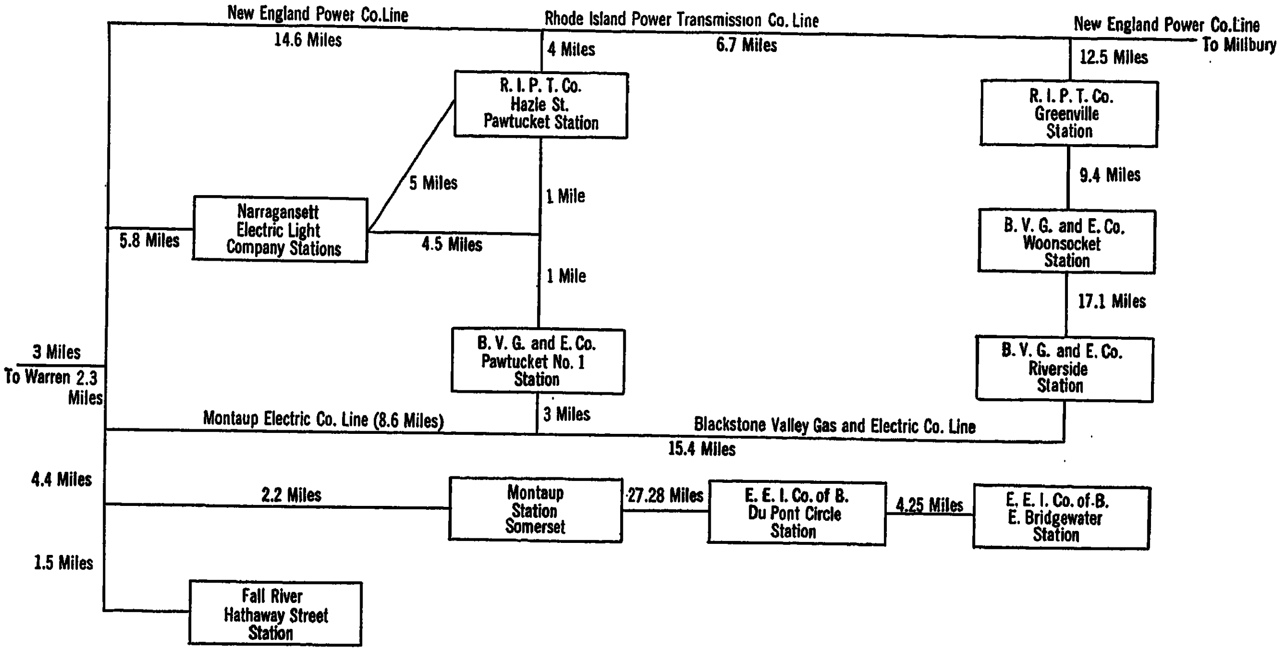
New England Power Co.Line
14.6 Miles
Rhode Island Power Transmission Co. Line
4 Miles
6.7 Miles
New England Power Co.Line
To Millbury
12.5 Miles
R. I. P. T. Co. Hazle St. Pawtucket Station
R. I. P. T. Co. Greenville Station
9.4 Miles
5 Miles
1 Mile
Narragansett Electric Light Company Stations
5.8 Miles
4.5 Miles
1 Mile
B. V. G. and E. Co. Woonsocket Station
17.1 Miles
3 Miles
To Warren 2.3 Miles
B. V. G. and E. Co. Pawtucket No. 1 Station
B. V. G. and E. Co. Riverside Station
Montaup Electric Co. Line (8.6 Miles)
3 Miles
Blackstone Valley Gas and Electric Co. Line
15.4 Miles
4.4 Miles
2.2 Miles
Montaup Station Somerset
27.28 Miles
E. E. I. Co. of B. Du Pont Circle Station
4.25 Miles
E. E. I. Co. of B. E. Bridgewater Station
1.5 Miles
Fall River Hathaway Street Station

FIG. 4—MONTAUP ELECTRIC COMPANY AND ALLIED UTILITIES

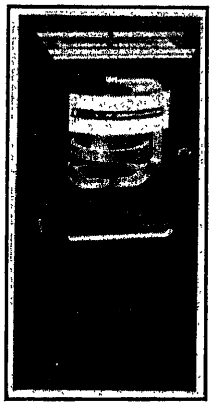

FIG. 5—INDICATING DEVICE VARYING FREQUENCY IMPULSE TELEMETER

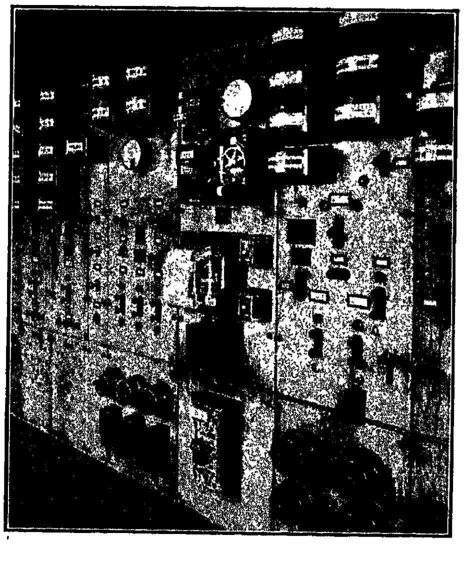

FIG 6.—VARYING FREQUENCY IMPULSE TELEMETER

a. Demand meter
b. Indicating and recording device
c. Transmitting device

At the receiving stations, an equal number of instruments is installed. All receiving instruments are frequency meters identical in construction, but each has a scale the same as the corresponding instrument at the remote station. A selector associated with the receiver is arranged to connect each receiving instrument to the channel during the period at which the corresponding transmitting instrument is connected to the telemetering apparatus by the transmitter selector. Thus each pair of instruments, one transmitting and one receiving, is given possession of the telemetering channel once in each complete cycle which includes all of the instruments. The receiving instruments employed have pointer movements of the type which retain their position when deenergized. Thus, when operated in the above manner, each instrument indicates the reading shown when last connected to the telemetering circuit. If we assume that each instrument is energized for a second, which time is sufficient for it to come to rest at its correct reading, each reading will be checked about every ten seconds if there are ten instruments. With other numbers of meters, the frequency of correction will vary accordingly.

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[3] An automatic instrument selecting device.

Clearly it is necessary to synchronize the selector at the receiving station with that at the transmitter. A feature of this scheme which is of special interest is that, unlike many existing methods of synchronizing, it does not require a special synchronizing channel. Synchronizing of the selectors is automatic when the apparatus is started from rest. Thus it is no more difficult to operate multiple instrument telemetering over carrier or radio than over a wire line.

The vacuum tube type is readily applicable to such special problems as: the totalizing of load indications at various points on a network, graphic recording, integrating, etc. Methods of carrying out these special applications involve some very interesting features, which may be dealt with more appropriately on another occasion.

VI. EXAMPLE OF APPLICATION OF VARYING FREQUENCY IMPULSE TYPE OF TELEMETER

An automatic telemetering equipment of the varying frequency impulse type, operating successfully on the Montaup Electric Company's system, is shown in Figs. 4, 5, and 6.

The Montaup Electric Company and its allied companies take power from the New England Power Company's lines at three different points. The primary purpose of the telemetering equipment is to indicate and record at Montaup Generating Station the total power taken from the New England Power Company. It is desirable to know the instantaneous transmission of power between the various systems and to have a record of the fifteen-minute demands.

In detail, the functions of the telemetering system may be listed as follows: (see schematic sketch in Fig. 4).

(1) To record at the Riverside St. Substation the maximum 15-minute demand of the Greenville and Riverside St. Substations.

(2) To indicate and record at Pawtucket No. 1, the total power taken from the New England Power Company at the Greenville Station and at the Pawtucket No. 1 Station.

(3) To record at the Pawtucket No. 1 Station the maximum 15-minute demand of the Pawtucket No. 1 Station.

(4) To record at the Fall River Station the maximum 15-minute demand of the Fall River Station.

(5) To indicate at the Montaup Station, Somerset, the total power taken from the New England Power Company at the Greenville and Pawtucket No. 1 Substations.

(6) To indicate at the Montaup Station, Somerset, the total power taken from the New England Power Company at the Fall River Station.

(7) To indicate and record in the dispatcher's office at the Montaup Station, Somerset, the total power taken from the New England Power Company by the Greenville, Pawtucket No. 1, and Fall River Substations.

(8) To record at the dispatcher's office at the Montaup Station the totalized maximum 15-minute demand of the Greenville, Pawtucket No. 1, and Fall River Substations from the New England Power Company; and to actuate an alarm if this totalized maximum demand exceeds a certain value.

Appendix A

TRANSMITTER CONSTANT

Let us assume the transmitter to be of the watt-hr. meter type with a meter constant K equal to the watt-hours required to turn the meter shaft through one complete revolution; that is,

$$K = \frac{\text{watt-hours}}{\text{revolutions}}$$

In turning through a single revolution the contact sends out N impulses, or

$$N = \frac{\text{impulses}}{\text{revolutions}}$$

The impulse constant, I, or the number of kilowatt-hours per impulse, is therefore

$$I = \frac{K}{1000\,N} = \frac{\text{kilowatt-hours}}{\text{impulses}}$$

From the above equation

$$\text{kilowatts} = I \times \text{impulses per hour.}$$

$$= 3.6\,\frac{K}{N} \times \text{impulses per second}$$

Appendix B

RECEIVER CONSTANT

The impulse frequency that should give the full scale or maximum deflection of the receiver may be determined as follows:

Let the desired full-scale reading of the telemeter be M kilowatts. Then

$$\text{impulse constant } I = \frac{\text{kilowatt-hours}}{\text{impulses}}$$

and,

$$\text{frequency of impulses} = \frac{M}{I} = \frac{\text{impulses}}{\text{hours}}$$

so that,

$$\text{impulses per second} = \frac{M}{3600\,I}$$

Bibliography

Automatic Transmission of Power Readings, by B. H. Smith and R. T. Pierce, A. I. E. E. JOURNAL, TRANS., Vol. XLIII, February 1924, pp. 303-310 inclusive.

"Power Limiting and Indicating System of the Chicago, Milwaukee, and St. Paul," by J. J. Linebaugh, *General Electric Review*, April 1920, Vol. XXIII, No. 4.

Discussion

S. C. Jacoby: The Montaup telemetering equipment was installed for two purposes: first, to record the fifteen-minute system demands of each of our member companies, and second, to determine the power received from the New England Power Company System at our points of interconnection.

The equipment used to record the fifteen-minute demands of our member companies is to be used for billing purposes. It has not been in continuous operationg long enough to determine the value of its records. The equipment on our power company connections is used to assist operation. It has been in service about six months. It consists of the following instruments:

1. Two indicating receivers.
2. One graphic receiver.
3. One demand meter.
4. One distance dial.

1. One indicating receiver indicates the power received from the New England Power Company System at Fall River; the other, the power received from the New England Power Company System at Pawtucket and at Woonsocket. These instruments are mounted on the main switchboard directly in front of the governor motor controls so that the switchboard operator may determine the effects on the New England Power Company ties of a change of the Montaup System load or a change of the Montaup Station output. These instruments require approximately ten seconds for the pointers to assume their proper position after a change of load has occurred. An appreciable change of load is noticed immediately because of the ticking sound of the rotary relay release. This ticking sound at first appeared objectionable, but has since proved useful. Any change of load causes a change in the sound of the instrument and is noticed at once, whereas a visible indication alone might not be noticed for an appreciable length of time. This is of special advantage during transmission troubles because, at that time, an immediate indication may avoid an interruption.

2. The graphic receiver indicates and records our total power company takings. This instrument is mounted in the dispatcher's room. An adjustable contact on the instrument operates an alarm when the demand limit is reached so that, in case the operator and the dispatcher do not notice an increase of total takings, the alarm will call it to their attention. To prevent the ticking noise of the rotary relay release from interfering with the dispatcher's telephone conversations, the relay is mounted in the control room and drives the receiver through a selsyn generator and motor. This makes the dispatcher's instrument practically noiseless.

3. The demand meter prints the total fifteen-minute demand of our power company takings.

4. The distance dial integrates the total power received.

Reliability. Outages have been caused chiefly by commutator failure, by the sticking of the differential relay, and occasionally by telephone fuses blowing at Greenville. These outages, we believe, can be prevented as they are not caused by defects in the principle of the varying frequency impulse equipment.

Accuracy. The accuracy of the indicating instruments compares favorably with the accuracy of regular switchboard instruments and is limited chiefly by the transmitting watthour meter. False readings on the indicating instruments are rarely encountered as failure of equipment usually causes the instruments to hunt or to stop altogether.

Advantages of the Telemeter to Operation. The advantages we have obtained by the use of our telemetering equipment are:

1. Improved service by avoiding interruptions. Where systems are connected by more than one tie, there is danger that, if one tie is lost during a disturbance, the increase of load on the remaining ties may cause their loss also. If all ties are lost, an interruption may result, but if the load on the remaining tie is reduced to a safe limit at once, its loss can be prevented. The telemeter provides us with an immediate indication, both audible and visible, of a loss of either major tie at a point where the load on the remaining tie can be controlled.

It is not advisable to have standing orders to pick up load during trouble without the telemeter indication because, in case of local trouble resulting in a loss of load, picking up load on our generators would aggravate the condition and might result in further trouble.

2. Smoother operation. On an interconnected system, correct frequency is not an indication of normal conditions as variations of system load or generation affect the load on the interconnecting tie lines and not necessarily the frequency. The telemeter indicates to the dispatcher whether or not he has anticipated his load correctly and, if not, steps can be taken to correct same without being informed by telephone. This is of special advantage when an unexpected change of load occurs, such as a change due to sudden darkness on one section of our territory.

During normal load change periods, the Montaup operator is instructed to keep the load on the power company ties within certain limits. In this way, excessive exchanges of load between systems is avoided.

3. Checking of switching operations. Much of our routine switching on the power company ties is done on schedule and not on direct order. The telemeter provides our dispatcher with a check on this switching.

4. Determining the source of disturbance. The graphic telemeter, used with the frequency recorder, supplies our dispatcher at once with valuable data concerning disturbances.

5. Economy. Last but not least, economy. Usually, the price of purchase power differs from the price of local generation. It may cost more or less than local generation, but is rarely the same price. There are two reasons for this:

1. The increment cost of local generation varies because it depends on the units in service and on the load at which these units are operating.

2. The cost of purchased power depends on variable conditions on the power company system.

During primary periods, our maximum rate of purchasing power is limited by our primary demand. During secondary periods, it is limited by the capacity of the ties or by the power available. We find that, with the telemeter in service, we can keep 1500 kw. hr. per hour closer to our desired takings than when we depend upon the men at the receiving points watching the load and notifying us of variations by telephone.

To put a commercial value to this, assume a difference of two mils per kw. hr. between our increment cost and the cost of purchased power, the resulting saving will be three dollars per hour. On a basis of 15 hr. per day and 200 days per year, this saving amounts to $9000 per year. This saving is not clear profit, but it does tend to put the telemeter on a paying basis.

Chester Lichtenberg: The subject of telemetering is relatively new; as a matter of fact, it is an art which has only been recognized as such for about four years. Initially the attempt was made to move instruments to a relatively great distance from the shunts or other sources from which they derived power. That means long leads and eventually large leads with the usual voltmeters, ammeters, wattmeters, and the like. When the distances became more than two or three miles, the usual types of instruments were not sufficiently delicate and the usual sources of power such as shunts, current, and potential transformers were not of sufficient rating to generate energy to operate the instruments efficiently. But the main trouble was to get large enough wires for long enough distances. The urge for telemetering came through the centralizing of the control of power in load dispatchers or power directors.

Mr. Jacoby has pointed out one use; such a use was really started about 1912 by the Chicago, Milwaukee & St. Paul

Railway, where the load demand had to be kept to a certain minimum because there wasn't enough water power to furnish all that the railroad required. In the last few years, the demand for telemetering has increased quite rapidly. Centralizing the control of power has reduced the number of operators in urban systems.

During the last four years, the tendency has been to use telephone wires or their equivalent for connecting power stations or substations or the sources of power supply to the central points at which the power directors have been located. The idea has been to use the existing telephone cable systems and, therefore, the amount of energy which might be used has been limited. At first the idea was to use either current or voltage in the usual fashion for transmitting the instrument readings, but it was soon found that inductive interferences and leakage caused an appreciable error. In looking around for other means to transmit instrument readings, it was finally decided that frequency gave the best means of transmitting electrical indications.

The number of wires which may be used for telemetering has sometimes given concern because each wire costs money, either to install or to maintain or to rent. Some systems are now available whereby as many as three instruments may be read continuously over one pair of telephone wires, but there does not seem to have been much demand for such designs. Instead, as Mr. Stewart pointed out, one pair of wires is being used for reading as many as 24 instruments successively,—not continuously, but successively.

There is a number of other systems which have not been described. Those which have been described merely give an idea of this new art of telemetering and present to the central station and industrial engineers as well as to the railway engineers a vision of what they may expect in the way of development so that in laying out new systems and in consolidating present systems they can take advantage of this new art for economical operation.

C. E. Stewart: No mention has been made of the application of telemetering in connection with automatic supervisory equipment or equipment for the remote control and indication of power apparatus. There are several phases of this particular application which are of interest.

The application of telemetering by itself requires an independent channel used for no other purpose except possibly that of telephone communication. The circuit under that condition is a continuous one and the telemetering readings are of a continuous or semi-continuous nature.

When we use telemetering with automatic supervisory equipment, however, it is possible to select and obtain readings from a large number of station circuits. These readings may be transmitted to the dispatcher over the same wires which are used for the supervisory equipment provided a momentary reading is satisfactory. This arrangement is only recommended for momentary readings because it necessarily holds the supervisory equipment on the selected circuit and does not permit it to function in case of the automatic operation of other supervised apparatus.

A preferable method is to use an independent channel for telemetering. The supervisory equipment may then be used to selectively connect this channel to the desired substation circuit. The supervisory equipment is thus left free to perform other functions.

When telemetering of a multiplicity of circuits is required the best practise is to use a supervisory equipment entirely for that purpose. Such an installation is soon to go into service in the city of New York in the Laconia Avenue Substation of the Bronx Gas and Electric Company. The combined supervisory and telemetering equipment operating over four line wires, provides the dispatcher with means of reading the a-c. current and a-c. voltage on twelve power circuits. The dispatcher can obtain these readings with as little effort and as promptly as if he were located in the substation itself.

The rectified-current type of telemetering and the synchronous-selector type of supervisory equipment are used on this intallation. An independent supervisory equipment provides the dispatcher with supervision of the substation equipment.

Totalizing of Electric System Loads

BY P. M. LINCOLN*

Fellow, A. I. E. E.

Synopsis.—The importance of measuring an electric system load at any distance from that load is emphasized. The use of thermal wattmeters in conjunction with thermal couples is suggested. Time of response of such devices is discussed; temperatures attained are also discussed. Characteristics of circuits for transmitting the thermal e. m. f. from the load to the point of measurement are discussed in detail. A list of users together with data pertaining to the measurement-transmitting circuits is given.

When all the power of a given electric system is generated in a single plant, the totalizing of the entire load is relatively an easy problem. A number of methods is known and used for accomplishing the desired result of making available the information as to the total of the load that is being supplied. These totalizers may be of the indicating or the graphic recording type as may be desired.

When two or more power plants are used to supply a given system, and these plants are located at a distance from each other (and this is invariably the case in modern electric systems of any size), the problem of totalizing the entire load of the system becomes more difficult. If to this situation we add the further complication of purchase and sale of energy to one or more contiguous systems—at points perhaps many miles removed from each other—the problem becomes much more complicated. Yet this last situation is exactly where many, if not the majority, of the modern large electric service supply systems find themselves at the present time.

Large electric supply systems are rarely fed from a single power generating plant. Moreover, a consideration of the economics of the situation usually dictates that the various power plants serving a given territory shall be widely separated within that territory. Further, the purchase and sale of power from contiguous territories naturally occurs at the boundaries of a given territory. These purchase and sale agreements between neighboring properties are becoming more and more common. The term "superpower" has been coined to cover this modern tendency and it is generally accepted as being both economic and logical This situation, however, makes more acute the problem of load dispatching.

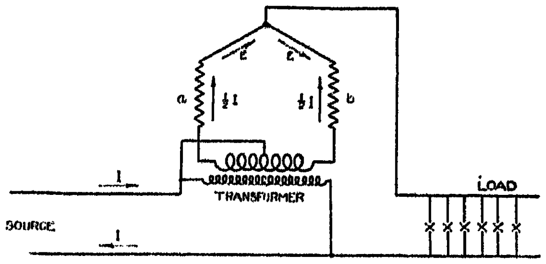

FIG. 1—ELEMENTARY DIAGRAM OF THERMAL WATTMETER

The problem of load dispatching requires that the load dispatcher shall know at all times not only the aggregate load on his own system but also how much load he is supplying to other systems and how much they are supplying to him. When the generating stations are widely separated and when the points of contact with contiguous systems are still more widely separated, as is the case in all large systems, how is the load dispatcher to obtain this essential information?

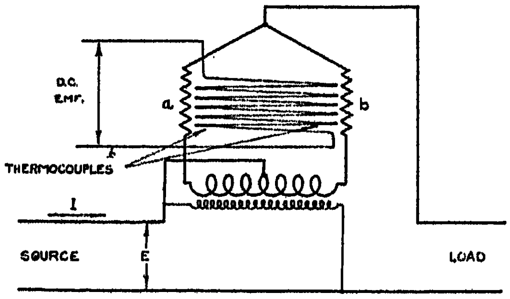

FIG. 2—ELEMENTARY DIAGRAM OF THERMAL WATTMETER USING THERMOCOUPLES AS MEASURING PRINCIPLE

It is the object of this brief paper to indicate one solution of this problem. The fundamental idea in the solution proposed is the same as that in the instrument described by the writer in a paper before the A. I. E. E. in October 1915.[1] Fig. 1 herewith is a reproduction of Fig. 2 of that paper and shows the elements of a thermal wattmeter. In the appendices of the 1915 paper is submitted a mathematical proof that, with circuit connections as shown in Fig. 1, the difference in temperature between resistances *a* and *b* of Fig. 1 is proportional to watts.

In the 1915 paper above referred to, a number of methods is proposed for measuring the difference in temperature between these two resistances *a* and *b*—among them the use of thermocouples. Still another method is described in the present author's paper[2] read before the A. I. E. E. on Feb. 15, 1918. This latter method has been incorporated into a commercial demand meter and has found considerable use among public utilities, particularly in Canada. Although this latter method is excellent for use in a demand meter, it does not lend itself readily to a solution of the totalizing problem outlined above. The use of thermocouples, however, does so lend itself in an ideal manner. As shown in the 1915 paper, the difference in temperature of the resistances *a* and *b*, Fig. 1, is proportional to watts. If thermocouple junctions are associated with these resistances, the resulting thermal e. m. fs. are then proportional to watts for each individual thermal wattmeter. By connecting these thermal e. m. fs. in series, and measuring the resultant total, the sum of all of the readings of the individual thermal wattmeters may easily be obtained at any desired point.

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*Director, School of Electrical Engineering, Cornell University, Ithaca, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

1. *Rates and Rate Making*, P. M. Lincoln, A. I. E. E. TRANS., 1915, p. 2279.

2. *The Character of the Thermal Storage Demand Meter*, P. M. Lincoln, A. I. E. E. TRANS., 1918, p. 189.

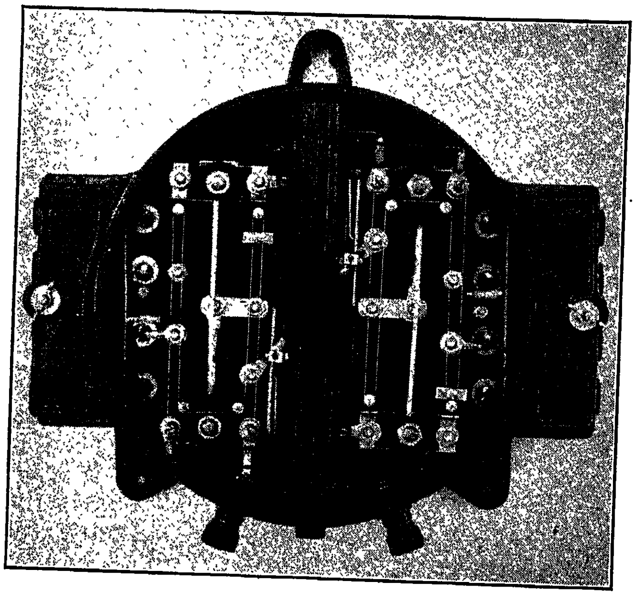

Fig. 3—Thermal Converter with Cover Removed Showing Adjustments

It should be noted that the thermal wattmeter shown in Fig. 1 is reversible. If the power flow is in one direction, resistance *a* becomes hotter than resistance *b* by a given amount. If the direction of wattage flow is reversed, resistance *b* becomes hotter than *a* by exactly the same amount—assuming, of course, that the watt value is the same in each case. In one case, therefore, the resultant thermal e. m. f. is in one direction and in the other case in the opposite direction. These thermal wattmeters may therefore be used to subtract a given load from the total as well as add its value; the thermal wattmeter is completely and accurately reversible.

All that is necessary to obtain a total indication is an electrical circuit connecting the thermal wattmeters to be totalized and some means of reading the total resultant thermal e. m. f.—preferably a potentiometer. This briefly is the basis of the method that is now proposed for load totalization. There is a number of points that warrant further discussion.

The Thermal Converter

For the sake of convenience, the instrument incorporating the metering circuit shown in Fig. 1, together with the necessary thermocouples and adjustments, has been called a "thermal converter." Fig. 2 shows such a thermal converter in diagram and Fig. 3 is a photograph of two such thermal converters mounted in a single case and comprising a complete polyphase wattmeter—one converter in each phase. Fig. 4 shows a diagrammatic section of one of these thermal converters. The form and arrangement of parts in Fig. 4 is due to the suggestions of Mr. H. S. Baker, Meter Supervisor of the Hydro-Electric Power Commission of Ontario at Niagara Falls.

The experimental work necessary to convert Baker's suggestion into a successful commercial instrument was carried out by Louis A. Paine of the Lincoln Meter Co. Ltd., Toronto, with numerous suggestions by Mr. Perry A. Borden, at that time an engineer in the laboratories of the Hydro-Electric Power Commission of Ontario, now an engineer with the Bristol Co., Waterbury, Conn.

Referring to Fig. 4, *A* is a heavy metallic plate, brass or copper. Around this plate, and insulated from it by pure sheet mica on the sides and by a grooved bakelite tube top and bottom, are the thermocouples *T*. Over these thermocouples is placed another layer of pure sheet mica and then the heaters *R*. After still another layer of insulation, the plates *P* are applied and firmly clamped together by suitable bolts. The terminal *B* . *B* of the heaters are associated in close thermal contact (but of course electrically insulated from each other), thereby eliminating errors that might be due to thermal conduction between leads and heaters.

Fig. 5 shows in diagram the adjustments that are provided. With either line current alone in the heaters or current from the voltage transformer alone, the two heaters must obviously attain exactly the same temperature. This may be accomplished by proper adjustment of the small resistances labeled "current balance adjustment" and "potential balance adjustment" in Fig. 5. After these adjustments have been made, there is necessary a still further adjustment so that with a given value of watts applied to the thermal converter there will be a given thermal e. m. f. from the thermocouples. It is of course necessary that all of the thermal converters used to totalize a given load shall have the same thermal e. m. f. per watt; this final adjustment is secured by the shunting resistance marked "shunt"

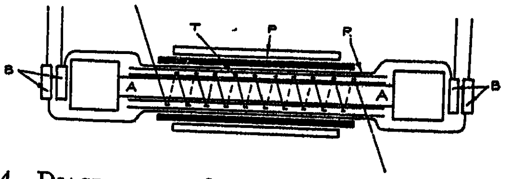

Fig. 4—Diagrammatic Section of Thermal Converter

in Fig. 5. The thermal e. m. f. that has been adopted and found suitable in practise is ten millivolts per meter element; that is, with full normal load on the metering element of a thermal converter, the resulting thermal e. m. f. is ten millivolts. For a polyphase unit, where two thermal converters are used (one in each phase), the resulting thermal e. m. f. at normal full load is, of course, 20 millivolts. The relation between watts and the resulting thermal e. m. f. is a perfectly straight line; the thermal e. m. f. is always proportional to watts input. It might be pointed out here that this perfectly straight line relationship is secured by properly designed compensation. It is well known that the thermal e. m. f. of most couples increases slightly with increasing temperature; it is also well known that heat emissivity and heat conductivity decrease with increasing temperature—at least within the temperature range used in this device. By proper combination of these two tendencies complete compensation is secured. Also, the device is relatively free from temperature error, being less than one-tenth per cent per degree centigrade. This temperature error is positive, the thermal e. m. f. per watt increasing slightly as the atmospheric temperature rises.

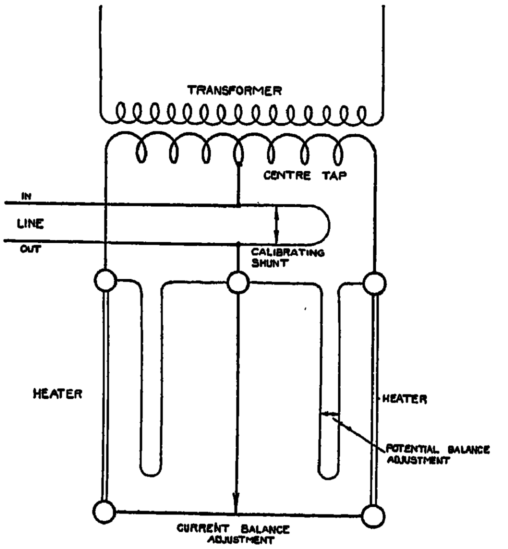

Fig. 5—Diagram showing Adjustments on Thermal Converter

The total thermal e. m. f. that is produced at the totalizing point is, of course, dependent on the number of thermal converters that are used in series on any particular project, as well as upon the load on each individual thermal converter. There is no limit to the number of thermal converters that may be thus connected in series. The maximum number so far actually used on a single installation is 19 thermal converters used by the Hydro-Electric Power Commission of Ontario to measure the power supplied to the Toronto Hydro Electric System.

Time of Response

No thermal device can respond instantly to the action of the currents that do the heating. This principle is utilized in the thermal demand meter and the sizes of the various parts are so adjusted as to secure the desired time of response. The most widely used time of response for demand meters is ten minutes to reach 90 per cent of the final indication. The thermal converter shown in Figs. 2, 3, and 4 has a very much shorter time, *viz.*, between 8 and 9 sec. to reach 90 per cent of its final reading. The reasons for this much shorter time may be worthy of further discussion.

As shown in Appendix 3 of the writer's paper[1] referred to previously, the instantaneous difference in temperature of two heaters arranged as in Fig. 1 is given by the expression

$$\theta_1 - \theta_2 = \frac{H_1 - H_2}{S\,E + 2\,Q}\left(1 - \epsilon^{-\frac{(S\,E + 2\,Q)\,t}{M}}\right) \quad (1)$$

where

θ_1 = instantaneous temperature of heater a above environment

θ_2 = instantaneous temperature of heater b above environment

H_1 = rate in gram calories per sec. at which heat is applied to a

H_2 = rate in gram calories per sec. at which heat is applied to b

S = surface area in sq. cm. of a or b (similar)

E = heat emissivity of a (or b) in gram calories per sec. per deg. cent. per sq. cm. of surface

Q = thermal conductivity between a and b in gram calories per sec. per deg. cent. of temperature difference

M = amount of heat in gram calories stored in a (or b) per deg. cent. of temperature rise

t = time in seconds after first application of heat to a and b

ϵ = base of Naperian logarithms

In Appendix 2 of this same paper, it is shown that $H_1 - H_2$ is proportional to watts when the arrangement is that shown in Fig. 1. From Equation (1), it is evident that the instantaneous temperature difference between a and b, Fig. 1, rises along an exponential curve and that it finally becomes proportional to watts, when t becomes infinite. It is further evident that the time of response is governed by the coefficient of t, *viz.*, the quantity $\frac{S\,E + 2\,Q}{M}$. The temperature difference between a and b will reach 90 per cent of its final value (assuming a steady load application) when

$$t = \frac{2.302\,M}{S\,E + 2\,Q} \quad (2)$$

Fig. 6 shows a photograph of the parts and the assembled element of a demand meter having a time of response of ten minutes (to reach 90 per cent of final value). Comparing Fig. 6 with Fig. 4 and comparing them with reference to the relation shown in Equation

(2), the reasons for the much shorter time of response of the structure of Fig. 4 becomes apparent. The value of M, Fig. 4, is very much less than that of Fig. 6—less than 10 per cent. The value of Q in Fig. 6 is nearly zero, while in Fig. 4 it is comparatively very large. These two factors alone are sufficient to account for the fact that the structure shown in Fig. 4 will respond to the application of power in approximately one-seventieth of the time of that shown in Fig. 6. It is obvious, further, that the time of response of the structure shown in Fig. 4 (or 6) may be modified if desired by modifying the design. However, the 8½ sec. (approximately) time of response of the structure shown in Fig. 4 has been found to be quite suitable for the purposes of a totalizing meter.

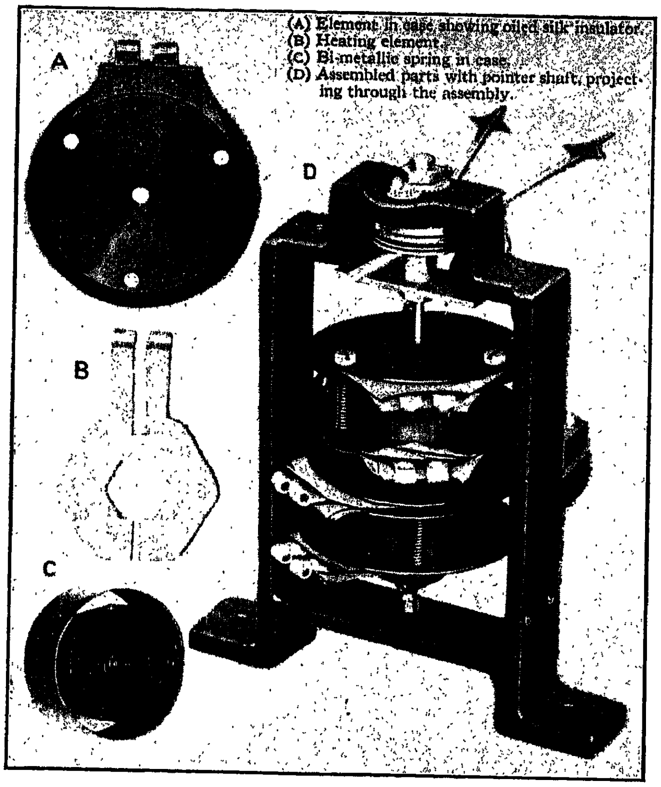
(A) Element in case showing oiled silk insulator.
(B) Heating element.
(C) Bi-metallic spring in case.
(D) Assembled parts with pointer shaft, projecting through the assembly.
A
B
C
D

Fig. 6—Parts and Assembly of the Measuring Elements of a Demand Meter (Thermal Storage Type)

Temperature Attained During Operation

The total number of couples used in series in each thermal converter is usually 21. The microvolts per couple per degree of temperature difference is approximately 42. To attain a total of 10 millivolts at full load, the resulting difference in temperature of the two elements a and b, Fig. 1, is therefore approximately 11 deg. cent. The actual rise in temperature of the hotter element above the surrounding air depends on power factor. The lower the power factor the higher will be the resulting temperature rise, since the total heat dissipated in both elements dictates the maximum temperature rise and this in turn is dictated by the total current flowing in resistances a and b. This total current obviously increases as power factor decreases. With the designs used in practise, the maximum temperature attained at 50 per cent power factor does not exceed 60 deg. cent. above the surrounding air at full load in watts. Since the insulation used is pure mica, there is obviously ample margin for safety in so far as temperature rise is concerned.

Circuit Characteristics

There are two characteristics of the circuit connecting the thermal converters with the central point of measurement that should be considered. These are:

1. Line resistance in series with thermocouples.
2. Possibility of stray e. m. fs. in connecting line.

The first of these is of relatively minor importance while the second is of vital importance.

1. *Resistance in series with thermocouples.*

The importance of the circuit resistance depends on the method of measuring the resulting e. m. f. While other methods might be possible, the only method worthy of serious considerations is the potentiometer. This is the method that has been used in all installations made to date and has been found entirely satisfactory. If the potentiometer method of measurement be used, the maximum permissible circuit resistance depends upon the amount of current that must flow to give an observable or operable deflection in the galvanometer. While the potentiometer method of measurement is theoretically a "null" method—that is, no current flows when exact balance is obtained—it does require a flow of current to detect an absence of balance. It is the value of this current that dictates the maximum permissible circuit resistance. This in turn depends on the characteristics of the galvanometer used in connection with the potentiometer, the e. m. f. available, and the degree of sensitivity demanded.

First as to galvanometer characteristics, what is demanded is not a laboratory instrument but one that can be placed under the supervision of the usual plant operators and give reliable results. Also, in most cases (all cases so far actually installed) a graphic record on a clock or time driven chart is demanded. In the installations so far made, eminent success has been obtained by the use of a recording-potentiometer made available by a well-known American firm.[3] This firm has available a line of recording potentiometers with galvanometer resistances varying from 11 ohms to 2300 ohms. These instruments record the potential of thermocouples on a clock or time driven chart approximately ten inches wide, the chart scale being divided into 100 divisions.

The galvanometers used in connection with these potentiometers are of the D'Arsonval type. This type has the advantage that it may be critically damped—a very important consideration in the successful operation of the recording mechanism. Two styles are available, one having a permanent magnet field in the galvanometer and the other an electromagnetic field. The sensitivity of the latter style is considerably

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3. Leeds and Northrup Co., Philadelphia, Pa.

higher than the former. The sensitivities and resistances of these two styles are given in the following table.

TABLE I

Galvanometer resistance ohms	Sensitivity—microamperes per step	
	Perm. mag.	Electro mag.
11	2.	1.2
22	3.	1.8
100	1.6	1.1
1180	0.25	0.17
2300	0.21	

Sensitivity (columns 2 and 3, Table I) may be defined as the amount of current that must flow through the galvanometer to make its pointer just engage the operating lever. Such an engagement will cause the pen to move approximately ¼ per cent of full scale. If this movement is not sufficient to cause the current in the galvanometer to reduce below the engaging point, movement of the pen continues until such a reduction in galvanometer current does occur.

The e. m. f. available to cause current to circulate depends on the balance between the e. m. f. of the thermal converter under measurement and the e. m. f. within the potentiometer itself. As above indicated, the e. m. f. at full load on one polyphase thermal converter is 20 millivolts. Therefore, in case of a departure from perfect balance, the e. m. f. available to cause galvanometer current is that departure's proportion of 20 millivolts. For instance, a one per cent departure from perfect balance—no matter in what part of the scale—would cause one per cent of 20 millivolts, or 200 microvolts, to become available for circulating galvanometer current. To determine the per cent departure to cause operation, we must consider not only the e. m. f. available but also the total resistance. If we assume for the moment that we have zero line resistance, the only resistance to the galvanometer current will be the galvanometer itself and the thermal converters. Each polyphase thermal converter has a resistance of approximately 30 ohms. With these data at hand it is quite easy to determine the per cent departure from perfect balance that will cause each galvanometer listed in Table I to begin to respond. When a single thermal converter is used these percentages are given in the following table.

TABLE II

Galvanometer resistance ohms	Total resistance (1 polyphase thermal converter)	Per cent departures from perfect balance to cause response (zero line resistance)	
		Perm. mag.	Electro mag.
11	41.	0.41	0.25
22	52.	0.78	0.47
100	130.	1.04	0.72
1180	1210.	1.51	1.03
2300	2330.	2.38	

From Table II, the sacrifice in sensitivity caused by any given amount of line resistance when using any given galvanometer combination may be easily determined. An increase in the total resistance (column 2, Table II) will result in exactly the same relative increase in the percentage of departure from perfect balance (columns 3 and 4, Table II) to cause response.

Table II holds for a single thermal converter. When a number of such converters is used in series (as is invariably the case), a much better performance is secured than that indicated in Table II. Each additional thermal converter adds only about 30 ohms to the resistance of the galvanometer circuit,—in most galvanometer combinations, an insignificant proportion. The voltage available from each converter with a given departure from perfect balance is exactly the same whether there be one or a hundred converters in series. Therefore the more converters in series, the less will be the per cent departure from perfect balance to cause response. To take a specific example, let us assume that we have a system using ten thermal converters in series, that we are equipped with the 1180-ohm permanent magnet type galvanometer, that our total line resistance is 1000 ohms. From the above data, it may be easily determined that such an installation would respond to a departure of 0.31 per cent of the total possible load, no matter in what converter that departure took place. If we should add another thousand ohms to the line resistance, the per cent departure would rise from 0.31 per cent to 0.435 per cent; if we eliminated all line resistance it would fall to 0.285 per cent. The electromagnetic style of galvanometer would give considerably better performance.

It is obvious from this discussion that for the usual case, line resistance is of no great moment; the distance between converters may be as great as any possible case will call for, and the conductors may be made as small as practicable and still secure a line resistance that will give entirely acceptable results.

One other point should not be overlooked in determining this line resistance. To obtain successful operation, it is highly important that the galvanometer be damped. Critical damping is secured with certain values of resistance in series. With the line of potentiometers indicated above, these values of resistance for critical damping vary from 25 ohms (galvanometer resistance 11 ohms) to 6000 ohms (galvanometer resistance 2300 ohms). Laboratory tests have shown that these values of resistance may be considerably exceeded (by perhaps 75 per cent), and still secure reasonably good results in so far as damping is concerned.

The methods of determining the maximum permissible value of circuit resistance are indicated in the above paragraphs. The size and character of the conductors to be used will depend on this maximum resistance and also upon the considerations to be discussed in the following paragraphs.

2. *Possibility of stray voltage in connecting line.*

The matter of stray voltage on the transmitting conductor is of much more importance. However, the only stray voltage that need be seriously considered is a direct (or continuous) voltage. The means of detecting and measuring the resultant thermal e. m. f. consists, as has been mentioned, of a galvanometer of the

D'Arsonval type. Such a galvanometer recognizes direct currents only and a superposed alternating voltage is of no particular moment, unless it becomes unduly large. The filtering out of these superposed alternating voltages may be quite readily accomplished.

Neighboring circuits may affect the transmitting circuit either by induction or by leakage. Induction may be either electrostatic or electromagnetic; either of these forms of induction, however, can induce only alternating e. m. f. into the transmitting circuit and therefore this source of stray e. m. f. is of no particular moment. The same is true of conduction effects so long as the conducted current is purely alternating. If, however, this conduction is of such a nature as to cause a direct-current e. m. f. to appear in the transmitting circuit, an error in the transmitted reading will be caused equal in amount to the proportion that the parasitic e. m. f. bears to the thermal e. m. f. of the thermal converters under measurement. The thermal e. m. f. is, at full load on each thermal converter, 20 millivolts per polyphase thermal converter. The total useful e. m. f. to be measured varies therefore from perhaps one or two millivolts as a minimum to perhaps 500 millivolts as a maximum, depending on the number of thermal converters used and the loads on each. It is obvious, therefore, that a relatively small amount of leakage from an external source would be fatal to the accuracy of this method of measurement. The transmitting circuit *must* be properly protected from leakage. However, with proper construction and proper precautions there is no need to anticipate any difficulty from this source. Perhaps the best proof of this statement is to enumerate existing installations and indicate the length and character of the transmitting lines. This information is given in Table III.

TABLE III

Company	Location	No. thermal converters (polyphase)	Total conductor miles	Total resistance ohms	Kind of conductor	Began operation	Max. power measured
Hydro-Elec. Power Commission..	Toronto, Ont	19	25	530	Paper ins.—lead covered	Sept. 1926	166,000 kw.
Ottawa Gas and Elec. Co........	Ottawa, Can.	3	3.5	74	Style B telephone wire rubber ins-duplex	May 1926	20,000 kw.
London Public Utilities.........	London, Ont	4	1.9	10	No. 10 B and S overhead wires on poles	?	20,000 kw.
Shawinigan Water and Power Co.	Three Rivers, Quebec	4	Within a single station	No data	Inside wiring	Aug. 1927	40,000 kw.
Wayagamac Pulp and Paper Co..	Three Rivers, Quebec	4	0.57	No data	No data	July 1927	8.400 kw.
Shawinigan Water and Power Co.	Thetford Mines, Quebec	3	2.5	53	No. 12 Style B copper weld overhead	Sept. 1927	7,000 kw.
Montreal Light, Heat and Power Company..................	Montreal, Quebec	2	Within a single substation		Inside wiring	Oct. 1927	No data
Price Bros.....................	Isle Maligue, Quebec	3	1.5	65	No. 16 style B	?	No data
Port Arthur Public Utilities.....	Port Arthur, Ont.	4	10	198	Paper ins., lead-covered cables	Oct. 1927	24,000 kw.
Windsor Hydro Elec. System....	Windsor, Ont.	5	2.5	20	Underground control cables	?	20,000 kw.

Fig. 7 shows about four hours record of one of these totalizers on the Toronto installation. This particular portion of the chart has been selected to show one of the advantages of this system of totalization, *viz.*, the ability to suppress the zero and thus increase the virtual scale length. On the left hand portion of this chart, the top represents 100,000 kw. and the bottom 250,000; the zero has been suppressed to the extent of 100,000 kw. At the right the zero suppressor has been removed and now the top is zero and the bottom 150,000 kw. With this type of measuring device it is a very simple matter thus to suppress the zero; it can be done to any extent desired.

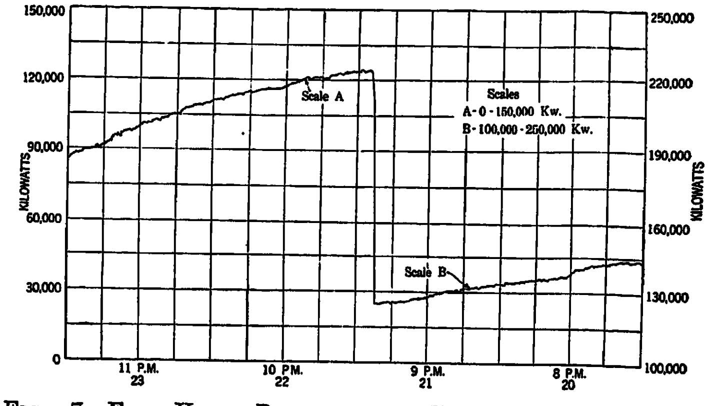

Fig. 7—Four Hours Record of the Toronto Installation showing Zero Suppression

In conclusion, while actual experience with this new totalizing system in practise has been limited to only about three years, nothing has developed to lead to the anticipation of any difficulty. Direct current leakage into the transmitting wires is the only thing that need be zealously guarded against and experience thus far indicates that with proper installation, no fear need be entertained from this source.

Discussion

H. B. Brooks: Mr. Lincoln points out that the potentiometer method of measurement, rigorously speaking, is not a true null method. While this distinction is not usually significant for laboratory work, it may become so for special cases like his, and it should be borne in mind.

In determining the value of total resistance in the galvanometer circuit, one should include the resistance introduced by the potentiometer itself. This may be of importance in the case of relatively few converters and low line resistance.

Table II shows that the galvanometers of higher resistance and better current sensitivity are inferior in performance for the case cited, namely, a single converter of 30 ohms resistance and no resistance in the line. For galvanometers of a given type which differ only in the size of the wire with which the coil is wound, the greatest sensitivity under working conditions will be obtained when the net resistance of the galvanometer coil is equal to the resistance of all the rest of the circuit. Thus for the case of a single converter of 30 ohms resistance, with zero line resistance and negligible resistance of the potentiometer and of the galvanometer suspensions, the galvanometer coil should be wound to have a resistance of 30 ohms. In practise, one cannot always expect to get a galvanometer coil specially wound for each case, but the principle just mentioned will assist in selecting the best stock galvanometer for the purpose.

I should like to suggest an exception to Mr. Lincoln's statement that the only instrument worthy of consideration for measuring the resultant e. m. f. of the converters is the potentiometer. Wherever a graphic record is desired, the recording potentiometer which he mentions is possibly the only suitable instrument available. However, it is bulky and expensive, and where it is desired merely to indicate the load in magnitude and direction of energy flow, an indicating instrument may well be used, if some expedient be provided which will render the indications free from error caused by variations in the total resistance. Methods for doing this were given on pages 540-541 of the TRANSACTIONS, Vol. 39, 1920, and an instrument embodying still another procedure was described by Harrison and Foote in the same volume, pages 371-402.

A simpler method which seems applicable to the case of load totalizing is as follows: A suitable pyrometer millivoltmeter with its scale figured in kilowatts is provided with an adjustable rheostat forming part of its series resistance. The remainder of the series resistance is non-adjustable. The resistance values are such that with the adjustable rheostat set to one-half of its maximum resistance the millivoltmeter reads the total power correctly for an average value of line resistance, and the range of adjustment of the adjustable rheostat permits the maintenance of the proper total value of resistance (of instrument plus converters and line) for all temperature conditions to which the line is exposed. A push button on the instrument, when depressed, inserts additional series resistance equal to this proper total value. If the adjustable rheostat is properly set, pushing the button at any time will reduce the reading to one-half its value, and if the line resistance has changed since the adjustable rheostat was last set, the latter is adjusted until the pushing of the button cuts the reading to one-half.

As to the feasibility of using such an instrument, the first installation given by Mr. Lincoln in Table III, with 25 mi. of conductor in the line, may be taken as an example. The full-load e. m. f. of the 19 polyphase thermal converters in series is 380 millivolts, their total internal resistance is 570 ohms, and the line resistance is 530 ohms. A particular stock switchboard pyrometer millivoltmeter gives full-scale deflection with a current of 200 microamperes. Such an instrument, with its internal resistance adjusted to 800 ohms, would give full-scale deflection if connected to the Toronto line with full load on all the thermal converters. If we assume that the resistance of the line and the converters is all of copper, to take an extreme case, and that the temperature may vary from − 20 deg. cent. to + 40 deg. cent., there will be a variation of 132 ohms on either side of the average resistance at 10 deg. cent., and if this is not compensated, it will affect the readings by 7 per cent. For some rough purposes this might not be objectionable, but it is relatively easy to provide the instrument with the means above described for eliminating this error.

It may seem that a pivoted instrument requiring only 200 microamperes for full-scale deflection is too delicate for the purpose. The instrument taken as an example is made for use in industrial pyrometry where average conditions are at least as severe as in a power plant or substation. In an article in *Die Messtechnik* for April and June, 1928, summing up the status of remote metering, Dr. G. Keinath of the Siemens and Halske Co. states that d-c. indicating instruments may be used for this purpose which give full-scale deflection on as little as 50 microamperes. The matter contained in this article, with some additional matter, is given on pages 166-185 of Vol. II of the third edition of Dr. Keinath's book, "Die Technik der Elektrischen Messgeräte."

F. L. Lawton: It may be of interest to mention that very satisfactory results have been obtained with the thermal converters used by Price Brothers & Company and the Duke-Price Power Company at Isle Maligne, Quebec. These converters are used for totalizing the load of two large paper mills of the former company, the mills being 23 mi. apart. The total load is about 100,000 hp.

Our experience indicates several distinct advantages of this method of totalizing power demands, such as:

(1) Ease and reliability with which an indication of the total load on a group of feeders or transmission lines can be obtained at a remote point.

(2) Flexibility of the method—equally good results can be had at points remote from, or close to, the load-measuring instruments.

(3) High accuracy of the thermal converters. Readings from different groups of converters, totalizing the same circuits, agree very closely.

(4) Absence of moving parts means that the thermal converters can be placed in almost any location, close to the instrument transformers, thus cutting the burden on the transformers, and giving low wiring costs.

(5) The zero suppressor makes possible the recording of occasional loads in excess of the normal range, or increasing the range, with only a moment's work on the part of the station operator.

(6) A very open graphic record is obtained.

Bela Gati: By replacing the resistances and thermocouples with barretter wires, that is, wires 1 to 3 micrometers in diameter, we can get the following results: The Lincoln wattmeter's time response will be reduced to 0.00005 sec. instead of 7 sec. The sensitiveness will be one hundred times greater. I proved this in the *Physik Nalische Zeitschrift* before the War. Telephone current and voltage can be measured with barretter wires; thus the Lincoln thermo-wattmeter could be transformerd into a speech-wattmeter, an instrument which has been sought for many years.

D. A. McKenzie: The theory on which this system is developed has been well treated by Professor Lincoln. However, I might point out some of the special features of this system which appeal particularly to the operating man.

1. The thermal converters contain no moving parts. Since our experience dates back to the first of these devices ever placed in service, we naturally have had some trouble with them but these devices are now an exceptionally reliable commercial product.

2. The scheme is very flexible inasmuch as not only may these converters be connected up to add or subtract different loads but the converters themselves are reversible and are therefore suitable

for use on tie feeders. It is therefore possible to take care of almost any conceivable scheme of system connections or supply.

3. The apparatus in the transmitting station is very simple and requires a minimum of space. Since it requires no regular inspection it may be located wherever it is found convenient to connect into the measuring circuits. In fact the converter itself may be given its regular periodical calibration test from any point where it may be found convenient to connect the testing equipment into the necessary circuits.

4. By suitably arranging the transmitting circuits, it is possible at any central point to obtain the load on any desired station or section, after which the groups may be totalized.

5. An exceptionally high accuracy may be reliably obtained. These devices are regarded by the operators as absolutely accurate. The Toronto Totalizer has a scale spread of 15,000 kw. per in.

Mr. Lincoln dismisses the matter of the transmitting circuit rather summarily, but it is our belief that this part of the equipment requires very careful attention. The London equipment shown as using No. 10 B. & S. overhead wires carried on porcelain insulators on the same pole circuit as two of the 13.2 kv. feeders never operated satisfactorily even though special means of protecting this circuit were evolved and applied. As a matter of fact, the recorder was only permanently placed in service after an underground cable connection was obtained. A considerable portion of the difficulties encountered with the Toronto installation is attributed to the fact that the Toronto Hydro-Electric System does not use cables in which the wires are arranged in twisted pairs, which makes the position of each conductor in the cable with reference to all other conductors a matter of somewhat indefinite and variable quantity. Also the transmission of other quantities on other conductors in the cable has caused some interference, though it is believed that this could be considerably reduced by special arrangements. The trouble may be said to be more of electrostatic than electromagnetic interference.

So far the recorder used in connection with this system has been of one type, *viz*, a recording potentiometer of a well known make which, since it had been in commercial use for a number of years, was expected to be suitable for use as it then was. It has been found that great care must be used, as indicated by Mr. Lincoln, in selecting a recorder with a galvanometer whose characteristics are suited to the remainder of the circuit including the transmission circuit and resistance of the converter elements as the damping in this circuit plays a very great part in the character of the record obtained. Due to the slight time lag in the thermal converter, it is naturally to be expected that a somewhat smoother load curve should be obtained from these devices than from direct-acting meters. It might also be mentioned that by careful adjustment and maintenance, it has been possible to obtain a performance from these recorders which is believed to surpass materially the expectations of the manufacturers.

C. G. Brown: I should just like to ask Professor Lincoln whether in his tests on the wattmeter as he showed it, he has any results as to the accuracy of the wattmeter element with, for instance, half voltage and double current.

A. R. Wells: The recording potentiometer as used in the Toronto installation was found to be designed to record a number of temperature indications at widely different portions of the scale. It was found the meter was designed to jump from, say, 10 per cent to 70 per cent of scale for the next indication. That does not suit the case where there are small variations of load, all of which are required to be recorded, so it was necessary to modify the speed of the pen across the paper so that it moves the full width of the paper in approximately 40 sec.

There are three essential adjustments in all graphic potentiometers, (1) zero adjustment of the galvanometer, (2) the adjustment of the slide-wire current, and (3) the adjustment of the pen zero so that it coincides with the chart zero line.

It was considered necessary in view of the importance of the record, to have a record on each day's chart that all of these adjustments were correct. This was accomplished by using the fact that as long as the galvanometer pointer remains at zero the pen will make a line on the chart parallel to side of the chart.

The galvanometer zero is recorded by disconnecting it from the circuit and short circuiting it through a resistance of the proper value for damping.

The correctness of the slide-wire current is recorded by connecting the galvanometer in the usual manner for this test. If the adjustment is correct the galvanometer will be on zero and the pen will make a line parallel to the edge of the paper.

The fact that the pen zero is correct is recorded in the same way as is normal in other graphic meters, by disconnecting the meter from the external circuit, but in this case the instrument must be short-circuited through a resistance.

All of these features have proved their value in service because of the magnitude of the load and because this meter must agree with the sum of the indications of the loads which are totalized.

P. M. Lincoln: There is one point on which I expected some question concerning the method of totalizing which I have described. That was on the relatively low voltages which were generated by the so-called thermal converters. We use, as I indicated in the paper, 10 millivolts per single phase element, which will mean 20 millivolts per polyphase element, where there are two single-phase elements used to measure the total polyphase energy. Twenty millivolts per converter, even with the 23 such thermal converters used on the system of the Hydro-Electric Power Commission, will still make a total of considerably less than one volt at the potentiometer. It might be claimed that if there is any d-c. leakage into such a circuit, the error will be very considerable, which is true. It might be contended that if the individual converter voltages were higher there would be an improvement. I would question such a contention. If there is going to be any d-c. leakage, there will be an error and the error is going to be the ratio that the parasitic voltage bears to the normal voltage. If such a parasitic voltage occurs, it is, I submit, much better to have such a voltage apparent at once to the operator than to have the parasitic voltage be so small a percentage of the measured voltage that it cannot be readily distinguished. I, therefore, submit that it is preferable to have a total measured voltage which is low so that parasitic voltages can be instantly detected, rather than to have the measured voltage higher.

Mr. Brooks has indicated that there are methods of measuring the voltage other than the potentiometer which is described. It has been our experience that in all cases we have come in contact with, a graphic record is demanded and the recording potentiometer which we have been using gives such a graphic method. If we are willing to dispose of the graphic chart and simply wish an indication, there are many other methods which might be accepted and adopted and one that Mr. Brooks has mentioned is certainly one that would be considered if the graphic record were not required.

Mr. Brown asked concerning the accuracy of these thermal wattmeters with other voltages and other currents applied. He asked specifically if we took one of these thermal wattmeters which had been adjusted for normal voltage and normal current and put on half voltage and double the current, would there be any error. I can say no. There is no observable error with a change of such a nature. The observable error is less than can be read. In other words, the thermal meter is just as truly a wattmeter as the ordinary electrodynamic instrument with which we are all familiar.

A New High Accuracy Current Transformer

BY M. S. WILSON[1]

Associate, A. I. E. E.

Synopsis.—This paper is presented for the purpose of describing a new method of compensation, by the use of which the ratio and phase-angle errors of current transformers are materially reduced.

There are several methods by which the ratio and phase-angle errors of current transformers may be reduced. The following are those which are most commonly used:

1. By changing the number of secondary turns, which reduces ratio errors only. This method is universally used and operates satisfactorily under various conditions of burden, frequency, etc.

2. By the use of a larger number of ampere-turns, a large cross-section of core, or both.

An increase in either one of these factors improves the accuracy characteristics of the transformer under the different conditions of burden, frequency, or secondary current.

3. By the use of a high-permeability low-loss steel, which improves the transformer characteristics for moderate values of secondary burdens. The errors increase rapidly under overload conditions.

4. By the use of the two-stage type transformer, which operates satisfactorily under the various conditions of burden and frequency. Slightly special equipment is required, however, for power or energy measurements.

5. By the use of compensating shunts across the primary or secondary windings of the transformer. This method operates satisfactorily only under fixed conditions of burden, frequency, etc.

6. By a new method, in which the secondary turns enclose a section of the magnetic circuit and a short-circuited band encircles the same section. By the use of this method both ratio and phase-angle errors are reduced for the various conditions of burden, frequency, and secondary current.

The change in ratio and phase-angle characteristics between light and full load is considerably reduced, resulting in more nearly straight ratio and phase-angle curves. This is a desirable feature in view of the demand for meter and transformer combinations which must operate satisfactorily over large ranges of current.

The overload characteristics of the current transformer are not injured by the use of this method of reducing the transformer errors.

The usual insulation between the primary and secondary can readily be maintained.

The use of this compensating feature does not in any way impair the strength of the transformer to withstand over-current shocks.

No changes in the physical dimension of the transformer are required by the application of this compensating method.

INTRODUCTION

As is quite well known, the ratio and phase-angle errors inherent in current transformers are due to the exciting current which is required to maintain the flux and supply the losses in the core. There is a number of methods by which these errors may be reduced or compensated for. The following are most commonly used and operate with varying degrees of effectiveness for the various conditions of burden, frequency, etc., to which the transformer may be subjected.

1. The changing of secondary turns. This does not affect the losses, but raises or lowers the ratio approximately the same amount for the different values of secondary current. It has no effect on the phase angle. This method, which is universally used, operates equally well under the various conditions mentioned above.

2. The use of a large number of ampere-turns, a large cross section of core, or both. For a first approximation the errors in a current transformer are inversely proportional to the square of the number of ampere turns and to the cross section of the magnetic circuit, assuming the mean length to remain constant. Therefore, an increase in either one or both of these factors will improve the operating characteristics of the transformer under all conditions. The use of these two methods is limited, however, by the physical dimensions of the transformer.

3. The use of a high permeability low-loss steel.[2] By using a steel which has a higher permeability and lower losses than the ordinary transformer steel, the exciting current may be diminished, thus reducing the transformer errors. A material having these characteristics, however, usually saturates at quite low densities, so that for moderate burdens or overload conditions the ratio and phase-angle errors increase very rapidly.

4. The use of the two-stage type of transformer.[3] This transformer is built in two stages or cores. The function of the second stage is to supply a current which is equal in phase relation and magnitude to the exciting current required by the first stage. This type of transformer works well under any of the operating conditions to which it may be subjected. Somewhat special equipment is necessary, however, for power or energy measurements.

5. The use of compensating shunts across the primary or secondary windings of the transformer. This method of correction operates satisfactorily only for fixed conditions of burden, frequency, etc.

6. A new method has recently been developed. When used in conjunction with the first method as described above, it furnishes a simple and effective means by which the ratio and phase-angle errors of current transformers are materially reduced. This type of correction operates in such a way as to effect an alteration of the flux conditions in the magnetic circuit, so that the product of the flux and secondary turns will vary in a direct proportion to the primary current.

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[1] Standardizing Laboratory, General Electric Co., West Lynn, Mass.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

[2] *Current Transformers with Nickel-Iron Cores*, by Thomas Spooner, A. I. E. E., Trans., 1926, p. 701.

[3] *Two-Stage Current Transformer*, by H. B. Brooks and F. C. Holtz, A. I. E. E., Trans., 1922, p. 382.

This tends to produce a constant ratio between the primary and secondary currents for different percentages of load. The resulting constant error may then be reduced by the usual method of changing secondary turns.

In the new method of correction, the phase-angle errors of the current transformer are reduced by the use of a lag band on a section of the magnetic circuit. The function of this band is to lag the flux slightly, so as to throw the secondary current nearer in phase opposition to the primary current. The effect of this compensation varies with the percentage of load in such a way that a large correction is obtained in the light load region, where the errors are large, and a smaller correction as full load is approached, where the errors are less. This combination of ratio and phase angle corrector operates well under the various conditions of burden, frequency, and load, as well as under various overload conditions. The transformer is also entirely standard, in so far as the use of external apparatus in either the primary or secondary circuits is concerned.

DETAILS OF THE NEW METHOD OF REDUCING THE RATIO AND PHASE ANGLE ERRORS OF CURRENT TRANSFORMERS

The principle upon which this new type of ratio corrector operates is, in effect, that of automatically changing the effective secondary turns as the load increases or decreases.

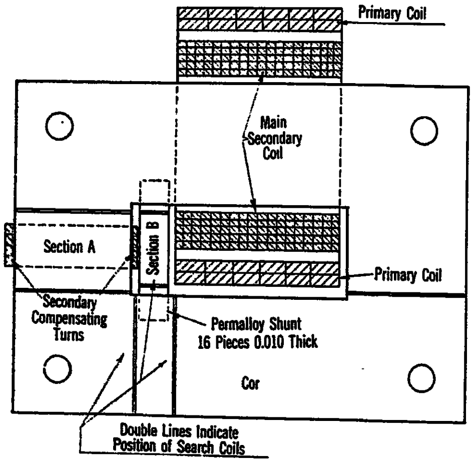
Primary Coil
Main Secondary Coil
Section A
Section B
Primary Coil
Secondary Compensating Turns
Permalloy Shunt 16 Pieces 0.010 Thick
Cor
Double Lines Indicate Position of Search Coils

Fig. 1—Sectional View of a Current Transformer

Illustrating location of compensating turns and permalloy shunt. Double lines indicate position of search coils

For example, assume a ratio correction factor of 1.010 for 10 per cent load and of 1.000 for 100 per cent load. This indicates that at 10 per cent load the secondary current will be 1 per cent too small and at 100 per cent load it will be correct. Therefore, if it were possible to increase the number of effective secondary turns by the proper amounts as the current increased from light load to full load, the ratio of the full-load point would be the same as that of the light-load point.

The resulting constant error, which in the case mentioned above would be 1 per cent, could then be eliminated by reducing the number of secondary turns 1 per cent.

In order to determine a method by which the effective secondary turns of a current transformer could be automatically changed for different load conditions, tests and calculations were made on a transformer in which the magnetic circuit and secondary turns were arranged as illustrated in Fig. 1. Here will be noted that the secondary windings are arranged in two sections on the magnetic circuit, with the larger number of turns on the first or main section, and the remainder, usually one or two per cent of the total, on the second section. Since all of these turns are wound in the same direction, a correction in the ratio would be obtained if the flux could be shunted around the smaller number of turns for the lower values of secondary current and gradually caused to flow through them in the proper direction as the current increases. This would then have the effect of adding secondary turns as the current or load increased, which in turn would increase the ratio and straighten out the ratio curve. With the proper number of turns on both sections, the five-ampere or full-load point would have approximately the same ratio as the 0.5-ampere point. This is graphically illustrated in Fig. 2.

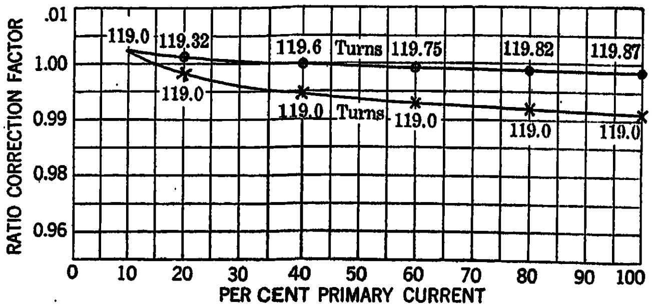

Fig. 2—Effect of Changing Partial Secondary Turns on a Current Transformer

x Standard transformer
● Compensated transformer

In order to afford a means by which the amount of flux linking the smaller number of turns could be altered, a shunt was employed which was made up of permalloy strips inserted as illustrated in Fig. 1. (Any material which has a low reluctance at low flux densities may be used for this shunt.)

The secondary turns around section A produce a magnetomotive force, the action of which is vital to the operation of this corrective method. This magnetomotive force causes a circulating flux, as illustrated in Fig. 3, which is superimposed on the main or primary flux in sections A and B. This results in a change in the relative flux densities of these two sections. Since the circulating flux varies with the secondary current, the relative flux densities in the two sections will also vary. This variation in density will cause a change in the relative reluctances of the two sections so that at higher currents, the reluctance of section B will be considerably larger than that of section A, thus causing a relatively larger portion of the main flux to flow in the latter section.

Therefore, since the proportion of main or primary flux, linking the secondary turns on section A, increases with the secondary current, the desired effect of an increase in effective secondary turns will be produced. This tends to maintain a nearly constant ratio for different values of secondary current.

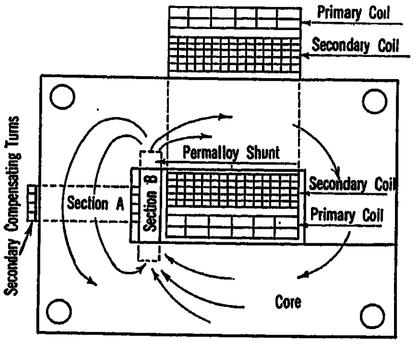

FIG. 3—REDISTRIBUTION OF FLUX IN CORE OF COMPENSATED CURRENT TRANSFORMER

From the arrangement of the turns on section A, and the arrangement of the magnetic circuit, it is seen that it is merely necessary to have a portion of the primary flux change from section B to section A as the secondary current increases, in order that the secondary turns on the larger section may become more effective and increase the ratio of transformation. This change will occur when the flux density of section B is such that the reluctance increases more rapidly than the reluctance of section A. In order to produce this condition, it is necessary to operate section B on the descending side of the permeability curve so that as the secondary current increases, the reluctance of this section will increase at a faster rate than that of Section A, which operates lower down on the permeability curve. The reluctance of the latter section may actually decrease as the flux density increases, due to the increase in secondary current or burden.

Although the ratio errors may be very materially reduced by the use of this type and construction of the magnetic circuit, it was considered impractical from a manufacturing standpoint for use in standard type of current transformers. Consequently, calculations, tests, and flux measurements were made on a transformer in which the parallel magnetic circuits were formed by an opening or hole through the laminations, as indicated in Fig. 4. The flux measurement proved that, with the proper number of secondary turns on the parallel section of the magnetic circuit, it was possible to produce flux conditions in this region which had the same effect on the ratio characteristics as was obtained by the use of the permalloy shunt. This construction proved to be satisfactory both from an electrical and mechanical standpoint and has been adopted by the General Electric Company in the manufacture of transformers using this method of compensation.

It will be noted in Fig. 4 that the secondary compensating turns enclose the narrow section of the magnetic circuit. The results obtained with the turns on either the narrow or wide section of the magnetic circuit are practically identical, providing the proper direction of winding is maintained. The direction of winding should be such that the instantaneous direction of the flux in the wider section of the parallel magnetic circuit, due to the current in these turns, will be opposite to the main flux. If these flux relations are maintained, a change in the main flux from sections S or T to R or U, respectively, as the flux density increases, will produce practically identical results regardless of whether the compensating turns enclose the wide or narrow section of the parallel magnetic circuit.

The flux densities in sections T and U are plotted on a characteristic d-c. permeability curve for D"X steel in Fig. 5, to illustrate the difference in the relative permeabilities of the two sections for 0.5 and 2.5 amperes secondary current.

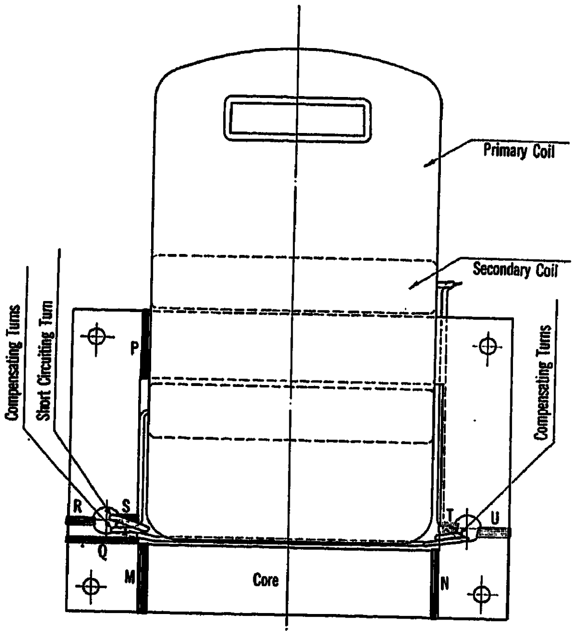

FIG. 4—15,000-VOLT COMPENSATED CURRENT TRANSFORMER

(Letters indicate position of search coils used in making flux measurements.)

It will be noted that until the flux densities indicated for both sections with 0.5 ampere secondary current are doubled, there will be but a slight change in the relative permeabilities and consequently the reluctances of the two sections. Thus, under these conditions, the compensating turns would start functioning to correct the ratio at approximately 1.0 ampere secondary current, as illustrated by the ratio curves in Fig. 7.

It is realized that the flux relations as illustrated on a d-c. permeability curve are not strictly true, but are considered satisfactory from the point of view of a qualitative analysis.

The corrective idea described thus far concerns the reduction or elimination of the ratio errors of current transformers, and has practically no effect on the phase angle. The phase angle for most current transformers, except those having a large number of ampere turns or having a large leakage reactance, are generally positive at lower current values and decrease as the current increases, until at full load rating the phase angle may be zero, or even negative. Since a positive angle indicates that the secondary current reversed leads the primary current, it is evident that if the secondary current were lagged by the proper amount, the phase angle could be reduced to zero. Also, if this lagging effect were diminished as the current is increased, it would be possible under proper conditions to correct for the large phase-angle errors at light loads or low currents and the small phase-angle errors occurring at full load.

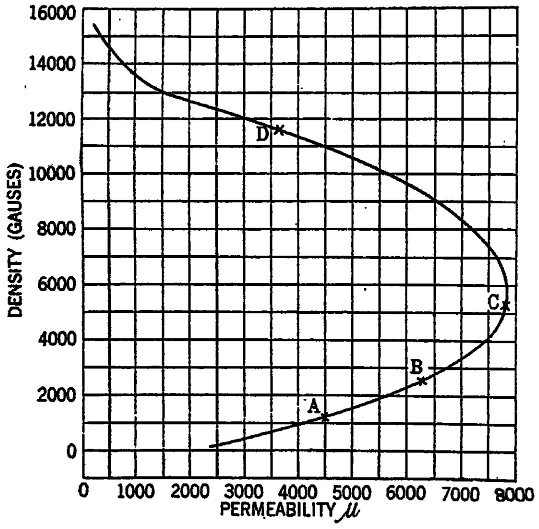

Fig. 5—Permeability Curves for *DX* (Silicon) Steel

Illustrating the flux density in the parallel magnetic circuits of a compensated current transformer

A. Section U—Secondary current 0.5 ampere
B. Section T—Secondary current 0.5 ampere
C. Section V—Secondary current 2.5 amperes
D. Section T—Secondary current 2.5 amperes

The method used to reduce the phase-angle errors involves the use of a lag band which encloses the same section of the magnetic circuit as does the secondary corrective turns. The action of the current in this band causes the main flux to lag slightly, consequently lagging the secondary current and throwing it nearer in phase opposition to the primary current, thus reducing the angle between the two currents. The action of this lag band may be likened to that of the lag band of a watt-hour meter, in which case the potential flux is caused to lag sufficiently to produce the quadrature relation between the flux and voltage of the potential element.

In the case of the compensated transformer the magnitude of the angles involved are so small that it is practically impossible to make flux measurements of sufficient accuracy to determine the exact manner in which the secondary current is caused to lag, also the reason for the change in the lagging effect as the flux density changes. Precision measurements of the phase angle accurately indicate the resulting effect of the lag band, and it is, of course, these results which are of primary importance.

In Tables I and II are given the flux densities and phase relation of the flux for the various sections of the magnetic circuit illustrated in Fig. 4, taken both with and without the compensation. In connection with these data it is interesting to note the effect of the leakage flux on the distribution of the mutual flux in the core. The density in the external member of the magnetic circuit is more than twice that of the inside member. There is also a change in phase relation of approximately 25 deg. between the fluxes in these two sections.

TABLE I

Flux measurements taken on a 1500-ampere-turn, 15,000-volt current transformer (cross-section of core 19.5 sq. cm. mean length of core 50 cm). Secondary burden 0.68 ohms resistance and 980 microhenrys inductance (19.4 volt-amperes at 0.88 power factor). Normal transformer, no secondary corrective turns or short circuited turns. (See Fig. 4.)

Section encircled by search coil	Sec. amperes	Induced voltage in search coils	Angle betw. flux in search coils and sec. current	Total flux in search coils (max. value)	Flux density gausses
P	0.498	0.0224	124°48′	841	42
R	"	0.0228	158°41′	1077	141
S	"	0.0231	159°52′	865	196
Q	"	0.0523	159°32′	1960	100
M	"	0.0546	160°48′	2046	104
N	"	0.0557	160°44′	2085	106
U	"	0.0298	159°44′	1115	146
T	"	0.0240	160°18′	900	204
P	2.506	0.1146	124°57′	4300	219
R	"	0.1447	158°35	5430	715
S	"	0.1204	159°21′	4510	1025
Q	"	0.267	159°5′	10020	512
M	"	0.278	159°49′	10420	532
N	"	0.285	160°19′	10700	547
U	"	0.1525	159°47′	5720	753
T	"	0.1256	159°33′	4710	1070
P	4.98	0.228	125°12′	8530	436
R	"	0.282	158°38′	10560	1390
S	"	0.244	159°46′	9130	2080
Q	"	0.536	159°18′	20100	1025
M	"	0.557	160°3′	20900	1065
N	"	0.572	160°36′	21450	1095
U	"	0.288	160°25′	10770	1417
T	"	0.267	159°44′	10030	2280

TABLE II

Flux measurement taken on a 1500-ampere-turn, 15,000-volt current transformer (cross-section of core 19.5 sq. cm. mean length of core 50 cm.). Secondary burden 0.68 ohms resistance and 980 microhenrys inductance (19.4 volt-amperes at 0.88 power factor). Three secondary corrective turns around Sections *S* and *T*. One short circuited turn around Section *S*. (See Fig. 4.)

Section encircled by search coil	Sec. amperes	Induced voltage in search coils	Angle betw. flux in search coils and sec. current	Total flux in search coils (max. value)	Flux density gausses
P	0.499	0.0263	127°42′	987	50.
R	"	0.0450	209°58′	1685	221
S	"	0.0458	106°32′	1718	390
Q	"	0.0566	157°40′	2132	109
M	"	0.0588	158°35′	2208	113
N	"	0.0608	159°32′	2284	117
U	"	0.248	334°12′	− 9300	1225
T	"	0.306	155°11″	11470	2610
P	2.5	0.1296	128°23′	4860	248
R	"	0.253	206°53′	9480	1246
S	"	0.222	99°33′	8320	1800
Q	"	0.285	158°24′	10680	546
M	"	0.295	159°8′	11060	565
N	"	0.306	160°7′	11460	585
U	"	1.073	346°15′	−40250	5290
T	"	1.365	164°42′	51200	11630
P	4.97	0.249	127°39′	9320	476
R	"	0.512	207°48′	19180	2520
S	"	0.445	98°16′	16680	3790
Q	"	0.556	158°46′	20840	1064
M	"	0.580	159°37′	21720	1107
N	"	0.596	160°23′	22320	1139
U	"	1.208	354°40′	−45300	5960
T	"	1.769	169°48′	66000	15000

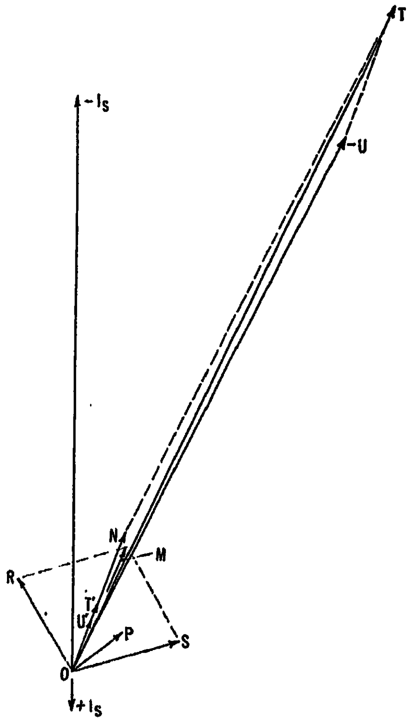

FIG. 6—VECTOR RELATIONS OF FLUX IN A COMPENSATED 15,000-VOLT CURRENT TRANSFORMER

0.5 ampere secondary current

Letters refer to sections enclosed by search coils in Fig. 4. Primes indicate measurements made without compensating turns on sections *S* or *T*

A vector diagram illustrating flux conditions in the magnetic circuit of the compensated transformer is given in Fig. 6. It will be noted on this diagram that the flux in Section *U* for the 0.5-amperes secondary point is more than eight times the amount in the uncompensated transformer for the same section. The sign is reversed which indicates a reversal of the flux in this section. This alternation of the flux condition is, as was the case when the permalloy shunt was used, due to the secondary turns on Section *T*.

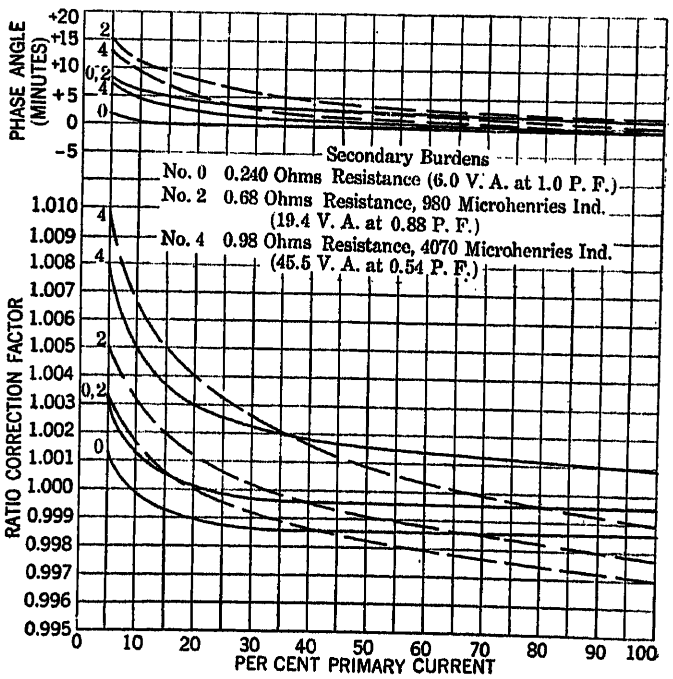

FIG. 7—RATIO AND PHASE-ANGLE CURVES

60 cycles frequency

——————Compensated type, 15,000-volt current transformer

——— ———Same transformer without compensation

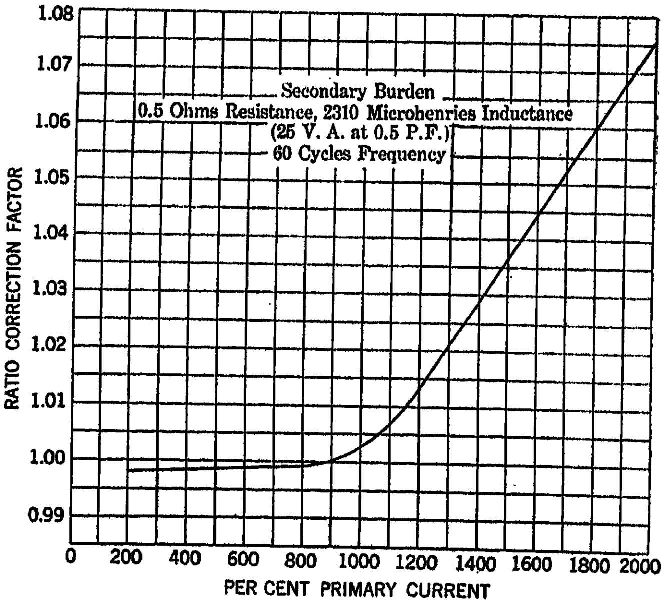

FIG. 8—OVERLOAD CURVE ON 15,000-VOLT, 20-AMPERE, 4:1 RATIO, COMPENSATED CURRENT TRANSFORMER

It should be noted that inasmuch as the flux measurements were made by the use of the a-c. potentiometer there are possibly some errors in the data due to wave form distortion. This distortion is confined, however, to the section of the magnetic circuit in the vicinity of the compensating turns, and has no appreciable effect on the normal operation of the transformer as a whole.

The phase-angle errors except for low power factor measurements are practically negligible on 60 cycles, for burdens up to 45 volt-amperes.

The use of the lag band on one of these parallel magnetic paths again alters the flux relations in the circuit by causing a large phase displacement between the flux in the two paths as is represented by vectors $O\,S$ and $O\,R$ in Fig. 6.

From the tabulated data it will be seen that similar conditions exist in the phase relation and distribution of the flux for the 2.5- and 5.0-ampere secondary current values.

RESULTS

The curves in Fig. 7 indicate comparative results obtained on the high grade metering type of transformer both with and without the new compensating feature.

These results show the straight line ratio characteristics, obtainable with the new method of compensating current transformers. This is considered a valuable feature in view of the demand for long range metering equipments, since by the use of this method of compensation the transformer range has been considerably extended in the light load regions. Although the ratio errors for a secondary burden of 0.24 ohms resistance (6.0 volt-amperes at 1.0 power factor) are in the order of only 0.15 per cent, and for most high grade metering considered negligible, a practically perfect over-all meter-transformer calibration may be obtained for values of secondary current from 10 per cent to 100 per cent load by merely adjusting the full-load calibration of the meter to take care of this error. For a secondary burden of 0.68 ohms resistance and 980 microhenrys inductance (approximately 20 volt-amperes at 0.9 power factor) the ratio of transformation is practically perfect from 15 to 100 per cent load. From 5 to 15 per cent load the error is still small, approximately 0.25 per cent.

Variations in accuracy characteristics between transformers of the same type and rating are principally caused by variations in the quality of the steel used in the cores. While the compensating method does not directly reduce these errors, it does, by considerably reducing the greater errors, make these smaller errors more important, and undoubtedly will result in a greater effort to improve the quality and uniformity of the steel used in current transformers.

The overload characteristics for this type of transformer are illustrated in Fig. 8. Tests indicate that there is practically no difference in the overload performance of the compensated and uncompensated transformers of the same type and rating.

The successful operation of protective methods employing differential and balanced relays largely depend on the similarity and uniformity of the current transformers characteristics under overload conditions. Since this compensating method can be applied to transformers of standard construction, both the compensated and uncompensated types can be used interchangeably for this service.

Accuracy tests indicate that there is practically no change in ratio or phase-angle characteristics for changes of 25 deg. cent. in ambient temperature.

The use of this new compensating feature in no way impairs the strength of the transformer to withstand over-current shocks.

The usual insulation value for breakdown between primary and secondary and primary to core can readily be maintained.

Discussion

I. F. Kinnard: The scheme of compensation described by Mr. Wilson was first conceived by him as a simple and direct method of obtaining the effect of variable secondary turns by utilizing the varying permeability of a magnetic shunt.

After much painstaking experimental work he discovered that the scheme not only worked but worked well. He found that the same results could be obtained by dispensing with the shunt and utilizing a parallel magnetic circuit in a small section of the core. The arrangement, although simple in application, is difficult of exact analysis, due largely to the local circulating flux set up. Mr. Wilson has indicated that it is not the modification in this internal secondary burden that produces the beneficial results but rather that the main flux divides according to the admittances of the paths at all times. This is probably the correct point of view.

Undoubtedly a more rigid analytical study can be made but the author has demonstrated very clearly the practical importance of the scheme and has given a clear and logical explanation of its functioning.

J. B. Gibbs: The new method which Mr. Wilson outlines gives very excellent curves and would be very valuable for current transformers if we were confined to silicon steel cores. We have found, however, for the smaller current transformers which make up the bulk of production, that equally good results can be obtained by the use of hypernik cores of proper proportions as mentioned in Mr. Wilson's paper. Comparing the transformer on the hypernik core with the former standard transformer on the silicon-steel core, we use about the same amount of copper, about one-third the weight of core, and considerably smaller dimensions. Comparing the transformer on the hypernik core with the transformer with the new compensation, we get about the same performance and smaller weights and dimensions.

A. M. Wiggins: The method of correcting for the ratio and phase angle errors in current transformers, as described by Mr. Wilson in his paper, seems to be applicable where the characteristics of the core material cannot be used to the same advantage. From the formulas derived by Dr. Agnew in the Bureau of Standards publication, Vol. 7, No. 3, the per cent ratio and phase angle of current transformers may be stated:

$$\text{Per cent ratio} = \left[\frac{n}{r} + \Delta'\left(\frac{w}{V\,A}\right)\right]100 \qquad (1)$$

$$\text{Phase angle} = \tan^{-1}\left(\Delta''\frac{w}{V\,A}\right) \qquad (2)$$

Where,
n = turn ratio, secondary to primary
r = nominal, or name-plate, ratio
w = weight of core material in lb.
VA = volt-ampere burden on the secondary, including external and internal burdens
$\Delta' = P \cos \varphi + Q \sin \varphi$
$\Delta'' = Q \cos \varphi - P \sin \varphi$
P = core loss in watts per lb. of core material
Q = magnetizing volt-amperes per lb. of core material
φ = power factor angle of secondary circuit

Now if in (1) and (2) a combination of design values can be found such that $\left(\Delta' \dfrac{w}{VA}\right)$ and $\left(\Delta'' \dfrac{w}{VA}\right)$ are constant, or nearly so, then the flat, or straight line, characteristic for ratio and phase angle curves obtained by Mr. Wilson can be realized without the compensating windings. Unfortunately, with the present commercial silicon steels this cannot be done, practicably. However, the proper use of a certain nickel-iron alloy results in nearly constant values for both these quantities over a considerable range. This alloy is described in the paper by Spooner, cited in the references in Mr. Wilson's paper.

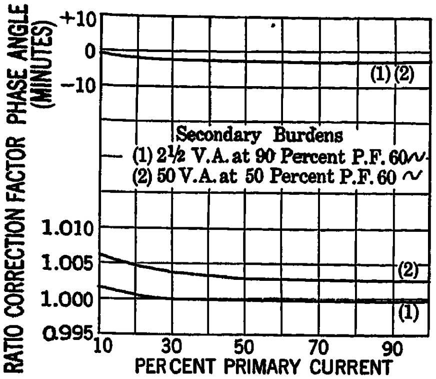

FIG. 1—RATIO AND PHASE ANGLE CURVES

Nickel-Iron Alloy Core
7500-Volt Design

Finding the correct combination for this condition is more of an experimental proposition than a calculation, as the calculations are at best only qualitatively reliable, due to the difficulty in calculating the effect of the leakage flux. As one example, in a 7500-volt design the combination which seems to give the best results employs 1500 to 1600 ampere-turns and rather high flux densities, approximately 2000 gausses, in the core section under the secondary winding with an external secondary burden of 25 volt-amperes, 0.80 power factor, 60 cycles. The weight of this core is only 3.5 lb. In all cases the flux densities which produce most satisfactory results are from 2 to 4 times those of a similar silicon-steel design, for the same burden, ampere-turns, etc. From this it might be supposed that such transformers would have very poor ratio characteristics at overcurrents, since nickel-iron alloys saturate at lower values than do silicon steels. Such is not the case, however. Figs. 1 and 2 herewith show typical performance curves for a 7500-volt design for both normal and overcurrents. From a comparison of these curves with the ones given by Mr. Wilson, it is evident that almost identical results have been obtained, but by somewhat different methods.

M. S. Wilson: The type of compensation described in the paper furnishes a simple and reliable means of improving the performance of standard current transformers.

There is, of course, a number of different methods by which the transformer characteristics may be improved. The use of a core material which reduces the exciting current obviously will reduce the ratio and phase-angle errors.

Due to the lack of a complete statement as to the secondary burden at which the overload curve presented by Mr. Wiggins was made, a discussion of this curve must be based on certain assumptions. It is the Institute practise to state the secondary burden of instrument transformers in ohms and henries, which are maintained constant regardless of the frequency or percentage of primary current.

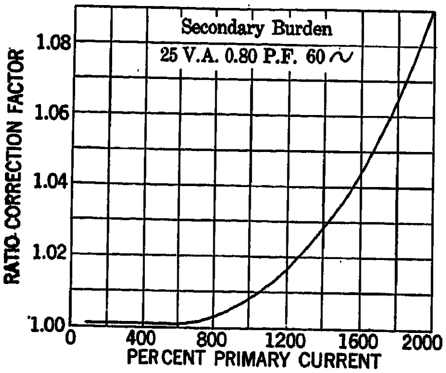

FIG. 2—OVERCURRENT CURVE

Nickel-Iron Core
7500-Volt Design

From Mr. Wiggins' description of the transformer and from the character of the overload curve shown in his Fig. 2, it would appear that the impedance of the secondary burden was adjusted for different percentages of current, so that the volt-ampere value was maintained constant. On this basis the impedance external to the transformer would be 1.0 ohm for 100 per cent current and only 0.0025 ohms for 2000 per cent current, whereas in the compensated type 1 ohm impedance was held constant throughout the entire range of primary current as shown by the curve in Fig. 8 of the paper.

On this assumption the overload curves for the two types are not comparable.

132-Kv. Shielded Potentiometer
for Determining the Accuracy of Potential Transformers

BY C. T. WELLER[1]
Member, A. I. E. E.

Synopsis.—The potentiometer principle is well understood. Its application to the determination of the ratio and phase angle accuracy of potential transformers is not new, but has been limited in unshielded form to about 66 kv. and in shielded form to about 33 kv. The paper describes a shielded potentiometer for 132 kv. which is designed for ultimate extension to 220 kv.

INTRODUCTION

THE first potentiometer used in the Schenectady Laboratory for determining the ratio and phase angle accuracy of potential transformers was described in 1909. This "deflection" potentiometer was rated at 33 kv. and remained in service until 1920. By that time many 66-kv. potential transformers had been built so that it became necessary either to extend the voltage range of the potentiometer or to abandon the early policy of having equipment available for testing at full rated voltage. A second potentiometer rated at 66 kv., which differed from the first principally in the type of resistors used, was therefore built and put into operation; but even this greater range soon became inadequate. The accumulated experience showed that the general scheme employed was entirely satisfactory but that a further extension of the voltage range would necessitate radical changes in the construction and arrangement of the resistors, principally to secure better dissipation of heat and to permit shielding against capacitance to ground.

In order to make further extension possible, the author proposed in 1924 to construct 100,000-ohm resistor units of fairly large dimensions rated at 11 kv. each, and to maintain the respective unit shields at the proper potentials by means of a special auto-transformer (in separate sections) of ample rating with suitable taps. Four of the units and one auto-transformer section constitute a complete 44-kv. group; the number of series groups necessary is determined by the voltage range desired. To the best of his knowledge, the proposed arrangement and method of shielding had not been utilized before. The 132-kv. potentiometer herein described incorporates the proposed features.

OBJECTIVE

A potentiometer was desired, which would (1) cover the range from 6.6 to 132 kv. at first and to 220 kv. by future extension and (2) conform as closely as practicable to an ideal potentiometer over the entire range.

An ideal potentiometer should incorporate the following features:

a. The d-c. resistance of the unit(s) obtained by means of a low voltage bridge should remain constant and should equal the a-c. effective resistance at operating voltage.

b. The current in all parts of the resistor(s) should be exactly proportional to and in phase with the line voltage.

c. The flexibility required should be obtainable without excessive complications and without sacrificing convenience of operation.

SUMMARY

The installation of three 44-kv. series groups and accessories at Pittsfield was completed in 1928. The groups alone occupy about 380 sq. ft. (42 ft. by 9 ft.) of floor space, have a height of approximately 12 ft. and weigh about 65 tons (the first potentiometer complete weighed less than one ton). A "General Description" of the potentiometer is given under that title.

Potential transformers have been tested for accuracy over the range from 6.6 to 132 kv. at 25 and at 60 cycles (see comments under "Test Results").

Compared with the ideal outlined:

a. The d-c. resistance of the units is practically constant and agrees with the a-c. resistance within 0.01 per cent.

b. The current in the units is directly proportional to the line voltage at all points but small phase displacements exist. The "net" phase angle of the current at 60 cycles ranges from approximately $-1'$ (leading) at 6.6 kv. to $-7'$ at 44 kv. in the first group, to $-6'$ at 88 kv. with two groups and to $-4'$ at 132 kv. with three groups; the corresponding corrections have the same signs.

c. Transformers rated between 6.6 and 44 kv. in 1.1-kv. steps (with a corresponding increase in ratio) and between 44 and 132 kv. in 5.5-kv. steps can be readily tested; this includes practically all standard ratings. Odd ratings differing less than 4 per cent from those indicated can usually be tested without special adjustments, but other odd ratings require an adjust-

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[1] General Engineering Laboratory, General Electric Company, Schenectady, N. Y.

[2] *Electrical Measurements on Circuits Requiring Current and Potential Transformers*, by L. T. Robinson, A. I. E. E. TRANS., Vol. 28, 1909, p. 1005.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

ment of the resistance. The ratio (correction factor) is read directly (or by inter-polation) for standard ratings but must be calculated for odd ratings; the phase angle is calculated from instrument deflections in all cases.

The complications are not excessive and the operation is reasonably convenient; provision is made also for operating with the mid-point instead of with the lower end of the potentiometer grounded.

The twelve resistor units and the three auto-transformer sections or shielding transformers are respectively interchangeable.

CONCLUSIONS

The accuracy obtainable is sufficient to permit certifying results to within 0.1 per cent in ratio and to within five minutes in phase angle over the entire range; these limits are in accord with our best previous practise.

The potentiometer can be extended to 220 kv. without making radical changes in any of the groups whenever the demand justifies it.

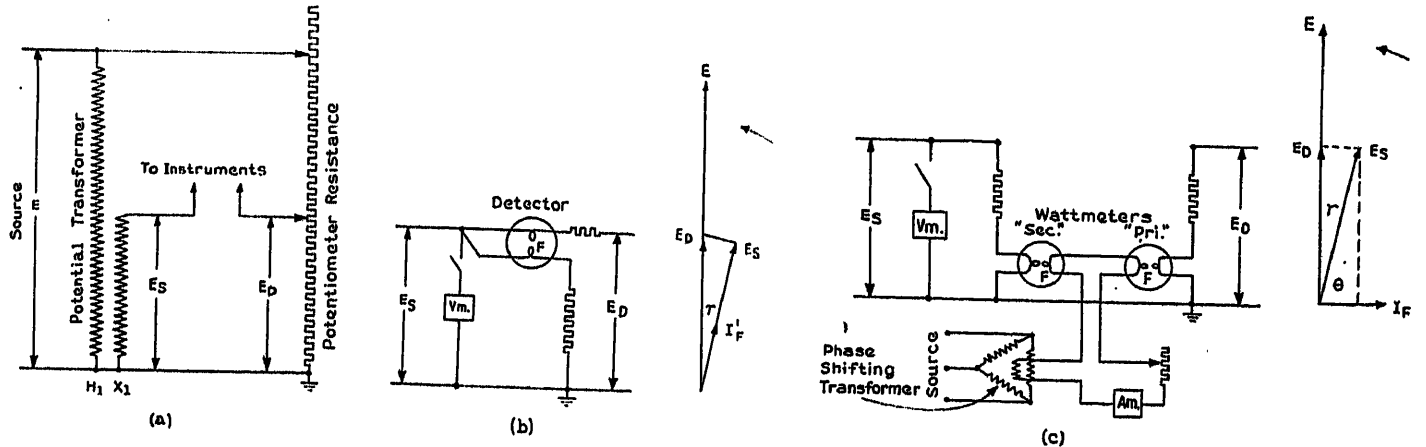

FIG. 1—PRINCIPLES OF OPERATION OF "DEFLECTION" POTENTIOMETERS

(a) Ratio and phase angle
(b) Ratio only
(c) Phase angle only

PRINCIPLES OF OPERATION

The principles of operation of our "deflection" potentiometers, which utilize deflections as well as null or zero instrument readings in comparing voltages, have been described[3] but are outlined again here. These principles will be apparent from a study of Fig. 1.

In the general diagram Fig. 1a, which applies to the determination of the ratio and the phase angle between E and E_S, the primary H_1 winding of the potential transformer under test and the potentiometer resistance at the proper tap are connected in multiple and supplied with rated voltage E from a grounded sine wave source. The transformer secondary X_1 voltage E_S is compared with the voltage drop E_D across a known fraction of the potentiometer resistance in use.

The potentiometer resistance chosen is 9.09 ohms per volt (1000 ohms for each 110 volts). The ratio between the total potentiometer resistance in use and the normal known fraction thereof is the same as the marked or nominal ratio of the transformer under test (60:1 or 60,000:1000 ohms for a 6600:110-volt transformer).

In Fig. 1b, which applies only to the determination of the ratio between E and E_S, a "balance" is obtained by interpolation between the 0.1 per cent steps (not indicated) in the fractional resistance; the corresponding ratio accuracy (ratio correction factor) is read from a dial scale; the correction due to the phase angle γ is usually negligible (equals only $+$ 0.015 per cent for a γ of $\pm$ 1 deg.).

The fractional resistance is made up of a 950-ohm "fixed" resistor and a 100-ohm "dial" resistor with one-ohm steps and a contact arm. Since the normal resistance value is 1000 ohms, this gives a range of $\pm$ 5 per cent in 0.1 per cent steps in the drop voltage E_D. Each 0.1 per cent step is marked directly in "ratio correction factor,"

$$\left(\text{or } R.\,C.\,F.;\text{ this is the } \frac{\text{true ratio}}{\text{marked ratio}} \text{ or } \frac{T\,R}{M\,R}\right).$$

The field F of the ratio detector is supplied (with I_F') by E_S, while the moving element is supplied by the vector difference between E_S and E_D; series resistors are used in both detector circuits. If a balance between E_S and E_D, as shown by a zero detector reading, is not obtainable on any step in the "dial" resistor, interpolation between any two adjacent steps is readily made by means of the two corresponding deflections. The ratio between the total potentiometer resistance in use and the fractional resistance at the "balance" point is evidently the true ratio of the potential transformer under test ($T\,R = M\,R \times R.\,C.\,F.$). The correction necessary for occasional large transformer phase angles γ is due to the fact that γ causes the hypotenuse E_D of the voltage triangle to exceed slightly the leg E_S in magnitude.

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[3] L. T. Robinson, *Loc. cit.*

In Fig. 1c, which applies only to the determination of the phase angle between E and E_S, the phase angle accuracy (γ, in minutes) is obtained by the two-wattmeter method; the phase of the excitation I_F is shifted until the "primary wattmeter" reads 0, when the "secondary watts" W_S divided by the volt-amperes equals $\cos \theta$ or $\sin \gamma$; that is:

$$\text{Sin } \gamma = \frac{W_S}{E_S I_F}$$

γ is read from a table of sines; the correction for the phase angle Δ (not indicated) of the potentiometer resistance in use is applied. A positive γ (E_S lags E) is indicated in the figure.

The fields F of the two practically identical wattmeters are supplied in series (with I_F) from the secondary of the phase shifting transformer, the primary of which is supplied from the main polyphase source. The field current I_F is set at a convenient value as shown by the 10-ampere ammeter *Am*. The moving element of the primary wattmeter is supplied by E_D, while that of the secondary wattmeter is supplied by E_S; the series resistors in the two circuits are also practically identical.

Five essential instruments are indicated in Fig. 1b and c, of which three (detector and wattmeters) are high-sensitivity, Type A L-2, astatic reflecting dynamometers and two are Type P-3 portable instruments. These instruments constitute an effective secondary burden of 0.5 volt-ampere; the single 150-volt voltmeter *Vm* is connected only when setting E_S and only one dynamometer at a time is supplied by E_S.

Although differing in many respects from its predecessors, the 132-kv. potentiometer incorporates only one important additional principle; that of shielding against capacitance of the resistance circuit to ground.

EFFECT OF CAPACITANCE

All potentiometer resistance used has been of manganin wire non-inductively wound and immersed in oil. Troublesome resistance changes and appreciable inductance were thus eliminated.

In the first potentiometer (33-kv.), the 0.010-in. manganin wire was wound on mica cards. The capacitance of the card sections to the grounded metal oil tank and the "offsetting" capacitance between the sections were both small.

In the second potentiometer (66-kv.), the 0.010-in. manganin wire was wound on brass tubes. Owing to the comparatively large amount of metal used and to the doubled-voltage range, the capacitance of the tube sections to the tank was comparatively large. The "offsetting" capacitance between the sections was also larger than before. The importance of these capacitances (particularly the former) at high voltages is indicated by the fact that at 60 cycles the "net" phase angle (and correction) at 22 kv. was + 3′ (lagging) while at 66 kv. it was + 51′.

In outlining the effects of capacitance, it is convenient, although not exact, to consider the distributed "internal" capacitance between resistor sections and the distributed "external" capacitance between the resistor sections and the grounded tank as independent "lumped" capacitances.

The "internal" capacitance between resistor sections acts as a shunt to the sections and causes the current to lead the voltage slightly. In Fig. 2a, the distributed capacitance between sections is represented by one capacitor C, which is shunted across part of the potentiometer resistance. The effect is indicated in the vector diagram. The current I in the circuit and the voltage drop E_D in the fractional resistance lead the supply voltage E by the angle Δ (the fractional resistance is actually separate from the other resistance and has a small "net" positive phase angle Δ' of its own—this has been omitted for clearness). The negative angle Δ represents the potentiometer phase angle and correction.

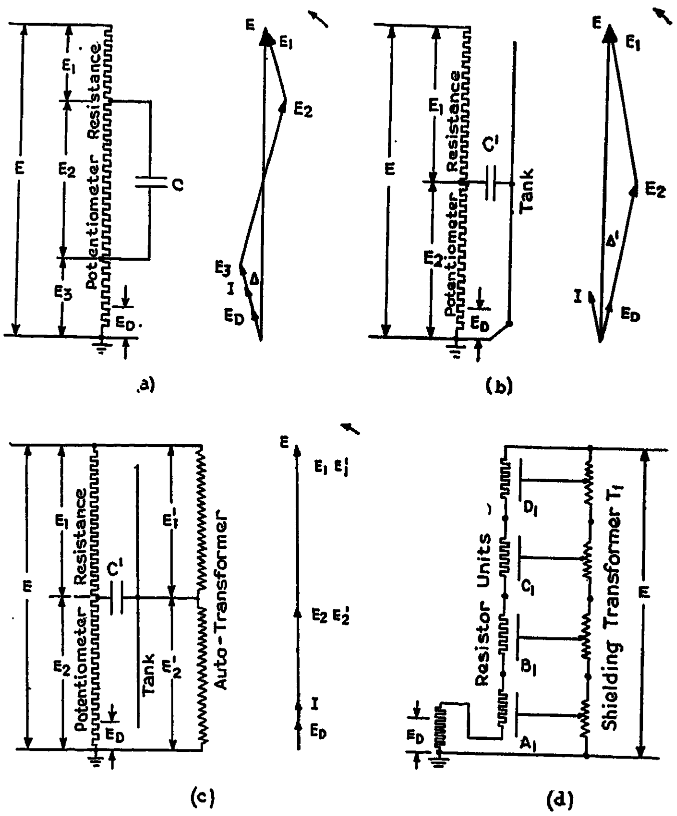

FIG. 2—EFFECTS OF CAPACITANCE

(a) "Internal" capacitance between sections
(b) "External" capacitance to tank (ground)
(c) Elimination of effect of (b) by means of an auto-transformer
(d) Schematic arrangement of 44-kv., Group No. 1

The "external" capacitance between the resistor sections and the grounded tank acts as a shunt to the grounded end of the potentiometer resistance. In Fig. 2b the distributed capacitance to the tank is represented by one capacitor C', which is shunted across the grounded half of the potentiometer resistance.

The effect is indicated in the vector diagram. The current I in the circuit leads the supply voltage E but the voltage drop E_D lags E by the angle Δ', which is now positive.

It will be evident that if the potential of the tank is raised to that of the mid-point of the potentiometer resistance, no current will flow in the capacitor C'. In Fig. 2c this is accomplished by means of an auto transformer with a middle tap. In the vector diagram, I and E_D are now in phase with E.

This, in general, is the scheme adopted for the 132-kv. potentiometer. The schematic arrangement of the first 44-kv. group is indicated in Fig. 2d. The tanks of the four resistor units are maintained at mid-unit potential by means of taps on the auto or shielding transformer; the fractional and accessory resistances are taken out of and are separate from the first unit. The "net" phase angle is leading ($-7'$ at 60 cycles) at the highest resistance tap (44 kv.) due to the effect to the "internal" capacitance; at lower taps this is partly offset by the effect of the "external capacitance to the tank(s) which is "inserted" by supplying the shielding transformer on a higher than normal tap (at 6.6 kv. $\Delta = -1'$ instead of $-7'$).

All unit tanks in use are maintained at mid-unit potential for taps above 44 kv. in the second and third groups. The "basic" negative angle ($-7'$) is offset to some extent by the effect of the small direct capacitance of the resistance circuit, (principally the external connecting leads) to the grounded group area screen ($\Delta = -6'$ at 88 kv. and $-4'$ at 132 kv.); this capacitance effect can be practically eliminated by additional shielding, if desired.

General Description

The 132-kv. shielded potentiometer combines potentiometer resistance suitable for alternating current with shielding transformers and other special auxiliary equipment. It is conveniently arranged for use, in connection with standard power-supply equipment, in determining the ratio and phase-angle accuracy of potential transformers rated from 6.6 to 132 kv., 25 to 60 cycles and 25 to 1000 volt-amperes, respectively.

The potentiometer is shown schematically in Fig. 3a and b and directly in Figs. 6 and 7. The three 44-kv. series operating groups, each consisting of four 100,000-ohm resistor units and one shielding transformer, occupy about 380 sq. ft. (42 ft. by 9 ft.) of floor space and have a maximum height of about 12 ft., 3 ft. of which is due to the Herkolite insulating cylinders. The group area contains about 900 sq. ft. (52 ft. by 17.5 ft.); part of the grounded wire mesh, which acts as an electrostatic screen for the groups, is buried under the floor, while the remainder forms a fence 20 ft. high on the four sides of the area. The supply voltage is brought in from the testing area through high voltage bushings. The metal and glass operating booth is about 16 ft. square and 8 ft. high. It contains the fractional and accessory resistances and all instruments and control equipment. Approved safety devices are used wherever necessary.

Resistors. The potentiometer has a total resistance of 1,200,000 ohms and is rated at 0.11 ampere. The resistance is divided into three 400,000-ohm operating groups, which are subdivided into four 100,000-ohm units. From the construction standpoint the 100,000-ohm interchangeable units are basic; they are made up of ten 10,000-ohm sections, which are in turn made up of twelve 833.3-ohm tubes each. The arrangement and ratings are summarized in Table I. A "Detailed Description," which includes the fractional (normal value—1000 ohms) and accessory resistances, is given under that title.

TABLE I
ARRANGEMENT AND RATINGS OF POTENTIOMETER RESISTORS

Item	Rating		Number Required	
	Ohms	Kv.	Per Step	Total
Tube........	833.3*	0.092	12 per Section	1440
Section......	10,000†	1.1	10 per Unit	120
Unit........	100,000	11	4 per Group	12
Group.......	400,000	44	3 for 132 kv.	3
3 Groups....	1,200,000	132		

*Approximate adjustment—rated at 0.11 ampere (9.09 ohms per volt) nominally, 0.125 ampere maximum.

†Adjustment correct within 0.02 per cent.

Shielding Transformers. The three interchangeable shielding transformers (one for each 44-kv. group) are rated as follows: 25 cycles–75 kv-a.–50,000: 500/500 volts. (The maximum voltage rating for the three primary windings in series is 150 kv.) The secondary and tertiary windings are considered as equalizing windings as their function is to equalize the flux density and voltage distribution in the transformers when either two or three are operating in series; the connections are indicated in Fig. 3a and b. Each transformer has eight primary taps and nine bushings, one of which supports the somewhat complicated tap switch. The four unit tanks are connected to four of the bushings and from them to the proper transformer taps by means of the switch and thus maintained at predetermined potentials.

Operation. In operating, the total resistance used in each case is obtained by setting the tap switch on the 100 per cent tap in each of the 100,000-ohm resistor units up to the unit in which the final setting is made; all or only part of the resistance of this unit may be required (ten equal steps are obtainable with the tap switch but only the 100 per cent and 50 per cent taps are, in general, used above group No. 1); the tap switch is set on the zero tap in each of the 100,000-ohm units beyond the unit in which the final setting is made. The correctness of the setting is then checked by a bridge measurement; this procedure is independent of the number of units used, since all units are connected in series; however, connection is made to the upper end of the "highest" group involved. The connections for 600,000 ohms or 66 kv. are indicated in Fig. 3a; the connection Z between the resistance and shielding transformer circuits is opened for the bridge measurement; the 0.2-ampere ammeter (Am) gives an approximate indication of the operating current.

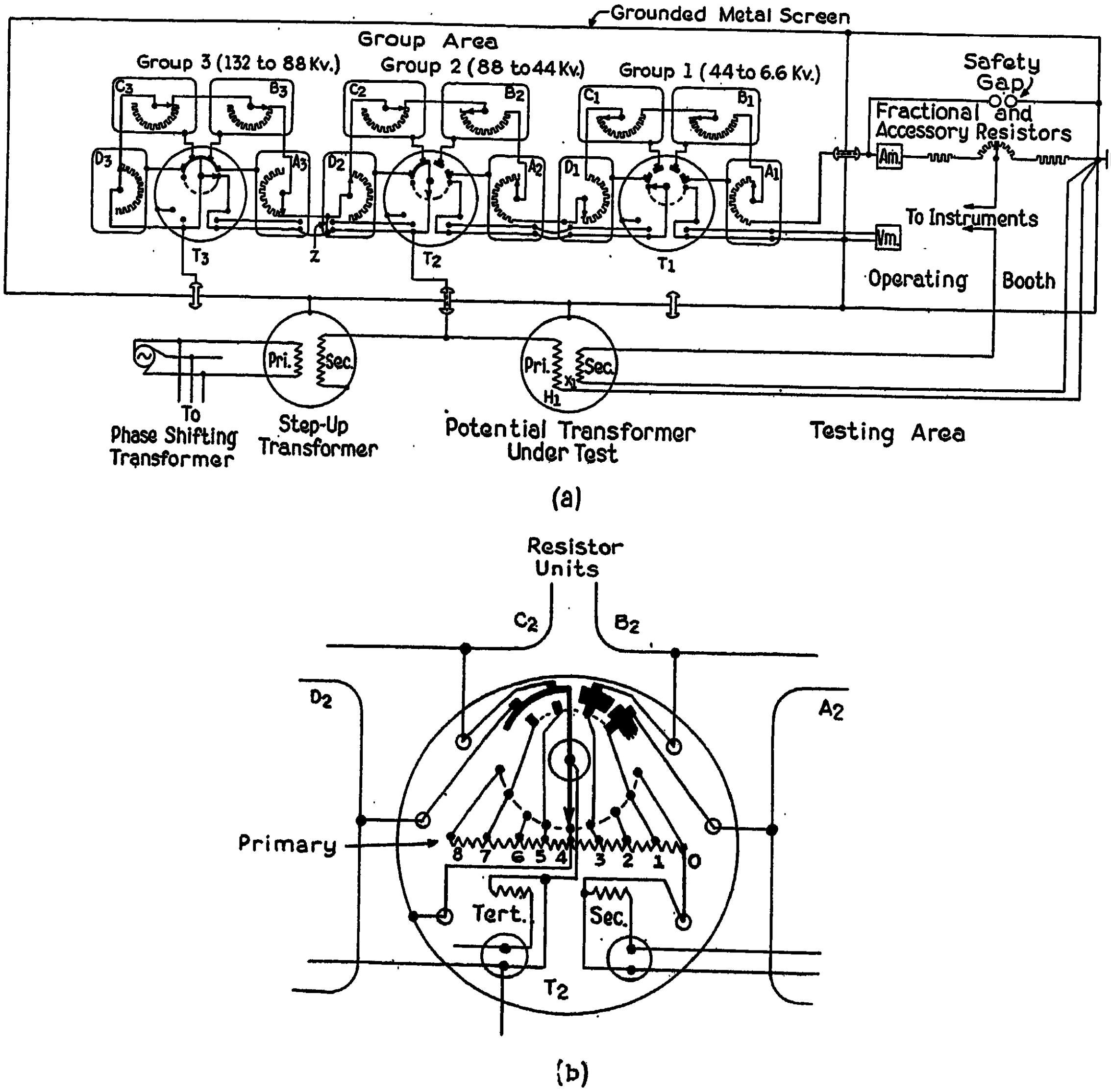

FIG. 3—132-KV. POTENTIOMETER—66-KV. CONNECTION (SCHEMATIC)

(a) General outline

(b) Shielding transformer T_2 in Group No. 2

The shielding transformers are excited magnetically only when one or more resistor units in their respective groups are in use. In group No. 1, transformer T_1 is excited on tap No. 6 (nominally 33 kv.) for voltages between 6.6 and 30.8 kv. to obtain a minimum phase angle; taps No. 7 and 8 are used for higher voltages up to 44 kv. One lead between the tertiary of T_1 and the secondary of T_2 is opened (similarly between T_2 and T_3); the other lead assures electrostatic excitation of group No. 2 (and No. 3) at line voltage; the tap switch of T_2 (and T_3) is set on the zero tap. Transformer T_2 is also excited magnetically for voltages between 44 and 88 kv. and transformer T_3 for voltages between 88 and 132 kv.; the "highest" transformer tap used in either group No. 2 or group No. 3 has, in general, the same nominal voltage rating as the corresponding tap (100 per cent or 50 per cent only) of the "highest" resistor unit in use. The correctness of the shielding transformer tap setting is checked in all cases by comparing the reading of a 600-volt voltmeter (Vm) supplied from the secondary of T_1 with the reading of a 150-volt voltmeter supplied from the secondary of the potential transformer under test; the proper ratio between the two readings is predetermined for each marked or nominal potential transformer ratio. The connections for 66 kv. are indicated in Fig. 3a and b.

The distribution of voltage in the resistor units and in the shielding transformers for the 132-kv. connection is summarized in Table II. In this case the indicated current in the resistor circuit would be approximately

0.11 ampere, while the ratio between the secondary voltages of T_1 and of a 132-kv:110-volt (1200:1 ratio) potential transformer under test would be approximately 440:110 or 4:1.

After obtaining the proper settings as outlined, the desired secondary burden is set (by means of dial switches) and verified by instrument readings. The ratio and phase angle readings are then taken as indicated under "Principles of Operation."

TABLE II
DISTRIBUTION OF VOLTAGE AT 132 KV. IN THE THREE 44-KV. GROUPS

Designation			Shielding Transformer Tap		
Group No.	Resis. Unit	Shield Trans.	No.	Kv.	Connected to
			0	0	Ground
			1	5.5	Tank of 1st Unit (A_1)
	A_1		2	11 *	—
			3	16.5	Tank of 2nd Unit (B_1)
1	B_1	T_1	4	22 *	—
			5	27.5	Tank of 3rd Unit (C_1)
	C_1		6	33 *	—
			7	38.5	Tank of 4th Unit (D_1)
	D_1		8	44 *	Tap No. 0 of T_2
			0	44	Tap No. 8 of T_1
			1	49.5	Tank of 5th Unit (A_2)
	A_2		2	55 *	—
			3	60.5	Tank of 6th Unit (B_2)
2	B_2	T_2	4	66 *	—
			5	71.5	Tank of 7th Unit (C_2)
	C_2		6	77 *	—
			7	82.5	Tank of 8th Unit (D_2)
	D_2		8	88 *	Tap No. 0 of T_3
			0	88	Tap No. 8 of T_2
			1	93.5	Tank of 9th Unit (A_3)
	A_3		2	99 *	—
			3	104.5	Tank of 10th Unit (B_3)
3	B_3	T_3	4	110 *	—
			5	115.5	Tank of 11th Unit (C_3)
	C_3		6	121 *	—
			7	126.5	Tank of 12th Unit (D_3)
	D_3		8	132 *	Line

*Voltage at corresponding resistor unit; the resistor and transformer circuits are joined at "Ground" and "Line" only.

DETAILED DESCRIPTION

The principal items described are: (1) 100,000-ohm resistor units, (2) fractional and accessory resistors, (3) operating panel and burdens, (4) shielding transformers, and (5) connections; a partial list of auxiliary equipment is included under (6).

1. *100,000-Ohm Resistor Units*

Construction details of the units are shown in Figs. 4, 5, 6, 9, and 10. Each unit is rated 11 kv., 0.11 ampere; its principal parts are: (a) 833-ohm resistor tubes, (b) 10,000-ohm resistor sections, (c) woodwork, (d) tap changing switch, (e) tank, (f) No. 10-C. Transil oil, (g) top shields, and (h) Herkolite supporting cylinders. The total of 100,000 ohms is made up of ten accurately adjusted (within 0.02 per cent) 10,000-ohm sections, each of which is made up of twelve approximately adjusted 833-ohm resistor tubes.

a. 833-Ohm Resistor Tubes. Several 833-ohm resistor tubes and one 10,000-ohm resistor section are shown in Fig. 4. Each resistor tube is made up of perforated sheet brass; it is 12 in. long with an outside diameter of 1.25 in. Treated paper is used for insulation between the tube and the first layer of wire and also between the first and second layers of wire. The two layers each have the same number of turns of 0.011-in. silk-covered manganin wire; the layers are connected in series (through the tube) with their respective magnetic fields in opposition (the small residual inductance is "offset" by the capacitance between layers and to the tube so that the "net" phase angle is very small). Three different black insulating compounds are used for impregnation; the compounds become oil-proof after suitable baking.

b. 10,000-Ohm Resistor Sections. Twelve tubes are assembled with insulating spacers, binders, and tie rods to make up a section approximately 36 in. by 8 in. The top binder, which contains the terminals, is made long enough to permit suspending the section between two parallel horizontal wooden supports. The section contains about the minimum insulating material consistent with proper insulation between tubes, as it is important to keep the overall dimensions as small as possible in order to minimize "leakage" and capacitance to the tank (see Preliminary Work). The arrangement of the section and its supports facilitates the dissipation of heat. The sections are interchangeable.

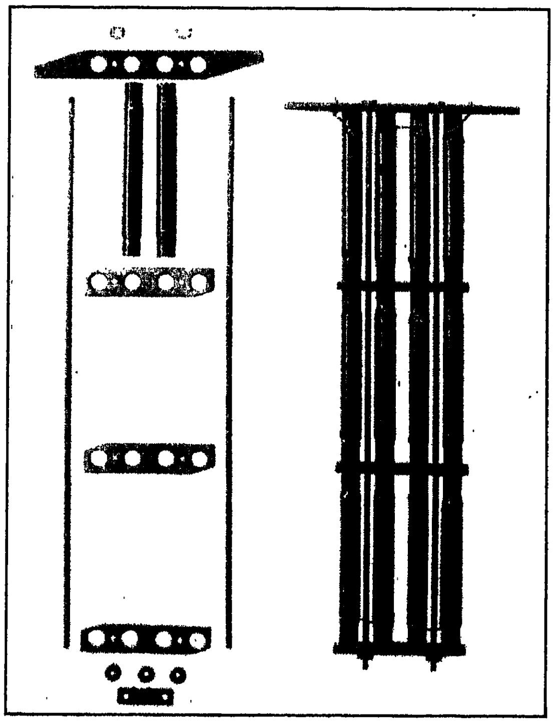

FIG. 4—PARTS AND ASSEMBLY OF A 10,000-OHM RESISTOR SECTION

(Approximately 36 in. x 8 in): 833-ohm resistor tubes; insulating spacers, binders, and tie rods

c. Woodwork. The treated woodwork is shown in Fig. 5. The framework for supporting the ten resistor sections is arranged to permit a zigzag assembly of the sections, which gives the maximum insulation and the minimum capacitance between adjacent terminals and tubes of opposite polarity; the sections can be readily removed.

The framework is supported by the cover, which is approximately 5 ft. by 3.5 ft. The cover also supports the tap changing switch; the bushing for the switch and the similar bushing for the "zero" resistance tap extend through the cover. The edges of the cover project slightly beyond the sides of the tank; the edges are cut to a 1-in. radius and are covered with sheet copper which extends back about 6 in. to assist in shielding the resistor sections and in preventing corona: the remaining surface is protected by a sheet of treated pressboard.

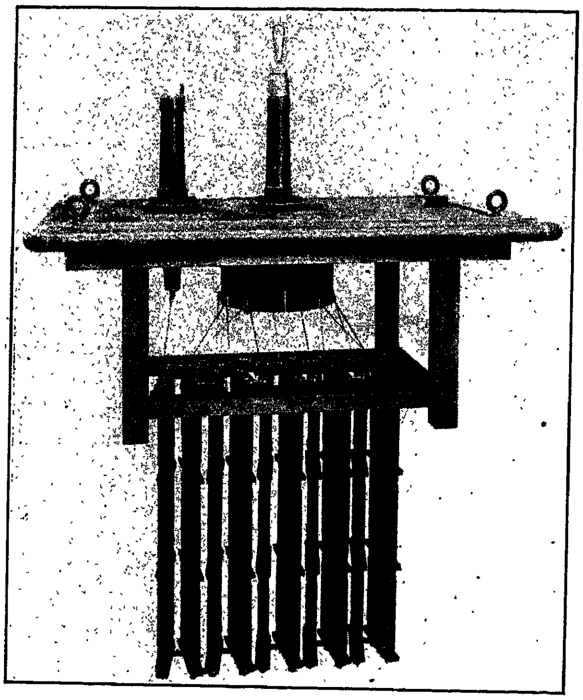

FIG. 5—100,000-OHM RESISTOR UNIT UNTANKED

With treated wood bushings cover (5 ft. x 3.5 ft.) and resistor supports; tap-changing switch; ten 10,000-ohm "zigzag" resistor sections. Welded steel tank (5.5 ft. deep) and two top shields (3.5 ft. high) not shown

d. Tap Changing Switch. The tap changing switch is shown in Fig. 5; it consists of an operating handle, position indicator, shaft and contact arm supported by the bushing (and cover); eleven contact points are mounted on the lower edge of an insulating cylinder which is supported by the cover and slotted to increase the creepage distance between the contact points. The number of sections in series can be readily changed from 0 to 10 by means of the switch.

e. Tank. The tank is shown in Fig. 10; it is approximately 5 ft. long, 3.5 ft. wide, and 5.5 ft. deep and is made up principally of sheet steel with welded seams. Four special internal lifting lugs, each of which has two holes to receive the top shields and a tapped hole for the cover "grounding bolt," and the angle pieces for supporting the cover are welded to the inside top edges of the tank. The outside corners and lower edges of the tank have a 2-in. radius to prevent corona up to 250 kv.

The tank contains the No. 10-C Transil oil and acts as a radiator of heat and as an electrostatic shield for the resistor sections on five sides. The "grounded" copper edges of the cover act as a partial shield for the top of the sections.

f. No. 10-C Transil Oil. The oil acts as a heat storage and transfer medium and as an insulator for the resistors; by its use, creepage between the resistor tubes or sections is reduced while corona is eliminated. There are about 530 gallons of oil in each tank; the oil level is just below the tap switch contacts.

g. Top Shields. The two top shields for each resistor unit are shown in Fig. 9; they are approximately 4.5 ft. and 3 ft. long, respectively, and extend about 3.5 ft. above the cover; they are made up of 1.75-in. diameter steel tubing welded at the joints; there are three parallel and two vertical bars; the latter set into the holes in the tank lifting lugs and are readily removable.

The two shields are located on the most exposed sides of each unit in the group and help to shield the top of the resistor sections and the terminals (bushings) and also the leads connecting the units. The diameter of the tubing is sufficient to prevent corona up to about 160 kv.

h. Herkolite Supporting Cylinders. The Herkolite cylinders are shown in Fig. 10; four cylinders, each 36 in. long with an outside diameter of about 18 in., support each unit and insulate it from ground and from adjacent units; the complete unit weighs about 2.8 tons.

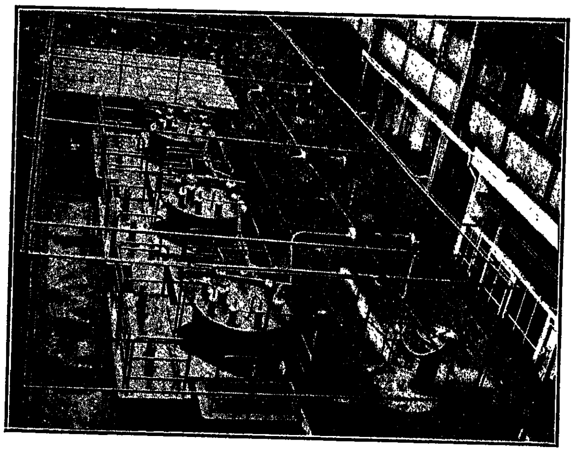

FIG. 6—132-KV. SHIELDED POTENTIOMETER: THREE 44-KV. GROUPS, GROUP AREA AND SCREEN; OPERATING BOOTH; TESTING AREA

2. Fractional and Accessory Resistors

A 2600-ohm adjustable resistor is shown in Fig. 8. There are two of these resistors; one is used as the 950-ohm "fixed" resistor, which, together with the 100-ohm "dial" resistor (not shown), makes up the fractional resistance; the other is used as an accessory resistor to make up the difference between the fractional resistance and the 2500 ohms (approximate) short-circuited in the first 10,000-ohm resistor section of the first unit A_1.

The general scheme of operation adopted necessitates "bringing out" the fractional resistance from unit A_1. The approximate value chosen (2500 ohms) makes possible a special maximum fractional resistance of 2000 ohms (instead of the normal 1000 ohms) for use in interchecking between potentiometer resistance taps and in testing special potential transformers with 220-volt secondaries.

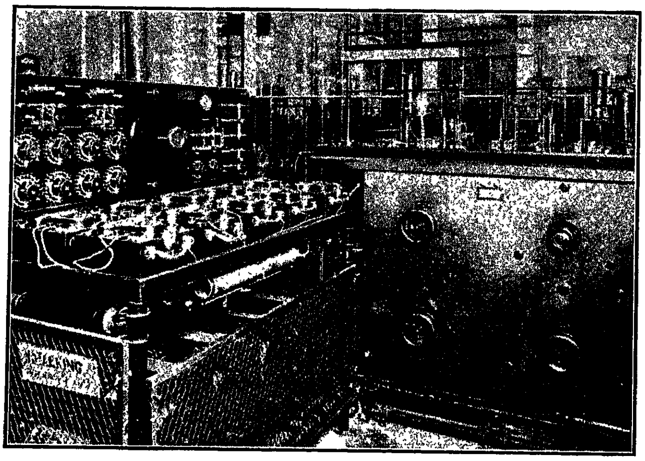

FIG. 7—INTERIOR OF OPERATING BOOTH: OPERATING PANEL, INSTRUMENTS, AND BURDENS; PIER FOR REFLECTING DYNAMOMETERS; FIELD CONTROL PANEL

Each resistor consists of fifteen resistor tubes with the necessary taps; the corresponding terminals are conveniently arranged to give resistance values from "zero" to 2611 ohms in 0.1-ohm steps. The tubes, which are of insulating material, are wound differently from the 833-ohm tubes. There are two "opposing" windings of manganin wire in one layer; the two windings are connected in multiple; this scheme permits tapping the joint winding at any point.

The container consists of an oil-filled glass jar set in a box of treated wood; the box is covered with sheet brass which is grounded. The purpose in view was to provide suitable "heat insulation" so that the heating of the 2600-ohm resistors would simulate that of the 100,000-ohm resistor units. The two 2600-ohm resistors are interchangeable.

3. Operating Panel and Burdens

The operating panel is shown at the left in Fig. 7. The switches at the right of the panel control the connections to the instruments (the switches on the horizontal panel on the pier are supplementary). The current in the potentiometer resistance circuit is read on the ammeter at the top. The center handwheel is attached to, but insulated from, the contact arm of the 100-ohm "dial" resistor (not shown); the ratio correction factor (or R. C. F.) for the "dial" step in use appears in the small slot at the right of the handwheel. All of the dial switches at the left control the connections to the secondary burdens. Most of the portable instruments in the tray are used in verifying the settings of the burdens.

The burdens obtainable range from "zero" up to about 1000 volt-amperes at 110 volts at unity power factor and also at about 0.05 power factor lagging at both 25 and 60 cycles; this range of values is obtainable in one volt-ampere steps. Part of the inductive burdens can be seen below the instrument tray; the coils are tapped; there are air gaps in the cores, which operate at comparatively low flux densities; separate sets of coils are used for each frequency.

4. Shielding Transformers

The shielding transformers are shown in Figs. 6, 9, and 10. Each transformer is rated 25 cycles-75 kv-a.-50,000 : 500/500 volts. It is convenient to consider it first as a straight auto transformer.

As indicated in Fig. 3b, the primary winding has eight taps; the "zero" tap is permanently connected to one bushing and the middle tap (No. 4) is connected to another bushing; these single conductor bushings are connected to the "joint secondary lead" (ground when operating group No. 1 only) and to the transformer tank, respectively; the tank is therefore maintained at mid-winding potential. The other four similar bushings are connected to four contacts of the tap changing switch and to the four resistor unit tanks, respectively; the corresponding switch contacts are connected to taps Nos. 1, 3, 5, and 7 respectively.

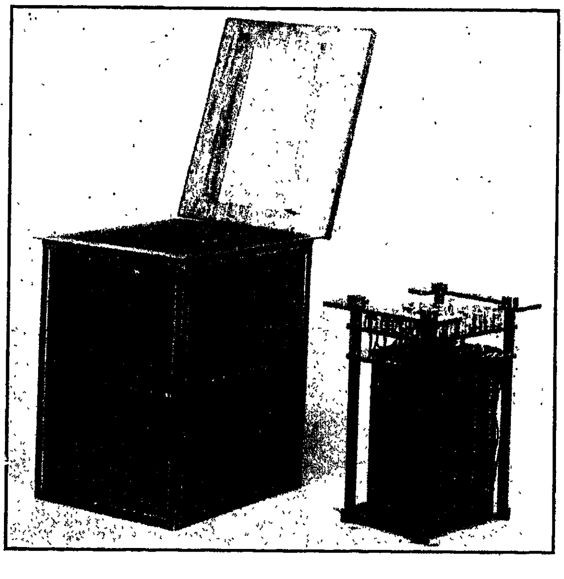

FIG. 8—2600-OHM ADJUSTABLE RESISTOR AND SHIELDED CONTAINER (FOR FRACTIONAL OR ACCESSORY RESISTANCE)

A larger bushing supports the handwheel and shaft of the tap changing switch; this bushing is connected to the "joint tertiary lead" (supply line when operating group No. 1 only). The switch connects this lead to any of the taps, and to the tanks of all resistor units not in use; it also connects the tanks of all units in use to the respective "mid-unit" winding taps.

It is evident that the secondary and tertiary windings would not be necessary to assure equal voltage distribution if group No. 1 only was to be operated, as indicated in Fig. 2d; they are essential, however, when operating more than one group; they require twin conductor bushings and heavy connecting leads (one "joint" connecting lead also carries the primary current). The three windings are interleaved to obtain minimum leakage reactance; the insulation between the windings themselves and between the windings and the core is designed to withstand a high-voltage test of 101 kv.

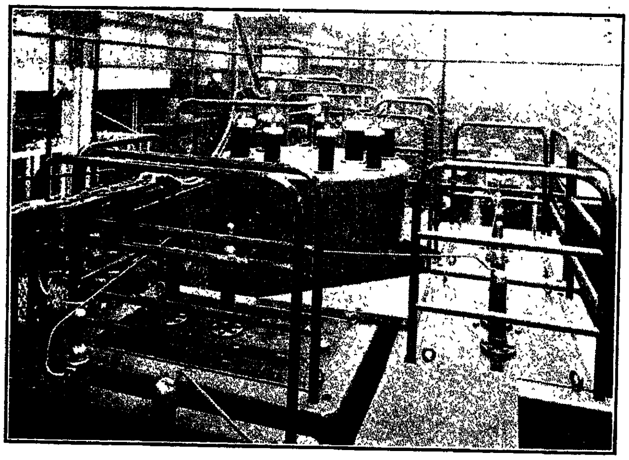

FIG. 9—UPPER PART OF 44-KV. GROUP NO. 3: RESISTOR UNITS; SHIELDING TRANSFORMER; TOP SHIELDS; CONNECTIONS; TREATED WOOD FLOOR SEGMENTS

The tank is about 7.8 ft. deep with an outside diamter of 5.3 ft.; the seams are welded; it contains about 925 gallons of No. 10-C Transil oil. The nine high-voltage bushings constitute the only projections from the tank; both edges of the tank have a radius of about 2 in. to prevent corona up to 250 kv. The core and windings are supported from the cover; the lifting lugs are screwed in only when required.

The two concentric Herkolite supporting cylinders are 36 in. long; the outer has an outside diameter of about 49 in. while that of the inner is about 13 in.; the complete transformer weighs about 10 tons.

5. *Connections*

The various connections are shown in Figs. 6 and 9. The minimum distance between any live part of the three 44-kv. groups and the grounded group area screen is 3 ft., while the minimum distance for any connection (or top shield) is 4 ft.; these comparatively wide spacings assist in reducing capacitance and in preventing corona, both of which are functions of distance: the spacing dimensions given do not, however, apply to the leads through the screen.

The connections between the 100,000-ohm resistor units are made with 0.375-in. diameter copper tubing; this comparatively small diameter is sufficient to prevent corona as the leads are fairly well shielded. The connection Z in Fig. 3a between the resistor and shielding transformer circuits is made with a flexible lead; this lead is shown at the extreme left in Fig. 9 in group No. 2; it is disconnected when group No. 3 is in use.

The connections between the unit tanks and shielding transformer bushings are made with 1.25-in. diameter copper tubing; this also applies to the connections between the shielding transformer bushings. This diameter is sufficient to prevent corona except at a few exposed points in group No. 3, where a diameter of 1.75 in. is used (the same as for the top shields).

The connections between unit A_1 and the fractional resistance in the operating booth and between the secondary winding of shielding transformer T_1 and the 600-volt voltmeter (one lead grounded) are made with 0.375-in. diameter copper tubing.

The connections between the tertiary of one shielding transformer and the secondary of the next are made with apparatus cable consisting of 1600 strands of No. 24 wire (646,400 cir. mils); to obtain minimum reactance the cables are tied together. The cables or leads from each transformer are brought to terminals mounted on a post insulator supported by the end top shield of the group. Connections between busbar "group" terminals are made with suitable links; the connections are shown in Fig. 9; one link is disconnected when group No. 3 is not in use.

FIG. 10—LOWER PART OF GROUPS: TANKS OF RESISTOR UNITS AND SHIELDING TRANSFORMER; HERKOLITE INSULATING AND SUPPORTING CYLINDERS

The connections between the group and testing areas are made with high-voltage single-conductor bushings; 1.25-in. diameter flexible leads are used in both areas for 44 and 88 kv., respectively, while a 1.75-in. diameter lead is used for 132 kv.

6. *Auxiliary Equipment*

Part of the auxiliary equipment is shown in Figs. 6 and 7; in general, this equipment is referred to in connection with other equipment which is not shown.

The power supply equipment includes a motor-generator set with suitable control panels and four single-phase step-up transformers. The set consists of a d-c. motor and a three-phase generator rated 60 cycles, 100 kv-a., 550 volts. The largest step-up transformer is shown in Fig. 6 in the testing area; it is rated 50 cycles, 100 kv-a., 220,000: 500/1100 volts. The load drawn by the complete potentiometer at its maximum voltage rating (150 kv.) is about 23 kw. at 25 cycles and 21.5 kw. at 60 cycles.

All voltages in the range of the potentiometer are dangerous to life. All entrances to the testing and group areas are provided with contacts operated by the gates. It is impossible to close the supply oil switch, if any of the gates are open; conversely, opening any gate when voltage is "on" will open the oil switch. The contacts are "supplemented" by several red lamps which light when voltage is "on;" the shielded lamps shown in Fig. 6 at the top of the group area screen are to warn the cranemen when voltage is "on."

An automatic temperature control system is installed in the operating booth. The practically constant temperature assures uniform accuracy for resistance measurements as the Wheatstone bridge is located in the booth.

PRELIMINARY WORK

An accuracy guarantee of within 0.1 per cent in ratio and within 5 minutes in phase angle means that the accuracy obtainable with the equipment under actual test conditions must be well within those limits in order to provide a reasonable "margin of safety" in the guarantee to cover slight differences between test and operating conditions. This in turn means that the sources of error in the equipment must be thoroughly investigated and that errors of the order of 0.01 per cent and 0.5′ are important.

An absolute accuracy better than within about 0.01 per cent cannot be expected for d-c. resistance values obtained by the best methods available; this limit must be somewhat increased for direct measurements with a high grade Wheatstone bridge. However, a much greater degree of comparative accuracy can be obtained by measuring the difference between two nearly equal resistors or between two nearly equal conditions of the same resistor. The latter method, which is also applicable to a-c. measurements, was therefore used whenever possible in investigating the sources of error.

The following investigations or tests on the completed equipment, preliminary to using it for routine testing, are outlined: leakage, heating and capacitance of units; characteristics of shielding transformers; effect of incorrect tank potentials and of corona and capacitance on operation (special conditions); determination of phase-angle corrections.

Leakage.

The relation between the d-c. resistance of the units obtained with a low-voltage bridge and the a-c. effective resistance at operating voltage was determined indirectly by investigating the corresponding characteristics of the insulation resistance between the different parts of the units.

The insulation resistance between the resistor circuit and the tank of one 100,000-ohm unit was determined (a) with a 1000-volt, 5000-megohm megger and (b) by calculation based on the change in the resistance of one unit, when part of it was shunted by the insulation resistance or leakage circuit the unit was compared with a similar unshunted unit at 11 kv., 60 cycles. The approximate values obtained are: (a) 5000 megohms, (b) 800 megohms; the ratio between (a) and (b) is in the order of 9:1 ((a) taken from Table III).

The results of a separate investigation indicated that the excess "leakage" with a-c. does not occur in the No. 10-C Transil oil, but is due to the presence of the solid insulation in the oil; this is one of the principal reasons for using a minimum of solid insulation in the oil.

The difference between the d-c. resistance of the units and the a-c. effective resistance, due to the effect of the combined leakages through the insulation between the different parts, does not exceed 0.01 per cent.

Heating.

A 36-hour heat run was made on the complete potentiometer at 150 kv., 60 cycles, principally to determine the change in resistance and the temperature rise of the 100,000-ohm resistor units.

The maximum resistance change (decrease) in any unit was found to be about 0.02 per cent; the average change was slightly less. This change does not necessitate a corresponding correction in the (ratio) test results, since the result appears as a ratio between the total potentiometer resistance and a fraction thereof.

The results of d-c. resistance measurements with different connections of the twelve units are given in Table III. The maximum difference (1669 ohms) obtained between "isolated" and "series" resistance measurements is only a fraction of the resistance of one section (10,000 ohms), so a "series" measurement assures the correctness of any desired tap setting. The insulation resistance changed from about 7000 to 5000 megohms per unit during the heat run.

TABLE III
D-C RESISTANCE MEASUREMENTS ON 12 UNITS IN HEAT RUN

Method	Ohms			Connections
	Before	After	Change	
Bridge	1,200,198	1,199,968	−230	Units isolated—sum of 12
Bridge	1,199,838	1,199,309	−529	12 in series—ungrounded
Bridge	1,199,513	1,198,299	−1214	12 in series—grounded
Megger	600×10^6	400×10^6	-200×10^6	12 in multiple to tanks

The maximum temperature rise in degrees centigrade indicated in the different parts of the units is as follows: oil at top, 20 deg.; side of tank near top, 15 deg.; side of tank near bottom, 7 deg. The maximum rise indicated in any part for the first eight hours was less than 15 deg.

The results of a separate investigation indicated a maximum rise of 6 deg. above the oil for the 833-ohm resistor tubes at the same current (0.125 ampere); this temperature rise occurs soon after applying voltage but in this case also, no (ratio) correction is necessary.

Capacitance.

The capacitance between the parts of the resistor units was determined by one or more of the following methods: capacitance bridge; calculation from the results of a volt-ampere test; calculation based on the change in the phase angle of one unit when part of it was shunted by the "capacitance circuit" (part of the "*b* test" under "Leakage"). These methods gave similar results; the choice of methods depended on the magnitude of the capacitance to be determined and on the normal operating voltage.

The capacitance between the resistor circuit and the tank and top shields of each 100,000-ohm unit is about 250 $\mu\mu$f. This capacitance supplements the capacitance between sections in producing the basic negative "internal" phase angle Δ of about 8 min. at 60 cycles (see Fig. 2). This negative angle would, in general, be obtained whenever the unit tanks in the first group are at mid-unit potential (as for the 44-kv. connection), except for the fact that it is partly offset by the positive "external" phase angle Δ' of about 1 min. in the fractional resistance; the "net" phase angle Δ under this condition is there about − 7 min. When the tank of the first unit is grounded (a special condition), its "net" phase angle Δ' becomes about + 1 min. which is practically the Δ' of the fractional resistance; in other words, the "internal" and "external" phase angles of the first unit (about 8 min. each) offset or cancel each other when the tank is grounded.

The capacitance of the three groups to ground is about 1400 μf. All or part of this capacitance, depending on the number of groups in use, constitutes a load on the shielding transformers in use and tends to produce phase displacements between the primary voltages and the line voltage.

The small direct capacitance of the potentiometer resistance circuit (principally the external connecting leads) to the grounded group-area screen is difficult to determine by the methods outlined because of the "intervening" tanks and top shields; however, it produces a positive shift in the potentiometer "net" phase angle Δ of about 3 min. from 44 kv. (− 7 min.) to 132 kv. (− 4 min.) at 60 cycles.

Shielding Transformers.

Since the three shielding transformers practically constitute parts of one auto-transformer, it was important to determine whether or not any appreciable discrepancies existed between them. Impedance, core loss, and ratio and phase-angle tests were made on each transformer and the transformers were checked against each other to determine the agreement between taps. The agreement between the transformers is very close in all respects. Some of the results are given in Table IV.

TABLE IV
TEST DATA ON SHIELDING TRANSFORMERS
Rated 25 Cycles–75 Kv-a.–50,000 : 500/500 Volts

Characteristic	Average Amount	Cycles	Volts	Amps.	P. F.	Winding
Short-circuit impedance......	1.37%	60	685	1.50	0.29	Prim. to sec.
Short-circuit impedance......	1.57%	60	7.85	150	0.27	Sec. to tert.
Coreless........	0.9 kw.	60	50,000	0.048	0.37	Primary
Coreless........	1.44 kw.	25	50,000	0.116	0.25	Primary

The equivalent load imposed at 132 kv., 60 cycles by the capacitance to ground under operating conditions is about 2.3 kv-a. This is based on the assumption that the load is concentrated in one capacitor at the 66 kv. tap (the estimated corresponding load at 220 kv. is 13.8 kv-a.).

This load does not appreciably affect the phase angle of shielding transformer T_1; that is, the phase angle of T_1 is practically the same at 44 kv. with only one group in operation as at 132 kv. with all three groups in operation, indicating that the three series primary voltages are in phase with the line voltage. However, calculations indicate that it requires only small phase angles between the shielding voltages and the line voltage to affect appreciably the potentiometer accuracy.

Tank Potentials.

The effect of incorrect tank potentials was determined by testing a 132-kv. potential transformer at 132 kv., 60 cycles (a) under normal conditions and (b) with the supply connected to tap No. 7 (instead of No. 8) on shielding transformer T_3. This raised all tank potentials, the range being from 5.5 kv. on resistor unit D_8 to 0.24 kv. on resistor A_1. (It also changed the secondary voltage of shielding transformer T_1 from 440 to 459 volts.) The differences between the results obtained under (a) and (b) are: + 0.16 per cent in ratio correction factor ((b) > (a)) and + 33′ in phase angle ((b) lags (a)); the signs of the differences are in accordance with expectations. This special condition indicates that the effect of incorrect tank potentials is cumulative.

Corona.

The effect of corona and added capacitance (of the resistance circuit) to ground was determined by testing a 132-kv. potential transformer at 25 and at 60 cycles (a) under normal conditions and (b) with six of the eight top shields removed from the units in each group. The differences between the results obtained under (a) and (b) are: − 0.005 per cent in ratio correction factor ((b) < (a)) and − 2.8′ in phase angle ((b) leads (a)) at 25 cycles and − 0.024 per cent and − 6.7′, respectively, at 60 cycles; the signs of the differences are in accordance with expectations. This special condition gives an indication of the efficiency of the top shields.

Phase Angle Corrections.

Phase angle corrections were determined in the past by testing several single and double-ratio (series—multiple primary) potential transformers on two or more potentiometer taps each, starting at a comparatively low voltage tap where the effect of capacitance to ground was inappreciable. (This was supplemented by calculating the transformer phase angles from impedance and core-loss data.) This procedure was also used for the present potentiometer and necessitated the design of a special multi-ratio potential transformer rated 25 cycles, 132 kv.

In order to obtain an independent check on the phase angle of the first unit (A_1), a "standard of phase angle" was made up and (A_1) was compared with it. It consists of a 50,000-ohm cord-wound "working" resistor in five shielded sections; the shields are maintained at the mid-potentials of the respective sections by means of taps from a similar parallel "shielding" resistor. The phase angle calculated from the dimensions is of the order of 0.1′ at 60 cycles. The resistance of the different taps in both resistors can be readily determined and the connections to the shields verified, thus assuring sustained accuracy.

The results obtained by the two methods agreed closely.

TEST RESULTS

Representative ratio and phase angle results obtained to date with the new potentiometer are not included as it is not the purpose of the paper to discuss the characteristics of potential transformers. It will be evident, also, that a tabulation of test results, although undoubtedly of interest in that connection, is not necessary to demonstrate the soundness of the general proposition that all potential transformers should be tested for accuracy at full rated voltage; this procedure is now possible up to 150 kv. maximum at from 25 to 60 cycles.

ACKNOWLEDGMENT

The development, design, construction, and testing of a previously untried combination of equipment of such magnitude, to operate under conditions for which little useful data were available, would obviously have been impossible without the cooperation and the suggestions of a large number of individuals.[4] Their assistance is greatly appreciated and the author regrets that a more direct acknowledgment is not possible here. It is a pleasure, however, to acknowledge particularly the assistance of Mr. C. W. LaPierre.

Discussion

W. B. Kouwenhoven: The development of the 132-kv. shielded potentiometer or voltage divider described by C. T. Weller is very interesting. A divider of this range is a real step forward.

It is a well known fact that, when a large resistance is placed across a high-potential alternating voltage, the currents flowing in the resistor are not in phase with the voltage. In the section of the resistor at the high-voltage end, the capacity currents are the largest and the voltage drop in that section leads the applied voltage in phase. In the grounded section of the resistor the current lags behind the applied voltage.

In order to bring the current in the resistor into phase with the applied voltage it is necessary to shield the resistance units and to apply the proper potential to each shield as pointed out by Mr. Weller.

The characteristics of such a shielded circuit may be studied by the use of Maxwell's telegraph equations. Orlich was the first to do this.

If such a resistor consists of but a single section or unit, then the solution of the equations gives the proper potential of the shield or ⅓ of the applied voltage. This will bring the current at the grounded end into phase with the applied voltage.

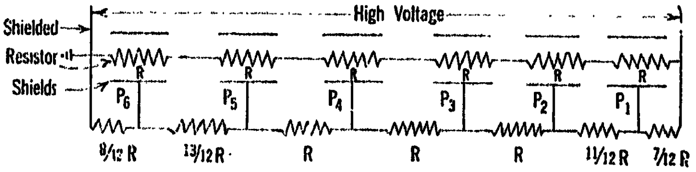

FIG. 1

If the resistance is divided up into a number of sections, each having the value of resistance R, then the theoretical values for the potentials on the shields as determined from Maxwell's equations are as shown in the accompanying Fig. 1.

Except in the two end sections the potentials applied to the shields are equal to the voltage at the midpoint of the resistance unit guarded. Actually no difference in phase between the high voltage and the current in the grounded section can be detected if the potential of each shield is brought to the midpoint of the resistance it guards.

Theoretically such a resistance voltage divider should have its resistance in the form of a straight wire running centrally through its shield. Such a construction is not practical and instead resistance cards are used in practise. These are so arranged, however, as to reduce the capacity C of the author's Fig. 2A to a minimum. In the construction adopted by Mr. Weller this capacity is fairly high.

A source of error will be present in the voltage divider if the phase angle of the voltage applied to the shields differs to any extent from the drop in the corresponding resistance unit. The value of the voltage applied to the shields may be represented by the relation $P_1 (1 + j\,\phi_1)$ for the first shield and by similar relations for the other shield potentials. The permissible value of the angle ϕ_1 depends upon the resistance of the unit R, the capacity between the unit and its shield C' (Fig. 2B of the author's paper) and 2 π times the frequency. The larger the values of R and C' the smaller the angle ϕ_1 for a given percentage of error. I should like to know the value of ϕ_1 that is present in Mr. Weller's voltage divider.

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[4] Principally in the General Engineering Laboratory at Schenectady and in the Pittsfield Works Laboratory and the General Transformer Engineering Department at Pittsfield.

In 1912, while a student at the Technische Hochschule at Karlsuche, Baden, Germany, I designed and built a voltage divider of this type with a maximum range of 28 or 30 kv. I did not use wire-wound resistances, but instead I used the so called manganin solution. This solution consists of 121.1 grams of mannitol, 41.2 grams of boric acid, and 0.06 grams of potassium chloride per liter of distilled water. This solution has a constant resistance over a temperature range of 5 deg. cent., and it has a high specific resistance. I made my resistances by placing it in Jena glass tubes each 1 meter in length and of 1.08 mm. inside diameter. Such a tube had a resistance of 24,000,000 ohms. The shields consisted of pieces of spouting and the capacity between a shield and its resistance unit was 1.25×10^{-12} farads. Similar construction was used in the resistor for providing shield potentials. The divider was used in checking a number of potential transformers and proved very satisfactory. It is inexpensive to construct.

A few years ago I constructed a voltage divider for use at Johns Hopkins University. In that instrument I used wire wound resistance cards. It is in almost daily use in our cable investigations.

Dr. Silsbee at the Bureau of Standards has thoroughly investigated the wire wound resistance divider type and has published his results in a Bureau of Standards paper.

Mr. Weller's voltage divider has the greatest voltage range of any divider that I know of. His construction is novel and I think that he is the first man to provide his shield potentials from a transformer primary.

A. R. Hand: A few points in connection with certain properties of transformer oils, resistance tubes, etc., such as were used in the new equipment, may be of sufficient general interest to warrant further comments.

When a series of resistance cards or tubes forming a high-resistance unit are used submerged in oil, as in an a-c. potentiometer circuit or potential divider, it is essential that an oil of high leakage resistance and low power factor be employed to keep at a minimum such stray currents as may leak off through the oil between the various parts of the resistor unit between which a difference of potential exists. As there has been considerable discussion at various times concerning the nature and extent of a-c. losses in dielectrics, such as oil, and as there were few data available on losses in transformer oils at moderate room temperatures and low frequencies, a series of direct measurements was made on various samples of oil at ordinary room temperatures and commercial frequencies to secure first hand information as to the losses and power factor likely to be encountered. A capacitor consisting of two 30-in. metal disks spaced ½ in. apart and provided with a shielding ring to minimize "fringe effects" was used. This capacitor had an air capacitance of approximately 300 μ μ f. and was used with r. m. s. voltage of 5500 volts between plates. The currents, voltages, and losses involved were measured on astatic reflecting electrodynamometers. A partial tabulation of test results is given in Table I. It is estimated that the measured losses and power factors are accurate to within approximately 20 per cent.

It was found that

1. The energy losses and power factors of high-grade oils were extremely low when the oil was new and uncontaminated. d-c. resistivities of about 60 mega-megohms per cu. cm. and power factors of about 0.0002 were noted.

2. An appreciable increase in losses with exposure of the oil to air and ordinary handling was evident. A given oil, however, appeared ultimately to settle down to a fairly stable electrical state. Oil which had been in use for some time showed practically constant resistivity and power factor at a given temperature.

3. With No. 10 Transil oil where it was possible to check the a-c. leakage resistance against the d-c. resistance (measured on a 1000-volt, 5000-megohm megger) it was found that the a-c. and d-c. leakage resistances were practically the same.

4. Treatment of No. 10 C Transil oil with Fuller's earth resulted in a temporary decrease in dielectric losses, but the improvement was short lived and in the course of several weeks both the treated and untreated oils showed practically identical properties.

It was concluded that No. 10 C oil, without special treatment, was suitable for use in the tanks of the resistor units because it embodied both high a-c. resistivity and moderate cost.

The accurate determination of high a-c. leakage resistances (of the order of 500 megohms) and small capacitances (of the order of 100 μ μ f.) under test conditions simulating actual high-voltage conditions, presents some rather interesting problems. Direct measurement with instruments such as the Evershed megger and capaciometer, or laborious calculations from physical dimensions, are often both impracticable and of questionable accuracy. It was found that the modified potentiometer scheme shown in Fig. 2 herewith served quite satisfactorily and yielded results of a good degree of accuracy.

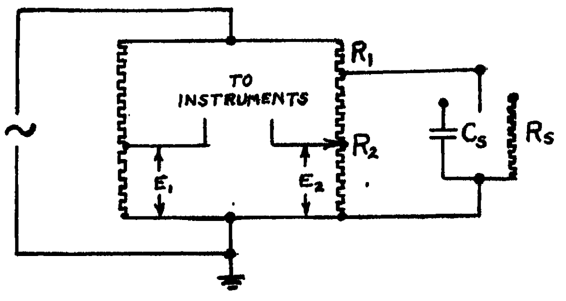

FIG. 2

Two similar 11-kv. potentiometer circuits were connected in parallel across a power source of the desired frequency. Suitable highly sensitive instruments were arranged to permit the accurate comparison of the relative magnitudes and phase angles of the respective fractional voltage drops E_1 and E_2. In practise, two 11-kv. units and two 1000-ohm fractional resistors were used to form the potentiometer circuits. One of the two 11-kv. 100,000-ohm units was provided with a tap lead at the 60,000-ohm point. The measured change in the voltage drop E_2 with respect to E_1 produced when a test resistance or capacitance (or combination of both) was shunted across the 60,000-ohm tap furnishes a ready basis for calculating the constants of the test piece. The change in magnitude ("ratio") of the voltage drop E_2 defined the "equivalent shunted resistance" of the shunting test piece, while the observed change in phase displacement defined its capacitance. It can be shown that the equivalent shunted resistance R_s, commonly known as the leakage resistance, is represented closely by the formula

$$R_s = \frac{R_1 R_2}{(R_1 + R_2)(P - 1)}$$

where P is the ratio of the magnitudes of the fractional voltage drop E_2 before and after connecting the test piece. It can likewise be shown that the value of shunted capacitance C_s is represented closely by the formula

$$C_s = \frac{(R_1 + R_2) \tan \theta}{\omega R_1 R_2}$$

where θ is the measured change in phase angle between the voltage drops before and after connecting the test piece, and ω has its customary meaning of 2 πf.

The above method is directly applicable to the measurement of the insulation resistance and capacitance between the resistance circuit of one unit of the potentiometer described by Mr. Weller and its containing tank when the unit is used as the test piece. It was determined that the a-c. leakage resistance had a value of approximately 800 megohms and the capacitance a value of approximately 250 μ μ f. The resistance of the leakage paths between various parts of the units are obtainable in the same manner. By combining the several leakage resistances, a fair estimate of the net effective leakage resistance shunting the metallic resistor circuit can be made. Calculations show that the a-c. effective resistance of the resistor circuit should deviate less than 0.01 per cent from the true d-c. resistance because of these effects.

High-grade manganin wire has long been recognized as a very satisfactory material for accurate "working standards" of resistance both because of its low temperature-resistance coefficient and its permanency of resistance after once being properly aged. Tests on representative samples of high-grade 11-mil manganin wire having a resistivity of 45 microhms per cu. cm. show an average resistance decrease from 25 deg. cent. to 35 deg. cent. of less than 0.01 per cent. Permanency of resistance (and calibration) of the high-resistance potentiometer circuit is thus assured for ordinary variations in operating temperatures. The choice of a relatively large working current (0.11 ampere) has been justified in that it has permitted the use of readily available materials without sacrificing accuracy; that is, the effects of leakage currents in particular have been relatively unimportant.

The employment of cylindrical tubes rather than the usual thin flat cards as forms on which to wind the resistance wire possesses a distinct advantage in that there are no sharp bends around edges where the wire may be weakened and easily broken in two. One of the chief disadvantages in the use of the old-style card-wound resistances for this purpose has been found to be repeated outages after several years' operation, due to the breaking of the resistance wire at such bends.

It may be of interest to point out that a new 44-kv. shielded potentiometer similar to the first group of the 132-kv. potentiometer described by Mr. Weller has been set up to replace the present 66-kv. unshielded potentiometer in the laboratory at Schenectady within the past year.

R. L. Ten Broeck: Some interest has been shown concerning the phase-angle calibration of the 132-kv. shielded potentiometer. The circuit used is shown in Fig. 3 herewith. The standard of phase angle was connected in parallel with the first unit A_1 of the potentiometer, but, in addition to other changes from the normal 11-kv. connection, the tank of A_1 was disconnected from the shielding transformer and grounded. The current through the working circuit of the standard went directly through the moving coil of the secondary dynamometer wattmeter. A resistance a equal to the resistance of the potential (moving-coil) circuit was put in series with the guard circuit to keep the shielding correct. The errors due to the inductance of the moving coil of the wattmeter and to the capacitance to ground of the lead from the "standard" to the wattmeters were calculated to be considerably less than 0.1 min. and were disregarded.

The maximum voltage that could be applied to the "standard" was 750 volts, so that in order to improve the sensitivity of the potentiometer at that low voltage, the drop voltage E_d point was changed to include 2000 ohms instead of 1000 ohms and the series resistance for the "primary" dynamometer wattmeter was reduced from 25,000 to 5000 ohms.

A common current was put through the current coils of the two wattmeter dynamometers and the phase angle between the currents in the potential circuits measured in the same manner as is explained under "Principles of Operation" in Mr. Weller's paper, except in this case the voltage E is the total voltage; this gave the phase angle of the primary circuit.

The phase angle of the secondary circuit was next measured with the working circuit of the standard in series with moving coil of the primary dynamometer and voltage E_s applied across the secondary leads as in Fig. 4 herewith.

The special correction for the first unit A_1 is then found by subtracting the phase angle of the secondary circuit from that of the primary circuit. This correction applies to the first unit with tank at ground potential, the fractional resistance of the potentiometer, 2000 ohms, and the series resistance for the primary dynamometer, 5000 ohms. Several potential transformers were tested for phase angle under this condition and then retested with the normal 11-kv. connection; that is, with the tank excited from the shielding transformer, the fractional resistance 1000 ohms, and the series resistance for the primary dynamometer 25,000 ohms. The average of the differences between these tests was added to the original "special" correction to obtain the correction for the first unit or the 11-kv. connection under normal conditions.

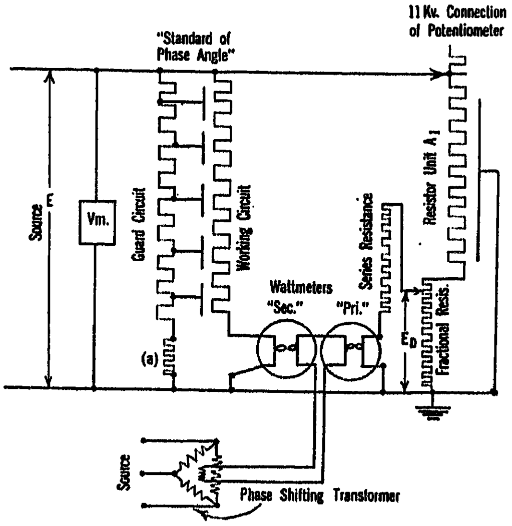

Fig. 3

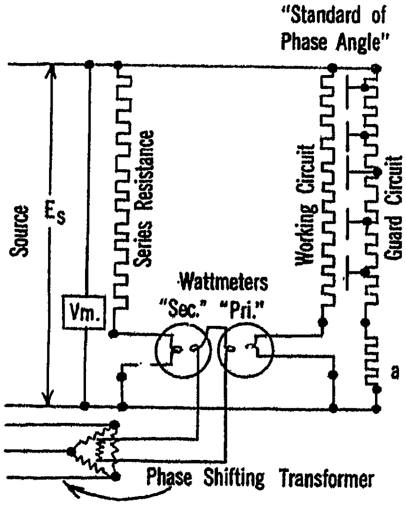

Fig. 4

The corrections for the other voltage connections or taps of the potentiometer were found by interchecking with convenient potential transformers. An 11-kv. potential transformer was tested on the 11-kv. tap A_1 which had just been calibrated and then on the 22-kv. tap B_1. The difference between the results of these two tests added to the correction for the 11-kv. connection gave the correction for 22-kv. potentiometer connection. The other corrections were obtained in a like manner. A 22-kv. potential transformer was tested on the 22-kv. tap and on the 44-kv. tap D_1; a 44-kv. potential transformer was tested on the 44-kv., 55-kv. A_2, 66-kv. B_2, and 88-kv. D_2 taps; a 66-kv. potential transformer was tested on the 66- and 132-kv. D_4 taps; and an 110-kv. potential transformer was tested on the 88-kv., 110-kv., and 132-kv. taps. This gave the correction for the 132-kv. tap in four or five steps from the first unit.

G. Thompson: Mr. Weller's paper on the 132-kv. shielded potentiometer represents the logical and perhaps the ultimate development of the pioneer work done some 20 years ago by Robinson and Craighead at the General Electric Company, by Sharp and Crawford at the Electrical Testing Laboratories, and by Agnew and Silsbee at the Bureau of Standards. In those days, 22 kv. was regarded as "high-voltage" and the discussions which followed Institute papers on methods of measuring ratio and phase angles of instrument transformers were devoted more to the relative merits of various kinds of detectors of balance than to errors inherent in the testing apparatus. The resistance-train potential divider was universally employed for the testing of potential transformers because of the comparative ease with which it could be constructed and measured. It was felt that the purity of the component resistance units did not affect the precision of the measurement provided the portion of the resistance train corresponding to the low-tension or secondary voltage of the test transformer is constructed in the same way as that across the high-tension winding.

As operating potentials became higher and the resistance trains correspondingly longer, the shunting capacitances began to assume troublesome values. In Scientific Paper No. 516 (June 16, 1925), Dr. Silsbee of the Bureau of Standards described a shielded resistor for use with transformers up to 30 kv. It is interesting to note the concern with which he viewed the difficulties of constructing a resistance train for higher voltages. In Silsbee's resistor, the shields for the component units of the resistance train were maintained at the different potentials by means of an auxiliary resistance train to which the shields were connected at intervals. With this arrangement the currents due to capacitance of shields to ground flow through the guarding train producing, for high voltages, a non-uniform distribution of voltage along the guard train. Mr. Weller has neatly solved this problem by substituting tapped transformers for the auxiliary train, the transformers being of such capacity that the displacement currents produce no effect on the distribution of voltage.

Those of us who are called upon to test voltage transformers looked with considerable apprehension at the size of the apparatus described in Dr. Silsbee's paper. What shall we say then to an equipment having the physical dimensions of that which Mr. Weller has constructed? And when it is further considered that the General Electric Company has only just now arrived at a point where 132-kv. transformers can be tested, when 250-kv. transformers are already a very real possibility, one wonders whether or not the testing equipment ultimately is not going to call for more space than the building in which the transformers are manufactured.

It would seem then that it behooves us to look about to see if some other method is available possessing the same accuracy as the resistance-train method but involving a smaller expenditure of money, consuming less power, and occupying less space than the imposing apparatus here described. In the paper referred to above, Dr. Silsbee rightly points out that ratio and phase angle have no physical dimensions but represent only ratios of alternating vector quantities which can be compared by null methods using two impedances in series across the high-tension terminals of the transformer. These impedances may be resistances, inductances, capacitances, or combinations of the same. The condenser represents an impedance which can be constructed with a purity equal to that of a resistance. It can be very effectively shielded from stray fields. It may be built for any voltage now commercially contemplated, will occupy much less space, requires comparatively little kilovolt-ampere capacity in the supply transformer, and probably represents a much smaller investment than the corresponding resistance train.

The Doyle-and-Salter transformer bridge for the measurement of the phase defect angle of a cable, described before the Institute in May of 1926, gives the ratio of two condensers in terms of the ratio of transformation of the transformer supplying the high-voltage. This bridge can be reversed and the ratio of transformation determined from the ratio of the two capacitances (see Fig. 5 herewith). Also the method which these two writers described for determining the relative phase defect angle of the two condensers can be used to determine the relative phase displacement of the primary and secondary voltages of the transformer.

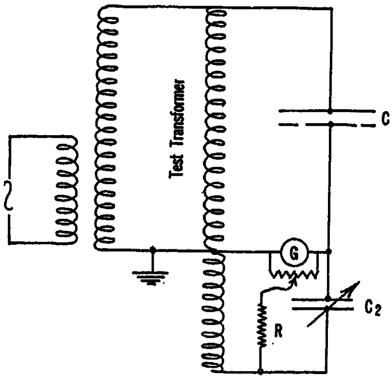

Fig. 5

In April, 1927, B. G. Churcher of the Metropolitan Vickers Company of Birmingham, described a similar circuit which had been used up to 66 kv. in testing potential transformers at the factory of that company. Churcher used a mutual inductance excited through a non-inductive resistance from the secondary winding of the test transformer to measure the relative phase displacement. (See Fig. 6 herewith.) The Churcher method of measuring phase angle is somewhat superior to that of the Doyle-Salter bridge, in that it is possible to adjust the circuit conditions so that phase angle may be read directly from the graduations of the mutual inductor. Neglecting phase displacements in the two condensers the ratio of the transformer is equal simply to C_2/C_1. Neglecting the self-inductance of the primary winding of the mutual inductor, the phase angle is given by tan $\varphi = (\omega M/R)(1 + C_1/C_2)$.

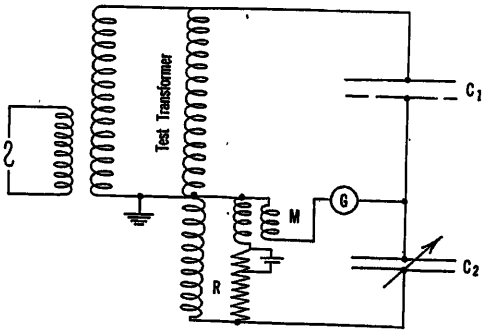

Fig. 6

The refinements which are necessary to secure results of the highest accuracy with the capacitance train method are in no way burdensome. The high-tension condenser, C_1 should be preferably a no-loss air condenser; the "low plate" should have a guard ring which in most cases can be connected directly to ground; the troublesome capacitance between the "low lead" and ground is thereby avoided. The low-tension condenser C_2 can be a good variable mica condenser in which the units of the principal dial have been adjusted to the marked values to better than 0.1 per cent. The phase angles of such a condenser are usually smaller than 5 min. If desired, exact balance can be secured with the aid of a small variable air condenser connected in parallel with the mica condenser. The self-inductance of the primary coil of the mutual inductor can be compensated by means of capacitance shunted across a part of the series resistance in the usual manner. The simple equations given above are then valid.

Furthermore, it is not necessary that C_1 should be an air condenser provided its phase defect angle at various voltages is known. At the Electrical Testing Laboratories we have had good results using one of our glass tube "standard dielectric load" condensers for C_1. The phase angle of these loads is of the order of 3 or 4 min. and is constant over a wide range of voltage. For precise work the phase defect angles of C_2 can be determined and the necessary correction applied.

The detector in the above circuit has a potential different from ground potential by the voltage induced in the secondary coil of the mutual inductor. If this is liable to introduce error in precision measurements the potential of the guard about the "low" lead and the detector may be brought to that of the test circuit by means of a second mutual inductor in the manner described by Churcher.

Two difficulties are met in the use of the capacitance-train method. If C_1 is fixed in value and must be used for a wide range of transformer ratios, C_2 may be inconveniently small for low-ratio transformers and inconveniently large for high ratios. However, the cost of constructing several air condensers to cover voltage ranges from, say, 30 kv. upwards, would not be prohibitive. The second difficulty is the determination of the capacitance of C_1. If the condenser is of a suitable geometric shape and well shielded its capacitance may be computed. We have used our large air condenser with a movable ground plate for C_1 and then determined the capacitance for the particular test condition either by the substitution method at low voltage using a bridge with resistance arms (Wien Bridge), or by calibration using a standardized voltage transformer of a lower voltage rating.

It would seem, therefore, that the capacitance-train method possesses characteristics which ought to make it very useful for testing voltage transformers from perhaps 20 kv. and upwards. Our experience has led us to set about 25 kv. as the point at which we change from the resistance train to the capacitance train. However, there may be features which we have overlooked and I should be very glad to have Mr. Weller tell us his opinion of the capacitance train method and the reasons why his company felt it desirable to make the investment represented in the apparatus described in his paper.

F. B. Silsbee: The construction of the truly monumental piece of apparatus which is described in this paper is a notable contribution to the art of precise measurements. It is pleasing to see that the attainment of accuracy is valued so highly by the electrical industry that a manufacturing corporation is justified in investing the large sum of money which this equipment must have cost solely for the purpose of maintaining the accuracy of its products at the high level demanded by its customers.

A detailed study of Mr. Weller's paper shows that the results obtained with his equipment are in good accordance with the theoretical relations governing the performance of such shielded resistors, which were published by the Bureau of Standards some time ago.[1] Thus Mr. Weller gives as the capacitance constant C of one of his units 250 $\mu\,\mu$ f. From Equation 45 of Scientific Paper 516 one would expect the change in phase angle of the first unit when the potential of its tank is changed from 5.5 kv. to 0 to be $\frac{\omega\, r\, c}{4} = 7.9$ min. Mr. Weller observes a change of 8 min. The value of the phase angle when the tank is grounded indicates that the constant A is equal to $C/32$. This relation is quite reasonable if much of the distributed capacitance between the resistor and the tank is located near the ends of the resistor.

For the experiment in which the supply voltage was connected to tap 7 instead of tap 8, the equations would indicate a change in phase angle of 30 min. The observed change was 33 min. The fact that the effective ratio was also changed by this procedure, implies that the tank potentials must have been slightly out of phase as well as abnormal in magnitude during this test. This effect is only one-sixth as great as the change in the in-phase component and is presumably the result of the unequal excitation of the auto transformers.

It would be interesting if more data were available on the regulation of the auto transformer primary circuits since it is this property which determines their ability to maintain the proper tank potentials in spite of the capacitance currents to ground.

It should be pointed out that the use of transformers to maintain the shield potentials has greatly extended the possibilities of the shielded type of resistor since the disturbing effects of capacitance are similar to those which would exist in a set-up in which the guard circuit had an impedance equal to the leakage reactance of the transformer (here perhaps 1400 ohms), while the load imposed on the supply circuit corresponds only to that of the open-circuit impedance (about 3×10^6 ohms). The gain in this case therefore may be by a factor of nearly 2000 in impedance and according to the 4th power relation indicated on page 511 of Scientific Paper No. 516 should permit the construction of shielded resistors for voltages nearly 7 times as high as can be handled by equipment in which there is no interaction between sections of the guard circuit.

C. W. La Pierre: (communicated after adjournment)

Fractional Resistors. The chief requirements of the fractional resistors are stability in the non-reactive resistance and convenience in obtaining taps. Residual inductance introduces no phase-angle error in the final result as its effect forms a part of the corrections, experimentally determined.

In winding these single-layer resistances with two wires in opposite directions but connected in parallel it is necessary to have two crossings per turn. For convenience in manufacture these crossings are made at one point instead of at two points diametrically opposite. An inevitable rotation of the crossing point about the spool occurs as the winding progresses so that one winding has effectively 6 or 8 turns per spool more than the other. The residual inductance due to these turns is negligible, especially since they have a wide spacing.

The taps are brought out to four sets of binding posts, each set arranged similar to a dial switch. A central binding post with connecting links to any of the other posts takes the place of the rotating member of a dial switch. The panel containing these connections is beneath the surface of the oil.

Shielded Resistor Phase Angle Standard. The five-unit 50,000-ohm shielded resistor for use as a standard of phase angle was inspired by a study of Dr. Silsbee's Bureau of Standards Paper No. 516 on "A Shielded Resistor for Voltage Transformer Testing." Dr. Silsbee demonstrates in a complete manner the effectiveness as well as the limitations of shielding by means of a resistance guard circuit. It is clear from a study of the quantitative relations involved in the shielded phase-angle standard that the shielding must be effectively perfect. The residual phase angle is due therefore entirely to the constants of each unit.

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[1] Bur. of Standards Sci. Paper No. 516 "A Shielded Resistor for Voltage Transformer Testing," by F. B. Silsbee, Dec. 1925.

The total capacitance of each resistance unit to its shield is approximately 26 $\mu\mu$f. By distributing this capacitance in accordance with the dimensions an approximate value for its effect may be obtained. The effect so determined at 60 cycles is less than 0.1 min. The card wound resistances have an inductive effect of less than 0.1 min. at 60 cycles as determined by approximate calculation. Since the two offset, the resultant comes out to be 0.01 or 0.02 min. Due to possible errors in computation the resultant has little or no value in considering the final angle. However, the small magnitude of the individual components is important in showing that an unlikely error in computation would have to be present in order to result in a phase angle larger than from one- to two-tenths of one minute.

Accuracy of the Equipment. The methods used in comparing the various points have been described by Mr. Ten Broeck. The magnitude of the errors involved in obtaining the phase angle of the first 11-kv. unit are fairly well understood to depend largely upon the reference standard. In transferring this result to the higher voltages a great deal depends on the precision of the results (differentiated from accuracy) or what might be termed more descriptively, their consistency.

Naturally in the course of comparing the various taps a great many data were obtained. From these I have arbitrarily chosen the results from the minimum number of tests necessary to obtain the phase angle of the 132-kv. tap in terms of the shielded resistor. The first step consists in comparing the shielded resistor with the first 11-kv. unit direct. The second to fifth steps compare the 11-kv. and 22-kv. taps and so on to 44, 66, and 132 kv. by means of potential transformers. Each comparison is made with five points at five different voltages. With no attempt at weighting the points in accordance with the variations in sensitivity at the different voltage, the average deviations from the mean of the five points in each step were (1) 0.053 min., (2) 0.043 min., (3) 0.084 min., (4) 0.043 min., and (5) 0.026 min. Of the 25 points four deviations from the mean of over 0.1 min. occurred, the maximum being 0.134 min.

The values obtained from all of the data available, agreed very closely with the values obtained with this single set of data. The differences ranged from a maximum of 0.2 min. at the 44-kv. tap to a minimum of 0.016 min. at the 132-kv. tap. These results are for 60 cycles. It should be emphasized, however, that the values just given do not represent the accuracy but are merely indicative of the precision of the results.

Corresponding results for ratio are not available for presentation. They are much more difficult to obtain due to the necessity of changing the value of the fractional resistance in going from tap to tap. The transfer thus depends upon resistance measurements which we have not been able to make to a degree of accuracy comparable to the sensitivity of the equipment. By an increase in the dynamometer excitation a sensitivity of 0.001 per cent for ration may be reached without any difficulty. The resulting burden on the transformer in test requires 2.5 volt-amperes instead of the usual output of 0.5 volt-ampere.

In connection with resistance measurements formulas have been derived for the correction for leakage when measuring a large number of units in series. These formulas are for one terminal of the resistance grounded. (The shields of each unit are grounded for all resistance measurements.)

$$\Delta R = \frac{R^2}{R_L}\left(\frac{4N^3 - N}{3}\right) \text{(ohms)}$$

where
ΔR = error in resistance
$2R$ = the resistance of one unit
R_L = the leakage resistance between the resistor of one unit and its shield
N = the number of units in series

For the bridge and measured circuit completely insulated from ground except of course through the leakages in each unit

$$\Delta R = \frac{2R^2}{R_L}\left(\frac{N^3 - N}{6}\right) \text{(ohms)}$$

A correction for ratio has been derived for use in tests involving comparatively large phase displacements. For ordinary use of the equipment the correction is negligible. The true ratio may be expressed for practically all cases by

$$K = \frac{R_t}{R_2} \times \frac{\cos\theta_2}{\cos\theta_t} \times \frac{\cos(\gamma - \theta_d - \phi_{fc})}{\cos(\theta_2 - \theta_d - \theta_t - \phi_{fc})}$$

where K = the true ratio
R_t = the total resistance
R_2 = the fractional resistance
θ_t and θ_2 = the corresponding phase angles
θ_d = Resultant angle of the detector potential coil circuit and the fractional resistance R_2.
ϕ_{fc} = The phase angle of the excitation applied to the detector field (dynamometer type of instrument)
γ = Phase angle of the potential transformer.

In the exact correction, the last term is much longer.

Extension to the High Voltages. Referring again to Dr. Silsbee's equations it is evident that such equipment as this paper describes has also a definite voltage limit beyond which it cannot be extended. In the design this limit was set at 220 kv. nominal (250 kv. maximum). The tests with the equipment so far completed indicate that no particular difficulty will be encountered in such an extension. It is quite likely, however, as anticipated, that for the most accurate work some flexibility will be sacrificed, and it may be necessary to use only the fifth group in its entirety. This, however, is not an important item.

The limiting factor in the extension lies of course in the leakage inductance and in the resistance of the shielding transformer windings. Establishing definite values for the effective inductance of the windings is complicated chiefly by the three windings and to a lesser extent by the comparatively large capacitance between windings.

Measurements of the capacitance between primary and secondary or tertiary indicated approximately 0.0048 microfarad with the tank connected to the midpoint of the primary. The measurements between secondary and tertiary windings indicated a capacitance of approximately 0.004 microfarad. (Measurements were made between two windings at a time and the effect of the third was not considered.) With the proper connections of secondary and tertiary in the final equipment, this results in a concentration of excitation current in the central portion of the primary or main winding.

Various impedance measurements were made on each winding with one or both of the others short-circuited. Measurements were also made on portions of the primary with the remainder or total short-circuited. In 15 such measurements under all conditions the phase angles ranged from 72.5 to 74.2 deg. with the average around 73 deg. The constancy of the phase angle indicates that for these short-circuit conditions at least the leakage inductance of each winding is a constant. Based on these results and the resistance measurements, the impedance of the primary at 60 cycles becomes approximately $55 + j\,180$ ohms and the impedance of the secondary and tertiary each $0.0081 + j\,0.0265$ ohms.

Such impedance values will result in a slight displacement of the shield voltages for five groups in series. The ratio error will be appreciable but less in magnitude than our present guarantee. A small correction will probably be necessary. This error is due to a displacement in the phase of the shield potentials.

For five groups in series it is expected that the phase-angle correction may change sign but not be larger in magnitude than those now occurring.

C. T. Weller: Professor Kouwenhoven's question in regard to angle ϕ_1 can be answered by stating that it is negligible in group No. 1 because of the small "capacitance load" and this is apparently true also for three groups as inferred in the paper under "Shielding Transformers;" Mr. LaPierre has calculated its approximate "overall" effect for five groups (220 kv.).

Mr. Thompson has described a "capacitance" potentiometer or voltage divider and has asked why we did not adopt capacitance in preference to resistance because of its relatively smaller size and lower cost. We investigated the possibilities of capacitance for this purpose about three years ago and arrived at the following conclusions:

(a) Capacitors of the types and ranges required were not at that time in the same class as resistors from the standpoints of stability and accuracy. These views were confirmed, in general, by Dr. H. L. Curtis of the Bureau of Standards;

(b) The impedance of resistance at different frequencies is constant while that of capacitance is inversely proportional to the frequency. This tends to amplify harmonics which are usually objectionable in precision testing equipment;

(c) We felt that we knew a great deal more about resistors than we did about capacitors for potentiometer purposes.

However, it is obviously desirable to keep the size and the cost of testing equipment at a minimum and it is hoped, therefore, that other laboratories will proceed with the necessary development work so that "capacitance" or "impedance" potentiometers will be available which are comparable with the 132-kv. shielded potentiometer in flexibility, stability, and accuracy as well as in voltage range.

Dr. Silsbee has shown that the results obtained are in general agreement with results calculated from the formulas in his Scientific Paper No. 516 of the Bureau of Standards. In this connection it should be noted that the capacitance and phase angle constants given for the 132-kv. potentiometer are approximate. He has given a multiplying factor of about 7 for the extension possible by means of shielding transformers; this would bring the maximum voltage to approximately 220 kv. He has concluded from the difference in ratio results that the shielding transformer primary (or shield) voltages were slightly out of phase with the line voltage, due apparently to "unequal excitation" of the shielding transformers, in the special test in which the supply voltage was connected to tap No. 7 instead of No. 8 on shielding transformer T_3. It is interesting to note that no similar difference in ratio was obtained between any "odd voltage" connection of the potentiometer (such as the 110-kv. connection for testing a 132-kv. potential transformer) and the nearest "standard voltage" connection (such as the 132-kv. connection for the same transformer), although a similar "unequal excitation" of the shielding transformers exists with all but the 132- and 88-kv. connections in groups 2 and 3; however, the unit tanks are at "mid unit" potentials for all standard voltages in groups 2 and 3. It has been suggested that the difference is due to the fact that the voltages across the various resistor units were out of phase with the line (and shield) voltage in the special test.

The limit for the extension of the voltage range of the potentiometer is set from a practical standpoint by the relation between the resulting potentiometer corrections and the "certificate guarantees" for the results of ratio and phase angle accuracy tests; it is preferable that the corrections should not greatly exceed the "guarantees."

The most important features of potentiometers for determining the ratio and phase angle accuracy of potential transformers are stability and flexibility; these insure precision and speed in testing. Our various potentiometers have incorporated these features. It is desirable, of course, to have the corrections as small as possible and in this respect the 132-kv. potentiometer represents an advance over its 66-kv. predecessor. It should be noted that the present range (up to 150 kv. maximum at from 25 to 60 cycles) takes care of practically all potential transformers now manufactured, including those for 220-kv. circuits.

A Precision Regulator for Alternating Voltage

BY H. M. STOLLER[1] and J. R. POWER[1]
Member, A. I. E. E. Non-member

Synopsis.—A recently developed precision voltage regulator for use with alternating current is described. It will maintain its output voltage constant to within 0.03 per cent over an input voltage range of 10 per cent and a load range of from zero to full load.

This regulation is accomplished by means of a small transformer inserted in one of the lines which boosts or bucks the impressed voltage by the required amount. The transformer is controlled by a vacuum tube circuit acting through an inductance bridge.

INTRODUCTORY

Modern industrial conditions are making it more and more imperative that the voltages used for testing purposes be held within precise limits. In some cases the requirements have far exceeded the regulation obtainable by the commonly used types of regulators.

This paper will describe a vacuum tube type of a-c. regulator which will maintain the output voltage within 0.03 of 1 per cent over the extreme range from maximum load minimum input voltage to minimum load maximum input voltage. This regulator is employed in the inspection of telephone equipment and is designed for operation on a line whose voltage fluctuates between 85 and 90 volts, 20 cycles.

For the great majority of tests, the line voltage limits are sufficiently precise, but there are frequent cases where a much higher degree of precision is necessary. It is therefore advisable to use a regulator of small power rating situated at the test position rather than at the power source because of line drop and because it would be uneconomical to regulate the entire alternator output so precisely.

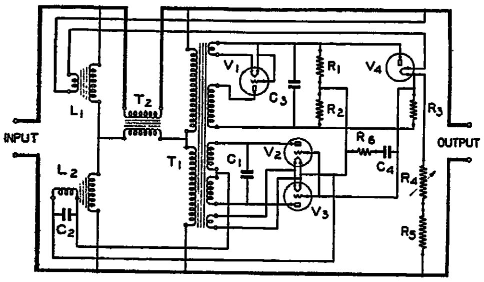

FIG. 1

APPARATUS

The circuit employs a small transformer whose secondary is in series with the line so that any voltage induced in it either aids or bucks the line voltage. The primary voltage supplied to this transformer is derived from a bridge circuit. Referring to Fig. 1, this circuit consists of the two halves of the primary winding of the power transformer T_1, the fixed retardation coil L_1, the variable retardation coil L_2, and the output transformer T_2. In the variable arm L_2 of this bridge, the inductance is varied by the commonly known method of changing the degree of magnetic saturation of the reactor by means of a direct current in an auxiliary winding. For this purpose a three legged core with a winding on each leg is used (Fig. 2). The two outer windings consist of an equal number of turns and are connected in series so as to aid each other in producing flux in the outer legs of the core. As they are very carefully balanced no flux is set up in the middle leg by these windings. Any flux set up by the middle winding divides equally between the two outer legs. If, then, a direct current is sent through the middle winding it tends to saturate the entire core and so reduce the impedance of the outer windings to alternating current. In the present instance the ratio of turns of the d-c. winding to the a-c. winding is large. By this means a direct current of a few milliamperes can control the impedance of a reactor capable of handling several amperes of alternating current.

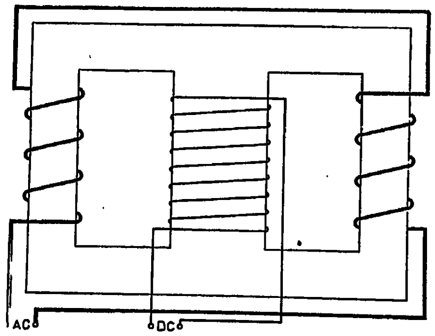

FIG. 2

The important feature of the fixed retardation coil (L_1, Fig. 1) is that it must be so designed that it will not saturate over the working range. This condition is met by the use of a large air-gap. An auxiliary winding used for compensation is placed on the same core and is connected in the regulating circuit.

The two element vacuum tube, V_4, is the primary source of control. It is supplied with an excess of plate potential (from the rectifier tube V_1) so that the space current is entirely controlled by filament emission.

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[1] Both of Bell Telephone Laboratories, 463 West St., New York, N. Y.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

The filament is made of tungsten[2] and is 0.001 inch diameter so as to have small time lag between current and temperature. This filament is connected in series with a resistance across the voltage output terminals so that a change in voltage produces a proportional change in filament current. A change of 0.1 per cent in filament current will cause about a 2 per cent change in the space current or, in other words, the tube has an effective amplification factor for current of about 20.

The tubes V_2 and V_3 are amplifier tubes supplying rectified current to the d-c. winding of the inductance L_2.

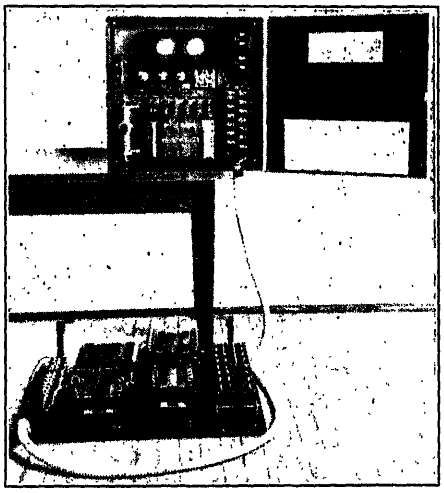

Fig. 3

The plate-voltage supply for these tubes is derived from the transformer T_1 which is connected across the output circuit of the regulator. Thus the tubes have a constant source of potential which is essential to satisfactory operation of the circuit.

The assembled apparatus is shown in Fig. 3.

PRINCIPLE OF OPERATION

Referring again to the circuit in Fig. 1, the secondary of the transformer T_2 is in series with the line. Its primary is connected across the mid-points of the bridge formed by the transformer T_1 and the retardation coils L_1 and L_2. When the impedance drop across L_2 is equal to that across L_1, the bridge is balanced, and no potential is applied to the output transformer and none, therefore, is induced in the line. Under this condition the output voltage is equal to the input voltage except for the impedance drop in the transformer. If the impedance of the variable reactor changes, this balance is disturbed and a voltage is induced in the line that either bucks or boosts the line voltage. As described above, the impedance of this reactor is a function of the direct current through it. This current is controlled by the vacuum tube circuit.

If the input voltage should increase, the instantaneous effect would be an increase in output voltage. This would cause the filament current of the regulator tube to increase and, as a result, its space current would increase at a much greater rate as shown in Fig. 4. The space current flowing through the coupling resistance R_3 would change the balance of the resistance bridge of which the tube and this resistance form one side. This change is in such a direction as to bring the potential of the junction between the tube and the resistance R_3 nearer to that of the junction between the resistances R_1 and R_2 which form the other side of the resistance bridge. By this means the negative bias on the grids of the amplifier tubes is reduced and their output increased. This increased output flowing through the winding of the variable reactor decreases its impedance which changes the balance of the inductance bridge and either reduces the aiding voltage supplied to the line by the transformer T_2 or increases the bucking voltage, depending upon previous conditions. This change in induced voltage will bring the output voltage almost back to its original value. In practise the operation of this sequence of events is sufficiently rapid to prevent the transient increases in output voltage from attaining noticeable proportions except on a very sudden and severe change in input voltage.

This reaction to an increase in input voltage would restore the output voltage almost, but not quite, to normal except for the action of the compensating auxiliary winding on L_1. With the aid of this winding the required increase in filament current through tube V_4 is secured without raising the output voltage at all. This effect is secured by making use of the increased voltage across L_1 due to the changed condition in the inductance bridge.

By using the proper number of turns in the auxiliary winding, the voltage may be kept constant or even be caused to rise. This feed-back effect does not cause hunting because there is a slight time lag in its action due to the high inductance of the d-c. winding on the variable retardation coil L_2.

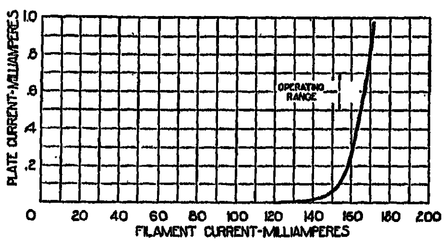

Fig. 4

In the case of load variation, the action of the regulator circuit is similar to its operation with voltage variation, an increase of load causing the same action as a decrease in applied line voltage and a decrease of load the same action as an increase in applied voltage.

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[2] The tungsten is pure and contains no thorium or other materials having thermionic activity.

There are two pieces of auxiliary apparatus that contribute to the precision of the regulator. They are the capacity C_1 and the a-c. by-pass circuit composed of the capacity C_4 and the resistance R_6. The by-pass circuit smooths out the pulsating current through the coupling resistance R_3. This pulsation is due to the cooling of the regulator tube filament between peaks of the alternating current through it. As there are two peaks per cycle the pulsation will be at twice the line frequency. If these pulses were in phase with the line voltage they would be permissible as the resultant pulsation in the grid potential of the amplifier would be in phase with its plate voltage and an acceptable output could be obtained from the amplifier. This phase relation does not exist, however, on account of the time lag introduced by the heating characteristics of the regulator tube filament. If capacity alone is used in the filter an oscillation is set up that causes the output voltage to hunt or, with an excess of capacity, the regulator is sluggish in its reaction to transients. In order to prevent these effects a small capacity is used and a resistance placed in series with it. The effect of this resistance is to damp the oscillation to such an extent that hunting is prevented but without causing a marked decrease in the filtering action.

The capacity C_1 aids the regulation with respect to load. The cause of a change in output voltage with a change in load is two fold. The first is a change in the impedance drop across the winding of the output transformer in series with the line. The other (the more important cause) is that the line current in the secondary of this transformer causes a corresponding current to flow in the primary. The path of this current is from the line through the fixed retardation coil, the primary of the output transformer and half of the primary of the power transformer to the other line. This current changes the balance of the bridge in such a direction as to reduce the output voltage. The condenser C_1 is placed across the high-voltage secondary of the power transformer. The volt-ampere input to this capacity is made large compared to the watts drawn by the transformer T_1 and results in a large leading component in its primary current. This adds a steady leading component to the line current through the output transformer. Any output current of the regulator adds vectorially to this steady current. This vectorial addition decreases the importance as viewed from the bridge circuit of all load variation on the regulator, especially lagging, and unity power factor loads. At the low frequencies employed in this case it was not considered probable that the leading component of the load would exceed 25 per cent of the rated full load of the regulator and the operation under these conditions was found to be satisfactory. Other important functions of this capacity are that of power factor correction for the entire regulator and the by-passing of harmonics in the amplifier circuit.

PERFORMANCE CHARACTERISTICS

The output characteristics of the regulator are shown in Fig. 5. The curves show the output voltage plotted against the input voltage with the load on the regulator as a parameter. These curves are plotted in per cent of normal voltage instead of in volts because the normal voltage may be adjusted over a range of several volts by means of the variable resistance R_4.

The regulator is not critical to frequency over a range of about ± 10 per cent of the normal frequency. Also the normal frequency can be adjusted over a range of about 30 per cent by adjustment of the by-pass circuit C_4 and R_6 and of the inductance L_1.

The distortion of wave form due to the regulator is slight. In the worst condition, that of low input voltage, the harmonics introduced are barely noticeable when viewed on a cathode ray oscillograph, and were estimated to be less than one-fourth of 1 per cent.

The limits of the regulating range with respect to input voltage are due largely to amplifier characteristics. The upper limit is fixed at the point where the grids of these tubes draw current. The lower limit is due to the ineffectiveness of a change in grid potential near the cut-off value and also to the insensitivity of the variable reactor at low values of direct current.

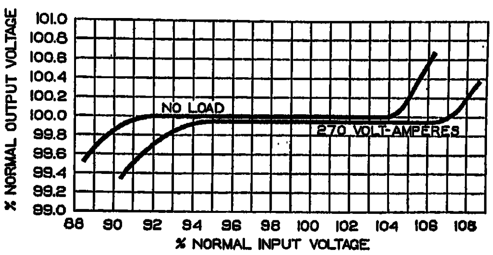

FIG. 5

Fig. 5 shows clearly the effect of a load on the regulator. It will be noted that this effect consists entirely of a shift of the regulating range and does not impair the regulation within this range. As a result, the maximum load that the regulator can carry depends upon the input voltage.

It is estimated that the life of the tubes should be about 3000 hr. It is necessary to readjust the resistance R_4 about once in 100 hr. of operation in order to compensate for the gradual reduction in the size of the filament of V_4 due to evaporation of the tungsten. The other apparatus in the circuit has a practically indefinite life.

It should be noted that no auxiliary sources of power supply are required for the plates or filaments of the tubes so that the circuit is well adapted for practical shop testing purposes.

DISCUSSION

The precision obtainable from a regulator of the type described is determined by the following factors:

1. The intrinsic precision of the voltage measuring device. (In this case, the emission characteristic of the tungsten filament of tube V_4).

2. The amplification between the measuring device and the controlling means.

3. The use of a suitable compensating means to offset the small change in output voltage which would otherwise be required to actuate the regulating circuit.

As regards the first factor, the tungsten filament gives a good intrinsic precision. If operated at a low emission it will give a long tube life.

The second factor is a matter of balancing cost against the value of the precision and range attained. In the circuit of Fig. 1 only one stage of amplification is employed. Another limiting factor in amplification is the tendency to hunting of the regulator. This tendency is reduced by reducing the time elements in the circuit between the measuring means and the control means. In this circuit the time factors are the heating lag in the tungsten filament, the time required to change the charge on the condenser C_4 across the grid circuit of tubes V_2 and V_3 and lastly, the time required to change the current in the highly inductive d-c. winding of L_2.

The third factor of compensation is essential in order to realize the full benefit of the circuit. For example, the circuit described would show a precision of only 0.70 of 1 per cent without compensation as compared with 0.03 of 1 per cent with compensation. The means of compensation must be such as to avoid impairing the stability of the circuit. This is most readily accomplished by introducing a small time element between the operation of the regulator and its compensation.

It may be concluded therefore that the limiting factors in the design of this type of regulator are economic rather than physical. By employing more amplification and suitably reducing the time factors in the circuit, it appears that a still higher degree of precision or a wider operating range could readily be obtained.

The circuit could of course be equally well designed to operate on 60 cycles or any other frequency of that order; in fact, the higher the frequency, the smaller and cheaper the apparatus would become.

Discussion

F. A. Byles: The voltage-regulator equipment described by Messrs. Stoller and Power is not only applicable for high grade laboratory service but may be very useful along many other lines. For many years the saturated-core transformer principle has been used to advantage in connection with various regulators. Not only has the vacuum-tube control been found to be very accurate, although rather complicated and expensive, but various vibrating-contact mechanical controls have also been found to be very satisfactory when used in combination with saturated core transformers.

I cannot say that I fully agree with the statement made in the first paragraph of their paper regarding the commonly used types of regulators.

A standard vibrating contact regulator will not usually give sufficiently accurate results for precision tests and measurements. However, with a few refinements such as jewel bearings, special windings, and resistors, a very fine degree of regulation may be obtained. I recall instances where I have noted regulation within 0.1-volt limits; that is, 0.05 volts either side of normal and continuing within these limits for days at a time on a 110-volt circuit.

In 1920 two installations were made where great accuracy was desired; one, in connection with nitrogen-fixation developments, and the other, for holding constant voltage for pipe insulation tests. Each of the regulators was designed to use buck-and-boost elements similar to those described in Messrs. Stoller's and Power's paper. The d-c. saturating windings were controlled by means of a very accurate magnetic mechanism with vibrating contacts. The results obtained came up to our expectations, not only from the standpoint of accuracy but also from the standpoint of rapid recovery under varying load conditions.

One of these regulators was designed to regulate a single-phase feeder circuit having a maximum capacity of 150 amperes at 110 volts. The voltage variation on the non-regulated incoming line was approximately 10 volts, the frequency varying between 58 minimum and 62 maximum.

A vacuum-tube regulator embodying the saturated-transformer principle for obtaining buck-and-boost regulation was built in our laboratory several years ago for meter testing. The precision results obtained with it augment the statements made in the paper under discussion.

Research and developmental work has been done, during the past few years, using similar principles of control, in connection with mercury-pool thyratron regulators and precision speed regulators, which has also proved the possibilities along this line of research.

Due to the comparatively high first cost, complicated construction, and the fact that vacuum tubes for this service are still in a more or less developmental stage, I have found that regulators of this general type have a somewhat limited application.

J. R. Power: In informal comment on this paper the use of a three-element tube as a rectifier has been questioned. This was used to reduce the number of types of tubes required. The rectifier tube is now identical with the amplifiers. There has also been comment on the fact that the grid of this tube is tied to the filament instead of the plate. This was done because of the high voltage across the tube during the reverse half wave. As it works in a high-impedance circuit the effect of the increased tube impedance due to this connection is negligible.

The regulator as shown in Fig. 3 is rather heavy and bulky. This is due to the fact that the frequency is low and is variable. It is designed for operation on from $16^2/_3$ to 20 cycles. If the frequency were increased the coils, of course, would become smaller and if the frequency were constant broadly tuned circuits could be used in some places to reduce still further the size of the apparatus. Another very important factor in the size and cost of the regulator is the range of input voltage. A very good index to the capacity of this type of regulator would be the product of the permissible input voltage variation and the maximum current. In this case such a rating would be 10 volts × 3 amperes or 30 regulating volt-amperes.

Bessel Functions for A-C. Problems

BY HERBERT BRISTOL DWIGHT*

Fellow, A. I. E. E.

***Synopsis.**—Tables of Bessel functions of zero and higher orders are given, for use in problems of skin effect and proximity effect in conductors carrying alternating currents. Formulas for deriving numerical values of the functions are given, in the form of series for large and for small values of the argument. The series are complete with their general terms.*

The application of the tables to the calculation of the skin effect resistance ratio of an isolated tubular conductor is described and an example is given.

THE behavior of alternating current in a round conductor or in groups of round conductors is calculated by means of Bessel functions, except for the case of infinitely thin tubes. Such problems are usually known as skin effect and proximity effect problems. The division of current between conductors connected in parallel, or in other words, the circulating currents in parallel conductors, the distribution of current over the cross section of the conductors, the watts loss in each conductor, the resistance drop, and the reactance drop, are characteristics which are desired to be known and which can be calculated for round wires.

Such problems occur in electrical engineering where heavy alternating currents of more than about 1000 amperes at 60 cycles are involved. When it is remembered that generators for power stations sometimes have a rating of 5000 amperes or more, and that electric furnaces up to 50,000 amperes capacity are built, it is evident that these problems occur in practical engineering. Small conductors carrying currents at radio frequencies require calculations similar to those for large conductors carrying 60-cycle current.

In this paper is presented a collection of available values of Bessel functions which are used in the classes of problems described.

In Table I are given values of ber x, bei x, ber$'$ x, bei$'$ x, ker x, kei x, ker$'$ x, and kei$'$ x. These are Bessel functions of the first and second kinds, order zero and argument $x\,i\,\sqrt{i}$.

The values of ber x, bei x, and ber$'$ x and the values of bei$'$ x from $x = 0.1$ to $x = 6.4$ were published by Professor A. G. Webster[9] in 1912. The corrected values of bei$'$ x from $x = 6.5$ to $x = 10$ were given in the same publication in 1916, and the tables of ker x, kei x, ker$'$ x, and kei$'$ x in 1915, by H. G. Savidge.[12] The writer has corrected the published value of bei 8.9 which contained a typographical error. Table I was published in *A Precise Method of Calculation of Skin Effect in Isolated Tubes.*[19]

Table II contains values of Bessel functions of the first kind and of higher orders than order zero. They were published in *Proximity Effect in Wires and Thin Tubes.*[20] They are used in problems of proximity effect of groups of wires or cables. See Bibliography, Nos. 14, 18, 21, 22, and 24.

Table III contains values of Bessel functions of the second kind and of higher orders than order zero. This table has not previously been published. The greater part of the values were calculated by W. H. Hastings[23] as thesis work in Massachusetts Institute of Technology in 1927. This table is used in calculating proximity effect in tubes.

It is sometimes required, in problems which arise, to have values of Bessel functions not given in the tables. The argument of the required function may be intermediate between those of the tables, or it may be outside the range of the tables. For such cases, formulas for deriving the values are required, and these formulas will now be given. It should be remembered that this paper deals with Bessel functions of argument $x\,i\sqrt{i}$, for problems in a-c. distribution. There are, of course, many other formulas and tables for other types of argument. For some intermediate values of x, the tables[9] of differences may be used.

For comparatively small values of x, the following series can be used:

$$\text{ber}\, x = 1 - \frac{\left(\frac{1}{2}x\right)^4}{(2\,!)^2} + \frac{\left(\frac{1}{2}x\right)^8}{(4\,!)^2} - \cdots + (-1)^{s-1}\frac{\left(\frac{1}{2}x\right)^{4(s-1)}}{(2s-2)\,!\,(2s-2)\,!} + \cdots \tag{1}$$

$$\text{bei}\, x = \frac{\left(\frac{1}{2}x\right)^2}{(1\,!)^2} - \frac{\left(\frac{1}{2}x\right)^6}{(3\,!)^2} + \frac{\left(\frac{1}{2}x\right)^{10}}{(5\,!)^2} - \cdots + (-1)^{s-1}\frac{\left(\frac{1}{2}x\right)^{4s-2}}{(2s-1)\,!\,(2s-1)\,!} + \cdots \tag{2}$$

$$\text{ber}'\, x = -\frac{2\left(\frac{1}{2}x\right)^3}{(2\,!)^2} + \frac{4\left(\frac{1}{2}x\right)^7}{(4\,!)^2} - \frac{6\left(\frac{1}{2}x\right)^{11}}{(6\,!)^2} + \cdots + (-1)^{s}\frac{2s\left(\frac{1}{2}x\right)^{4s-1}}{(2s)\,!\,(2s)\,!} + \cdots \tag{3}$$

$$\text{bei}'\, x = \frac{\frac{1}{2}x}{(1\,!)^2} - \frac{3\left(\frac{1}{2}x\right)^5}{(3\,!)^2} + \frac{5\left(\frac{1}{2}x\right)^9}{(5\,!)^2} - \cdots$$

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For numbered references see Bibliography.

TABLE I
BESSEL FUNCTIONS OF ORDER ZERO

x	ber x	bei x	ber′ x	bei′ x	x	ker x	kei x	ker′ x	kei′ x	x
0	+1.0	0	0	0	0	+ ∞	−0.785 398 2	− ∞	0	0
0.1	+0.999 998 438	+0.002 500 000	−0.000 062 500	+0.049 999 974	0.1	+2.420 474 0	−0.776 850 6	−9.960 959 3	+0.145 974 8	0.1
0.2	+0.999 975 000	+0.009 999 972	−0.000 499 999	+0.099 999 167	0.2	+1.733 142 7	−0.758 124 9	−4.922 948 5	+0.222 926 8	0.2
0.3	+0.999 873 438	+0.022 499 684	−0.001 687 488	+0.149 993 672	0.3	+1.337 218 6	−0.733 101 9	−3.219 865 2	+0.274 292 1	0.3
0.4	+0.999 600 004	+0.039 998 222	−0.003 999 911	+0.199 973 334	0.4	+1.062 623 9	−0.703 800 2	−2.352 069 9	+0.309 514 0	0.4
0.5	+0.999 023 464	+0.062 493 218	−0.007 812 076	+0.249 918 621	0.5	+0.855 905 9	−0.671 581 7	−1.819 709 8	+0.333 203 8	0.5
0.6	+0.997 975 114	+0.089 979 750	−0.013 498 481	+0.299 797 507	0.6	+0.693 120 7	−0.637 449 5	−1.456 538 6	+0.348 164 4	0.6
0.7	+0.996 248 828	+0.122 448 939	−0.021 433 032	+0.349 562 345	0.7	+0.561 378 3	−0.602 175 5	−1.190 943 3	+0.356 309 5	0.7
0.8	+0.993 601 138	+0.159 886 230	−0.031 988 623	+0.399 146 758	0.8	+0.452 882 1	−0.566 367 6	−0.987 335 1	+0.359 042 5	0.8
0.9	+0.989 751 357	+0.202 269 303	−0.045 536 553	+0.448 462 528	0.9	+0.362 514 8	−0.530 511 1	−0.825 868 7	+0.357 443 2	0.9
1.0	+0.984 381 781	+0.249 566 040	−0.062 445 752	+0.497 396 511	1.0	+0.286 706 2	−0.494 994 6	−0.694 603 9	+0.352 369 9	1.0
1.1	+0.977 137 973	+0.301 731 269	−0.083 081 791	+0.545 807 563	1.1	+0.222 844 5	−0.460 129 5	−0.585 905 3	+0.344 521 0	1.1
1.2	+0.967 629 156	+0.358 704 420	−0.107 805 642	+0.593 523 499	1.2	+0.168 045 6	−0.426 163 6	−0.494 643 2	+0.334 473 9	1.2
1.3	+0.955 428 747	+0.420 405 966	−0.136 972 160	+0.640 338 102	1.3	+0.123 455 4	−0.393 291 8	−0.417 227 4	+0.322 711 8	1.3
1.4	+0.940 075 057	+0.486 733 934	−0.170 928 324	+0.686 008 176	1.4	+0.085 126 0	−0.361 664 8	−0.351 055 1	+0.309 641 6	1.4
1.5	+0.921 072 184	+0.557 560 062	−0.210 011 017	+0.730 250 074	1.5	+0.052 034 9	−0.331 395 6	−0.294 181 6	+0.295 608 1	1.5
1.6	+0.897 891 139	+0.632 725 677	−0.254 544 638	+0.772 739 922	1.6	+0.026 029 9	−0.302 565 5	−0.245 114 7	+0.280 903 8	1.6
1.7	+0.869 971 237	+0.712 037 202	−0.304 838 207	+0.813 104 047	1.7	+0.004 691 1	−0.275 228 8	−0.202 081 8	+0.265 777 2	1.7
1.8	+0.836 721 794	+0.795 261 955	−0.361 182 125	+0.850 926 951	1.8	−0.014 696 1	−0.249 417 1	−0.165 942 4	+0.250 438 5	1.8
1.9	+0.797 524 167	+0.882 122 341	−0.423 844 516	+0.885 736 950	1.9	−0.029 661 4	−0.225 142 2	−0.134 128 2	+0.235 065 7	1.9
2.0	+0.751 734 183	+0.972 291 627	−0.493 067 125	+0.917 013 613	2.0	−0.041 664 5	−0.202 400 1	−0.106 601 0	+0.219 807 9	2.0
2.1	+0.698 685 001	+1.065 388 161	−0.569 060 755	+0.944 181 339	2.1	−0.051 106 5	−0.181 172 6	−0.082 823 4	+0.204 789 7	2.1
2.2	+0.637 690 457	+1.160 969 944	−0.652 000 244	+0.966 608 614	2.2	−0.058 338 8	−0.161 430 7	−0.062 337 3	+0.190 113 7	2.2
2.3	+0.568 048 926	+1.258 528 975	−0.742 018 947	+0.983 606 691	2.3	−0.063 670 5	−0.143 135 7	−0.044 747 9	+0.175 863 8	2.3
2.4	+0.489 047 772	+1.357 485 476	−0.839 202 721	+0.994 424 643	2.4	−0.067 373 5	−0.126 241 5	−0.029 712 3	+0.162 106 9	2.4
2.5	+0.399 968 417	+1.457 182 044	−0.943 583 409	+0.998 268 847	2.5	−0.069 688 0	−0.110 696 1	−0.016 929 8	+0.148 895 4	2.5
2.6	+0.300 092 000	+1.556 877 774	−1.055 131 815	+0.994 202 944	2.6	−0.070 825 7	−0.096 442 9	−0.006 135 8	+0.136 268 9	2.6
2.7	+0.188 706 304	+1.655 742 407	−1.173 750 173	+0.981 488 365	2.7	−0.070 973 6	−0.083 421 9	+0.002 904 3	+0.124 255 8	2.7
2.8	+0.065 112 108	+1.752 850 564	−1.299 264 112	+0.958 965 456	2.8	−0.070 206 3	−0.071 570 7	+0.010 399 0	+0.112 874 8	2.8
2.9	−0.071 367 826	+1.847 176 116	−1.431 414 136	+0.925 650 305	2.9	−0.068 939 0	−0.060 825 5	+0.016 534 2	+0.102 136 2	2.9
3.0	−0.221 380 249	+1.937 586 785	−1.569 846 632	+0.880 482 324	3.0	−0.067 029 2	−0.051 121 9	+0.021 476 2	+0.092 043 1	3.0
3.1	−0.385 531 455	+2.022 839 042	−1.714 104 430	+0.822 297 088	3.1	−0.064 678 6	−0.042 305 5	+0.025 373 8	+0.082 592 2	3.1
3.2	−0.564 376 430	+2.101 573 388	−1.863 616 954	+0.749 923 691	3.2	−0.061 984 8	−0.034 582 3	+0.028 360 3	+0.073 775 2	3.2
3.3	−0.758 407 012	+2.172 310 131	−2.017 680 096	+0.662 139 131	3.3	−0.059 032 9	−0.027 619 7	+0.030 555 4	+0.065 579 4	3.3
3.4	−0.968 038 995	+2.233 445 750	−2.175 405 175	+0.557 689 801	3.4	−0.055 896 6	−0.021 446 3	+0.032 066 2	+0.057 988 1	3.4
3.5	−1.193 598 180	+2.283 249 967	−2.336 050 130	+0.435 296 178	3.5	−0.052 639 3	−0.016 002 6	+0.032 988 6	+0.050 982 1	3.5
3.6	−1.435 305 322	+2.319 803 655	−2.498 252 527	+0.293 662 421	3.6	−0.049 315 6	−0.011 231 1	+0.033 408 7	+0.044 539 4	3.6
3.7	−1.693 259 984	+2.341 297 714	−2.660 778 962	+0.134 686 700	3.7	−0.045 971 7	−0.007 076 7	+0.033 403 0	+0.038 636 4	3.7
3.8	−1.967 423 273	+2.345 433 061	−2.822 163 850	−0.052 526 621	3.8	−0.042 646 9	−0.003 486 7	+0.033 040 0	+0.033 248 0	3.8
3.9	−2.257 599 466	+2.330 021 882	−2.980 743 427	−0.259 654 007	3.9	−0.039 373 61	−0.000 410 81	+0.032 380 46	+0.028 348 32	3.9
4.0	−2.563 416 557	+2.292 690 323	−3.134 653 964	−0.491 137 441	4.0	−0.036 178 85	+0.002 198 40	+0.031 478 49	+0.023 910 62	4.0
4.1	−2.884 305 732	+2.230 942 780	−3.281 821 353	−0.748 166 860	4.1	−0.033 084 40	+0.004 385 82	+0.030 381 79	+0.019 908 04	4.1
4.2	−3.219 479 832	+2.142 167 987	−3.419 051 224	−1.031 862 169	4.2	−0.030 107 58	+0.006 193 61	+0.029 132 42	+0.016 313 67	4.2
4.3	−3.567 910 863	+2.023 647 069	−3.546 519 744	−1.343 251 997	4.3	−0.027 261 77	+0.007 661 27	+0.027 767 30	+0.013 100 84	4.3
4.4	−3.928 306 621	+1.872 563 796	−3.658 765 306	−1.683 250 947	4.4	−0.024 550 89	+0.008 825 62	+0.026 318 68	+0.010 243 31	4.4
4.5	−4.299 086 552	+1.686 017 204	−3.753 081 326	−2.052 634 662	4.5	−0.021 999 88	+0.009 720 92	+0.024 814 54	+0.007 715 43	4.5
4.6	−4.678 356 937	+1.461 036 836	−3.828 010 348	−2.452 012 698	4.6	−0.019 595 03	+0.010 378 86	+0.023 279 08	+0.005 492 26	4.6
4.7	−5.063 885 587	+1.194 600 797	−3.878 239 739	−2.881 799 197	4.7	−0.017 344 41	+0.010 828 72	+0.021 733 00	+0.003 549 76	4.7
4.8	−5.453 076 175	+0.883 656 854	−3.900 599 216	−3.342 181 300	4.8	−0.015 248 19	+0.011 097 40	+0.020 193 91	+0.001 864 78	4.8
4.9	−5.842 942 442	+0.525 146 811	−3.891 060 511	−3.833 085 297	4.9	−0.013 304 90	+0.011 209 53	+0.018 676 61	+0.000 415 22	4.9
5.0	−6.230 082 479	+0.116 034 382	−3.845 339 473	−4.354 140 518	5.0	−0.011 511 73	+0.011 187 50	+0.017 193 40	−0.000 819 98	5.0
5.1	−6.610 653 357	−0.346 663 218	−3.758 900 943	−4.904 640 985	5.1	−0.009 864 74	+0.011 052 01	+0.015 754 36	−0.001 860 79	5.1
5.2	−6.980 346 403	−0.865 839 727	−3.626 966 748	−5.483 504 900	5.2	−0.008 359 11	+0.010 821 28	+0.014 367 57	−0.002 726 05	5.2
5.3	−7.334 363 435	−1.444 260 151	−3.444 527 187	−6.089 232 022	5.3	−0.006 989 28	+0.010 512 00	+0.013 039 35	−0.003 433 49	5.3
5.4	−7.667 394 351	−2.084 516 693	−3.206 356 389	−6.719 859 076	5.4	−0.005 749 13	+0.010 139 29	+0.011 774 46	−0.003 999 69	5.4
5.5	−7.973 596 451	−2.788 980 155	−2.907 031 958	−7.372 913 333	5.5	−0.004 632 16	+0.009 716 31	+0.010 576 33	−0.004 440 16	5.5
5.6	−8.246 575 962	−3.559 746 593	−2.540 959 318	−8.045 364 552	5.6	−0.003 631 56	+0.009 254 06	+0.009 447 17	−0.004 769 28	5.6
5.7	−8.479 372 252	−4.398 579 111	−2.102 401 197	−8.733 575 532	5.7	−0.002 740 38	+0.008 765 72	+0.008 388 18	−0.005 000 41	5.7
5.8	−8.664 445 263	−5.306 844 640	−1.585 512 096	−9.433 251 539	5.8	−0.001 951 58	+0.008 257 74	+0.007 399 67	−0.005 145 84	5.8
5.9	−8.793 666 753	−6.285 445 623	−0.984 382 394	−10.139 388 063	5.9	−0.001 258 12	+0.007 730 02	+0.006 481 21	−0.005 216 89	5.9
6.0	−8.858 315 966	−7.334 746 541	−0.293 079 967	−10.846 223 584	6.0	−0.000 653 04	+0.007 216 40	+0.005 631 71	−0.005 223 02	6.0
6.1	−8.849 080 413	−8.454 495 269	+0.494 289 242	−11.547 178 908	6.1	−0.000 129 53	+0.006 698 06	+0.004 849 57	−0.005 176 37	6.1
6.2	−8.756 062 474	−9.643 739 286	+1.383 522 213	−12.234 815 078	6.2	+0.000 319 05	+0.006 182 75	+0.004 132 75	−0.005 082 83	6.2
6.3	−8.568 792 503	−10.900 736 825	+2.380 248 300	−12.900 778 962	6.3	+0.000 699 12	+0.005 680 77	+0.003 478 86	−0.004 951 05	6.3
6.4	−8.276 249 873	−12.222 863 128	+3.480 851 325	−13.535 756 059	6.4	+0.001 016 83	+0.005 193 58	+0.002 885 23	−0.004 788 03	6.4
6.5	−7.866 890 928	−13.606 512 001	+4.717 382 012	−14.129 423	6.5	+0.001 278 080	+0.004 723 992	+0.002 348 995	−0.004 600 032	6.5
6.6	−7.328 687 885	−15.046 092 991	+6.067 462 487	−14.670 413	6.6	+0.001 488 446	+0.004 274 219	+0.001 867 130	−0.004 392 632	6.6
6.7	−6.649 176 464	−16.538 424 538	+7.544 180 362	−15.146 260	6.7	+0.001 653 215	+0.003 845 947	+0.001 436 521	−0.004 170 782	6.7
6.8	−5.815 515 115	−18.073 623 609	+9.150 973 359	−15.543 406	6.8	+0.001 777 354	+0.003 440 398	+0.001 053 990	−0.003 938 849	6.8
6.9	−4.814 556 200	−19.643 992 365	+10.890 503 759	−15.847 109	6.9	+0.001 865 512	+0.003 058 385	+0.000 716 382	−0.003 700 061	6.9
7.0	−3.632 930 243	−21.239 402 580	+12.764 522 560	−16.041 489	7.0	+0.001 922 022	+0.002 700 305	+0.000 420 510	−0.003 459 509	7.0
7.1	−2.257 144 280	−22.848 078 597	+14.773 723 174	−16.109 484	7.1	+0.001 950 901	+0.002 366 480	+0.000 163 267	−0.003 218 285	7.1
7.2	−0.673 695 379	−24.456 479 797	+16.917 584 633	−16.032 856	7.2	+0.001 955 861	+0.002 056 620	−0.000 058 386	−0.002 979 421	7.2
7.3	+1.130 799 653	−26.049 183 639	+19.194 204 342	−15.792 207	7.3	+0.001 940 312	+0.001 770 454	−0.000 247 403	−0.002 744 978	7.3
7.4	+3.169 457 312	−27.608 770 523	+21.600 120 535	−15.367 001	7.4	+0.001 907 373	+0.001 507 429	−0.000 406 628	−0.002 516 671	7.4
7.5	+5.454 962 184	−29.115 711 867	+24.130 124 710	−14.735 602	7.5	+0.001 850 888	+0.001 266 868	−0.000 538 787	−0.002 295 904	7.5
7.6	+7.999 382 494	−30.548 262 965	+26.777 964 473	−13.875 334	7.6	+0.001 800 431	+0.001 047 959	−0.000 646 478	−0.002 083 800	7.6
7.7	+10.813 965 476	−31.882 362 359	+29.531 637 360	−12.762 551	7.7	+0.001 731 326	+0.000 849 790	−0.000 732 165	−0.001 881 234	7.7
7.8	+13.908 911 711	−33.091 539 670	+32.382 176 399	−11.372 739	7.8	+0.001 654 654	+0.000 671 373	−0.000 798 170	−0.001 688 855	7.8
7.9	+17.293 127 645	−34.146 833 088	+35.314 428 336	−9.680 623	7.9	+0.001 572 275	+0.000 511 664	−0.000 846 677	−0.001 507 120	7.9
8.0	+20.973 955 611	−35.016 725 165	+38.311 325 701	−7.660 318	8.0	+0.001 485 834	+0.000 360 584	−0.000 879 724	−0.001 330 313	8.0
8.1	+24.956 880 800	−35.667 980 514	+41.352 754 078	−5.285 490	8.1	+0.001 396 782	+0.000 244 032	−0.000 899 210	−0.001 176 567	8.1
8.2	+29.245 214 706	−36.061 119 081	+44.415 316 208	−2.529 555	8.2	+0.001 306 386	+0.000 133 902	−0.000 906 891	−0.001 027 888	8.2
8.3	+33.839 755 432	−36.159 400 616	+47.472 094 831	+0.634 098	8.3	+0.001 215 743	+0.000 038 090	−0.000 904 388	−0.000 890 168	8.3
8.4	+38.738 422 961	−35.919 829 830	+50.492 416 438	+4.231 841	8.4	+0.001 125 797	−0.000 044 491	−0.000 893 190	−0.000 763 209	8.4
8.5	+43.935 872 751	−35.297 700 300	+53.441 018 430	+8.289 519	8.5	+0.001 037 349	−0.000 114 902	−0.000 874 656	−0.000 646 733	8.5
8.6	+49.423 084 977	−34.245 760 640	+56.280 822 496	+12.832 116	8.6	+0.000 951 070	−0.000 174 175	−0.000 850 022	−0.000 540 398	8.6
8.7	+55.186 932 009	−32.714 319 308	+58.966 717 374	+17.883 387	8.7	+0.000 867 511	−0.000 223 306	−0.000 820 407	−0.000 443 813	8.7
8.8	+61.209 725 224	−30.651 387 879	+61.451 354 510	+23.465 444	8.8	+0.000 787 120	−0.000 263 248	−0.000 786 819	−0.000 356 542	8.8
8.9	+67.468 740 848	−28.002 867 538	+63.681 960 575	+29.598 302	8.9	+0.000 710 249	−0.000 294 010	−0.000 750 159	−0.000 278 127	8.9
9.0	+73.935 729 857	−24.712 783 168	+65.600 770 999	+36.200 384	9.0	+0.000 637 164	−0.000 319 153	−0.000 711 231	−0.000 208 079	9.0
9.1	+80.576 411 145	−20.723 569 533	+67.144 889 467	+43.582 976	9.1	+0.000 568 055	−0.000 336 788	−0.000 670 745	−0.000 145 903	9.1
9.2	+87.349 952 674	−15.976 414 197	+68.246 178 293	+51.450 634	9.2	+0.000 503 046	−0.000 348 579	−0.000 629 326	−0.000 091 093	9.2
9.3	+94.208 443 358	−10.411 661 917	+68.831 185 381	+59.935 547	9.3	+0.000 442 203	−0.000 355 236	−0.000 587 517	−0.000 043 145	9.3
9.4	+101.096 359 718	−3.969 285 324	+68.821 113 743	+69.011 850	9.4	+0.000 385 540	−0.000 357 420	−0.000 545 789	−0.000 001 559	9.4
9.5	+107.950 031 881	+3.410 573 282	+68.131 840 035	+78.683 888	9.5	+0.000 333 029	−0.000 355 743	−0.000 504 544	+0.000 034 158	9.5
9.6	+114.697 114 173	+11.786 984 189	+66.673 089 017	+88.940 434	9.6	+0.000 284 604	−0.000 350 768	−0.000 464 122	+0.000 064 485	9.6
9.7	+121.256 066 255	+21.217 531 810	+64.353 071 286	+99.762 855	9.7	+0.000 240 168	−0.000 343 010	−0.000 424 806	+0.000 089 887	9.7
9.8	+127.535 651 521	+31.757 530 896	+61.069 692 033	+111.124 240	9.8	+0.000 199 598	−0.000 332 940	−0.000 386 830	+0.000 110 811	9.8
9.9	+133.434 460 262	+43.459 152 933	+56.719 839 030	+122.988 479	9.9	+0.000 162 751	−0.000 320 983	−0.000 350 379	+0.000 127 684	9.9
10.0	+138.840 465 942	+56.370 458 554	+51.195 258 394	+135.309 302	10.0	+0.000 129 466	−0.000 307 524	−0.000 315 597	+0.000 140 914	10.0

$$+ (-1)^{s-1} \frac{(2s-1)\left(\frac{1}{2}x\right)^{4s-3}}{(2s-1)!\,(2s-1)!} + \cdots \qquad (4)$$

$$\operatorname{ker} x = (0.1159315 - \log_e x) \operatorname{ber} x + \frac{\pi}{4} \operatorname{bei} x - \left(1 + \frac{1}{2}\right) \frac{\left(\frac{1}{2}x\right)^4}{(2\,!)^2} + \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \frac{\left(\frac{1}{2}x\right)^8}{(4\,!)^2} - \cdots + (-1)^s \left(1 + \frac{1}{2} + \frac{1}{3} \cdots + \frac{1}{2s}\right) \frac{\left(\frac{1}{2}x\right)^{4s}}{(2s)!\,(2s)!} + \cdots \qquad (5)$$

$$\operatorname{kei} x = (0.1159315 - \log_e x) \operatorname{bei} x - \frac{\pi}{4} \operatorname{ber} x + \frac{\left(\frac{1}{2}x\right)^2}{(1\,!)^2} - \left(1 + \frac{1}{2} + \frac{1}{3}\right) \frac{\left(\frac{1}{2}x\right)^6}{(3\,!)^2} + \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right) \frac{\left(\frac{1}{2}x\right)^{10}}{(5\,!)^2} - \cdots + (-1)^{s-1} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2s-1}\right) \frac{\left(\frac{1}{2}x\right)^{4s-2}}{(2s-1)!\,(2s-1)!} + \cdots \qquad (6)$$

$$\operatorname{ker}' x = (0.1159315 - \log_e x) \operatorname{ber}' x - \frac{1}{x} \operatorname{ber} x + \frac{\pi}{4} \operatorname{bei}' x - \left(1 + \frac{1}{2}\right) \frac{\left(\frac{1}{2}x\right)^3}{2\,!\,1\,!} + \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) \frac{\left(\frac{1}{2}x\right)^7}{4\,!\,3\,!} - \cdots + (-1)^s \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2s}\right) \frac{\left(\frac{1}{2}x\right)^{4s-1}}{(2s)!\,(2s-1)!} + \cdots \qquad (7)$$

$$\operatorname{kei}' x = (0.1159315 - \log_e x) \operatorname{bei}' x - \frac{1}{x} \operatorname{bei} x - \frac{\pi}{4} \operatorname{ber}' x + \frac{\frac{1}{2}x}{(1\,!)^2} - \left(1 + \frac{1}{2} + \frac{1}{3}\right) \frac{\left(\frac{1}{2}x\right)^5}{3\,!\,2\,!} + \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right) \frac{\left(\frac{1}{2}x\right)^9}{5\,!\,4\,!} - \cdots + (-1)^{s-1} \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2s-1}\right) \frac{\left(\frac{1}{2}x\right)^{4s-3}}{(2s-1)!\,(2s-2)!} + \cdots \qquad (8)$$

See Reference 10, p. 210.

The constant 0.1159315 is equal to $\log_e 2 - \gamma$, where γ is Euler's constant, 0.5772157. See Reference 15, pp. 23 and 254.

For higher orders, where n is a positive integer,

$$\operatorname{ber}_n x + i \operatorname{bei}_n x = J_n (x\, i \sqrt{i}) = u_n + i\, v_n \qquad (9)$$

TABLE II

BESSEL FUNCTIONS OF THE FIRST KIND, OF ORDERS HIGHER THAN ZERO

$$\operatorname{ber}_n x + i \operatorname{bei}_n x = J_n (x\, i \sqrt{i}) = i^n I_n (x \sqrt{i}),$$

$$\operatorname{ber}_n' x = \frac{d}{d\,x} \operatorname{ber}_n x.$$

x	1	2	3	4	5	6	7	8	9	10	x
$\operatorname{ber}_1 x$	−0.395 868	−0.997 078	−1.732 64	−1.869 25	+0.359 78	+ 7.462 20	+20.368 9	+32.506 9	+20.719 2	− 59.478	$\operatorname{ber}_1 x$
$\operatorname{bei}_1 x$	+0.307 557	+0.299 775	−0.487 45	−2.563 82	−5.797 91	− 7.876 67	− 2.317 2	+21.673 5	+72.054 3	+131.879	$\operatorname{bei}_1 x$
$\operatorname{ber}_1' x$	−0.476 664	−0.720 532	−0.635 99	+0.658 74	+4.251 33	+10.206 52	+14.677 5	+ 5.866 4	−37.108 0	−132.087	$\operatorname{ber}_1' x$
$\operatorname{bei}_1' x$	+0.212 036	−0.305 845	−1.364 13	−2.792 83	−3.327 80	+ 0.235 45	+12.780 7	+36.882 2	+61.749 0	+ 45.127	$\operatorname{bei}_1' x$
$\operatorname{ber}_2 x$	+0.010 411	+0.165 279	+0.808 37	+2.317 85	+4.488 43	+ 5.242 91	− 0.950 4	−22.889 0	−65.869 2	−111.779	$\operatorname{ber}_2 x$
$\operatorname{bei}_2 x$	−0.124 675	−0.479 225	−0.891 02	−0.725 36	+1.422 10	+ 7.432 44	+17.592 4	+25.438 9	+10.134 8	− 66.610	$\operatorname{bei}_2 x$
$\operatorname{ber}_2' x$	+0.041 623	+0.327 788	+1.030 93	+1.975 73	+2.049 97	− 1.454 56	−12.493 0	−32.589 1	−50.963 2	− 28.840	$\operatorname{ber}_2' x$
$\operatorname{bei}_2' x$	−0.248 047	−0.437 789	−0.286 47	+0.853 82	+3.785 30	+ 8.368 74	+11.015 1	+ 1.300 6	−38.551 6	−121.987	$\operatorname{bei}_2' x$
$\operatorname{ber}_3 x$	+0.013 788	+0.085 612	+0.130 44	−0.282 63	−2.094 35	− 6.430 04	−12.876 5	−15.420 4	+ 3.166 6	+ 72.253	$\operatorname{ber}_3 x$
$\operatorname{bei}_3 x$	+0.015 629	+0.144 210	+0.565 38	+1.437 76	+2.454 41	+ 1.901 46	− 4.407 2	−22.575 0	−54.538 7	− 81.423	$\operatorname{bei}_3 x$
$\operatorname{ber}_3' x$	+0.039 433	+0.093 575	+0.072 00	−0.914 09	−2.922 76	− 5.747 81	− 6.249 2	+ 3.979 6	+38.354 6	+104.463	$\operatorname{ber}_3' x$
$\operatorname{bei}_3' x$	+0.048 634	+0.239 418	+0.636 27	+1.073 55	+0.695 57	− 2.498 96	−11.222 9	−25.707 4	−35.563 4	− 7.513	$\operatorname{bei}_3' x$
$\operatorname{ber}_4 x$	−0.002 60	−0.040 97	−0.193 27	−0.493 10	−0.628 67	+ 0.648 3	+ 6.083 5	+19.094 7	+38.667	+ 46.579	$\operatorname{ber}_4 x$
$\operatorname{bei}_4 x$	−0.000 13	−0.008 30	−0.093 02	−0.499 85	−1.727 62	− 4.230 2	− 7.116 9	− 5.288 8	+14.082	+ 70.500	$\operatorname{bei}_4 x$
$\operatorname{ber}_4' x$	−0.010 40	−0.080 56	−0.234 32	−0.323 71	+0.248 34	+ 2.770 0	+ 8.745 2	+17.319 5	+19.140	− 12.148	$\operatorname{ber}_4' x$
$\operatorname{bei}_4' x$	−0.000 78	−0.024 83	−0.183 52	−0.716 65	−1.834 36	− 3.071 1	− 1.921 9	+ 7.703 5	+34.545	+ 80.465	$\operatorname{bei}_4' x$
$\operatorname{ber}_5 x$	+0.000 19	+0.006 80	+0.058 59	+0.273 08	+0.851 04	+ 1.830 5	+ 2.209 0	− 1.821 3	−18.619	− 58.722	$\operatorname{ber}_5 x$
$\operatorname{bei}_5 x$	−0.000 18	−0.004 84	−0.025 54	−0.033 53	+0.211 43	+ 1.475 6	+ 5.242 3	+12.812 8	+21.384	+ 15.193	$\operatorname{bei}_5 x$
$\operatorname{ber}_5' x$	+0.000 97	+0.017 84	+0.104 78	+0.360 76	+0.815 11	+ 1.007 4	− 0.847 2	− 8.623 9	−26.955	− 53.427	$\operatorname{ber}_5' x$
$\operatorname{bei}_5' x$	−0.000 87	−0.011 00	−0.028 32	+0.046 69	+0.565 64	+ 2.220 0	+ 5.589 6	+ 9.233 7	+ 5.504	− 24.511	$\operatorname{bei}_5' x$

TABLE III

BESSEL FUNCTIONS OF THE SECOND KIND, OF ORDERS HIGHER THAN ZERO

$$\mathrm{ker}_n\,x + i\,\mathrm{kei}_n\,x = \frac{\pi i}{2} H_n^1 (x\, i\, \sqrt{i}) = i^{-n} K_n (x \sqrt{i})$$

$$\mathrm{ker}_n'\,x = \frac{d}{d\,x}\,\mathrm{ker}_n\,x$$

x	1	2	3	4	5	6	7	8	9	10	x
$\mathrm{ker}_1\,x$	− 0.740 322	− 0.230 806	−0.049 898	+0.005 351 3	+0.012 737 4	+0.007 676 09	+0.002 743 59	+0.000 322 857	−0.000 355 78	−0.000 322 80	$\mathrm{ker}_1\,x$
$\mathrm{kei}_1\,x$	− 0.241 996	+ 0.080 049	+0.080 270	+0.039 166 0	+0.011 577 8	+0.000 288 35	−0.002 148 90	−0.001 566 975	−0.000 650 05	−0.000 123 52	$\mathrm{kei}_1\,x$
$\mathrm{ker}_1'\,x$	+ 0.887 604	+ 0.287 983	+0.100 178	+0.022 690 0	−0.002 318 3	−0.005 920 41	−0.003 660 46	−0.001 352 336	−0.000 185 34	+0.000 158 19	$\mathrm{ker}_1'\,x$
$\mathrm{kei}_1'\,x$	+ 0.794 742	+ 0.073 632	−0.038 005	−0.036 928 3	−0.018 366 4	−0.005 612 66	+0.000 156 61	+0.000 985 180	+0.000 748 45	+0.000 321 35	$\mathrm{kei}_1'\,x$
$\mathrm{ker}_2\,x$	+ 0.418 03	+ 0.261 472	+0.128 391	+0.048 134 2	+0.011 183 7	−0.001 088 3	−0.002 910 45	−0.001 819 91	−0.000 683 40	−0.000 101 28	$\mathrm{ker}_2\,x$
$\mathrm{kei}_2\,x$	+ 1.884 20	+ 0.309 001	+0.036 804	−0.017 937 6	−0.018 064 9	−0.009 093 7	−0.002 820 51	−0.000 149 65	+0.000 477 20	+0.000 370 64	$\mathrm{kei}_2\,x$
$\mathrm{ker}_2'\,x$	− 0.141 46	− 0.154 871	−0.107 070	−0.055 545 6	−0.021 666 9	−0.005 268 9	+0.000 411 05	+0.001 334 70	+0.000 863 10	+0.000 335 85	$\mathrm{ker}_2'\,x$
$\mathrm{kei}_2'\,x$	− 4.120 77	− 0.528 809	−0.116 579	−0.014 941 8	+0.008 046 0	+0.008 255 2	+0.004 265 37	+0.001 373 73	+0.000 102 03	−0.000 215 04	$\mathrm{kei}_2'\,x$
$\mathrm{ker}_3\,x$	+ 4.887 27	+ 0.298 022	−0.036 451	−0.052 071 1	−0.029 282 9	−0.011 449 9	−0.002 707 2	+0.000 267 67	+0.000 720 5	+0.000 456 3	$\mathrm{ker}_3\,x$
$\mathrm{kei}_3\,x$	− 6.269 71	− 0.886 821	−0.236 018	−0.060 518 2	−0.007 685 2	+0.004 511 5	+0.004 464 6	+0.002 263 32	+0.000 714 8	+0.000 047 3	$\mathrm{kei}_3\,x$
$\mathrm{ker}_3'\,x$	− 16.289 7	− 0.850 418	−0.080 360	+0.017 701 2	+0.022 435 5	+0.012 924 7	+0.005 212 6	+0.001 292 32	−0.000 094 4	−0.000 327 3	$\mathrm{ker}_3'\,x$
$\mathrm{kei}_3'\,x$	+ 17.772 4	+ 1.296 62	+0.300 78	+0.092 108 5	+0.025 293 0	+0.003 405 0	−0.001 977 0	−0.002 029 80	−0.001 059 0	−0.000 347 9	$\mathrm{kei}_3'\,x$
$\mathrm{ker}_4\,x$	− 47.753 1	− 2.774 90	−0.410 62	−0.057 09	+0.007 143	+0.012 375	+0.007 257	+0.002 878 3	+0.000 680 7	−0.000 072 2	$\mathrm{ker}_4\,x$
$\mathrm{kei}_4\,x$	+ 3.981 0	+ 0.940 03	+0.348 52	+0.137 36	+0.049 433	+0.014 000	+0.001 780	−0.001 192 6	−0.001 153 8	−0.000 584 3	$\mathrm{kei}_4\,x$
$\mathrm{ker}_4'\,x$	+191.990	+ 5.966 15	+0.740 16	+0.136 71	+0.020 426	−0.003 344	−0.005 361	−0.003 228 8	−0.001 317 5	−0.000 327 2	$\mathrm{ker}_4'\,x$
$\mathrm{kei}_4'\,x$	− 8.035	− 1.042 25	−0.323 58	−0.131 38	−0.054 819	−0.020 620	−0.006 088	−0.000 814 8	+0.000 516 8	+0.000 522 9	$\mathrm{kei}_4'\,x$
$\mathrm{ker}_5\,x$	+287.76	+10.209 4	+1.467 9	+0.327 07	+0.077 13	+0.012 982	−0.001 719	−0.003 146 2	−0.001 873 6	−0.000 746 0	$\mathrm{ker}_5\,x$
$\mathrm{kei}_5\,x$	+253.88	+ 6.076 6	+0.353 1	−0.052 99	−0.056 32	−0.029 378	−0.011 767	−0.003 455 3	−0.000 417 5	+0.000 324 1	$\mathrm{kei}_5\,x$
$\mathrm{ker}_5'\,x$	−1407.9	−24.226 0	−2.402 6	−0.465 59	−0.117 13	−0.029 468	−0.005 162	+0.000 774 4	+0.001 375 4	+0.000 837 2	$\mathrm{ker}_5'\,x$
$\mathrm{kei}_5'\,x$	−1306.0	−17.818 4	−1.125 3	−0.071 26	+0.026 42	+0.023 332	+0.011 279	+0.005 038 1	+0.001 529 2	+0.000 200 1	$\mathrm{kei}_5'\,x$

(Refer. 14, p. 622).

$$\operatorname{ber}_n x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p}}{p\,!\,(n+p)\,!} \left(\frac{1}{2}x\right)^{n+2p} \cos\frac{(n+2p)\pi}{4} \tag{10}$$

$$\operatorname{bei}_n x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p+1}}{p\,!\,(n+p)\,!} \left(\frac{1}{2}x\right)^{n+2p} \sin\frac{(n+2p)\pi}{4} \tag{11}$$

$$\operatorname{ber}_n' x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p}\left(\frac{n}{2}+p\right)}{p\,!\,(n+p)\,!} \left(\frac{1}{2}x\right)^{n+2p-1} \cos\frac{(n+2p)\pi}{4} \tag{12}$$

$$\operatorname{bei}_n' x = \sum_{p=0}^{\infty} \frac{(-1)^{n+p+1}\left(\frac{n}{2}+p\right)}{p\,!\,(n+p)\,!} \left(\frac{1}{2}x\right)^{n+2p-1} \sin\frac{(n+2p)\pi}{4} \tag{13}$$

$$\begin{aligned}
\operatorname{ker}_n x + i \operatorname{kei}_n x &= i^{-n} K_n (x\sqrt{i}) \\
&= \frac{\pi i}{2} H_n^{(1)} (x i \sqrt{i}) \\
&= \frac{\pi i}{2} [J_n (x i\sqrt{i}) + i\, Y_n (x i \sqrt{i})] \\
&= \frac{\pi i}{2} (\operatorname{her}_n x + i \operatorname{hei}_n x)
\end{aligned} \tag{14}$$

$$\begin{aligned}
\operatorname{ker}_n x &= -\frac{\pi}{2} \operatorname{hei}_n x \\
&= (0.1159315 - \log_e x) \operatorname{ber}_n x + \frac{\pi}{4} \operatorname{bei}_n x \\
&+ \frac{1}{2} \sum_{p=0}^{n-1} \frac{(-1)^{n+p}(n-p-1)!}{p\,!} \left(\frac{1}{2}x\right)^{2p-n} \cos\frac{(n+2p)\pi}{4} \\
&+ \frac{1}{2} \sum_{p=0}^{\infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{p} + 1 + \frac{1}{2} + \frac{1}{3} + \ldots \right. \\
&\left. + \frac{1}{n+p}\right) \frac{(-1)^{n+p}}{p\,!\,(n+p)\,!} \left(\frac{1}{2}x\right)^{n+2p} \cos\frac{(n+2p)\pi}{4}
\end{aligned} \tag{15}$$

$$\begin{aligned}
\operatorname{kei}_n x &= \frac{\pi}{2} \operatorname{her}_n x \\
&= (0.1159315 - \log_e x) \operatorname{bei}_n x - \frac{\pi}{4} \operatorname{ber}_n x \\
&+ \frac{1}{2} \sum_{p=0}^{n-1} \frac{(-1)^{n+p}(n-p-1)\,!}{p\,!} \left(\frac{1}{2}x\right)^{2p-n} \sin\frac{(n+2p)\pi}{4} \\
&- \frac{1}{2} \sum_{p=0}^{\infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{p} + 1 + \frac{1}{2} + \frac{1}{3} + \ldots \right. \\
&\left. + \frac{1}{n+p}\right) \frac{(-1)^{n+p}}{p\,!\,(n+p)\,!} \left(\frac{1}{2}x\right)^{n+2p} \sin\frac{(n+2p)\pi}{4}
\end{aligned} \tag{16}$$

See Reference 8, paragraph 3.

$$\begin{aligned}
\operatorname{ker}_n' x &= (0.1159315 - \log_e x) \operatorname{ber}_n' x - \frac{\operatorname{ber}_n x}{x} + \frac{\pi}{4} \operatorname{bei}_n' x \\
&+ \frac{1}{4} \sum_{p=0}^{n-1} \frac{(-1)^{n+p}(2p-n)(n-p-1)\,!}{p\,!} \left(\frac{1}{2}x\right)^{2p-n-1} \cos\frac{(n+2p)\pi}{4} \\
&+ \frac{1}{4} \sum_{p=0}^{\infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{p} + 1 + \frac{1}{2} + \frac{1}{3} + \ldots \right. \\
&\left. + \frac{1}{n+p}\right) \frac{(-1)^{n+p}(n+2p)}{p\,!\,(n+p)\,!} \left(\frac{1}{2}x\right)^{n+2p-1} \cos\frac{(n+2p)\pi}{4}
\end{aligned} \tag{17}$$

$$\begin{aligned}
\operatorname{kei}_n' x &= (0.1159315 - \log_e x) \operatorname{bei}_n' x - \frac{\operatorname{bei}_n x}{x} - \frac{\pi}{4} \operatorname{ber}_n' x \\
&+ \frac{1}{4} \sum_{p=0}^{n-1} \frac{(-1)^{n+p}(2p-n)(n-p-1)\,!}{p\,!} \left(\frac{1}{2}x\right)^{2p-n-1} \sin\frac{(n+2p)\pi}{4} \\
&- \frac{1}{4} \sum_{p=0}^{\infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{p} + 1 + \frac{1}{2} + \frac{1}{3} + \ldots \right. \\
&\left. + \frac{1}{n+p}\right) \frac{(-1)^{n+p}(n+2p)}{p\,!\,(n+p)\,!} \left(\frac{1}{2}x\right)^{n+2p-1} \sin\frac{(n+2p)\pi}{4}
\end{aligned} \tag{18}$$

By putting $n = 0$, Equations (1) to (8) are obtained. The function ber x = ber$_o$ x, etc.

For large values of x, an asymptotic series for $I_n(z) = i^{-n} J\, n\, (i z)$ is available. (Reference 16, para. 7.23) If n is a positive integer, putting $z = x\sqrt{i}$,

$$\sqrt[4]{i} = e^{\frac{i\pi}{8}} \text{ and } \frac{1}{\sqrt{i}} = e^{\frac{-i\pi}{4}} = \cos\frac{\pi}{4} - i \sin\frac{\pi}{4},$$

$$J_n\,(x\,i\,\sqrt{i}) = \text{ber}_n\,x + i\,\text{bei}_n\,x = i^n\,I_n\,(x\,\sqrt{i})$$

$$= \frac{e^{x/\sqrt{2}}}{\sqrt{(2\,\pi\,x)}}\,e^{i\left(\frac{x}{\sqrt{2}}-\frac{\pi}{8}+\frac{n\,\pi}{2}\right)}\,[L_n\,(x) + i\,M_n\,(x)] \qquad (19)$$

where $L_n\,(x) = 1 - \dfrac{4\,n^2 - 1^2}{1\,!\,8\,x}\cos\dfrac{\pi}{4}$

$$+ \frac{(4\,n^2 - 1^2)\,(4\,n^2 - 3^2)}{2\,!\,(8\,x)^2}\cos\frac{2\,\pi}{4} + \ldots$$

$$\ldots + (-1)^p\,\frac{(4\,n^2-1^2)(4\,n^2-3^2)\ldots\{4\,n^2-(2\,p-1)^2\}}{p\,!\;\;(8\,x)^p}$$

$$\cos\frac{p\,\pi}{4}\ldots \qquad (20)$$

$$M_n\,(x) = \frac{4\,n^2 - 1^2}{1\,!\,8\,x}\sin\frac{\pi}{4}$$

$$- \frac{(4\,n^2 - 1^2)\,(4\,n^2 - 3^2)}{2\,!\,(8\,x)^2}\sin\frac{2\,\pi}{4} + \ldots$$

$$\ldots + (-1)^{p+1}\frac{(4\,n^2-1^2)(4\,n^2-3^2)\ldots\{4\,n^2-(2\,p-1)^2\}}{p\,!\,(8\,x)^p}$$

$$\sin\frac{p\,\pi}{4} + \ldots \qquad (21)$$

Then,

$$\text{ber}_n\,x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\,\pi\,x)}}\left[L_n\,(x)\cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right.$$

$$\left.- M_n\,(x)\sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right] \qquad (22)$$

$$\text{bei}_n\,x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\,\pi\,x)}}\left[M_n\,(x)\cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right.$$

$$\left.+ L_n\,(x)\sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right] \qquad (23)$$

The derivative of the Bessel function is given by the reduction formula

$$2\,J_n'\,(x\,i\,\sqrt{i}) = J_{n-1}\,(x\,i\,\sqrt{i}) - J_{n+1}\,(x\,i\,\sqrt{i}) \quad (24)$$

This gives two series similar to (19), but they can be combined into one series as follows:

$$J_n'(x\,i\sqrt{i}) = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\,\pi\,x)}}\,e^{i\left(\frac{x}{\sqrt{2}}-\frac{5\pi}{8}+\frac{n\pi}{2}\right)}\,[S_n\,(x) + i\,T_n\,(x)] \qquad (25)$$

where

$$S_n\,(x) = 1 - \frac{4\,n^2 + 1 \times 3}{1\,!\,8\,x}\cos\frac{\pi}{4}$$

$$+ \frac{(4\,n^2 - 1^2)\,(4\,n^2 + 3 \times 5)}{2\,!\,(8\,x)^2}\cos\frac{2\,\pi}{4}$$

$$- \frac{(4\,n^2 - 1^2)\,(4\,n^2 - 3^2)\,(4\,n^2 + 5 \times 7)}{3\,!\,(8\,x)^3}\cos\frac{3\,\pi}{4} + \ldots$$

$$\ldots + (-1)^p\,(4\,n^2-1^2)(4\,n^2-3^2)\ldots\{4\,n^2-(2\,p-3)^2\}$$

$$\frac{\{\,4\,n^2 + (2\,p - 1)\,(2\,p + 1)\,\}}{p\,!\,(8\,x)^p}\cos\frac{p\,\pi}{4} + \ldots \qquad (26)$$

$$T_n\,(x) = \frac{4\,n^2 + 1 \times 3}{1\,!\,8\,x}\sin\frac{\pi}{4}$$

$$- \frac{(4\,n^2 - 1^2)\,(4\,n^2 + 3 \times 5)}{2\,!\,(8\,x)^2}\sin\frac{2\,\pi}{4}$$

$$+ \frac{(4\,n^2 - 1^2)\,(4\,n^2 - 3^2)\,(4\,n^2 + 5 \times 7)}{3\,!\,(8\,x)^3}\sin\frac{3\,\pi}{4} - \ldots$$

$$\ldots + (-1)^{p+1}\,(4\,n^2-1^2)(4\,n^2-3^2)\ldots\{4\,n^2-(2\,p-3)^2\}$$

$$\frac{\{\,4\,n^2 + (2\,p - 1)\,(2\,p + 1)\,\}}{p\,!\,(8\,x)^p}\sin\frac{p\,\pi}{4} + \ldots \qquad (27)$$

$$\text{ber}_n'\,x + i\,\text{bei}_n'\,x = \frac{d}{d\,x}\,(\text{ber}_n\,x + i\,\text{bei}_n\,x)$$

$$= i\,\sqrt{i}\,J_n'(x\,i\,\sqrt{i}) = e^{\frac{3\pi i}{4}}\,J_n'(x\,i\,\sqrt{i})$$

$$\text{ber}_n'\,x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\,\pi\,x)}}\left[S_n\,(x)\cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right.$$

$$\left.- T_n\,(x)\sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right] \qquad (28)$$

$$\text{bei}_n'\,x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\,\pi\,x)}}\left[T_n\,(x)\cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right.$$

$$\left.+ S_n\,(x)\sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right] \qquad (29)$$

An asymptotic series for

$$K_n\,(z) = i^n\,\frac{\pi\,i}{2}\,H_n^{(1)}\,(i\,z) \qquad (30)$$

also is given in Reference 16, paragraph 7.23. From this is obtained

$$\text{ker}_n\,x + i\,\text{kei}_n\,x = i^{-n}\,K_n\,(x\,\sqrt{i})$$

$$= \left(\frac{\pi}{2\,x}\right)^{\frac{1}{2}} e^{-x/\sqrt{2}}\,e^{-i\left(\frac{x}{\sqrt{2}}+\frac{\pi}{8}+\frac{n\pi}{2}\right)}\,[L_n\,(-\,x) + i\,M_n\,(-\,x)] \qquad (31)$$

$$\text{ker}_n\,x = \left(\frac{\pi}{2\,x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}}\left[L_n\,(-\,x)\cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8}\right.\right.$$

$$\left.\left.+ \frac{n\,\pi}{2}\right) + M_n\,(-\,x)\sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right] \qquad (32)$$

$$\text{kei}_n\,x = \left(\frac{\pi}{2\,x}\right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}}\left[M_n\,(-\,x)\cos\left(\frac{x}{\sqrt{2}}\right.\right.$$

$$\left.\left.+ \frac{\pi}{8} + \frac{n\,\pi}{2}\right) - L_n\,(-x)\sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} + \frac{n\,\pi}{2}\right)\right] \qquad (33)$$

$L_n\,(-\,x)$ and $M_n\,(-\,x)$ are obtained by changing x to $(-\,x)$ in (20) and (21).

For the derivative, the reduction formula can be used as was done for (25). The two series will combine into one series, as follows:

$$\mathrm{ker}_n' \, x + i \, \mathrm{kei}_n' \, x$$

$$= - \left(\frac{\pi}{2x} \right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} e^{-i\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2}\right)} [S_n(-x) + i \, T_n (-x)] \qquad (34)$$

$$\mathrm{ker}_n' \, x = - \left(\frac{\pi}{2x} \right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[S_n(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2} \right) + T_n(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2} \right) \right] \qquad (35)$$

$$\mathrm{kei}_n' \, x = - \left(\frac{\pi}{2x} \right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[T_n(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2} \right) - S_n(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} + \frac{n\pi}{2} \right) \right] \qquad (36)$$

$S_n \, (-x)$ and $T_n \, (-x)$ are obtained by changing x to $(-x)$ in (26) and (27).

Bessel Functions of Order Zero, with Large Values of x.

Putting $n = 0$, $J_0 \, (x \, i \, \sqrt{i}) = \mathrm{ber} \, x + i \, \mathrm{bei} \, x$

$$\mathrm{ber} \, x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left[L_0(x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) - M_0(x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) \right] \qquad (37)$$

$$\mathrm{bei} \, x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left[M_0(x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) + L_0(x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) \right] \qquad (38)$$

where

$$L_0(x) = 1 + \frac{1^2}{1 \, ! \, 8x} \cos\frac{\pi}{4} + \frac{1^2 . 3^2}{2 \, ! \, (8x)^2} \cos\frac{2\pi}{4} + \frac{1^2 . 3^2 . 5^2}{3 \, ! \, (8x)^3} \cos\frac{3\pi}{4} + \ldots + \frac{1^2 . 3^2 . \ldots . (2p-1)^2}{p \, ! \, (8x)^p} \cos\frac{p\pi}{4} + \ldots \qquad (39)$$

$$M_0(x) = - \frac{1^2}{1 \, ! \, 8x} \sin\frac{\pi}{4} - \frac{1^2 . 3^2}{2 \, ! \, (8x)^2} \sin\frac{2\pi}{4} - \ldots - \frac{1^2 . 3^2 . \ldots . (2p-1)^2}{p \, ! \, (8x)^p} \sin\frac{p\pi}{4} - \ldots \qquad (40)$$

$$\mathrm{ber}' \, x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left[S_0(x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) - T_0(x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) \right] \qquad (41)$$

$$\mathrm{bei}' \, x = \frac{e^{x/\sqrt{2}}}{\sqrt{(2\pi x)}} \left[T_0(x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) + S_0(x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) \right] \qquad (42)$$

where

$$S_0(x) = 1 - \frac{1 . 3}{1 \, ! \, 8x} \cos\frac{\pi}{4} - \frac{1^2 . 3 . 5}{2 \, ! \, (8x)^2} \cos\frac{2\pi}{4} - \frac{1^2 . 3^2 . 5 . 7}{3 \, ! \, (8x)^3} \cos\frac{3\pi}{4} - \ldots - \frac{1^2 . 3^2 \ldots (2p-3)^2 (2p-1)(2p+1)}{p \, ! \, (8x)^p} \cos\frac{p\pi}{4} - \ldots \qquad (43)$$

$$T_0(x) = \frac{1 . 3}{1 \, ! \, 8x} \sin\frac{\pi}{4} + \frac{1^2 . 3 . 5}{2 \, ! \, (8x)^2} \sin\frac{2\pi}{4} + \frac{1^2 . 3^2 . 5 . 7}{3 \, ! \, (8x)^3} \sin\frac{3\pi}{4} + \ldots + \frac{1^2 . 3^2 \ldots (2p-3)^2 (2p-1)(2p+1)}{p \, ! \, (8x)^p} \sin\frac{p\pi}{4} + \ldots \qquad (44)$$

It may be noted that $S_0 \, (x)$ has the same value, term for term, as $L_1 \, (x)$, and $T_0 \, (x)$ the same as $M_1 \, (x)$. This is in agreement with the fact that $J_0' \, (x) = - J_1 \, (x)$. However, $S_1 \, (x)$ has not the same value as $L_2 \, (x)$.

$$\mathrm{ker} \, x = \left(\frac{\pi}{2x} \right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[L_0(-x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) + M_0(-x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) \right] \qquad (45)$$

$$\mathrm{kei} \, x = \left(\frac{\pi}{2x} \right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[M_0(-x) \cos\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) - L_0(-x) \sin\left(\frac{x}{\sqrt{2}} + \frac{\pi}{8} \right) \right] \qquad (46)$$

where $L_0 \, (-x)$ and $M_0 \, (-x)$ are obtained by changing x to $-x$ in (39) and (40).

$$\mathrm{ker}' \, x = - \left(\frac{\pi}{2x} \right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[S_0(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) + T_0(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) \right] \qquad (47)$$

$$\mathrm{kei}' \, x = - \left(\frac{\pi}{2x} \right)^{\frac{1}{2}} e^{-\frac{x}{\sqrt{2}}} \left[T_0(-x) \cos\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) - S_0(-x) \sin\left(\frac{x}{\sqrt{2}} - \frac{\pi}{8} \right) \right] \qquad (48)$$

where $S_0 \, (-x)$ and $T_0 \, (-x)$ are obtained by changing x to $(-x)$ in (43) and (44).

Since the general terms of these series are given, any desired number of terms can be evaluated. However, the series for large values of x are semi-divergent, and there is a limit to the amount of precision which they will give, though this is usually appreciable only for the smaller values of x. It can be proved that the sum of a given number, greater than n, of terms of the series for I_n or K_n differs from the true value of the function by less than the last term included, and so the series can be used for numerical computation.[6]

Note that K_n is sometimes, particularly in older publications, used to represent a function corresponding to J_n instead of one corresponding to I_n. The term ker, however, has been used to represent only one function, namely, the function whose values are tabulated in this paper.

As a check on the correctness of these series, Equations (1) to (18) have been used to calculate the values for $x = 1$, with $n = 0$ and $n = 5$, and Equations (20) to (48) have been used to calculate the values for $x = 10$, with $n = 0$ and $n = 5$. The calculations were carried out to several significant figures, and they agreed with the independently obtained tabulated values.

Other series for large values of x, of which the general terms were not given, have been published by H. G. Savidge.[12] In his formula for η, the term

$$-\frac{21}{128\sqrt{2}x^3}$$

is evidently a typographical error, since it should be

$$+\frac{21}{128\sqrt{2}x^3}$$

in order to agree with his preceding calculation on p. 108, and also with his previous publication of the first terms of the same series in 1910.[7]

The values of Bessel functions of higher orders in Tables II and III are not near enough together to give intermediate values by interpolation. They were obtained from the values of functions of order zero by the following reduction formulas:[20]

Bessel Function Reduction Formulas for Functions of Higher Orders than Zero

Special

$$u_0 = \text{ber}\, x \tag{49}$$

$$v_0 = \text{bei}\, x \tag{50}$$

$$u_1 = \text{ber}_1\, x = +(1/\sqrt{2})(u_0' - v_0') \tag{51}$$

$$v_1 = \text{bei}_1\, x = +(1/\sqrt{2})(u_0' + v_0') \tag{52}$$

$$u_2 = -\frac{\sqrt{2}}{x}(u_1 - v_1) - u_0 = \frac{2\,v_0'}{x} - u_0 \tag{53}$$

$$v_2 = -\frac{\sqrt{2}}{x}(u_1 + v_1) - v_0 = -\frac{2\,u_0'}{x} - v_0 \tag{54}$$

$$u_2' = -(1/\sqrt{2})(u_1 + v_1) - \frac{2\,u_2}{x} = -u_0' - \frac{2\,u_2}{x} \tag{55}$$

$$v_2' = +(1/\sqrt{2})(u_1 - v_1) - \frac{2\,v_2}{x} = -v_0' - \frac{2\,v_2}{x} \tag{56}$$

General

$$u_{n+1} = -\frac{n\sqrt{2}}{x}(u_n - v_n) - u_{n-1} \qquad n \neq 0 \tag{57}$$

$$v_{n+1} = -\frac{n\sqrt{2}}{x}(u_n + v_n) - v_{n-1} \qquad n \neq 0 \tag{58}$$

$$u_n' = -(1/\sqrt{2})(u_{n-1} + v_{n-1}) - \frac{n\,u_n}{x} \qquad n \neq 0 \tag{59}$$

$$v_n' = +(1/\sqrt{2})(u_{n-1} - v_{n-1}) - \frac{n\,v_n}{x} \qquad n \neq 0 \tag{60}$$

These formulas are also suitable for Bessel functions of the second kind by changing u to ker and v to kei.

A practical problem using values from Table I is the calculation of the skin effect resistance ratio and reactance ratio of an isolated tubular conductor. The general solution of this problem was given by Heaviside.[3]

Let μ = permeability of the material of the tube (assumed = 1 for copper).

$\omega = 2\,\pi\,f$

f = frequency in cycles per second

ρ = resistivity of the material of the tube in absolute units (= 1724 for copper of 100 per cent conductivity at 20 deg. cent.).

If the conductor is a cable, with a rope core or hollow center, ρ is not the value for solid copper, but should be determined from the resistance per 1000 ft. of the cable and from its diameter.

$i = \sqrt{(-1)}$

$m^2 = \dfrac{4\,\pi\,\mu\,\omega}{\rho}$

r = outside radius of tube in centimeters

q = inside radius of tube in centimeters

R' = effective value of the resistance per cm. of the tube at frequency f

X' = effective value of the reactance per cm. of the tube at frequency f, due to flux inside radius r.

R = resistance for direct current per cm. of tube.

$$\frac{R' + i\,X'}{R} = i\,\frac{m\,r}{2}\,\frac{(r^2 - q^2)}{r^2}$$

$$\frac{\left[(\text{ber}\, m\,r + i\,\text{bei}\, m\,r) + \dfrac{(C + i\,D)}{(A + i\,B)}(\text{ker}\, m\,r + i\,\text{kei}\, m\,r)\right]}{\left[(\text{ber}'\, m\,r + i\,\text{bei}'\, m\,r) + \dfrac{(C + i\,D)}{(A + i\,B)}(\text{ker}'\, m\,r + i\,\text{kei}'\, m\,r)\right]} \tag{51}$$

$$\text{where } \frac{C + i\,D}{A + i\,B} = -\frac{(\text{ber}'\, m\,q + i\,\text{bei}'\, m\,q)}{(\text{ker}'\, m\,q + i\,\text{kei}'\, m\,q)} \tag{52}$$

For the derivation of these formulas, see Reference 19. The right-hand part of Equation (**51**) can be found as a numerical complex quantity. The real part is equal to R'/R, the skin effect resistance ratio of the tube, and the unreal part gives the change in inductance of the tube due to frequency.

The following example was given in Reference 19:

Example. Find the skin effect resistance ratio of a tube in which $t/d = 0.20$ and $m\,t = 3$, t being equal to $(r - q)$, the thickness of the tube in centimeters, and d being equal to $2\,r$, the outside diameter of the tube.

$$\begin{aligned} \frac{r-q}{2\,r} &= 0.2 \\ r - q &= 0.4\,r \\ q &= 0.6\,r \\ m\,(r-q) &= 3 \\ m\,r &= 3/0.4 = 7.5 \\ m\,q &= 4.5 \end{aligned}$$

From Equation (**52**) and Table I,

$$\frac{C + i\,D}{A + i\,B} = -\frac{\mathrm{ber}'\,4.5 + i\,\mathrm{bei}'\,4.5}{\mathrm{ker}'\,4.5 + i\,\mathrm{kei}'\,4.5}$$

$$= \frac{3.753681 + i\,2.052635}{0.02481454 + i\,0.00771543}$$

$$= 161.387 + i\,32.5401$$

From Equation (**51**),

$$Z'/R = i\,3.75 \times 0.64$$

$$\frac{(5.454962 - i\,29.115712 + 0.258938 + i\,0.264977)}{(24.130125 - i\,14.735602 - 0.012244 - i\,0.388061)}$$

$$= i\,3.75 \times 0.64\,\frac{(574.136 - i\,609.403)}{810.397}$$

$$= 1.80475 + i\,1.70031$$

Therefore, $R'/R = 1.80475$

This is seen at once to check with the curve for $t/d = 0.20$, reading at $m\,t = 3$, in Fig. 3, of Reference 13.

To check the result with Fig. 1 of Reference 17, we have

f/R_{dc} (in ohms per 1000 ft.)

$$= \frac{\pi\,(r^2 - q^2)}{\rho}\;\frac{f \times 10^9 \times 0.3937}{12000}$$

and
$$m^2\,t^2 = \frac{8\,\pi^2 f\,(r-q)^2}{\rho}$$

where the dimensions are in centimeters and ρ is in absolute units.

Therefore,

$$\sqrt{f/R_{dc}} = \sqrt{\frac{9 \times 10^9 \times 0.3937 \times 4}{8\,\pi \times 12000}} = 216.8$$

The value of R'/R_{dc} read from the curve is seen to be 1.805.

In order to save calculation, one may often use to advantage the curves published in Fig. 3, Reference 13, and Fig. 1, Reference 17, for obtaining skin effect ratios of isolated tubular conductors.

A problem involving the Bessel functions discussed in this paper which occurs very frequently is that of finding the skin effect resistance and reactance ratios of isolated round wire. A very complete table of these ratios is given in Bibliography 5, Table XXII. These tabulated values may be used with comparatively good accuracy for cables which do not have rope cores.

It should be remembered that, as previously stated, if the curves or tables are used for cable, ρ is not the value for solid copper, but should be determined from the resistance per 1000 ft. of the cable, and from the cable diameter.

It is desired to make acknowledgment of valuable suggestions from Mr. John R. Carson regarding the form of the Bessel function series.

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Some Notes on Transformer Design

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Member, A. I. E. E.

Synopsis.—A method for determining the most economical proportions for transformers is suggested. The method takes into account the densities of flux and of current in addition to the dimensions taken into account in previous discussions of this problem. Various shapes of transformers are considered.

WITH certain conditions which must be fulfilled there is some particular design of any machine which is the cheapest to build. These relations are usually so complicated that no attempt is made to predetermine the conditions or dimensions of the cheapest product except on the basis of experience and empirical constants. There is, however, among electrical apparatus, one, the transformer, which having no moving parts is so simple in its construction and relations, that considerable work has been done in the line of attempting to predetermine mathematically the shape and dimensions that give the lowest cost. Most of the work published has been done in England.[2] These methods have been carried further at the University of Illinois during the last several years with the help of several workers, principally Mr. J. K. McNeely, Mr. L. B. Archer, who has worked out several of the forms and graphs, and Mr. E. E. Perkins, who has done much of the checking and drawing. More variables have been brought into the equation and the necessary curves for numerous types of transformers have been worked out.

It is of course still necessary to make use of numerous empirical constants, most of which depend on the manufacturing conditions obtaining where the transformer is to be brought out. It seems that none of these should be particularly difficult to secure in any particular place and under a definite set of conditions. It has been hardest to find any accurate data on heating which can be used in a general method.

The method of design consists in setting up an equation for the cost of the active material of a transformer including labor in terms of the dimensions, and differentiating this equation for a minimum. Restraining equations for power and losses contain also the terms for densities of flux and current which are interrelated. It is in this development, due to Professor McNeely, that the density variables formerly assumed have been included in the equations and curves. The solution of these equations is rather tedious but their solutions may be reduced to curves once for all from which the necessary values may very readily be taken, and the result for any given case may be very quickly found.[3]

The flux density has been made an independent variable on which the heating depends. As the approximate value for this density is known from experience, for any given type and make of transformer but little work needs to be done to obtain dimensions giving suitable surface for heat radiation. An accurate knowledge of the magnetic quality of the iron to be used is of course presupposed and all the dimension proportions of the design are contingent upon these constants.

It may here be noted that for any flux density there is a fixed relation between the iron losses and copper losses for the cheapest transformer but that the best relation of flux and current densities is not the same for different types of transformers.

While it may never be possible or desirable to build any transformer exactly as figured by the following formulas, they should offer a ready means of checking up on how near a design comes to being the cheapest shape and proportions and to give a method which shows in advance just what should be striven for to give the cheapest design when the unit costs of iron and copper and the unit costs of winding, stacking, and insulating have been determined.

Every method of design has certain limitations and this is no exception. For instance, this method does not allow starting with a certain predetermined efficiency or reactance and working out the design with those factors specified. If, however, a flux density and desired core loss are decided upon, the best proportions are fixed and may be obtained by trial from the curves.

There develops necessarily an indefinite number of solutions, one for every density and that one giving the allowable heating must be picked out. Since the limiting temperature may first be reached in either the iron or the copper, a cheaper practical design may be obtained by varying the density either of flux or of current from that shown by the mathematical formula to give the least cost. Evidently a transformer having an infinite density of flux and current and therefore infinite losses would require no material and therefore be the cheapest to build.[4] If it were possible to place in the equation an expression fixing any quantity by a temperature limit, one solution only would result. This has not been found practicable. But if a basis of watts per square inch of surface can be used an analytic or more easily a graphic solution can be obtained. In regard to reaction it has been often pointed out that this characteristic may be varied within wide ranges without altering the core dimensions and small changes of core to adjust for exact reactions desired may be made with very small increases in cost, since fortunately the curves of cost at the minimum point are very flat.

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[1] Assistant Professor of Electrical Design, University of Illinois, Urbana, Illinois.

[2] Fleming and Hansen, 1903, and others at later dates. See bibliography.

[3] Since the writing of this paper Professor McNeely has published a bulletin using similar methods of obtaining a minimum cost transformer. Bul. 87, Eng. Exp. Sta., Iowa State College.

[4] The converse of this is not true though it has so been stated in print. A transformer of infinite dimensions does not give zero losses.

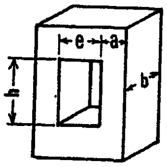


Fig. 1—Core Type

The theoretical derivation of this method for determining the dimensions of a transformer of minimum cost follows: As in practically all electrical machine design, the starting point is the fundamental e. m. f. equation.

$$E = 4.44 f \varphi N 10^{-8} \qquad (1)$$

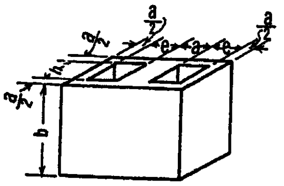


Fig. 2—Shell Type

where N = turns, E = voltage, f = frequency, φ = flux Multiply by I, the current in the coil.

$$E I = 4.44 f \varphi N I 10^{-8} \qquad (2)$$

Since there is a difference between the ampere-turns of the primary and secondary sides, we shall use the mean value of $N I$. But this difference being usually very small, its neglect would make only a slight difference.

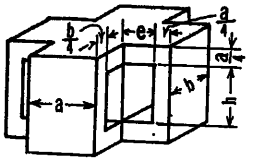


Fig. 3—Cruciform Type

Substituting for φ and $N I$ their values in terms of densities and areas, we have,

$$\text{kv-a.} = P = 0.001 E I$$

$$= 2.22 f \beta \text{ (stacked iron area) } F_i s h e F_c 10^{-5} \qquad (3)$$

F_i and F_c being the space factors for iron and copper respectively; β is kilolines per sq. in.; s is current density in kilo-amperes per sq. in.; $a\ b\ h\ e$ are dimensions for which see Figs. 1 to 7.

This "stacked iron area" is

$a\ b$ for forms 1 and 2

a^2 for form 3

$16.395\ p^2$ for form 4

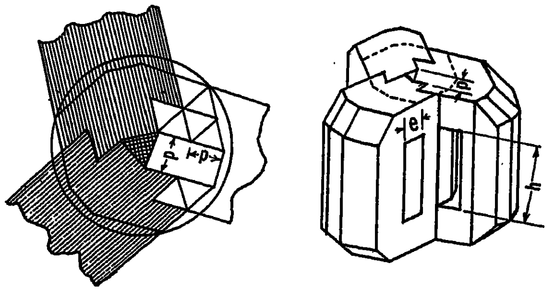


Fig. 4—Ring Coil Type and Cross-Section of Core

$0.618\ a^2$ for form 5

$0.668\ a^2$ for forms 6 and 7 (4)

For forms 5, 6, and 7 a is taken, for convenience, as the inside diameter of the insulating tube carrying the winding. (See Figs. 5, 6, 7.)

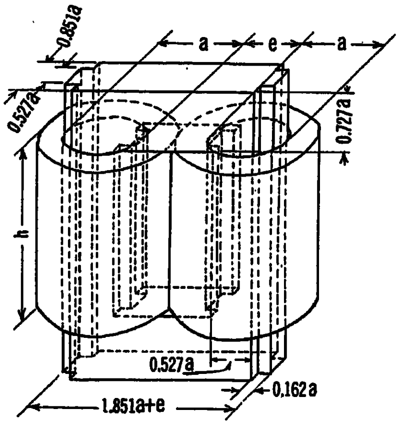


Fig. 5—Cruciform Core—Core Type Transformer

In each case k_3 is so chosen that,

$$P = k_3 \beta s a b h e \qquad (5)$$

in several cases $b = a$ and in the case of the ring coil type $a\ b$ becomes p^2.

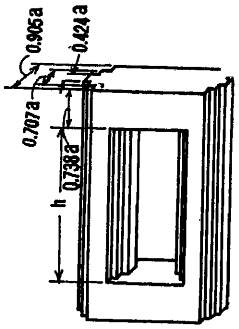


Fig. 6—Double Notched Core Circular Coil—Core Type

The losses in terms of volumes can be expressed thus:

$$L = L_i + L_c = b_1 \beta^2 k_i F_i V_i + c s^2 F_c V_c \qquad (6)$$

($b_1 \beta^2$ being a convenient form of expressing watts loss per lb. of iron—see under symbols, b_1)

The cost of active material,

$$C = C_i\,k_i\,F_i\,V_i + C_c\,k_c\,F_c\,V_c \qquad (7)$$

where C_i and C_c are the costs per pound and k_i and k_c the weights per cu. in. of iron and copper respectively.[5]

In every case, the volumes may be expressed in terms of constants and the four (or three) linear dimensions. The equation is then a third power equation and it will be noticed that this expression does not contain either β or s.

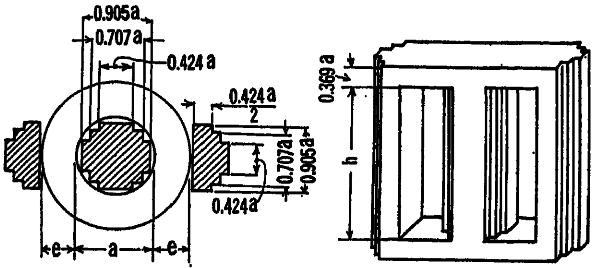


FIG. 7—DOUBLE NOTCHED CORE CIRCULAR COIL—SHELL TYPE CROSS-SECTION OF DOUBLE NOTCHED CORE CIRCULAR COIL—SHELL TYPE

The problem is to find the minimum value of C. To do this we may write:

$$\frac{\partial C}{\partial a} = 0 \quad \frac{\partial C}{\partial b} = 0 \quad \frac{\partial C}{\partial h} = 0 \quad \frac{\partial C}{\partial e} = 0 \qquad (8)$$

There are then six unknowns, the two densities and four linear dimensions, and six equations. In all cases the equations can be reduced to these forms, even if it be decided to increase the area of the yokes or outside legs, in order to decrease densities in these parts. This will only bring a further numerical constant into the quantity. In this discussion the flux density has been assumed uniform in all parts of the iron circuit.

As V_i and V_c will be a different function of $a\ b\ h\ e$ for each type of transformer, a separate solution for each type must be obtained. For the actual mathematical work involved see Appendix.

As might be anticipated these equations solve out as ratios. The values of h/e and b/e are obtained as functions of a/e or p/e. These are most easily determined by drawing curves between a/e or p/e and the other two ratios.

The proper proportioning of iron and copper, *i. e.*, the value of the ratio a/e or p/e will evidently depend on R the cost ratio of iron and copper and the ratio of iron and copper losses. That is, the shape of transformer will depend, not on actual values, but only on the ratios.

During the solution of the differential equations the quantity

$$(a_0\,\beta^{1.6} + b_0\,\beta^2)/(.8\,a_0\,\beta^{1.6} + b_0\,\beta^2)$$

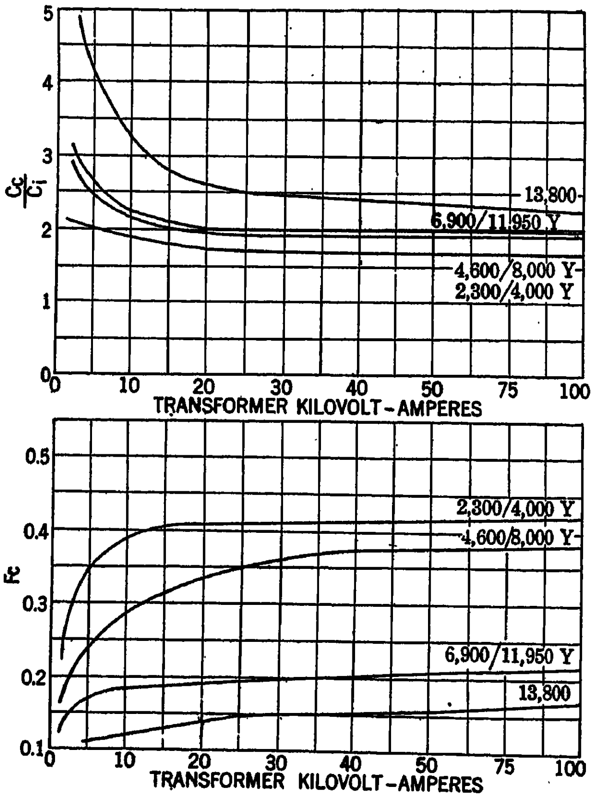
appears. This comes of course from the use of the Steinmetz formula for iron losses.

$$L_{1b} = a_0\,\beta^{1.6} + b_0\,\beta^2$$

and is for any quality of iron a definite function of β only. This quantity is called r which quantity later proves to be the ratio of iron to copper loss under the conditions of minimum cost. See Appendix, page 10.

$$\therefore r = \frac{k_6\,A}{k_7\,B} = \frac{L_i}{L_c} = 1.25 - b_0/(3.2\,a_0/\beta^{.4} + 4\,b_0) \qquad (9)$$

for the minimum cost transformer (but not for others) for any value of β.



UPPER HALF

RATIO OF COPPER COST TO IRON COST OF TRANSFORMER

$$\frac{C_c}{C_i} = \frac{\text{Cost per lb. of covered copper including winding cost}}{\text{Cost per lb. of assembled core}}$$

LOWER HALF

COPPER SPACE FACTOR OF TRANSFORMERS

$$\text{Approximate space factor } (F_c) = \frac{\text{Area of copper}}{\text{area of window}}$$

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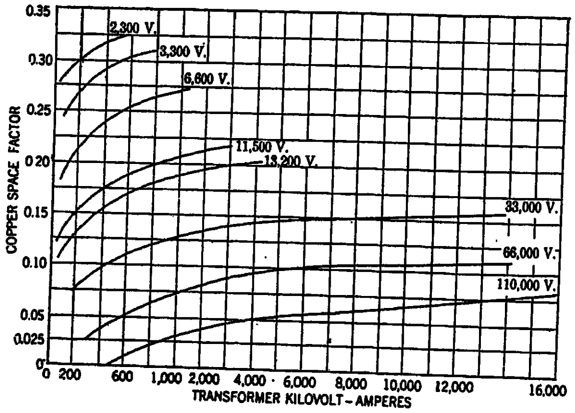
[5] It should here be noted that the costs must include the labor of putting the iron and copper in place in order to give a minimum cost transformer as manufactured. This necessitates getting unit costs,—costs per pound—at the factory concerned. The shape depends upon the necessary space required for the winding; therefore F_c must be a function of the voltage and of the type of insulation used in the factory. Also C_c must include insulation and labor costs and is therefore also a function of voltage. C_i, the cost of stacked iron per pound, will evidently vary with the proportions of the core, but as these proportions vary only through a narrow range in the region of transformers as now built, unit costs so obtained are quite accurate enough. It will develop further on, and in fact is quite evident by inspection, that the shape and proportions as regards costs are a function of the ratio of the costs and not of the absolute costs. Therefore curves of the ratios of C_c to C_i for various voltages and sizes are sufficient.

Such curves as obtained for certain average conditions from two manufacturers are given.

The curve, which was drawn from the data furnished by the manufacturer of the iron, did not show a_0 and b_0 to be constant as would have been the case if the Steinmetz Law were exactly true. The curves for b_1 and r presuppose an accurate curve showing the relation between flux density per square inch and watts loss per pound of iron. If a curve be used which gives constant values of a_0 and b_0, the mathematical results here given still hold. Points were taken in pairs at short intervals and values of a_0 and b_0 were calculated and tabulated. This was the only use made of a_0 and b_0.



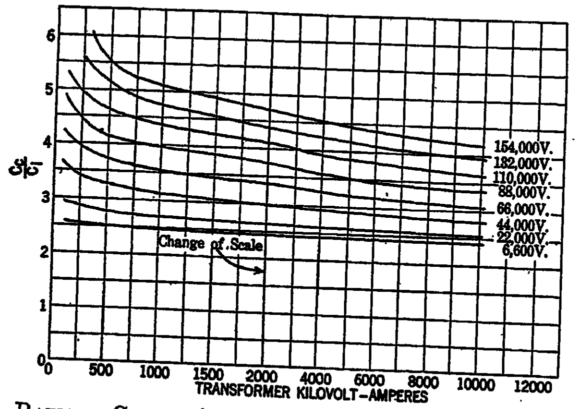
Space Factors of Transformers for Different Voltages and Sizes

$$\text{Space factor} = \frac{\text{Net area of copper}}{\text{Area of window}}$$

The value of R also appears in the form

$$R = (5\,Q - 8)/T\,(8 - 3\,r\,Q) = F_c\,C_c\,k_c/F_i\,C_i\,k_i \qquad (10)$$

Q and T are functions only of the linear dimensions. From this equation for any given values of R and r values of a/e and p/e may be obtained. Though this is a rather complicated function it may be reduced, by dividing numerator and denominator by e^4, to terms containing only the ratios a/e (or p/e) b/e, h/e, r, and constants. By means of the curves for b/e and h/e referred to above it may be reduced further to terms only of a/e (or p/e) and r and constants. Therefore, for any values of R and r curves may be drawn from which a/e (or p/e) may be found. (In actual computation it is easier to assume the value of a/e and compute R)



Ratio of Copper Cost to Iron Cost of Transformer

C_c = Cost per lb. of copper including insulation, winding, and assembly
C_i = Cost per lb. of assembled core

$$L_c = s^2\,c\,F_c\,V_c \qquad \text{whence}$$

$$s^2 = L_c/c\,F_c\,V_c = L_i/r\,c\,F_c\,V_c \text{ see (9)} \qquad (11)$$

and therefore s for the minimum cost transformer is also a function of β.

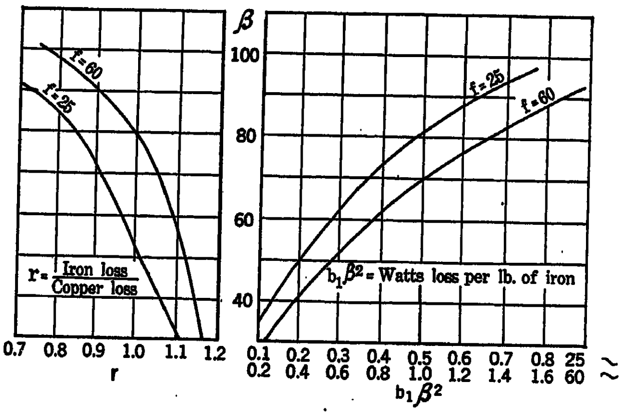
As an example, the simple core type transformer may be used to illustrate certain of the quantities involved and some of the results obtained. Equations (5), (6), and (7) are used, and for the,

1. Core Type Transformers

$$V_i = 2\,a\,b\,(2\,a + h + e) = 2\,A \qquad (112)$$

$$V_c = 2\,h\,e\,(a + b + e) = 2\,B \qquad (113)$$

This equation assumes that the coil has square corners which of course is not true. The mean length of turn might be written as $2\left(a + b + \pi\,\frac{e}{4}\right)$ instead of $2\,(a + b + e)$. This would be slightly too small due to the bulging out of the coil along the sides a and b. The actual value lies between the two but the longer value has been used in the calculations.



Values of v and b, β^2 are for Apollo Special Electrified Sheet Steel, Size 29–30¼ x 108 (1923)

Equation (5) becomes

$$P_1 = 2.22\,F_c\,F_i\,f\,\beta\,s\,a\,b\,h\,e\,10^{-5} \qquad (114)$$

Equation (6) becomes

$$L_1 = L_i + L_c = 2\,k_i\,b_1\,\beta^2\,F_i\,a\,b\,(2\,a + h + e) + 2\,c\,s^2\,F_c\,h\,e\,(a + b + e) \qquad (115)$$

and Equation (7) becomes

$$C = 2\,F_i\,C_i\,k_i\,a\,b(2\,a + h + e) + 2\,F_c\,C_c\,k_c\,h\,e(a + b + e) \qquad (116)$$

Putting the total derivatives of these three equations each equal to 0 and solving we finally obtain

$$h/e = (3\,a + 2\,e)/(a + e) \qquad (117)$$

$$b/a = (4\,a + 3\,e)/(2\,a + e) \qquad (118)$$

$$R = k_5/k_4 = (5\,Q - 8)/T(8 - 3\,r\,Q) \qquad (119)$$

From these equations the curves for a/e, b/e, and h/e have been computed.

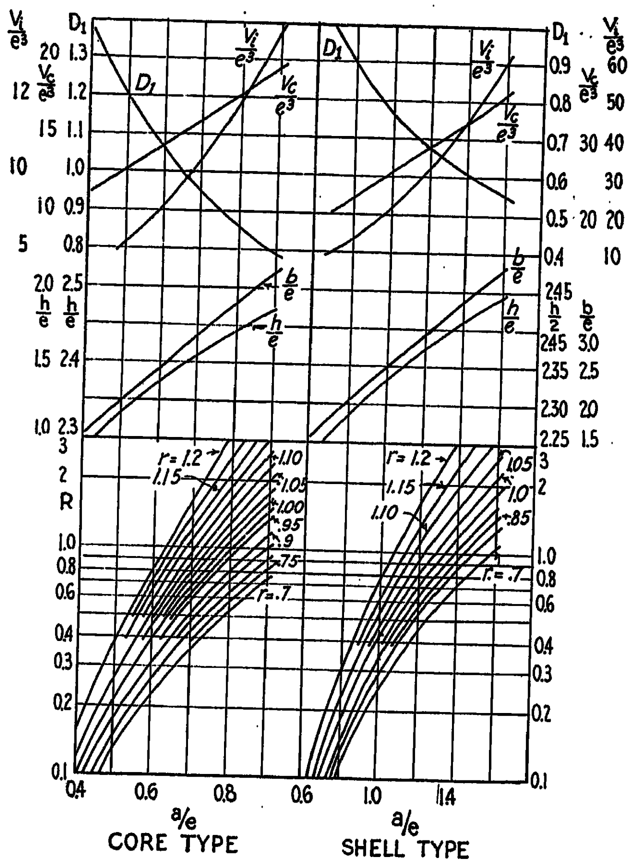
From (6) and (9)

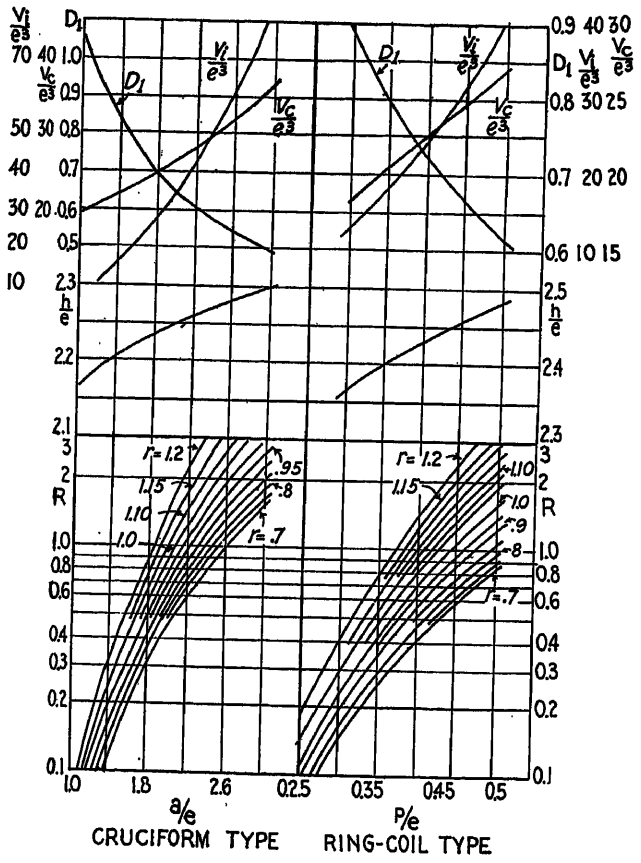
$$L_1 = (1 + r)\, L_c = \left(1 + \frac{1}{r}\right) L_i \qquad (120)$$

whence

$$L_1^2 = (1 + r)\left(1 + \frac{1}{r}\right) L_c\, L_i \qquad (121)$$

Squaring (114) and dividing by (121) into which the values of L_c and L_i have been substituted from (115) gives,





$$\frac{a\,b\,h\,e}{(2\,a + h + e)\,(a + b + e)}$$

$$= \frac{4\,(1 + r)\left(1 + \frac{1}{r}\right) c\, b_1\, F_c\, F_i\, k_i\, P_1^2}{k_3^2\, L_1^2} = S\, b_1\, P_1^2 / L_1^2 \qquad (122)$$

Where

$$S = 4\,(1 + r)\left(1 + \frac{1}{r}\right) c\, F_c\, F_i\, k_i / k_3^2 = 16\, c\, F_c\, F_i\, k_i / k_3^2$$

since for the quality of iron and the densities used it is found that $(1 + r)\left(1 + \frac{1}{r}\right)$ does not vary from 4 by more than 1 per cent, *i. e.*, when r varies from 0.8 to 1.25. This is a less error than that in the iron loss coefficient which is a determining factor.

Substituting into (121) values of L_1 from (115) and using the identities

$$a = \frac{a}{e}\, e, \qquad b = \frac{b}{e}\, e, \qquad h = \frac{h}{e}\, e$$

the equation may be solved for e in terms of one unknown β and the ratios a/e, b/e, and h/e. The resulting equation is

$$e = \sqrt[8]{\frac{S\, P_1^2\, [a/e + b/e + 1]}{4\left(1 + \frac{1}{r}\right)^2 k_i^2\, b_1\, \beta^4\, F_i^2\, (a/e)^3\, (b/e)^3\, (h/e)\, [\,2\, a/e + h/e + 1\,]}} \qquad (123)$$

By using this equation we may determine e for a few assumed values of β. Having e, the other dimensions and the losses are readily found.

This allows a somewhat different method of attack upon the problem from that usually employed, eliminating all cut and try features. A design is obtained for each assumed value of β and this requires but one substitution in the equations.

All the other types of transformers are solved for in exactly the same manner.

2. SHELL TYPE TRANSFORMERS

$$V_i = 2\, a\, b\, (a + h + e) = 2\, A \qquad (212)$$

$$V_c = 2\, h\, e\, (a + b + 2\, e) = 2\, B \qquad (213)$$

As in the core type here also the coils are assumed to have square corners.

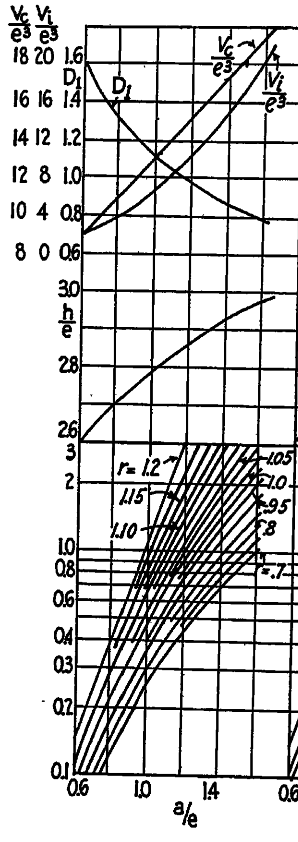
Using these forms for the volumes and substituting as before,

$$h/e = (3\,a + 4\,e)/(a + 2\,e) \qquad (217)$$

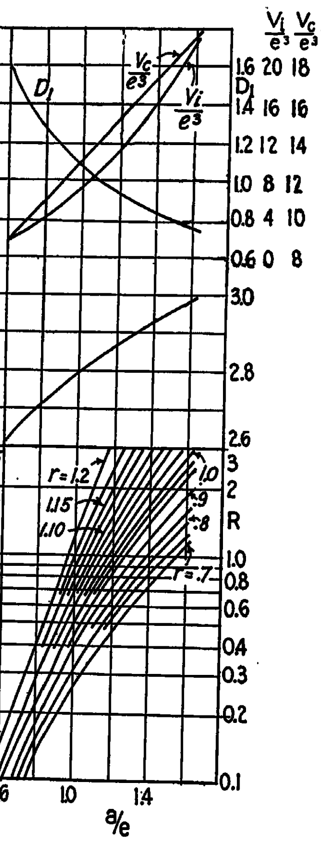
$$b/e = (2\,a + 3\,e)/(a + e) \qquad (218)$$

R has the same general form but the quantities Q and T are of course different. This is true in all subsequent types, and therefore the equation for R has not been repeated.

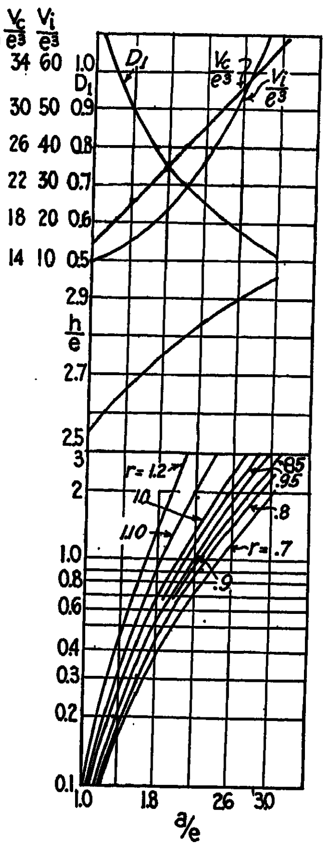
$$\frac{a\,b\,h\,e}{(a + h + e)\,(a + b + 2\,e)} = S\,b_1\,P_1^2/L_1^2 \qquad (222)$$



CRUCIFORM CORE, CORE TYPE



DOUBLE-NOTCHED CORE, CIRCULAR COIL—CORE TYPE



DOUBLE-NOTCHED CORE, CIRCULAR COIL—SHELL TYPE

when S has the same value as before, whence,

$$e = \sqrt[8]{\frac{S\,P_1^2\,(a/e + b/e + 2)}{4\left(1 + \frac{1}{r}\right)^2 k_i^2\,b_1\,\beta^4\,F_i^2\,(a/e)^3\,(b/e)^3\,(h/e)\,(a/e + h/e + 1)}} \qquad (223)$$

3. CRUCIFORM TYPE TRANSFORMER

$$V_i = a^2\,(2\,e + 2\,h + a) = A/2 \qquad (312)$$

$$V_c = 4\,e\,h\,(a + e) = 2\,B \qquad (313)$$

$$h/e = \frac{8\,e + 5\,a}{4\,e + 2\,a} \qquad (317)$$

$$a/e = b/e \text{ and } a = b \qquad (318)$$

We have one less unknown quantity and a simpler reduction with less equations on that account.

$$\frac{a^2\,h\,e}{(a + e)\,(2\,e + 2\,h + a)} = A\,S\,b_1\,P_1^2/L_1^2 \qquad (322)$$

$$e = \sqrt[8]{\frac{S\,P_1^2\left(\frac{a}{e} + 1\right)}{\left(\frac{a}{e}\right)^6 \frac{h}{e}\left(1 + \frac{1}{r}\right)^2 k_i^2 b_1 \beta^4 F_i^2\left(2 + 2\,\frac{h}{e} + \frac{a}{e}\right)}} \qquad (323)$$

4. RING COIL TYPE TRANSFORMER

$$V_i = 32.8\,p^2\,(h + e + 3\,p) = 32.8\,A \qquad (412)$$

$$V_c = h\,e\,(15\,p + \pi\,e) = B \qquad (413)$$

Here again there is a slight difference from actual volume for the sake of simplicity.

$$h/e = 2 + \frac{6.63\,p}{7.50\,p + \pi\,e} \qquad (417)$$

p will be taken as the equivalent of $a = b$ but it should be noted that this is a new definition for a and b and not the one used in the other types.

$$\frac{p^2\,h\,e}{(15\,p + \pi\,e)\,(h + e + 3\,p)} = S\,b_1\,P_1^2/L_1^2 \qquad (422)$$

$$e = \sqrt[8]{\frac{S\,P_1^2\left(15\frac{p}{e}+\pi\right)}{\left(1+\frac{1}{r}\right)^2 k_i^2\, b_1\, \beta^4\, F_i^2\, 32.8^2\left(\frac{p}{e}\right)^6 \frac{h}{e}\left(\frac{h}{e}+1+3\,\frac{p}{e}\right)}} \qquad (423)$$

5. CRUCIFORM CORE, CORE TYPE TRANSFORMER

$$V_i = 1.236\,a^2\,(h + e + 1.727\,a)^* = 1.236\,A \qquad (512)$$

$$V_c = \left(\frac{\pi}{2}\right) h\,e\,(2\,a + e) = \frac{\pi}{2}\,B \qquad (513)$$

$$\frac{h}{e} = \frac{2\,e + 3.5905\,a}{e + a} \qquad (517)$$

$$\frac{a^2\,h\,e}{(2\,a + e)\,(h + e + 1.727\,a)} = S\,b_1\,P_1^2/L_1^2 \qquad (522)$$

$$e = \sqrt[8]{\frac{S\,P_1^2\left(2\frac{a}{e}+1\right)}{\left(1+\frac{1}{r}\right)^2 k_i^2\, b_1\, \beta^4\, F_i^2\, 1.527\left(\frac{a}{e}\right)^6 \frac{h}{e}\left(\frac{h}{e}+1+1.727\,\frac{a}{e}\right)}} \qquad (523)$$

6. DOUBLE NOTCHED CIRCULAR COIL, CORE TYPE TRANSFORMER

$$V_i = 1.3356\,a^2\,(h + e + 1.738\,a)^* = 1.3356\,A \qquad (612)$$

$$V_c = (\pi/2)\,h\,e\,(2\,a + e) = \frac{\pi}{2}\,B \qquad (613)$$

$$h/e = \frac{2\,e + 3.607\,a}{a + e} \qquad (617)$$

$$\frac{a^2\,h\,e}{(2\,a + e)\,(h + e + 1.738\,a)} = S\,b_1\,P_1^2/L_1^2 \qquad (622)$$

$$e = \sqrt[8]{\frac{S\,P_1^2\left(2\frac{a}{e}+1\right)}{\left(1+\frac{1}{r}\right)^2 k_i^2\, b_1\, \beta^4\, F_i^2\, 1.785\left(\frac{a}{e}\right)^6 \frac{h}{e}\left(\frac{h}{e}+1+1.738\,\frac{a}{e}\right)}} \qquad (623)$$

7. DOUBLE NOTCHED CIRCULAR COIL, SHELL TYPE TRANSFORMER

$$V_i = 1.3356\,a^2\,(h + e + .869\,a)^* = 1.3356\,A \qquad (712)$$

$$V_c = \pi\,h\,e\,(a + e) = \pi\,B \qquad (713)$$

$$h/e = \frac{4\,e + 3.607\,a}{2\,e + a} \qquad (717)$$

$$\frac{a^2\,h\,e}{(a + e)\,(h + e + .869\,a)} = S\,b_1\,P_1^2/L_1^2 \qquad (722)$$

$$e = \sqrt[8]{\frac{S\,P_1^2\left(\frac{a}{e}+1\right)}{\left(1+\frac{1}{r}\right)^2 k_i^2\, b_1\, \beta^4\, F_i^2\, 1.785\left(\frac{a}{e}\right)^6 \frac{h}{e}\left(\frac{h}{e}+1+.869\,\frac{a}{e}\right)}} \qquad (723)$$

It can be seen that in all the cases the equation for e can be put into the form

$$e = \sqrt[8]{\frac{S\,P_1^2\,\frac{U}{e}}{K\left(1+\frac{1}{r}\right)^2 k_i^2\; b_1\, \beta^4\; F_i^2\left(\frac{a}{e}\right)^3\left(\frac{b}{e}\right)^3\left(\frac{h}{e}\right)\left(\frac{Z}{e}\right)}}$$

Substituting the values for S, which is, in every case, a product of constants, space factors, and frequency, this can be broken up into four factors as follows.

$$e = \sqrt[4]{P_1}\,.\,\sqrt[8]{\frac{1}{F_c\,f^2}}\,.\,\sqrt[8]{\frac{10^{10}}{\left(1+\frac{1}{r}\right)^2\, b_1\;\beta^4}}$$

$$\sqrt[8]{\frac{K'\,\frac{U}{e}}{\left(\frac{a}{e}\right)^3\left(\frac{b}{e}\right)^3.\,\frac{h}{e}\,.\,\frac{Z}{e}}} = A_1\,.\,B_1\,.\,C_1\,.\,D_1$$

Where K and K' are numerical coefficients differing with the type. These four quantities can be plotted in curves of which the first three are the same for all types of transformer core, the fourth is dependent on the dimensions and therefore there must be a separate curve for each type.

A_1 is evidently a function of P only and can best be obtained from the slide rule direct

B_1 requires a separate curve for each f. Each curve is a function only of F_c

C_1 for a given frequency is a function of β only

D_1 is a function of a/e or p/e only for any one type.

PROCEDURE IN THE DESIGN OF A MINIMUM COST TRANSFORMER

When the curves have been drawn as indicated in the previous discussion, the quality of iron and the values of space factors and unit costs being known, the procedure in the design of a minimum cost transformer will be as follows:

Given—Type, kilovolt-amperes, voltages, and frequency, and unit costs.

To find—Dimensions, Losses, Costs.

Assume β at values near those known to give allowable heating.

Calculate—L_i, L_c, C, and S for each value of β and draw curves using β as abscissas.

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*For dimensions of notched core giving maximum iron section inscribed in a circle, see Bohle and Robertson—"A Treatise on Transformers," p. 169.

To find these quantities the dimensions and then the volumes must be obtained and these in turn depend on several quantities, as indicated in the previous discussion.

Assume—$F_i = 0.9$ (or other such value as the factory test may indicate).

F_c, C_c/C_i and r are to be taken from curves or tables.

F_c the space factor for copper, as is evident from the equations, takes care of all space in the window not occupied by copper. This includes space for ventilating ducts.

C_c the cost per pound of copper in place and including insulation, and also

C_i the cost per pound of iron core assembled, must be given by the cost records of the concern making the apparatus.

r can be determined only from the loss curves of the particular quality and thickness of iron to be used. See pages 823 and 824.

R the cost ratio of a cubic inch (gross) of copper to iron

$$= \frac{C_c F_c k_c}{C_i F_i k_i}$$

Having determined these quantities a/e or p/e may be taken from the curves and with this value b/e and h/e are obtained.

$$e = \sqrt[3]{S\ P_1^2 \times f_1\ (\beta) \times f_2\ (a/e)} = A_1 \,.\, B_1 \,.\, C_1 \,.\, D_1$$

See page 7 and preceding formulas.

P_1 = kv-a. + $L_1/2000$ (see page 2) but L is to be determined from the volumes, and will actually vary with each value of β. However, $L_1/2000$ is always very small compared to P_1, usually not exceeding 2 per cent, and it may therefore be assumed with sufficient accuracy for all practical purposes. In fact it may generally be neglected.

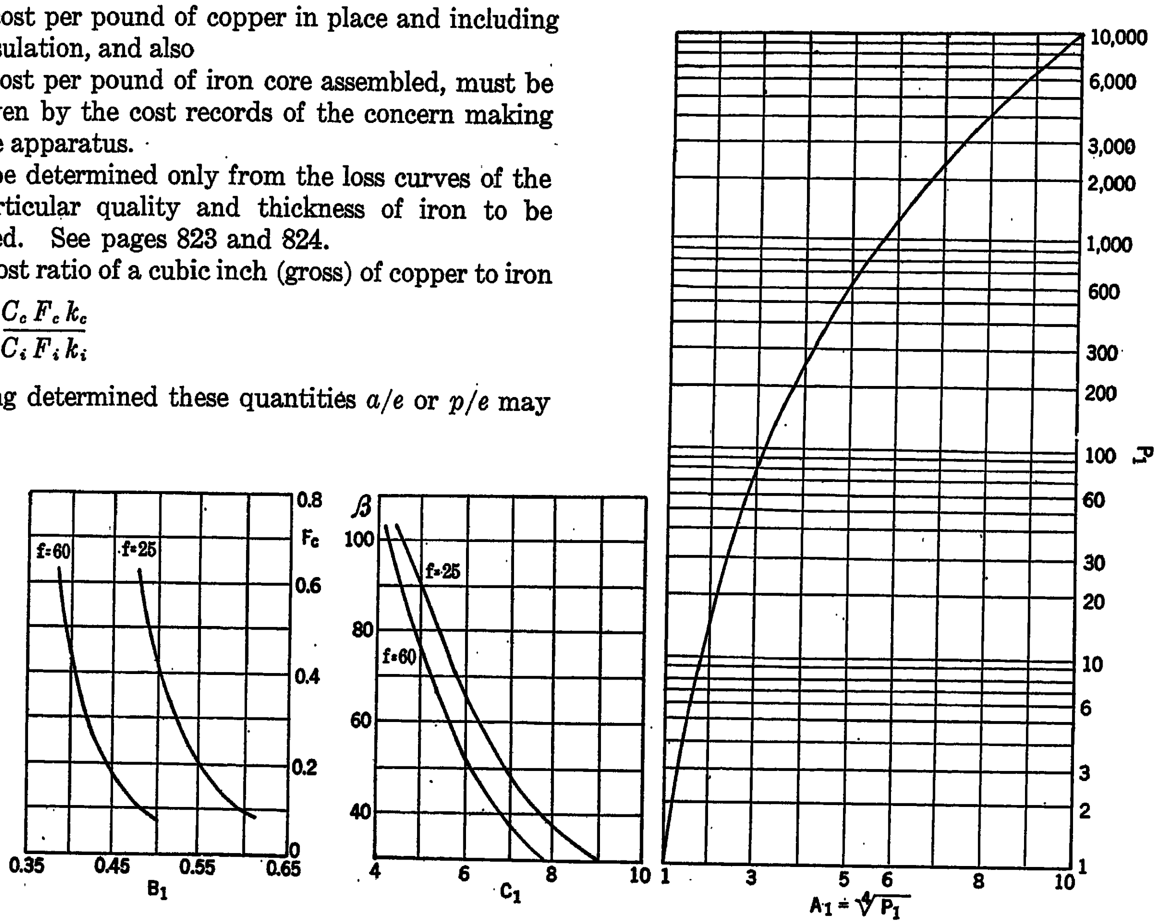
V_i and V_c see formulas given under the various types $L_i = 0.271\ b_1\ \beta^2\ F_i\ V_i$ (see curves of iron loss and explanations of b_1, p. 833).

$L_c = L_i/r$ for the minimum cost transformer. In other calculations the following formula must be used.

$$L_c = s^2\, c\, F_c\, V_c$$

$$s = \sqrt{L_c/0.82\, F_c\, V_c}$$

From the various dimensions and losses obtained as above it is easy to plot curves for "watts per square inch of surface" of both iron and copper losses. From these, according to the constants determined for any form and make, the proper density may be picked out to give the working temperature desired.



P_1 = Mean Kv-a. of Pri, and Sec. Windings
A_1 = A Factor Used in Determining Dimension e
B_1 and C_1 are Factors Used in Determining the Dimension e

Distribution Transformer

It will be found that the minimum cost transformer works out so that the iron loss and copper loss are nearly equal and therefore the maximum efficiency comes at full load. This may be quite satisfactory for a "power" transformer but is not the proportion desired for a "distribution" transformer which is to be on the line continuously but loaded only a small part of the time. Here it is of first importance to have the core losses as small as possible and on the other hand a considerable copper loss is not so serious a disadvantage.

Evidently the core loss may be decreased by picking out a low density. This will increase cost and decrease the temperature rise. This is simply underrating the transformer to gain efficiency.

If it is permissible to increase copper loss, material and costs may be saved by increasing s and decreasing β, to a point where copper temperature limits further increase of s. But better, the amount of iron may be decreased and the *amount*, *i. e.*, the number of turns, of copper increased in proportion. In this way L_1 may be practically constant, and the heating and temperature also, if β and s are kept as before. This method proves a means of decreasing core loss very materially with but small increase in cost and with practically constant total losses provided the changes are made as indicated below. And such a transformer is quite comparable with the minimum cost transformer, as output, losses, and temperature rise are alike.

Since the product φN must be constant and β and s are to be kept constant the product $a\,b\,h\,e$ must be constant. Call this value M, as before $L_i/L_c = k_6 A/k_7 B$ and $C = k_4 A + k_5 B$. Taking the differentials of P and C, as heretofore, putting the differentials of L_i/L_c in place of L' in matrix (7) (see Appendix p. 13) by similar methods of elimination the first four rows of matrix (8) may be obtained just as in the original solution for the minimum cost transformer. From these rows the relation of the ratio a/e to b/e and h/e were obtained. Therefore, these same curves still held with the condition of fixed β and s, which of course might have been anticipated.

As an example of the procedure and of the values obtained, let us consider a 25-kv-a., shell type, 2300-volt, 60-cycle distribution transformer.

If our curves are drawn from the proper manufacturing conditions as to cost, iron, and space factors, we may take at once from the curves the following values:

$F_i = 0.9 \quad F_c = 0.41 \quad C_c/C_i = 1.7$ whence $R = 0.918$ for $\beta = 75$

$r = 1.02$ and $b_1 B^2 = 1.11 \quad A_1 = 2.25 \quad B_1 = 0.402 \quad C_1 = 5.03 \quad a/e = 1.23$

$b/e = 3 \quad h/e = 2.38 \quad D_1 = 0.665 \quad V_i/e^3 = 34.3 \quad V_c/e^3 = 29.5$

From these values we immediately obtain:

$e = A_1 B_1 C_1 D_1 = 3.02 \quad V_i = 944 \quad V_c = 812$ whence $L_i = 255$ and $L_c = L_i/r = 250$.

Therefore we may take approximately,

$a = 3\frac{3}{4}$ in. $b = 9$ in. $h = 7\frac{1}{4}$ in. and $e = 3$ in. and $a\,b\,h\,e = 730$

This is all that is required to determine a minimum cost transformer if we know that at 75,000 lines the heat may be properly dissipated.

If now we wish to reduce the iron loss to one-half of the copper loss, the values again taken from the curves are $a/e = 0.76$ $b/e = 1.93$ and $h/e = 2.273$. Since we are keeping the densities the same

$a/e \times b/e \times h/e \times e^4 = 730$ whence $e = 3.84$ in. say $3\frac{7}{8}$ in.

Getting the other values (except L_c) as before, we would find the iron loss 190 watts. The total losses would be increased about 10 per cent. Other types than the shell type, with one exception, would show less change of total loss in this method of keeping densities constant.

If iron costs 10 cents per lb., then the cost of the active material in the minimum cost transformer would be \$41.25 and for the distribution transformer, with 25 per cent less core loss \$44.40, an increase of 7½ per cent.

To determine then the best proportions for any transformer becomes a very simple matter and the increased cost for any change from these dimensions is readily found.

It works out that for a given density the least expensive transformer is that having the iron divided into the most parts in the yoke, the core type being the most expensive. It is possible, however, to use a different density in the different forms as the allowable heating may be different.

On the same basis the cruciform type has also the lowest core losses and total losses but the shell type the greatest. The differences in general are small.

Since in simple core and shell type transformers with rectangular cores it always works out to be more economical to have sections where b is greater than a, it seemed reasonable to think that an elliptical coil with notched core fitted in would be cheaper than a round coil. Core and shell types of this shape were worked out and show a decreased cost as expected. It is not large, in the neighborhood of 3 per cent, and would probably never pay to give up the advantages of the round coil.

In this connection, however, it may be worth while pointing out an advantage which, so far as the writer knows, has been utilized by only one concern. It was shown years ago that the maximum iron section would be obtained in a circular coil where steps of the notches both ways were equal, that is, with a cross section symmetrical about two axes at right angles. Therefore, it might naturally be assumed that this is the most economical shape to make the core. This is not true, however, for the same reason that an elliptical coil and core is more economical than circular, the mean length of the iron circuit is decreased. By slightly increasing the diameter of the circular coil, the same amount of iron may be put in the core section in a shape giving a material decrease in dimensions a with a corresponding increase in other dimensions and therefore a material decrease in the iron volume. In fact, by careful adjustments practically the same results may be obtained as with an elliptical coil. In addition, this gives better cooling, for the surface of iron core perpendicular to the lamination is materially increased. The relative amounts of iron and copper for the minimum cost circular coil and elliptical coil transformer, however, is not the same, and a type using a circular coil and flattened core is cheaper if proportions are used nearer those of the elliptical coil type.

The complete mathematical solutions of a number of forms are given in the appendix.

Appendix

MATHEMATICAL SOLUTION OF THE EQUATIONS FOR FINDING A MINIMUM COST TRANSFORMER

On p. 822 the following six equations in six unknowns were given:

$$P_1 = k_3 \beta\, s\, a\, b\, h\, e$$

$$L_1 = L_i + L_c = b_1 \beta^2 k_i F_i V_i + c\, s^2 F_c V_c$$

$$\frac{\partial C}{\partial a} = 0 \qquad \frac{\partial C}{\partial b} = 0 \qquad \frac{\partial C}{\partial h} = 0 \qquad \frac{\partial C}{\partial e} = 0$$

The partial derivatives of C are taken in order to solve for minimum values with the other two equations giving restraining conditions. The two equations are necessary since fixing the size (P_1) introduces two new variables (β, s). Indeterminate multipliers will be used. To do this the total derivative of P_1 is multiplied by λ_1 and that of L_1 by λ_2. For convenience let

$$P_a' = \frac{\partial \beta}{\partial a} \qquad L_a' = \frac{\partial L}{\partial a} \qquad C_a' = \frac{\partial C}{\partial a}, \text{etc.}$$

$$\lambda_1 (P_a'\, d a + P_b'\, d b + P_h'\, d h + P_e'\, d e + P_\beta'\, d \beta + P_s'\, d s) = \lambda_1 d P_1 = 0 \qquad (1)$$

$$\lambda_2 (L_a'\, d a + L_b'\, d b + L_h'\, d h + L_e'\, d e + L_\beta'\, d \beta + L_s'\, d s) = \lambda_2 d L_1 = 0 \qquad (2)$$

$$C_a'\, d a + C_b'\, d b + C_h'\, d h + C_e'\, d e = 0 \qquad (3)$$

Adding these three equations there results:

$$(\lambda_1 P_a' + \lambda_2 L_a' + C_a')\, d a + (\lambda_1 P_b' + \lambda_2 L_b' + C_b')\, d b + (\lambda_1 P_h' + \lambda_2 L_h' + C_h)\, dh + (\lambda_1 P_e' + \lambda_2 L_e' + C_e')\, d e + (\lambda_1 P_\beta' + \lambda_2 L_\beta')\, d \beta + (\lambda_1 P_s' + \lambda_2 L_s')\, d s = 0 \qquad (4)$$

Since there are six variables and two fixed conditions, four of the variables are independent; a, b, h, and e may be selected. This leaves β and s as dependent variables. λ_1 and λ_2 may be so chosen that the coefficients of $d\beta$ and $d s$ vanish identically.

That is,

$$\lambda_1 P_\beta' + \lambda_2 L_\beta' = 0 \text{ and } \lambda_1 P_s' + \lambda_2 L_s' = 0$$

Since b_1 varies with β the iron loss in watts per lb. must be expressed in the form of the Steinmetz equation, or

$L_i = (a_o \beta^{1.6} + b_0 \beta^2)\, k_i F_i V_i$ before differentiating.

Then

$$\lambda_1 k_3 s\, a\, b\, h\, e + \lambda_2 (1.6\, a_0 \beta^{.6} + 2\, b_0 \beta)\, k_i F_i V_i = 0 \qquad (5)$$

$$\lambda_1 k_3 \beta\, a\, b\, h\, e + 2\, \lambda_2 c\, s\, F_c V_c = 0$$

Multiplying the first equation by β and the second by s, and subtracting the second from the first,

$$\lambda_2 (1.6\, a_0 \beta^{1.6} + 2\, b_0 \beta^2)\, k_i F_i V_i - 2\, \lambda_2 c\, s^2 F_c V_c = 0 \qquad (6)$$

$$(0.8\, a_0 \beta^{1.6} + b_0 \beta^2)\, k_i F_i V_i = c\, s^2 F_c V_c = L_c$$

$$(a_0 \beta^{1.6} + b_0 \beta^2)\, k_i F_i V_i\, (0.8\, a_0 \beta^{1.6} + b_0 \beta^2)/(a_0 \beta^{1.6} + b_0 \beta^2) = L_c$$

$$L_i = r\, L_c$$

where r is the reciprocal of the last two terms on the left hand side of the above equation and equals the ratio of the iron loss to the copperl oss. (See p. 823).

From Equations (1), (2), and (3) the following matrix is set up for determining the relations among the variables which will satisfy the equations.

$$\begin{vmatrix} P_a' & L_a' & C_a' \\ P_b' & L_b' & C_b' \\ P_h' & L_h' & C_h' \\ P_e' & L_e' & C_e' \\ P_\beta' & L_\beta' & 0 \\ P_s' & L_s' & 0 \end{vmatrix} = \begin{vmatrix} k_3 \beta\, s\, b\, h\, e & k_6 A_a' + k_7 B_a' & k_4 A_a' + k_5 B_a' \\ k_3 \beta\, s\, a\, h\, e & k_6 A_b' + k_7 B_b' & k_4 A_b' + k_5 B_b' \\ k_3 \beta\, s\, a\, b\, e & k_6 A_h' + k_7 B_h' & k_4 A_h' + k_5 B_h' \\ k_3 \beta\, s\, a\, b\, h & k_6 A_e' + k_7 B_e' & k_4 A_e' + k_5 B_e' \\ k_3 \beta\, s\, a\, b\, h\, e & (k_6)_\beta' A & 0 \\ k_3 \beta\, a\, b\, h\, e & (k_7)_s' B & 0 \end{vmatrix} \qquad (7)$$

From this matrix any three rows may be selected to form third order determinants, each of which gives a relation among the variables. Not all of these are independent relations and the useful ones must be found by trial. Rules for the reduction of determinants can be applied to this matrix in order to simplify the calculations. Divide the first column by P_1. Multiply the second column by k_5 and subtract from it k_7 times the third column.

Multiply the third column by k_6 and subtract from it k_4 times the second column. There results:

$$\begin{vmatrix} 1/a & (k_6 k_5 - k_7 k_4) A_a' & (k_6 k_5 - k_7 k_4) B_a' \\ 1/b & (k_6 k_5 - k_7 k_4) A_b' & (k_6 k_5 - k_7 k_4) B_b' \\ 1/h & (k_6 k_5 - k_7 k_4) A_h' & (k_6 k_5 - k_7 k_4) B_h' \\ 1/e & (k_6 k_5 - k_7 k_4) A_e' & (k_6 k_5 - k_7 k_4) B_e' \\ 1/\beta & k_5 (k_6)_\beta' A & -k_4 (k_6)_\beta' B \\ 1/s & k_5 (k_7)_s' B & -k_4 (k_6)_s' B \end{vmatrix} = \begin{vmatrix} 1 & a A_a' & a B_a' \\ 1 & b A_b' & b B_b' \\ 1 & h A_h' & h B_h' \\ 1 & e A_e' & e B_e' \\ 1 & \dfrac{k_5\, 2 L_i}{\Delta r} & -\dfrac{k_4}{\Delta} \dfrac{2\, L_i}{r} \\ 1 & \dfrac{k_5}{\Delta} 2\, L_c & -\dfrac{k_4}{\Delta} 2\, L_c \end{vmatrix} \qquad (8)$$

The transformation to the matrix on the right is made by multiplying the rows by a, b, etc., and dividing the second and third columns by $(k_6 k_5 - k_7 k_4)$. Let this latter expression be equal to Δ and from (5) and (6) it can be shown that $\beta\, (k_6)_\beta' A = 2\, L_i/r$ and $s\, (k_7)_s' B = 2\, L_c$. For the ring coil transformer p is used instead of a and b, and for some of the other types, a alone is used.

CORE TYPE TRANSFORMER

From page 14 of the symbols: $A = a\, b\, (2 a + h + e)$ $B = h\, e\, (a + b + e)$

$A_a' = b(4a + h + e)$ $A_b' = a(2a + h + e)$ $A_h' = A_e' = a\, b$

$B_a' = B_b' = h\, e$ $B_h' = e(a + b + e)$ $B_e' = h(a + b + 2\, e)$

Form a third order determinant using the first three rows of the last matrix given above.

$$\begin{vmatrix} 1 & a\, b\, (4\, a + h + e) & a\, h\, e \\ 1 & a\, b\, (2\, a + h + e) & b\, h\, e \\ 1 & a\, b\, h & h\, e\, (a + b + e) \end{vmatrix} = \begin{vmatrix} 0 & 2\, a & a - b \\ 0 & a + e & -(a + e) \\ 1 & h & a + b + e \end{vmatrix} = 0$$

The transformation to the second form is made by dividing the second and third columns by $a\, b$ and $h\, e$ respectively and then subtracting the second row from the first and the third from the second. Expanding the second order determinant in the upper right hand corner, it is found that:

$$2\, a(a + e) = -\, (a - b)\, (2\, a + e)$$

From which $b/a = (4\, a + 3\, e)/(2\, a + e)$

From the first, second, and fourth rows:

$$\begin{vmatrix} 1 & ab(4a+h+e) & ahe \\ 1 & ab(2a+h+e) & bhe \\ 1 & abe & he(a+b+2e) \end{vmatrix} = \begin{vmatrix} 0 & 2a & a-b \\ 0 & 2a+h & -(a+2e) \\ 1 & e & a+b+2e \end{vmatrix} = 0$$

$$-2a(a+2e) = (a-b)(2a+h)$$

This result, using the relation just found between b and a, reduces to:

$$h/e = (3a+2e)/(a+e)$$

From the first, second, and fifth rows:

$$\begin{vmatrix} 1 & ab(4a+h+e) & ahe \\ 1 & ab(2a+h+e) & bhe \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & \frac{k_4}{\Delta}\frac{2L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & 2a^2b & he(a-b) \\ 0 & abZ-\frac{k_5}{\Delta}\frac{2L_i}{r} & bhe+\frac{k_4}{\Delta}\frac{2L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & -\frac{k_4}{\Delta}\frac{2L_i}{r} \end{vmatrix} = 0$$

$$2a^2b\left(bhe+\frac{k_4}{\Delta}\frac{2L_i}{r}\right)$$

$$= he(a-b)\left[ab(2a+h+e)-\frac{k_5}{\Delta}\frac{2L_i}{r}\right]$$

The solution is somewhat lengthy, although perfectly simple algebraically. The result is:

$$\frac{k_5}{k_4} = R = \frac{a^2(4a+3e)(2a^2+6ae+3e^2)(6a^2+3ae-e^2)}{e^2(3a+2e)(6a^2+6ae+e^2)[2(2a^2+6ae+3e^2)-3r(2a^2+3ae+e^2)]}$$

By dividing numerator and denominator by e^7, R is made a function of a/e and r. By substituting values of these variables into the equation, values of R can be obtained and curves drawn to show the relation of a/e to R for various values of r. (See Curve Sheet.)

SHELL TYPE TRANSFORMERS

$A = ab(a+h+e)$ $A_a' = b(2a+h+e)$ $A_b' = a(a+h+e)$ $A_h' = A_e' = ab$

$B = he(a+b+2e)$ $B_a' = B_b' = he$ $B_h' = e(a+b+2e)$ $B_e' = he(a+b+4e)$

Following the same procedure as was used for the core type, there results from the first three rows of (8), p. 830:

$$\begin{vmatrix} 1 & ab(2a+h+e) & ahe \\ 1 & ab(a+h+e) & bhe \\ 1 & abh & he(a+b+2e) \end{vmatrix} = \begin{vmatrix} 0 & a & a-b \\ 0 & a+e & -(a+2e) \\ 1 & h & a+b+2e \end{vmatrix} = 0$$

$$-a(a+2e) = (a-b)(a+e) \text{ or } b/a = (2a+3e)/(a+e)$$

From the first, second, and fourth rows of (8):

$$\begin{vmatrix} 1 & ab(2a+h+e) & ahe \\ 1 & ab(a+h+e) & bhe \\ 1 & abe & he(a+b+4e) \end{vmatrix} = \begin{vmatrix} 0 & a & a-b \\ 0 & a+h & -(a+4e) \\ 1 & e & a+b+4e \end{vmatrix} = 0$$

$$-a(a+4e) = (a-b)(a+h) \text{ or } h/e = (3a+4e)/(a+2e)$$

From the first, second, and fifth rows of (8):

$$\begin{vmatrix} 1 & ab(2a+h+e) & ahe \\ 1 & ab(a+h+e) & bhe \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & -\frac{k_4}{\Delta}\frac{2L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & a^2b & (a-b)he \\ 0 & abZ-\frac{k_5}{\Delta}\frac{2L_i}{r} & bhe+\frac{k_4}{\Delta}\frac{2L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r}-\frac{k_4}{\Delta}\frac{2L_i}{r} & \end{vmatrix} = 0$$

$$a^2b\left(bhe+\frac{k_4}{\Delta}\frac{2L_i}{r}\right)$$

$$= he(a-b)\left[ab(a+h+e)-\frac{k_5}{\Delta}\frac{2L_i}{r}\right]$$

$$\frac{k_5}{k_4} = R = \frac{a^2(2a+3e)(a^2+6ae+6e^2)(3a^2+3ae-2e^2)}{e^2(3a+4e)(3a^2+6ae+2e^2)[2(a^2+6ae+6e^2)-3r(a^2+3ae+2e^2)]}$$

CRUCIFORM TYPE TRANSFORMERS

$A = 2ab\left(2h+2e+\frac{a}{2}+\frac{b}{2}\right)$ $A_a' = 2b\left(2h+2e+a+\frac{b}{2}\right)$ $A_b' = 2a\left(2h+2e+\frac{a}{2}+b\right)$ $A_h' = A_e' = 4ab$

$B = he(a+b+2e)$ $B_a' = B_b' = he$ $B_h' = e(a+b+2e)$ $B_e' = h(a+b+4e)$

From the first three rows of (8):

$$\begin{vmatrix} 1 & 2ab\left(2h+2e+a+\frac{b}{2}\right) & ahe \\ 1 & 2ab\left(2h+2e+\frac{a}{2}+b\right) & bhe \\ 1 & 4abh & he(a+b+2e) \end{vmatrix} = \begin{vmatrix} 0 & \frac{a}{2}-\frac{b}{2} & a-b \\ 0 & 2e+\frac{a}{2}+b & -(a+2e) \\ 1 & 2h & a+b+2e \end{vmatrix} = 0$$

$$\left(\frac{a}{2}-\frac{b}{2}\right)(a+2e)-(a-b)\left(2e+\frac{a}{2}+b\right) = 0$$

$$(a-b)\cdot(2a+2b+6e) = 0$$

Since $(2a+2b+6e)$ is a positive real number, $(a-b)$ must equal zero and $a = b$. Using this relation $A = 2a^2(2h+2e+a)$, $B = 2he(a+e)$, $P_a' = 2k_3\beta s a h e$, $A_a' = 4a\left(2h+2e+3\frac{a}{2}\right)$ and $B_a' = 2he$

Then from the first, third, and fourth rows of (8):

$$\begin{vmatrix} 2 & 4a^2(2h+2e+3a/2) & 2ahe \\ 1 & 4a^2h & 2he(a+e) \\ 1 & 4a^2e & 2he(a+2e) \end{vmatrix} = \begin{vmatrix} 0 & 2e+3a/2 & -(a+2e) \\ 0 & (h-e) & -e \\ 1 & e & a+2e \end{vmatrix} = 0$$

$$-e(2e+3a/2)+(h-e)(a+2e) = 0$$

$$h/e = (5a+8e)/(2a+4e)$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & 4a^2h & 2he(a+e) \\ 1 & 4a^2e & 2he(a+2e) \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & -\frac{k_4}{\Delta}\frac{2L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & 4a^2(h-e) & -2he^2 \\ 0 & 4a^2e-\frac{k_5}{\Delta}\frac{2L_i}{r} & 2he(a+2e)+\frac{k_4}{\Delta}\frac{2L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2L_i}{r} & -\frac{k_4}{\Delta}\frac{2L_i}{r} \end{vmatrix} = 0$$

$$4a^2(h-e)\left[2he(a+2e)+\frac{k_4}{\Delta}\frac{2L_i}{r}\right]$$

$$+2he^2\left(4a^2e-\frac{k_5}{\Delta}\frac{2L_i}{r}\right) = 0$$

$$\frac{k_5}{k_4} = 4R = \frac{2a^2(a^2+9ae+12e^2)(3a^2+2ae-4e^2)}{e^2(5a+8e)(a+e)[2(a^2+9ae+12e^2)-3r(a+2e)^2]}$$

RING COIL TYPE TRANSFORMERS

$A = p^2(h+e+3\,p) \quad A_p' = p(2\,h+2\,e+9\,p) \quad A_h' = A_e' = p^2$
$B = h\,e(15\,p+\pi\,e) \quad B_p' = 15\,h\,e \quad B_h' = e(15\,p+\pi\,e) \quad B_e' = h(15\,p+2\,\pi\,e) \quad P_p' = 2\,k\beta\,s\,p\,h\,e$

From the first, third, and fourth rows of (8):

$$\begin{vmatrix} 2 & p^2(2\,h+2\,e+9\,p) & 15\,p\,h\,e \\ 1 & p^2 h & h\,e\,(15\,p+\pi\,e) \\ 1 & p^2 e & h\,e\,(15\,p+2\,\pi\,e) \end{vmatrix} = \begin{vmatrix} 0 & 2\,e+9\,p & -(15\,p+2\,\pi\,e) \\ 0 & h-e & -\pi\,e \\ 1 & e & 15\,p+2\,\pi\,e \end{vmatrix} = 0$$

$$\pi\,e\,(2\,e+9\,p) = (h-e)\,(15\,p+2\,\pi\,e)$$

$$h/e = 1 + \pi\,(2\,e+9\,p)/(15\,p+2\,\pi\,e)$$
$$= (43.3\,p + 12.57\,e)/(15\,p + 6.28\,e)$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & p^2 h & h\,e(15\,p+\pi\,e) \\ 1 & p^2 e & h\,e(15\,p+2\,\pi\,e) \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_4}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & p^2(h-e) & h\,e\,(-\pi\,e) \\ 0 & p^2 e - \frac{k_5}{\Delta}\frac{2\,L_i}{r} & h\,e(15\,p+2\,\pi\,e) + \frac{k_4}{\Delta}\frac{2\,L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_4}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = 0$$

$$p^2\,(h-e)\,[h\,e\,(15\,p+2\,\pi\,e) + 2\,k_4\,L_i/\Delta\,r]$$
$$+ \pi\,h\,e^2\,(p^2\,e - 2\,k_5\,L_i/\Delta\,r) = 0$$

$$\frac{k_5}{k_4} = \frac{R}{32.8}$$

$$= \frac{p^2\,(45\,p^2 + 77.1\,p\,e + 18.85\,e^2)\,(135\,p^2 + 15\,p\,e - 6.28\,e^2)}{e^2(43.3\,p+12.57\,e)(15\,p+3.14\,e)[2(45\,p^2+77.1\,p\,e+18.85\,e^2)-3r(45\,p^2+33.85\,p\,e+6.28\,e^2)]}$$

CRUCIFORM CORE, CORE TYPE TRANSFORMER

$A = a^2\,(h+e+1.727\,a) \quad A_a' = a\,(2\,h+2\,e+5.18\,a) \quad A_h' = A_e' = a^2 \quad B = h\,e\,(2\,a+e)$
$B_a' = 2\,h\,e \quad B_h' = e\,(2\,a+e) \quad B_e' = 2\,h\,(a+e) \quad P_a' = 2\,k_3\beta\,s\,a\,h\,e$

From the first, third, and fourth rows of (8), p. 830

$$\begin{vmatrix} 2 & a^2(2\,h+2\,e+5.18\,a) & 2\,a\,h\,e \\ 1 & a^2 h & h\,e\,(2\,a+e) \\ 1 & a^2 e & 2\,h\,e\,(a+e) \end{vmatrix} = \begin{vmatrix} 0 & 2\,e+5.18\,a & -(2\,a+2\,e) \\ 0 & h-e & -e \\ 1 & e & 2\,(a+e) \end{vmatrix} = 0$$

$$(h-e)\,(2\,a+2\,e) = e\,(2\,e+5.18\,a)$$
$$h/e = (3.59\,a+2\,e)/(a+e)$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & a^2 h & h\,e\,(2\,a+e) \\ 1 & a^2 e & 2\,h\,e\,(a+e) \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_4}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & a^2\,(h-e) & -h\,e^2 \\ 0 & a^2 e - \frac{k_5}{\Delta}\frac{2\,L_i}{r} & 2\,h\,e\,(a+e) + \frac{k_4}{\Delta}\frac{2\,L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_4}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = 0$$

$$2\,a^2\,(h-e)\,[h\,e\,(a+e) + k_4\,L_i/\Delta\,r]$$
$$+ h\,e^2\,(a^2\,e - 2\,k_5\,L_i/\Delta\,r) = 0$$

$$\frac{k_5}{k_4} = \frac{R}{0.787}$$

$$= \frac{a^2\,(1.73\,a^2 + 6.32\,a\,e + 3\,e^2)(5.18\,a^2 + a\,e - e^2)}{e^2(3.59\,a+2\,e)(2\,a+e)[2(1.73\,a^2 + 6.32\,a\,e + 3\,e^2) - 3\,r\,(1.73\,a+e)\,(a+e)]}$$

DOUBLE NOTCHED CORE, CIRCULAR COIL, CORE TYPE

$A = a^2(h+e+1.738\,a) \quad A_a' = a(2\,h+2\,e+5.214\,a) \quad A_h' = A_e' = a^2 \quad B = h\,e(2\,a+e)$
$B_a' = 2\,h\,e \quad B_h' = e(2\,a+e) \quad B_e' = 2\,h(a+e) \quad P_a' = 2\,k_3\beta\,s\,a\,h\,e$

From the first, third, and fourth rows of (8), p. 830

$$\begin{vmatrix} 2 & a^2\,(2\,h+2\,e+5.214\,a) & 2\,a\,h\,e \\ 1 & a^2 h & h\,e(2\,a+e) \\ 1 & a^2 e & 2\,h\,e\,(a+e) \end{vmatrix} = \begin{vmatrix} 0 & 2\,e+5.214\,a & -2\,(a+e) \\ 0 & h-e & e \\ 1 & e & 2\,(a+e) \end{vmatrix} = 0$$

$$2\,(h-e)(a+e) = e\,(5.214\,a + 2\,e) \text{ or } h/e = \frac{3.61a+2e}{a+e}$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & a^2 h & h\,e\,(2\,a+e) \\ 1 & a^2 e & 2\,h\,e\,(a+e) \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & \frac{k_4}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & a^2\,(h-e) & -h\,e^2 \\ 0 & a^2 e - \frac{k_5}{\Delta}\frac{2\,L_i}{r} & 2\,h\,e\,(a+e) + \frac{k_4}{\Delta}\frac{2\,L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_4}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = 0$$

$$2\,a^2\,(h-e)\,[h\,e\,(a+e) + k_4\,L_i/\Delta\,r]$$
$$+ h\,e^2\,(a^2\,e - 2\,k_5\,L_i/\Delta\,r) = 0$$

$$\frac{k_5}{k_4} = \frac{R}{0.85}$$

$$= \frac{a^2\,(1.74\,a^2 + 6.35\,a\,e + 3\,e^2)\,(5.21\,a^2 + a\,e - e^2)}{e^2(3.61\,a+2\,e)\,(2\,a+e)\,[2\,(1.74\,a^2 + 6.35\,a\,e + 3\,e^2) - 3\,r\,(1.74\,a+e)\,(a+e)]}$$

DOUBLE NOTCHED CORE, CIRCULAR COIL, SHELL TYPE

$A = a^2(h+e+0.869\,a) \quad A_a' = a(2\,h+2\,e+2.607\,a) \quad A_h' = A_e' = a^2 \quad B = h\,e(a+e)$
$B_a' = h\,e \quad B_h' = e(a+e) \quad B_e' = h(a+2\,e) \quad P_a' = 2\,k_3\beta\,s\,a\,h\,e$

From the first, third, and fourth rows of (8), p. 830

$$\begin{vmatrix} 2 & a^2\,(2\,h+2\,e+2.607\,a) & a\,h\,e \\ 1 & a^2 h & h\,e\,(a+e) \\ 1 & a^2 e & h\,e\,(a+2\,e) \end{vmatrix} = \begin{vmatrix} 0 & 2\,e+2.607\,a & -(a+2\,e) \\ 0 & h-e & -e \\ 1 & e & a+2\,e \end{vmatrix} = 0$$

$$(h-e)\,(a+2\,e) = e\,(2.607\,a+2\,e)$$
$$h/e = (3.607\,a+4\,e)/(a+2\,e)$$

From the third, fourth, and fifth rows of (8):

$$\begin{vmatrix} 1 & a^2 h & h\,e(a+e) \\ 1 & a^2 e & h\,e(a+2\,e) \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_4}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = \begin{vmatrix} 0 & a^2\,(h-e) & -h\,e^2 \\ 0 & a^2 e - \frac{k_5}{\Delta}\frac{2\,L_i}{r} & h\,e\,(a+2\,e) + \frac{k_4}{\Delta}\frac{2\,L_i}{r} \\ 1 & \frac{k_5}{\Delta}\frac{2\,L_i}{r} & -\frac{k_4}{\Delta}\frac{2\,L_i}{r} \end{vmatrix} = 0$$

$$a^2\,(h-e)\,[h\,e\,(a+2\,e) + 2\,k_4\,L_i/\Delta\,r]$$
$$+ h\,e^2\,(a^2\,e - 2\,k_5\,L_i/\Delta\,r) = 0$$

$$\frac{k_5}{k_4} = \frac{R}{0.425}$$

$$= \frac{a^2\,(0.869\,a^2 + 6.35\,a\,e + 6\,e^2)(2.61\,a^2 + a\,e - 2\,e^2)}{e^2(3.61\,a+4\,e)(a+e)[2(0.869\,a^2 + 6.35\,a\,e + 6\,e^2) - 3\,r\,(0.869\,a+e)\,(a+2\,e)]}$$

LIGHTING TRANSFORMERS

In order to determine the dimensions of a transformer having minimum cost and a lower iron loss than the one selected on the basis of allowable temperature rise, β and s will again be selected as the dependent variables and will be held constant. The equations for the power and for the ratio of iron loss to copper loss will now be the restraining conditions. Taking the derivatives as in the previous development (p. 830), the following equations are obtained.

$$P_a'\,da + P_b'\,db + P_h'\,dh + P_e'\,de = 0$$
$$r_a'\,da + r_b'\,db + r_h'\,dh + r_e'\,de = 0$$
$$C_a'\,da + C_b'\,db + C_h'\,dh + C_e'\,de = 0$$

Since β and s are constants, the use of indeterminate multipliers is not necessary. A matrix can be set up as before using the coefficient of the differentials in the above equations.

$$\begin{vmatrix} P_a' & r_a' & C_a' \\ P_b' & r_b' & C_b' \\ P_h' & r_h' & C_h' \\ P_e' & r_e' & C_e' \end{vmatrix} = \begin{vmatrix} k_3\beta s b h e & k_6(A_a' B - A B_a')/k_7 B^2 & k_4 A_a' + k_5 B_a' \\ k_3\beta s a h e & k_6(A_b' B - A B_b')/k_7 B^2 & k_4 A_b' + k_5 B_b' \\ k_3\beta s a b e & k_6(A_h' B - A B_h')/k_7 B^2 & k_4 A_h' + k_5 B_h' \\ k_3\beta s a b h & k_6(A_e' B - A B_e')/k_7 B^2 & k_4 A_e' + k_5 B_e' \end{vmatrix}$$

$$\text{Since } r = \frac{k_6\,A}{k_7\,B} \quad \frac{\partial r}{\partial a} = (k_6\,A_a'\,k_7 B - k_6\,A\,k_7\,B_a')/k_7^2 B^2$$
$$= k_6(A_a' B - A\,B_a')/k_7 B^2, \text{ etc.}$$

Divide the first column by $P = k_3\,\beta\,s\,a\,b\,h\,e$ and multiply the second column by $k_7\,B^2/k_6$.

$$\begin{vmatrix} 1/a & A_a' B - A B_a' & k_4 A_a' + k_5 B_a' \\ 1/b & A_b' B - A B_b' & k_4 A_b' + k_5 B_b' \\ 1/h & A_h' B - A B_h' & k_4 A_h' + k_5 B_h' \\ 1/e & A_e' B - A B_e' & k_4 A_e' + k_5 B_e' \end{vmatrix} = \begin{vmatrix} 1/a & (k_5 B + k_4 A)A_a' & (k_5 B + k_4 A)B_a' \\ 1/b & (k_5 B + k_4 A)A_b' & (k_5 B + k_4 A)B_b' \\ 1/h & (k_5 B + k_4 A)A_h' & (k_5 B + k_4 A)B_h' \\ 1/e & (k_5 B + k_4 A)A_e' & (k_5 B + k_4 A)B_e' \end{vmatrix}$$

The matrix on the right is derived from the one on the left by adding A times the third column to k_5 times the second and by subtracting k_4 times the second column from B times the third. Divide the second and third columns by $k_5 B + k_4 A$ and multiply the rows by a, b, h, and e respectively.

$$\begin{vmatrix} 1 & a\,A_a' & a\,B_a' \\ 1 & b\,A_b' & b\,B_b' \\ 1 & h\,A_h' & h\,B_h' \\ 1 & e\,A_e' & e\,B_e' \end{vmatrix}$$

This matrix is in the same form as the first four rows of the right-hand member of matrix (8) p. 830 and will yield the same values of b/a and h/e, as the ones found for the various types in the preceding pages of this Appendix. Thus the same curves between a/e and b/e, h/e can be used for new assumed values of a/e, which are to be used in reducing the iron volume and loss.

TRANSFORMER DESIGN NOTES

Symbols.

a = width of iron core in inches, (see Figs. 1, 2, 3) or the diameter of the circle circumscribing notched cores, (see Figs. 5, 6, 7)
a_0 = a constant of the Steinmetz formula for iron loss. Its product with $\beta^{1.6}$ gives watts hysteresis loss per lb. of iron
A = a letter replacing a combination of symbols. See p. 834
$A_1 = P_1^{\frac{1}{4}}$
b = depth of iron core in inches except in Ring Coil Types, see Figs. 1-7.
b_0 = a constant of the Steinmetz formula for iron loss. Its product with β^2 gives watts eddy current loss per lb. of iron.
b_1 = a coefficient by which β^2 is multiplied to obtain watts iron loss per lb. of iron: determined empirically from the iron loss curve for various values of β. (See p. 823.)
B = a letter replacing a combination of symbols. See p. 834
$B_1 = \sqrt[8]{1/F_c f^2}$ see p. 827 also see curve
c = resistance, in microhms, measured from face to face of a copper cube of one inch edge = (0.82)
C = cost of the active iron and copper in the transformer
C_c = cost in cents per lb. of copper (wound in place with insulation)
C_i = cost in cents per lb. of iron (stacked in place)
$C_1 = \sqrt[8]{10^{10}/\left(1 + \frac{1}{r}\right)^2 b_1 \beta^4}$ See p. 827 also see curve
D_1 = see p. 834, and curves
e = width of window (See Figs. 1-7)
E = high-tension volts
f = frequency in cycles per second
F_c = space factor of copper (see curves)
F_i = space factor of iron (taken as 0.9)
h = height of window (see Figs. 1-7)
I = high tension current in amperes at rated kv-a.
k_i = weight in lb. of one cu. in. of iron net = (0.271)
k_c = weight in lbs. of one cu. in. of copper net = (0.321)
k_3, k_4, k_5, k_6, k_7 letters replacing combinations of symbols. See p. 834
L = total loss (in watts)
L_c = watts, copper loss
L_i = watts, iron loss
L_1 = any definite numerical value of L
N = high tension turns in series
p = a dimension of Ring Coil Transformer. See Fig. 4
P = mean kv-a.
P_1 = any definite numerical value of P
Q = a letter replacing a combination of symbols. See p. 834
$r = L_i/L_c$
R = ratio of cost of a cu. in. of copper to one of iron (gross)
s = current density in kilo-amperes per sq. in.
S, T, U = letters replacing combination of symbols. (See p. 834)
V_c = gross volume of copper in cu. in.
V_i = gross volume of iron in cu. in.
Z = a letter replacing a combination of symbols. (See p. 834)
β = flux density in core (kilo-lines per sq. in. of net iron section)
φ = total flux in core.

SYMBOLS AND THEIR EQUIVALENTS

1. *Core Type.*

$A = a\,b\,(2\,a + h + e) = V_i/2$
$B = h\,e\,(a + b + e) = V_c/2$

$$D_1 = \sqrt[8]{\frac{3.38\,U/e}{\left(\frac{a}{e}\right)^3\left(\frac{b}{e}\right)^3 \cdot \frac{h}{e} \cdot \frac{Z}{e}}}$$

$k_3 = 2.22\,F_i\,F_c\,f\,10^{-5}$
$k_4 = 2\,F_i\,C_i\,k_i$
$k_5 = 2\,F_c\,C_c\,k_c$
$k_6 = 2\,k_i\,b_1\,\beta^2\,F_i$
$k_7 = 2\,c\,s^2\,F_c$
$Q = 1 + (1 - h/Z)/(1 - b/U)$
$S = 16\,c\,F_c\,F_i\,k_i/k_3^2$

$$T = \frac{e}{a}\,\frac{h/Z}{b/U} = B/A$$

$U = a + b + e$
$Z = 2\,a + h + e$

2. *Shell Type.*

$A = a\,b\,(a + h + e) = V_i/2$
$B = h\,e\,(a + b + 2\,e) = V_c/2$
D_1 = Same form as core type
k_3, k_4, k_5, k_6, k_7 = Same form as core type
Q, S, T = Same form as core type
$U = a + b + 2\,e$
$Z = a + h + e$

3. *Cruciform Type.*

$$A = 2\,a\,b\left(2\,h + 2\,e + \frac{a}{2} + \frac{b}{2}\right) = 2\,a^2(2\,h + 2\,e + a) = 2\,V_i$$

$B = h\,e(a + b + 2\,e) = 2\,h\,e\,(a + e) = V_c/2$

$$D_1 = \sqrt[8]{\frac{13.5\,U/e}{\left(\frac{a}{e}\right)^6 \cdot \frac{h}{e} \cdot \frac{Z}{e}}}$$

k_3, k_5, k_7 = Same as Core Type
$k_4 = 0.5\,F_i\,C_i\,k_i$
$k_6 = 0.5\,k_i\,b_1\,\beta^2\,F_i$

$$Q = 1 + \left(1 - \frac{4\,h - \omega}{Z}\right)/(1 - a/U)$$

S = Same as Core Type

$$T = \frac{e}{a}\,\frac{h/Z}{b/U} = \frac{e\,h\,U}{a^2\,Z} = B/A$$

$U = a + b + 2\,e = 2\,(a + e)$

$$Z = 2\left(2\,h + 2\,e + \frac{a}{2} + \frac{b}{2}\right) = 2\,(2\,h + 2\,e + a)$$

4. *Ring Coil Type.*

$A = p^2\,(h + e + 3\,p) = V_i/32.8$
$B = h\,e\,(15\,p + \pi\,e) = V_c$

$$D_1 = \sqrt[8]{\frac{0.000382\,U/e}{\left(\frac{p}{e}\right)^6 \cdot \frac{h}{e} \cdot \frac{Z}{e}}}$$

$k_3 = 36.4\,F_i\,F_c\,f\,10^{-5}$
$k_4 = 32.8\,F_i\,C_i\,k_i$
$k_5 = F_c\,C_c\,k_c$
$k_6 = 32.8\,k_i\,b_1\,\beta^2\,F_i$
$k_7 = c\,s^2\,F_c$

$$Q = 1 + \frac{\left(1 + \frac{1.50\,p - h}{Z}\right)}{(1 - 7.50\,p/U)}$$

$S = 131.1\,c\,F_c\,F_i\,k_i/k_3^2$

$$T = e/p\,\frac{h/Z}{p/U} = B/A$$

$U = 15\,p + \pi\,e$
$Z = h + e + 3\,p$

5. *Cruciform Core, Core Type.*

$A = a^2\,(h + e + 1.727\,a) = V_i/1.236$
$B = h\,e\,(2\,a + e) = 2\,V_c/\pi$

$$D_1 = \sqrt[8]{\frac{11.21\,\frac{U}{e}}{\left(\frac{a}{e}\right)^6 \frac{h}{e} \cdot \frac{Z}{e}}}$$

$k_3 = 1.372\,f\,F_i\,F_c\,.\,10^{-5}$
$k_4 = 1.236\,F_i\,C_i\,k_i$
$k_5 = 1.5708\,F_c\,C_c\,k_c$
$k_6 = 1.236\,b_1\,\beta^2\,k_i\,F_i$
$k_7 = 1.5708\,c\,s^2\,F_c$
$S = 7.768\,c\,F_c\,F_i\,k_i/k_3^2$
$U = 2\,a + e$
$Z = h + e + 1.727\,a$

6. *Double Notched Circular Coil, Core Type.*

$A = a^2\,(h + e + 1.738\,a) = V_i/1.3356$
$B = h\,e\,(2\,a + e) = 2\,V_c/\pi$

$$D_1 = \sqrt[8]{\frac{8.89\,\frac{U}{e}}{\left(\frac{a}{e}\right)^6 \frac{h}{e} \cdot \frac{Z}{e}}}$$

$k_3 = 1.482\,f\,F_i\,F_c\,.\,10^{-5}$
$k_4 = 1.3356\,F_i\,C_i\,k_i$
k_5, k_7 = Same as cruciform core, core type
$k_6 = 1.3356\,b_1\,\beta^2\,k_i\,F_i$
$S = 8.4\,c\,F_c\,F_i\,k_i/k_3^2$
$U = 2\,a + e$
$Z = h + e + 1.738\,a$

7. *Double Notched Circular Coil, Shell Type.*

$A = a^2\,(h + e + 0.869\,a) = V_i/1.3356$
$B = h\,e\,(a + e) = V_c/\pi$

$$D_1 = \sqrt[8]{\frac{17.78\,\dfrac{U}{e}}{\left(\dfrac{a}{e}\right)^6 \dfrac{h}{e} \cdot \dfrac{Z}{e}}}$$

k_3, k_4, k_6 = Same as double notched core type above
$k_5 = 3.1416\, F_c\, C_c\, k_c$
$k_7 = 3.1416\, c\, s^2\, F_c$
$S = 16.78\, c\, F_c\, F_i\, k_i/k_3^2$
$U = a + e$
$Z = h + e + 0.869\, a$

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Recent Developments in Telephone Construction Practises

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Synopsis.—In this paper are described recent developments in telephone construction practises which react to preserve the integrity of the sheath of lead covered cable, thus decreasing insulation troubles due to moisture seeping through armor breaks and warding off serious service interruptions.

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INTRODUCTION

GROWTH of the telephone plant in the last few years has naturally followed the tremendous increase in the use of this method of communication. The outside plant which has been built to satisfy these increased requirements is very largely either aerial or underground cable.

The extension of the toll cable network, wherein several hundred telephone and telegraph messages are carried within a lead-antimony cable sheath of approximately 2⅝-in. diameter, and the development of exchange cables in sizes up to 1818 pairs of wires, have necessitated new construction practises intended to lessen service interruptions caused by the opening of this lead sheath.

In this paper a description of recent developments in the construction of telephone outside plant in so far as they affect maintenance and service reliability will be presented. They are the result of development work throughout the country and it is felt that they represent a decided advance in building plant which will be subject to fewer troubles and appreciably less interruptions to service.

The paper is divided into three sections: First, the development of gas pressure testing which has made it possible to determine the location of sheath openings before they result in failure. Second, recent advances made in the methods of erecting aerial cable, which it is hoped will eliminate bowing, buckling, and other movements of the cable which in time cause a fracture of the sheath. Third, the application of catenary construction to telephone cables which has resulted in the possibility of locating cable routes over rivers and rugged terrain which, a few years ago, could not have been followed except where the cables could be attached to bridges or other crossing structures.

GAS PRESSURE TESTING

General. Sheath breaks occur in both aerial and underground cable plants and can usually be attributed to one of the following:

Aerial Cable

Ring cuts
Breaks due to buckling and bowing
Bullet holes
Splicing defects
Manufacturing defects

Underground Cable

Breaks at duct entrances
Breaks due to poor arrangement in manholes
Snagging cable while pulling
Electrolysis
Foreign companies' workmen
Splicing defects
Manufacturing defects

With the extension of the toll cable network, it became necessary to provide a definite plan of preventive maintenance by means of which the above potential troubles might be eliminated.

The plan for eliminating these defects was set up along the following lines:

1. Gas pressure testing
2. More care in racking in manholes
3. Educating employes in the handling of cable
4. Extensive electrolysis surveys
5. Frequent manhole inspections
6. Filling sleeves with paraffin (in the case of exchange cables)
7. Frequent inspection of aerial routes
8. Close observation of work being done by foreign companies' workmen along routes paralleling telephone plant
9. Pulling and splicing cable at such tensions as will insure the elimination of trouble due to bowing
10. Erecting strand at proper tensions in order to procure proper sags
11. Engineering on such a basis as to warrant cable's being free from trees, fire hazards, and other sources which might cause interruptions.

The introduction of gas under pressure into a cable is not a new departure, but the kind of gas and the apparatus for applying it have undergone considerable development and improvement in the past three years.

Briefly, the earlier methods consisted of injecting compressed air, which had been dried with calcium chloride, into the cable, and watching the previously

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installed pressure gages for a drop in the internal sheath pressure which would indicate a hole or holes in the sheath.

In using this method on telephone cable considerable difficulty was experienced in removing the moisture from the air completely enough to insure that the serviceability of the insulation would not be affected. As a result of this experience it was deemed necessary to find a gas of a lesser moisture content and this led to the use of extra dry carbon dioxide for this purpose. While from a moisture standpoint this gas was well suited for testing purposes, its physical characteristics were such as to curtail, to a certain degree, it use by the field construction forces.

A special dried nitrogen gas has come into widespread use for testing telephone cables. This gas is extremely dry and has the advantage of requiring very little associated equipment to regulate its flow into the cable.

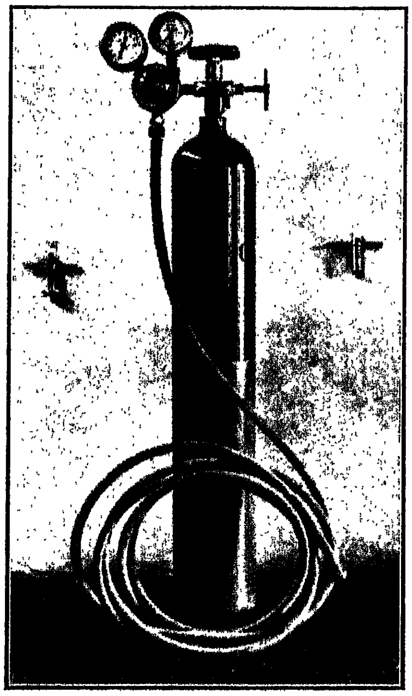

Fig. 1—Small Cylinder Gas Pressure Equipment

Showing regulating equipment valves and awl.

Equipment. Inasmuch as oil pumped nitrogen has almost universally supplanted carbon dioxide in cable testing work, we will, in this paper, describe only equipment and tests which use this gas.

Two types of cylinders are used for supplying gas to the field forces. The larger size is the usual 55-in. cylinder which the manufacturer uses to send the gas from his plant. These contain approximately 15 lb. of gas at a pressure of 2000 lb. per sq. in., their total weight being 150 lb. Containers of this type are usually employed in large construction projects where there are thousands of feet of cable to be tested and where it is usually convenient to haul them to the job by means of trucks.

A cylinder of this type will test approximately 6000 ft., or one loading section of full sized cable.

In order that small amounts of gas may be available for testing single splices, as is the case when a splicer has worked on an existing cable, a small 25-in. cylinder, (see Fig. 1) containing sufficient gas to test eight or nine 4½-in. sleeves, is used. These tanks are filled from a battery of the larger 2000-lb. cylinders, until their internal pressure reaches 1600 lb. per sq. in., as shown in Fig. 2. The small tanks are located at the splicer's supply base and are ordered by him in much the same manner as he would order sleeves, solder, etc.

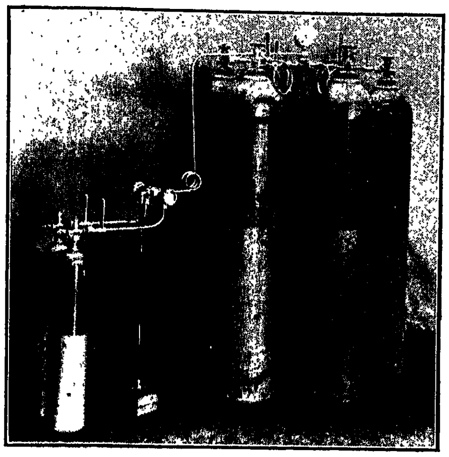

Fig. 2—Manifold Equipment

For transferring gas from large to small cylinder

The regulating equipment for both cylinders is practically the same. This apparatus is also shown in Fig. 1 and consists of a regulating valve for controlling the ingress pressure, a 50-lb. gage for reading this pressure, and a 3000-lb. gage for registering the cylinder pressure. The scale of the latter gage is also calibrated in cubic feet at atmospheric pressure in order to indicate the amount of gas in the tank at all times. The rubber hose provides a means of conducting the gas from the tank into the cable.

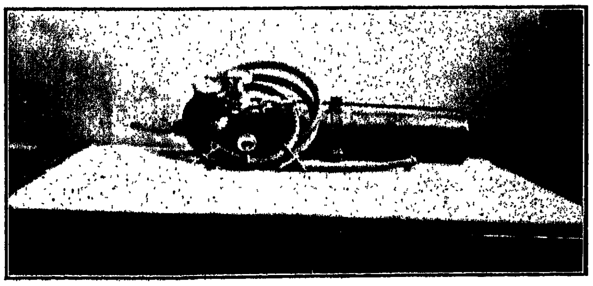

Fig. 3—Portable Pressure Testing Equipment

In the splice is installed a 30-lb. gage and an ingress stem, both designed so as to screw into a hole made with an awl. Each of these items is equipped with a rubber gasket so as to make an air-tight fit with the sleeve of the cable, without employing the use of solder.

A carrying case has been developed in which is packed equipment to be used in placing the cable or splice under pressure and inspecting the metal work after this has been done, (see Fig. 3). The only other equipment necessary to carry on these tests is carried by the splicer as part of his regular kit.

Single Splice Tests. In the case of single splice tests, two holes must be made with the awl, one near each end of the sleeve. These are made at this point as it is here that the possibility of injuring the wire work is a minimum. The valve stem and 30-lb. pressure gage are then screwed into these holes. The splice will then look like Fig. 4.

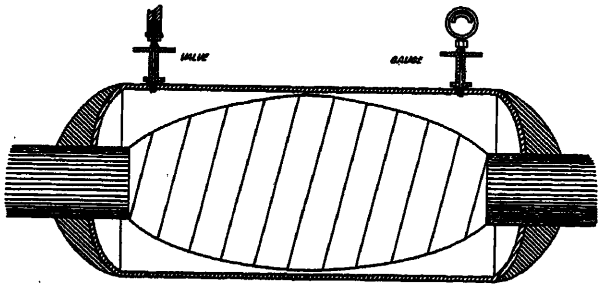

Fig. 4—Single Splice Testing Arrangements

Valve and gage locations

The regulator equipment is then attached to the small cylinder and the hose connected to the valve stem in the splice.

Before the gas is turned on, the entire sleeve and both joints are covered with a heavy coating of soapsuds. After this has been done, the splice is ready to receive the gas. The cylinder valve is then released and the regulator set until the ingress pressure reaches 40 lb., the rate at which the gas is allowed to flow into the cable.

Before the sleeve is placed on the splice, the conductors and muslin are boiled out with paraffin heated to a temperature of 375 deg. to 390 deg. fahr., in order to remove any moisture which may have been deposited on the conductors by perspiration from splicers' hands or manhole dampness. This paraffin usually runs back under the armor of the cable for a distance of six or eight inches. When cold, this paraffin forms a temporary dam which holds the gas in the splice long enough to make the test.

When the pressure within the splice reaches 5 lb. as indicated on the screw-in pressure gage, the gas is cut off at the cylinder. The sleeve and joints are then examined with the magnifying mirror with a view to finding bubble formations which indicate leaks. In the case of large leaks, it is often possible to hear the hiss caused by the escaping gas.

If no leaks are found, the gage and valve stem are removed, the gas permitted to escape, and the holes carefully soldered shut. Should a leak be detected in the joints, they are rewiped and the tests repeated. All splices which are tested by means of gas are stamped "G. T." (gas tested) in order that we can check from time to time on the effectiveness of this type of maintenance.

On new cable projects where long lengths of cable are to be tested, the usual procedure is to splice the cable in loading sections of 6000 ft. and to test these sections as a unit. After all electrical testing has been completed and before the loading is cut in, the ends of the sections are capped and gages installed at each end. At the loading point where the two ends of different sections appear, valve stems are fastened into the lead caps and a large cylinder of gas is connected to the cable by means of a regulator and two-way hose connection. The set up for this type of test would be quite similar to that shown on Fig. 5.

After the equipment is connected to the cable, the gas is turned on and its flow regulated in exactly the same manner as outlined for the single splice tests. Nitrogen is permitted to flow in at a 40-lb. rate until the meters at the far extremities of the section register 5 lb., at which time the tank is disconnected and the pressures at all meters recorded as soon as possible.

In making tests of this nature it is possible to classify the defects to be found into two groups; those at the point of splicing which includes all exposed cable in the manhole in the case of underground cable; and those which fall in the sections between the splices or which are between the manholes (in the duct line).

Troubles of the first type are detected by soaping of sleeves in the sections under test. Special attention is given to seams, wipes, and bends. Leaks, of course, are indicated by the formation of bubbles.

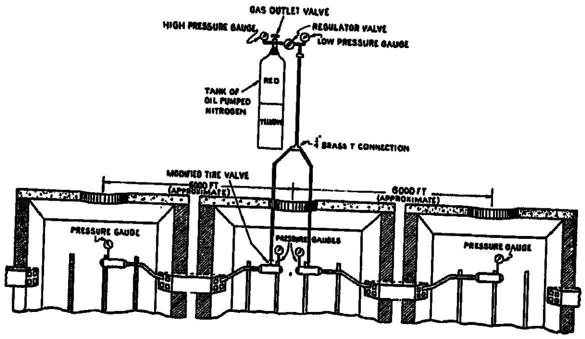

Fig. 5—Section Testing

Method of setting up equipment when pressure testing long sections of cable

In the case of sectional defects more elaborate means of detection are employed. After the gas has been disconnected from the cable, the readings at the point of application will be considerably higher than those at the ends of the section; but within three hours the pressure should equalize throughout the sections. It is usually customary to leave the gas pressure in the cable over night when practicable. The ability of the cable to hold the pressure is a positive indication that the sheath is in good condition. Inability to get a pressure reading at the end of the section under test is an indication of an extremely large leak, which must be repaired before the test can be continued.

If the reading at one end is appreciably different from that at the other, or if they are equal but lower than the average at the time the tank was disconnected, it is probable that one or more defects exist, and that they are in the duct line in the case of underground cable, or in the section between the splices in the case of aerial cable.

Several methods of locating this type of trouble have been used with fair degrees of success.

On aerial cable where it is possible to ride the entire section under test, the first step is usually to cover the section with the intention of hearing the hiss caused by escaping gas and to inspect the cable from the ground for any bends, kinks, or flaws which would be soaped to locate the trouble.

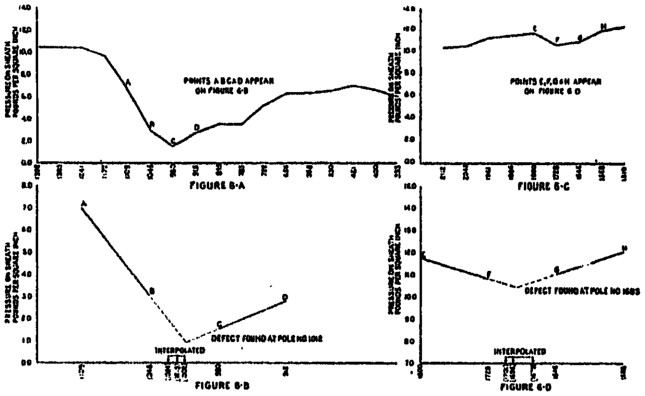

FIG. 6—GRAPHIC METHOD OF LOCATING SHEATH BREAKS

Typical pressure curve for section of aerial cable having two sheath breaks

Note: One 200-cu. ft. cylinder of nitrogen is sufficient to test a 6000 ft. section of cable

The next step would be to install a gage in each splice in the section. As full size cable is usually put up in 750-ft. sections, there would be seven splices in a 6000-ft. section in which meters would have to be installed. It is necessary that the valve test points be equally spaced along the cable. This would make nine points at which the pressure of the cable can be read.

The pressure readings taken to the tenth of a pound are then recorded on cross-section paper laying off the pressure as ordinates and the test points as abscissas.

Theoretically, were the cable to be free of defects, these points would lie in the same straight line. The presence of defects would be indicated by low points on the curve.

The curves shown in Fig. 6 are an example of the method of locating trouble in a ten-mile section. From curve 6-*A* low spots or defects exist in the vicinity of poles 980, 785, and possibly near 530. Since the condition at pole 980 is most clearly defined, the defect at that point will be considered first.

The plan used in the location is to extend the converging slopes of the curve at the low point and the defect will be found close to the location at which the converging lines intersect. Thus the readings at poles 1109, 1046, 980, and 915 shown in Fig. 6A are laid off on a larger scale on Fig. 6B. Lines drawn through these points intersect near pole 1012 and the trouble was found in a leaky sleeve at that point.

In Fig. 6C the defect was indicated between poles 1728 and 1646. The pressure readings at poles 1800, 1728, 1646, and 1568 are laid off on Fig. 6D and the extended lines on this curve intersect at pole 1695. The defect was found to be a bad sleeve at pole 1693.

This same procedure is used in locating defects in underground cable where the sheath break occurs in the duct line. The defect will be located between the manholes at which the lowest pressures are recorded.

After the defect has been located between two manholes, the splices at those points are filled with paraffin, care being taken to heat the armor so as to have the paraffin run back into the cable and fill all voids. When this wax cools, it forms a dam, and is reasonably effective in preventing the flow of gas during a short period. This length of cable is then again pumped up and a check test made by means of comparative pressure readings. Should these readings again fall, the section is replaced and all electrical and capacity unbalance tests remade at which time the gas pressure tests are repeated.

Still another method of locating sectional defects on underground cable is to segregate the defective sections by means of the paraffin dams just described. The first dam would be placed at the final capacity unbalance splice with gages fitted into the armor on each side. The two half sections are then pumped up and the pressure will fall in the half section having the sheath break. This half section is then redivided into two sections by placing a dam in its midpoint, and so on until the leak is narrowed down to one duct length between two manholes. The trouble is cleared in the same manner as mentioned in the previous case.

Constant Pressure Testing. So effective has this system of pressure testing been that quite a number of the more important toll cables have been placed under constant pressure.

After the cable has been cut into service, all lateral taps and both terminal ends are sealed with dams or plugs to prevent the escape of gas.

In addition to plugging the cable at the ends and lateral taps, intermediate plugs are installed at approximately ten-mile intervals, in order to sectionalize the cable for use in locating sheath breaks. Valve stems are located at the loading splices which give them a 6000-ft. spacing.

In testing underground cable on this principle, the stems are located above the water line of the manholes and connected to the cable by means of copper tubing.

When the section is ready, gas is introduced into the cable at the usual 40-lb. rate at each of the valve stems along the ten-mile section. By injecting from 180 to 200 cubic ft. of nitrogen for each mile of cable, it is possible to gage by the cubical content registered on the regulator equipment just how much gas should be put in to result in an equalized pressure of 15-20 lb.

After the gas has been allowed to equalize, which in this case requires several days, the pressure reading at each point is taken with a gage made to fit over the valve stems, and is plotted on cross-section paper. As before, the low points are then taken as indications of sheath trouble. When the trouble has been narrowed down to one section, the methods described for sectional defects are used to locate the trouble.

In order to have a means of detecting loss of pressure after the section has been inflated for some time, electrical indicating gages associated with an alarm circuit are placed in each ten-mile section. The alarm circuit is usually a non-loaded pair stubbed out to a terminal, the tail of which has been plugged. The gage and terminal are placed in a box on the pole and the gage connected to the cable by copper tubing. This line is equipped with a cock for shutting off the gas when working on the gage. The alarm is set so as to operate when the cable's internal pressure falls below 12 lb. In the central office or repeater station the alarm circuit is terminated with a relay whose local contacts operate a lamp signal or bell. When the alarm operates, its contacts automatically cross the pair to which the gage is attached.

The testboard man takes a loop resistance measurement to the cross and knows immediately what ten-mile section is in trouble. He then dispatches a cable man to make the readings at the mile points in the section in trouble and by the method of plotting pressures he finally locates the trouble.

Alarm gages are inspected and tested once a month to insure their operating in the event of lowering pressure.

Pressure Testing Results. The results of gas pressure testing have been very gratifying, both from a maintenance and cost standpoint. The number of troubles found by the use of gas varies with the type of plant tested, *i. e.*, underground or aerial, new or old cable.

The rather limited results procured so far show about 6 per cent of the splices on underground cable to be defective. Of this number 3 per cent have been attributed to poor tinning of the joint, cracks, and blow holes. The other 3 per cent showed porous joints due to the use of cold or coarse metal. On new aerial cable work, preliminary tests have shown 4 per cent of the splices tested were found to leak.

These figures are averages, and may be exceeded in some cases while in others fewer troubles will be found. Considering the large number of splices occurring in, say, a 100-mi. length of cable, the importance of insuring the integrity of the wiped joints in order to avoid potential failures in long toll cables, is readily apparent.

It is expected that the number of troubles found will be materially reduced as the splicing and engineering forces are, in some cases, actually able to determine what causes the defects. One of the things which have been noted so far is that in a great many cases the joint at one end of the sleeve is good, and at the other end is porous. This is undoubtedly due in most cases to the fact that the solder was not sufficiently hot when the second joint was wiped.

It is of interest to note that on a recent underground toll cable project, 16 mi. in length, there were 19 troubles found in the splices and two in the duct line. Of the 19 defective joints, 18 occurred in the first five miles of cable tested, and only one in the remaining eleven miles. This is, of course, an extreme case, but it illustrates the fact that as the men had never before had their metal work checked, they were rather negligent in this particular phase of their job. As soon as the pressure testing began to reveal these delinquencies they took more pains with the wiping, and produced better joints.

Clearing Trouble. Another important advantage of the nitrogen gas in maintenance work is the drying effect of the gas when admitted in either side of a cable fault which is wet. In certain instances, pulling in and cutting over a new section of cable has been avoided and service restored within three or four hours after failure. In these latter cases, the gas was applied and the drying out process continued up to the time when the rodding and pulling had been completed and the sections ready to cut over.

Drying under these conditions not only restored practically all the telephone service, but materially aided the splicers in toning through the defective section when cutting over. It is reasonably certain that only in those cases where the cable is partially wet can it be dried out completely with gas.

Cases where the cable is thoroughly saturated with water will have to be handled in the usual manner, either by replacing the section or by cutting in a short section of cable. The gas is, however, found to be helpful in many cases in avoiding the replacement of cable.

AERIAL CABLE CONSTRUCTION

General. New methods have recently been developed for placing and splicing aerial cable which, it is felt, will decrease troubles and lower maintenance costs. Results have shown that the erection of a pole line according to specifications is a comparatively simple problem and few troubles result from faulty construction; but the placing of a suspension strand and cable presents a more difficult problem.

Cause and Theory of Bowing. The causes of bowing n full sized aerial cable supported on 16-M suspension strand have been determined. Measures for the prevention of bowing have been worked out theoretically and have been put into practise in the field with satisfactory results.

Observations locally have coincided with observations of other sections in that bows are continually increasing and decreasing in size under the influence of daily and seasonal temperature variations, thereby causing the cable to move up and down in the rings or in and out relative to the pole. This motion contributes in some measure to ring cuts, chafing of the cable against pole attachments, sheath cracks at wipes, and breakage of terminal taps. It also sets up varying stresses in the cable sheath which change its structure at bowed points and eventually cause it to crack. Fig. 7 is a typical example of a bow which has developed in a large cable near a splice and which will eventually cause a sheath crack at the wipe.

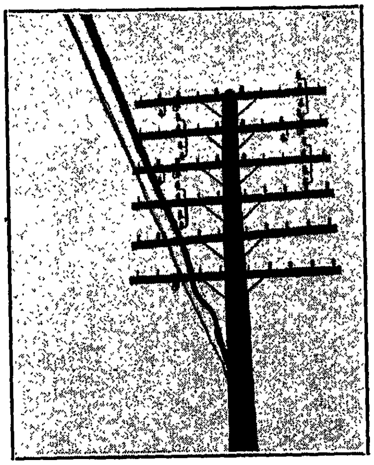

Fig. 7—Typical Bow in a Large Cable

This condition will eventually cause a sheath break at the wipe

The generally accepted theory of the cause of bowing is as follows:

Due to a difference in the coefficients of expansion of suspension strand and cable, the cable expands more than the strand under the influence of a rise in temperature. The length of strand and cable in any span is assumed to be equal at the time the cable is spliced. As the temperature rises above the splicing temperature, the cable expands faster than the strand and, as it cannot move along the line, compression is set up within the cable. When this compression exceeds the resistance of the cable to bending, the cable bows out of line. The amount of compression and bowing depends upon the extent to which the temperature has risen above the splicing temperature.

Under the influence of a drop in temperature, the cable will contract faster than the strand and when the temperature falls below that at which the cable was spliced, tension is developed in the cable. The tension which will develop in the cable at the minimum temperature of the locality depends on the temperature at which the cable was spliced. If the cable was spliced at a high temperature, the tension developed during extremely cold weather may stretch the cable permanently to some extent. As a result, when the temperature rises again to the original splicing temperature, the cable instead of being once more free of stress, will be under compression and as the temperature rises still further, bowing will occur sooner than it would have if the cable had not been permanently stretched.

Construction Methods to Minimize Bowing. As a result of studies and field trials of bowing in full sized cables supported on 16-M strand, the following construction methods have been recommended for placing new aerial cable so as to minimize bowing:

1. *Reduced strand tension.*

The first method for the prevention of bowing in new aerial cable is the only one to date which has been included in the specifications for aerial cable construction, namely, to reduce the initial strand tension from 6400 lb. (60 deg. fahr.) to 3600 lb. (60 deg. fahr.). The theory of this method is that a high initial strand tension reduces to a considerable extent the normal (or unstressed) elongation or contraction of the strand with temperature change. Therefore, if the initial strand tension is reduced, the tension existing in the strand when the cable is in place will be reduced. This decrease in loaded strand tension will permit an increased elongation and contraction of the strand with temperature change. As a result, the changes in strand length become more nearly equal to the changes in cable length, and the tendency for the cable to bow is reduced. The initial tension in 16-M strand was not reduced below 3600 lb. because of the greater sags which would result. Field trials which have been under way for several years show that the reduction in strand tension mentioned above has practically eliminated all bowing in those cases where the cable was spliced at a temperature of approximately 40 deg. fahr. or above.

2. *Placing strand at correct tension.*

In order to permit the field forces to string suspension strand more easily and more nearly at the correct initial tension, an instrument called the "strand dynamometer" has been developed. This instrument may be placed on the suspension strand at any point without cutting or otherwise injuring the strand and will indicate the tension existing in the strand at that point. This instrument consists essentially of a means of deflecting the strand out of line and a means of measuring the force tending to restore the strand to a straight line. This restoring force is proportional to the tension existing in the strand. An indicator dial gives the amount of this restoring force and the actual tension existing in the strand is obtained from a calibration curve.

3. Placing cable so that it is free from waves.

In order to prevent bowing it is necessary to have the cable the same length as the strand when it is first placed in the rings. Formerly the methods used for placing cable sometimes resulted in a series of small waves developing in the cable, particularly in that section of the cable which is nearest the center of the cable reel. The use of the "aerial cable guide and straightener" has eliminated, in large measure, these waves. This tool consists of a sheet iron shoe terminating in a steel tube of slightly larger diameter than the cable. The cable is pulled over this shoe and through the tube before it reaches the rings. Any waves which may be in the cable as it comes off the reel are thus removed as the cable passes over the shoe and through the tube.

4. Preventing the cable from being spliced while the strand is in a stretched condition.

Splicing is ordinarily done from platforms which support one or two workmen and their equipment. This extra weight will stretch the strand if no measures are taken to prevent it. As a result, after the cable is spliced and the weight of the splicers and their equipment is removed, the strand will contract and some excess cable will remain in that span. This stretching of the strand has been prevented by placing a ladder alongside the splicing point and tying the strand to the ladder before the splicing platform is erected. In this way the ladder is used to support a major part of the weight of the splicers and their equipment and stretching of the strand is largely prevented.

5. Placing tension in the cable.

At temperatures of 40 deg. and below it has been found that the above measures are not sufficient to prevent the cable from bowing when the temperature subsequently rises to the neighborhood of 100 deg. fahr. Accordingly the practise has been developed of placing tension directly in the cable before splicing and holding it there until the splice is completed.

An instrument called the "tension splicing tool" has been developed for placing tension in cables. This tool takes hold of the ends of adjacent reel lengths of cable at the splicing point, pulls them together until the amount of tension required by the existing temperature has been set up in the cable, and holds this tension in the cable until the splice is completed.

The effect of this tension is to reduce the amount by which the cable would otherwise expand or contract and thereby bring the amount that the cable will expand under a given temperature rise closer to the amount that the strand will expand. In this practise the amount of tension placed in the cable varies with the temperature existing at the time the splice is made. For temperatures just under 40 deg. the tension is small, but for temperatures as low as 20 deg. below zero the tension is about 1500 lb. Where this practise has been conscientiously followed, bowing has been prevented in cables spliced at temperatures as low as 5 deg. below zero.

Removing Bows in Old Cable. Two methods have been investigated for removing the bows in cable which has been erected and spliced for some time. They are:

1. Pull the excess length of cable to concentration points and cut it out.
2. Cut suspension strand at intervals, introduce excess strand at these points, and slack off until bows disappear.

Field trials have revealed that bows can be completely removed by either of the above two methods, although the bows removed by Method 1 tend to return in a few years while those removed by Method 2 have not returned in the same length of time.

Where the bows are removed by cutting out the excess cable length, some tension remains in the cable after being pulled to the concentration points but the strand tension is not materially reduced. Therefore, during hot weather, the tension in the cable tends to reduce the bowing while the high initial strand tension tends to promote it. Consequently the results obtained in this manner are not as successful as the results obtained by slacking off the suspension strand at various intervals, which method places the cable in tension and reduces the strand tension simultaneously.

Another advantage of Method 2 over Method 1 is that slacking off the strand does not require that the working cable be opened, whereas in pulling the slack out of the cable it must be cut to remove the excess length. Since the comparative costs of the two methods are approximately the same, slacking off the strand appears to be superior to cutting excess length out of the cable, as a method for removing bows from cables supported on 16-M strand. Cutting the excess length out of bowed cables has its application to those cases where the strand tension is low, the cable is badly bowed, and the existing clearance above ground is such that no material increases in sag can be permitted.

The danger encountered in removing bows by slacking off the strand is that the tension introduced into the old, badly bowed cable may cause sheath breaks.

Analysis of results from a small number of cables indicated that where the cable was not more than four years old there was little danger of causing sheath cracks by slacking off the strand. On cable which was about eight years old when bows were removed, a number of sheath cracks was caused by the resulting tension. An analysis of the sheath cracks caused by tension compared to those which were caused by bowing in another section of the same cable, showed that the sheath breaks caused by tension started at a high rate, but have decreased rapidly in each year since that time. The sheath breaks due to bowing started at a lower rate but have been increasing subsequently. The total number of sheath breaks caused by tension during a three year period was equal to the number caused by bowing during the same period, but it appeared that from then on sheath breaks due to tension would be considerably less than those due to bowing.

Therefore, in determining whether or not the bows should be removed in a certain cable, it becomes necessary to consider its age and to estimate the length of time that it will remain in service. Where cables are expected to remain in service for a considerable period of time, it is undoubtedly economical to remove the bows, but on cables where the remaining life is limited, an investigation is necessary to determine whether the expenditure involved in removing the bows is justified by the saving due to an eventual decrease in the number of sheath breaks.

The application of these methods has been investigated most thoroughly on cable supported by 16-M strand, and it seems that no difficulties should be encountered in applying these same methods for removing bows to cables supported with 10-M and 6-M strand and the same success should be attained.

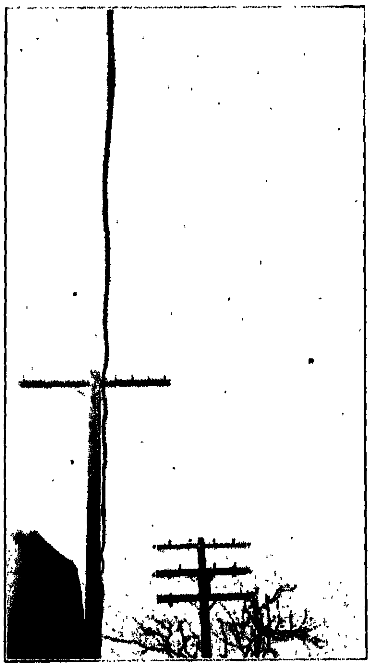

Fig. 8—Bowing in a Comparatively New Cable

This cable was spliced lying loose in the rings and, undoubtedly, tension would have reduced the bowing considerably.

Bowing in Small Cables. Up to the present time, the only results published on the prevention of bowing in aerial cable have been methods which are recommended only for full sized cables supported on 16-M strand. In the plant of the Cincinnati and Suburban Bell Telephone Co. the percentage of cables supported on 16-M strand is small compared to the percentage supported on 6-M strand and, therefore, it has been felt that measures should be taken to prevent bowing of the smaller cables.

The methods used in attempting to prevent bowing in the smaller cables supported on 6-M strand would necessarily be similar to those used on full size cable supported on 16-M strand.

Applying the theory of reduced initial strand tension to 6-M strand, it can be logically deduced that the benefits derived from this method would not be nearly as great when applied to the smaller strands as when applied to the 16-M strand. The coefficients of expansion of the 6-M, 10-M, and 16-M strands are practically the same, and inasmuch as the specification initial strand tension of 6-M strand is approximately 1100 lb. (span—130 ft.; 60 deg. fahr.), reducing this initial tension might not materially affect the change in length of the strand with temperature.

Therefore, applying tension to the cable before splicing has been resorted to since 1920 with reasonable success. Inasmuch as there was no precedent as to the amount of the tension which should be applied to the cables, no specific instructions were issued to the construction forces. Since then, the judgment of experienced foremen has been depended upon and as yet no troubles have ever developed due to an excess of tension in the cables. Another point to be noted is that tension is being applied at all temperatures. The method that is being used to apply tension to the cable, is as follows: The cable is pulled from both ends by means of blocks and tackle and clamped at the ends to the suspension strand by means of grade clamps. After splicing, these grade clamps are removed. To place tension in a cable which pulls around a corner, the cable is blocked out from the pole before applying tension. After the cable is in tension, it is clamped to the strand on each side of the pole by means of grade clamps and the blocks are then removed, thus holding it away from the pole. These grade clamps are never removed.

The trouble encountered due to the exchange cables being in tension, is the difficulty of working in a tight cable when necessary to open it for various purposes. As data become available regarding the minimum tensions which can be applied to the cables and still minimize bowing, this trouble will lessen, since it is a certainty that more tension is now being applied to some of the cables than is necessary. However, after eight years' experience our local supervisors are convinced that the additional difficulty of working in a tight cable is more than compensated by results obtained in the elimination of bowing. However, the difficulty of working in a tight cable when necessary to open it for various purposes may be overcome by the use of the tension splicing tool. This tool may be applied at any point where it is desired to open the cable and after the sheath is stripped off, the conductors may be ballooned to any extent desired by operating the tension splicing tool. Fig. 8 is a typical example of a comparatively new cable which was spliced lying loose in the rings. Probably a small amount of tension would have reduced the bowing considerably. Fig. 9 illustrates a badly bowed section of cable, the bowing of which can be partly attributed to construction methods. If tension in the cable is undesirable, the construction forces should at least be required to pull all of the kinks out of the cable when placing it.

Experimental Results. Various other problems and questions have arisen in regard to observations on placing tension in the smaller cables. In order to find answers to these questions and to verify theories regarding the field observations, experimental spans were erected using 50-pair 22-gage cable and 100-pair 22-gage cable supported with 6-M strand. The physical data regarding these cables are:

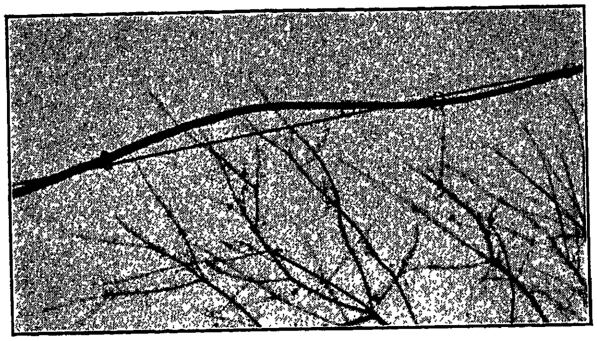

FIG. 9—EXCESSIVE BOWING PARTLY CAUSED BY POOR CONSTRUCTION

The small bows and kinks caused by unreeling the cable were not removed before splicing

TYPE B S A (22 ga)

Pairs	Mean outside diameter (inches)	Sheath thickness (inches)	Lb. per ft.
51	0.73	0.070	0.95
101	0.99	0.080	1.58

The suspension strands were erected using the specification initial strand tension of 1100 lb. (span.—130 ft. 60 deg. fahr.) and all tensions were obtained by means of a deflection dynamometer. The problems to which solutions were sought were:

1. Is the total tension applied to the cable transferred to the strand at the grade clamps, thereby increasing the strand tension?
2. Effect of cable tension on the sag.
3. Proportionality of the cable elongation to the applied tension.
4. Effect of the slip of the sheath over the copper when tension is applied.
5. Permanent set.

To determine the increase in the strand tension caused by clamping a cable which is in tension, to the strand at corner poles, tensions were taken with the cable inert and with the cable in tension. The tensions applied to the cables varied from 300 to 750 lb. and in no case were the strand tensions increased by more than 150 lb. The strand tensions on each side of the grade clamp differed approximately by the amount of the stress in the cable. These results indicate that the additional tension placed in the suspension strand, between the clamp and the pole, is negligible.

The effect of the tension in the cable is to reduce the sag. This reduction in sag depends on the tension applied and the span. The sag in a 140-ft. span carrying a 100-pair 22-gage cable was decreased 3½ in. when a tension of 750 lb. was applied.

The sag in a 140-ft. span carrying a 50-pair 22-gage cable was reduced 1¾ in. when 300-lb. tension was placed in the cable.

In any size of cable, elongation is directly proportional to the stress applied within certain limits only. For full sized cable this limit has been set at about 3000 lb. In the smaller cables, there is a tendency for the sheath to slip over the copper when tension is applied at the ends of the cable by means of wire grips. The 100-pair 22-gage cable had a sheath slip of 3/8 in. when 500-lb. tension was applied and a 3¾-in. sheath slip at a 750-lb. tension. Therefore, in attempting to regulate the tension which should be applied to the cable, by means of the stretch, the sheath slip must be eliminated. The discrepancy which enters the measurement of the total stretch due to the sheath slipping over the copper is small at tensions below 500 lb. and can be entirely eliminated by cutting away the sheath at the end and applying tension by pulling on the copper only.

The slipping of the sheath over the copper is practically confined to the final 10 ft. of the pulling end of the cable and the weakening effect caused by the cable sheath thinning out is negligible at tensions below 500 lb.

In locating points of maximum elongation due to the tension applied, it was noted that the sheath slippage is practically confined to the five or six feet of cable on each side of the poles.

The immediate permanent set of the 100-pair cable at 500 lb. tension was negligible but at 750 lb. was 1⅛ in. in a total length of 260 ft. The 50-pair cable showed no immediate permanent set at a 300-lb. tension. The measurement of permanent set in a 100-pair cable of considerable length suspended on strand is liable to considerable error for the following reasons. When the cable is pulled certain portions of it move toward the point from which the pull is made and friction in the rings may prevent this cable from going all the way back to its original position when the pull is released. On full sized cable a 1500-lb. pull placed on a 750-ft. length of cable will cause the cable to come in toward the pulling point about six in. and when this pull is released the cable goes back only about three inches. This remaining three inches is partly due to the fact that the cable has been pulled closer to the rings at poles and closer to the pole at corners, and partly due to the fact that some tension still remains in the cable because contraction of the cable is resisted by ring friction. This tension contributes to the remaining three inches by the amount of elongation and the decrease in sag due to that tension.

The general conclusion to be drawn from these data is that no physical harm whatever is done to the smaller cables when a tension of 500 lb. or less is applied, and inasmuch as local experience has proved that applying tension to the smaller cables minimizes bowing, it seems that this practise should be more generally followed.

Long Span Construction

General. A "long span" in telephone plant refers to a considerably shorter span than in some other types of construction. This is due to the fact that telephone line long spans differ from others such as power conductors, in that the power wires support their own weight while a telephone cable cannot. Power conductors in such installations are, as a rule, essentially a copper covered steel strand, uninsulated, while telephone cable is formed of a number of small copper wires, each wire being covered with paper insulation and the whole assembly of wires enclosed in a lead-antimony sheath. This combination of numerous small wires and soft sheath naturally causes the entire cable to possess low tensile strength. Thus, while a power conductor may be suspended from steel towers and carry its own weight over long spans, in the case of telephone cable a means of support must be found, outside of the cable itself.

This problem is solved by using long span construction which may take either of two forms: first, the catenary type where suspension guy relieves the cable and its messenger strand of a great part of the load. The suspension guy is attached to the crossing poles on each side of the span and then pulled up until the sag is correct as determined from tables prepared for different spans and cable loadings. The theory of the loaded catenary is not new but its application to the telephone plant is of recent development. The second type of long span construction is simply an elongated form of the usual span wherein a high strength steel messenger strand is used to support the cable which is attached directly to it.

Economic considerations are largely the determining factor in deciding the length of span which can be justified in a given case. Catenary construction is used in our long toll cable plant largely as a matter of convenience and as a means of carrying the cable suspension strand across streams or other barriers continuously without dead-ending it. On the other hand, a long span was built about a year ago and more are projected where the length of the span is in excess of 1000 ft. and in which the cable suspension strand alone carries the entire stress and no catenary construction is used. These particular long span crossings are cable spans inserted in open wire lines at river crossings and the cable terminates at the crossing poles.

Construction of Catenary Spans. After the suspension guy (upper strand) is in place, the messenger is extended across the span and fastened to a point on each crossing pole a normal distance above the ground, the same as on the other poles back from the crossing. The messenger is then attached to the suspension guy at the midpoint by three-bolt clamps and at the quarter points, sixth points, or eighth points, according to the length of span, by hangers of the proper length to make the messenger extend in an approximate straight line across the span, these lengths being given in the tables referred to previously. The rings are then placed on the messenger and the cable pulled through the rings to complete the crossing.

While the curve of the suspension guy is a true catenary before the load in the form of the cable is applied, after loading at a number of points it assumes very nearly the shape of a parabola, modified by the fact that the load is applied to the suspension guy as a series of concentrated loads through the hangers. It approximates a parabola closely enough to make the use of the general span formula for parabolas allowable in making up the tables. This formula is: $d = \frac{wl^2}{8t}$ where d = sag in feet; w = resultant force in pounds per foot due to the combined weight of cable, messenger, suspension strand plus the maximum ice formation thereon acting in a vertical direction, and the wind pressure acting in a horizontal direction: l = length of span in feet; and t = allowable tension in pounds in suspension strand. This formula is for a parabolic curve, which assumption is very nearly true. The tables are also calculated to allow for the work being done at various temperatures, and the proper sag is given for each of those assumed.

Limits. The length of cable span which can be permitted without special construction depends upon several factors, of which the size of cable, length of span, size of suspension strand, and unbalanced loads on the pole at the end of the long span are to be considered. From the standpoint of strength of suspension strand, it is permissible to use spans as long as 200 ft. without special construction in the case of full size cable carried on 16-M strand installed at an initial tension of about 3600 pounds. For long span crossings of full size cables much over 600 ft. in length special consideration may be required such as larger size strand, possibly steel towers, etc. Spans of practically any length can be constructed if the economics of the situation indicate that it would be desirable.

Advantages. Long span construction, under certain conditions, has distinct advantages, especially where there is some extensive obstacle to clear, as a river, or a deep, narrow valley.

The simpler type of long span involving the high strength messenger with the cable attached directly to it has several advantages over the catenary type. Theoretical considerations appear to indicate that a long span without the catenary pick-ups would result in less buckling of the cable than a span with pick-ups. The simpler type of long span crossing also permits construction with somewhat smaller sags. As a matter of fact, with the same total number of steel strands used slightly smaller sags could be obtained by hanging the strand and cable in one long span supported only at the ends than if catenary construction were used. This is for the reason that the cable suspension strand itself will carry a greater proportion of the weight of the cable in a long span.

In this discussion we have been dealing with spans where the crossing poles have been approximately at the same level. Where this is not the case, certain obvious differences will occur, but the principle is the same throughout.

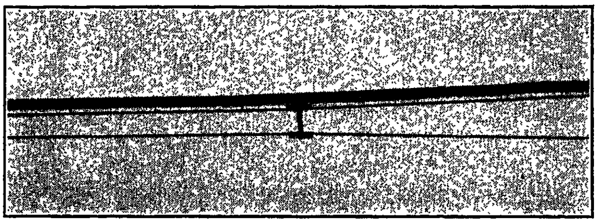

Fig. 10—Hanger Details

Showing horizontal position of cable at pick-up points

Long span construction—typical projects; Cleves-Elizabethtown cable at Lost Bridge.

The Cleves-Elizabethtown section of the Cincinnati-Lawrenceburg toll cable crosses the Great Miami River at Lost Bridge by a series of spans, each of which is a complete catenary in itself. The total length of the crossing is 759 ft., but this has been divided into three shorter spans of 253 ft. each, corresponding to the number of truss systems forming the bridge itself.

While plans for the construction of the cable crossing were under way, it was observed that vehicular traffic on the bridge set up severe vibration in the structure, and it was agreed that if the proposed cable were attached to the bridge itself, or allowed to come in contact with it in any manner such that this excessive vibration might be imparted to the cable itself it was likely that serious damage might be done to its sheath and insulation from abrasion. The problem was how to make use of the advantages afforded by the bridge crossing at this point, and still protect the cable from injury. A plan was finally worked out whereby it would be supported at two intermediate points of the crossing, these supports to be steel towers approximately 12 ft. high and 6 ft. wide at the base, formed of angle irons bolted together, and each tower resting on a pier of the bridge and bolted securely to the concrete,—the inner side of each tower being about one foot from the outer part of the bridge truss. During construction great care was taken to put the crossing poles on either shore accurately in line with these two towers, so that the cable would maintain an equal distance from the bridge at all points of the span, for if the end poles were out of line toward the structure, the wind blowing down the river might cause the cable to swing against the truss midway of the end spans, and this frequent bumping and rubbing would severely damage the soft armor of the cable in a short time. (Fig. 11.)

As the completed crossing now stands, it is a series of three catenary spans, the cable leaving the southeast shore from a fiber duct about four feet below ground line at the bridge abutment, extending to the tower on the first pier, thence to the tower on the second pier, and from there to a similar fiber duct on the northwest shore, the only contact with the bridge being through the piers, which will transmit little, if any, vibration from the bridge to the towers.

The 6000-lb. messenger strand supporting the cable is secured on each end to an eye-bolt passing through the 18-in. abutment wall, which is prevented from pulling through by a 3-in. channel 12 in. long acting as a washer. The 16,000-lb. suspension guy, which supports the messenger and cable and is secured to the towers about one foot from their tops, is dead-ended at each shore on a wood pole directly adjacent to the abutment wall and securely fixed in place by a concrete toe-block 6 ft. below the surface of the ground. The upper end of each pole is secured by two guy strands running to a log anchor firmly set in the ground. The fiber ducts carrying the cable pass as close as possible beside the foot of the crossing pole on each shore, to maintain the cable alinement.

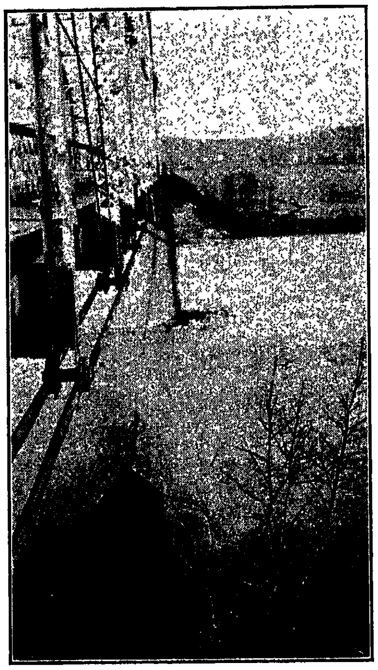

Fig. 11—River Crossing at Lost Bridge

The messenger strand is clamped to the suspension guy at the midpoint of each span, and also secure to it at the quarter points by hangers, thus reducing the greatest actual span of the cable between any two supporting points to 63 ft. 6 in. By this means, very little sag is produced in the cable.

It should be noted here that this project resulted in breaking up a very long span into three shorter spans, each of which forms a complete catenary in itself.

However, the effect of breaking up a cable into shorter spans between suspension points is conducive to buckling rather than tending to reduce it, and it would be better from a buckling standpoint if longer spans between pick-up points were used.

Cable over Licking River at Falmouth, Kentucky. The cable crossing over the Licking River at Falmouth, Kentucky, is a good example of catenary construction which must extend across a long span without any intermediate supports whereby the length might be shortened. It was found necessary to cross the Licking at this point with a 50-pair 19-gage cable, and due to the low, marshy nature of the ground on the north shore, it was decided to set the north crossing pole back from the river a distance nearly one and one-half times the width of the river itself, thus obtaining for the pole a position out of reach of high water and affording an elevation for the cable approximately the same as that on the other poles back from the river. The south bank has a more abrupt slope, and it was only necessary to place the crossing pole back about 100 ft. in this case to obtain the desired elevation.

In this manner local conditions caused a span of over 600 ft. to be built over a stream normally only about 200 ft. wide. This, of course, required long span construction.

The two crossing poles are Class "A" Cedars 45 ft. long, the suspension guys two 16,000 lb. strands, and the messenger a 6000 lb. strand. The messenger and cable are attached to the suspension guys at the sixth points by standard hangers and by three-bolt clamps at the midpoint of the span.

Before the span was built, a mathematical analysis was made of the problem in order to determine the sag of the suspension guys after the load was applied. The basic formula used was the one previously referred to, namely $d = \frac{w l^2}{8t}$. An extract of the development showing the significant steps follows:

Material Loads.

Material	Weight per foot
Two 16,000-lb. guys	0.88 lb.
One 6000-lb. messenger	0.22 lb.
50-pair—19-gage cable	1.50 lb.
Hangers and rings (approx.)	0.25 lb.
Total material load per ft. of span	2 85 lb.

Ice Loads.

½ in. radial deposit assumed.
No shielding effect considered.

1—6000-lb. messenger	0.23 lb.
2—16,000-lb. guys	1.16 lb.
1—50-pr.—19-gage cable	0.91 lb.
Total ice load per ft. of span	2.30 lb.

Wind Loads

Wind loads figured at 8 lb. per foot of projected area of span with ½ in. radial deposit of sleet covering same.

1—6000-lb. messenger	0.86 lb.
2—16,000-lb. guys	1.92 lb.
1—50-pair—19-gage cable	1.29 lb.
Total wind pressure per ft. of span	4.07 lb.

Resultant force = 6.56 lb. per ft. of span.

Formula

$d = \frac{w l^2}{8t}$ Where:

t = Tension............ 16,000 lb.
w = Force per ft. of span... 6.56 lb. 1 ft.
l = Length of span....... 621 ft.—0 in.
D = Sag............... ?

With two 16,000-lb. strands supporting the entire load with a factor of safety of two, the allowable tension in the catenary becomes $\frac{2 \times 16{,}000}{2} = 16{,}000$ lb. $= t$.

$d = 19.8$ ft.—Sag under application of full storm load, (cable, strands, ice, and wind).

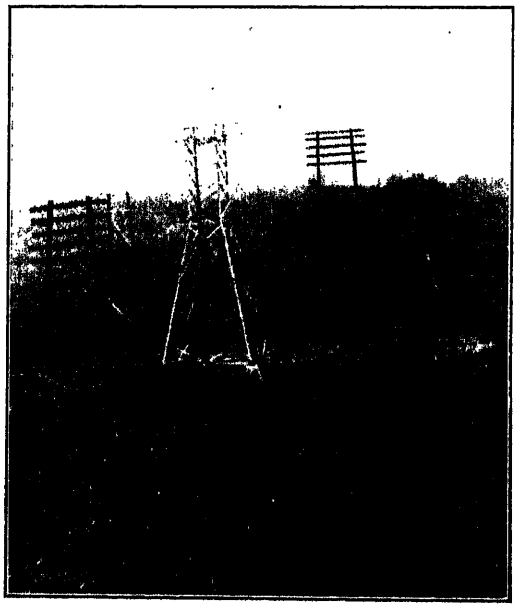

Fig. 12—Typical Long Span River Crossing

Employing the use of catenary suspension guys supported by steel tower

Steel Tower Construction. On a recently constructed toll cable line, it was necessary to cross a navigable stream with a long span. This span, which is a good example of steel tower-supported long span construction as shown in Fig. 12, is 623 ft. in length, and the ultimate load on the towers will be very heavy, as it is planned to install eventually one "full size" cable, weighing 7.85 lb. per lineal foot and two "over size" cables, each weighing approximately 11 lb. per lineal foot. The design of the long span for the first cable, which is full size cable already in place, is such as to place an unbalanced load on the tower of about 26,500 lb. under maximum assumed storm loading, and with all cable in place this load will increase to 107,000 lb. under full storm loading. Storm loading of ½-in. radial coating of ice, wind amounting to 8 lb. per square foot of projected area, and a temperature of 0 deg. fahr. were assumed in the design.

To withstand this heavy load, and still give the clearance over the river required by the Government, it was necessary to abandon the use of wood poles, customary for catenary construction over shorter spans and with lighter loads, and erect a large steel tower at each end of the proposed crossing. These towers are 85 ft. high by 30 ft. square, and each rests on four 5-ft. concrete piers. The ends of the suspension strands are attached 5 ft. below the top, and the messenger strand is secured 29 ft. below.

There are 7 pick-up points in the span, making the maximum unsupported span 78 ft.

CONCLUSION

Eliminating service interruptions with their resultant inconvenience to subscribers, embarrassment of business, loss of revenue, etc., due to cable failures, justifies far reaching measures.

It is felt that definite progress has been made towards building cable plant whose sheath continuity will be undisturbed except from extraneous influences. Further advances can probably be realized by utilizing the ideas presented.

The outstanding features of these methods have been shown to be arrangements for determining the location of sheath breaks before complete failure, minimizing sheath movement through the rings, and new construction methods for long span crossings.

Discussion

C. A. Jaques: I want to ask Mr. Wagner whether they have taken into consideration the possibility of damage to the lead sheath from the 40-lb. internal pressure, and if so, whether care is taken as to the outside diameter of the sheath, or the alloy sheath, or the thickness of the sheath.

E. B. Shand: I should like to ask one question. In bringing up the cables is it necessary to bring them through insulated gas-proof joints?

B. S. Wagner: Forty-pounds pressure, as the maximum internal pressure, has been carefully determined. The lead-antimony alloy sheath that is used on the cable will safely stand pressures below 50 lb. Due to the movement of the cable with temperature variation, and due to movement in handling—even after the cable is in service, of course, either under ground or aerial, it is necessary that it be handled—incipient cracks may be developed inside which will eventually cause trouble with pressure over 50 lb.; so that 40 lb. ingress pressure has been determined to be the maximum safe pressure.

Answering the second question, the wires are brought out into a special terminal which has a pressure-tight cover, so that the entire device is pressure-tight. There are no plugs made, either in the main cable or in the lateral cables, which serve to stub out the wires. The entire arrangement of the alarm device is in a pressure-tight chamber at the present time.

It has been found that it is very difficult to place any kind of a plug in either a main cable or a lateral which will hold its gas. You can pour in hot beeswax or hot paraffin or anything of that sort, and it will plug for some hours, but the difference in the expansion of the copper and the besswax or paraffin, finally causes a crack between the sheath and whatever is inside of it; so that it has been very difficult to get a dam that would really hold the gas.

Lighting of Airways and Airports

BY H. E. MAHAN[1]
Associate, A. I. E. E.

Synopsis.—*The Department of Commerce under the Air Commerce Act is responsible for the installation, operation, and maintenance of the national airways. The lighting facilities along the airways consist of revolving beacons at 10-mi. intervals and intermediate landing fields approximately every 30 mi. At the intermediate landing fields, in addition to the revolving beacon there are provided an illuminated wind cone, boundary, approach, and obstruction lights. The Department of Commerce is also responsible for the rating of airports, although their establishment, maintenance, and operation are matters of municipal or private concern. The rating of an airport is a measure of the facilities available. A preferred rating for lighting facilities requires the installation of an airport beacon to locate the airport from distant points, an illuminated wind direction indicator, white or yellow lights marking the boundary of the landing area, red lights on all obstructions, green range lights marking approaches, floodlighted hangars or other buildings as a measure of altitude and to illuminate identifying markings, a searchlight for measuring ceiling height and suitable floodlighting for the landing area.*

In view of the regulating authority vested in the Department of Commerce, the practise in lighting airways and airports follows very closely established standards. The floodlighting of the landing area proper offers the greatest opportunity for original thought and ideas. There have developed two schools of opinion regarding the fundamentals of field floodlighting systems. One school favors the use of a single unit or a group of units at one location for floodlighting the field, which system is referred to as the centralized system. The opposing school, advocating two or more light sources placed at different locations about the field, is known as the distributed system. Both systems have in common the use of lighting units delivering a fan shaped beam of light only a few degrees wide in the vertical plane and varying from 45 deg. to 180 deg. in the horizontal plane.

The present lighting facilities are found very satisfactory during fair weather but the problem of aiding the flier during fog still faces the engineer and research scientist as a problem.

* * * * *

LAST year was celebrated the twenty-fifth anniversary of the first airplane flight of the Wright Brothers at Kitty Hawk, N. C. Today the Department of Commerce reports that there are 15,128 miles of airways operating and 1330 established airports in the country exclusive of Army and Navy fields. Of this number 9341 miles of airways, 274 intermediate fields, and 74 other types of fields are lighted for night flying. By July 1, 1929, it is expected to have 11,270 miles of lighted airways. This is truly a remarkable record of progress and a glance at the map (Fig. 1) indicates the present scope of organized air transportation.

Obviously, if such facilities are to be operated at the maximum of efficiency, the service must be continued throughout the twenty-four hours of the day, for it is the distance covered between the end of one business day and the beginning of the next one that gives air travel its greatest advantage over other forms of transportation. In other words, the movement of material and passengers during the night conserves daylight working hours, and the greater the distance that may be covered in this period, the greater the benefit realized. Thus arises the problem of providing adequate lighting that will permit after-dark operations at our airports and along our airways. The author advances no original ideas for the solution of this problem but merely outlines the practise followed at the present time in lighting and marking airports and airways for night flying.

AIRWAYS

An airway is defined by the Department of Commerce as "An air route between air traffic centers with landing facilities at intervals equipped with aids to air navigation and communication system for transmission of information pertaining to the operation of aircraft. The term 'airway' may apply to an air-route for either land planes, sea planes, or both." Under the Air Commerce Act of 1926, the Secretary of Commerce has authority to establish and maintain civil airways; operate and maintain intermediate landing fields, lights, signal and radio direction finding apparatus; and other structures and facilities (excepting airports) used as aids to navigation. In fulfillment of this obligation,

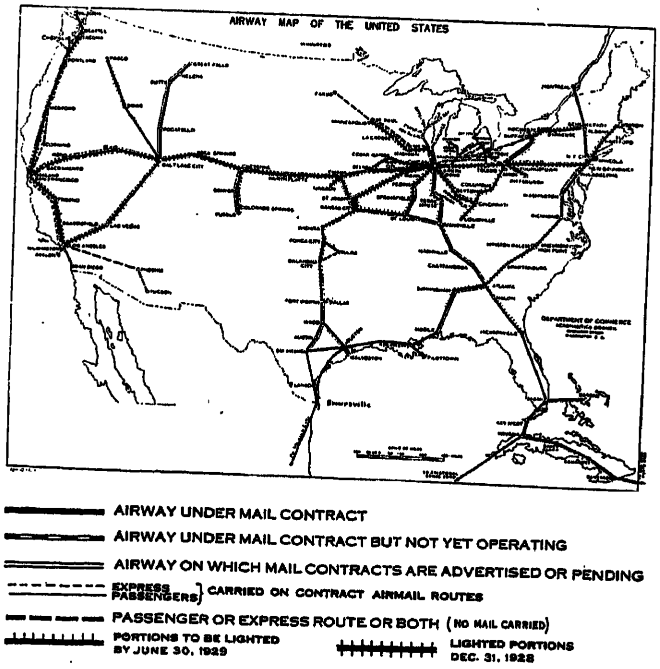
AIRWAY MAP OF THE UNITED STATES
Brownsville
AIRWAY UNDER MAIL CONTRACT
AIRWAY UNDER MAIL CONTRACT BUT NOT YET OPERATING
AIRWAY ON WHICH MAIL CONTRACTS ARE ADVERTISED OR PENDING
EXPRESS PASSENGERS } CARRIED ON CONTRACT AIRMAIL ROUTES
PASSENGER OR EXPRESS ROUTE OR BOTH (NO MAIL CARRIED)
PORTIONS TO BE LIGHTED BY JUNE 30, 1929
LIGHTED PORTIONS DEC. 31, 1928

FIG. 1—AIRWAY MAP OF THE UNITED STATES SHOWING ROUTES LIGHTED AND TO BE LIGHTED

the Department of Commerce has established along the airways, beacon lights at 10-mi. intervals and emergency landing fields at 30-mi. intervals.

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Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

It is desirable to mount the beacons at sufficient elevation to obtain the desired range of visibility and avoid obstructions. A steel tower (Fig. 2) approximately 50 ft. high usually meets requirements. The beacon (Fig. 3) consists of a searchlight having a 24-in. diameter parabolic mirror and a 1000-watt, 115-volt incandescent lamp. It is revolved at 6 rev. per min. by means of a 1/6-hp. motor and gearing mounted in the base of the searchlight. Many of these beacons operate in inaccessible places and are without regular attendants. As a safeguard against outages, an automatic lamp changer is employed for replacing a burned out lamp with one held in reserve. A series coil in the lamp changer holds open contact points on normal operation. Upon the failure of the lamp these contact points close, permitting current to flow in a shunt coil the solenoid of which trips a latch swinging the spare lamp into focus.

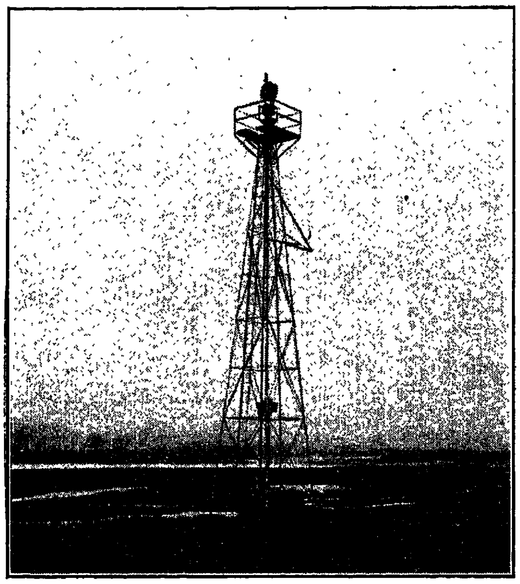

Fig. 2—Airway Beacon and Tower

Power is supplied for airway beacons from commercial circuits where available; otherwise a 2-kw. gasoline driven generator is provided. The beacon is supplemented by two "course" lights consisting of 14-in. projectors equipped with a 500-watt, G-40, 110-volt lamp and giving about a 15 deg. beam spread. One of these course lights faces in each direction along the airway. They are flashed by a mechanism integral with the beacon mechanism and synchronized with the rotation of the beacon so that the course light opposite to the direction of the beacon is flashed. The flashes indicate in code the number of the beacon which in turn indicates to the pilot his position on the course. These course lights are equipped with a red lens where there is no landing field and a yellow lens at intermediate landing fields. Intermediate fields are established at intervals of approximately 30 mi. and are equipped with a beacon, illuminated wind cone, boundary, approach, and obstruction lights.

Airports

The Department of Commerce requires that an airport in order to obtain the highest (A) rating for its lighting facilities, must have the following equipment:

(a) An airport beacon
(b) An illuminated wind-direction indicator
(c) Boundary lights
(d) Obstruction lights
(e) Hangar floodlights
(f) A ceiling projector
(g) Landing area floodlighting system.

Items (a) to (d) inclusive must operate all night.

The function of the beacon is to mark the airport from distant points. In order to accomplish this purpose effectively, a light must be sufficiently distinctive to contrast with surrounding lights; it must have a distribution which makes it visible at all points on the horizon and nearly to the zenith, and be of sufficient beam cp. to carry great distances. A 24-in. revolving beacon designed according to government specifications is shown in Fig. 3. The essential details of this beacon have been referred to under the subject of Airways. Any change in the distribution of light away from a symmetrical beam may be obtained by means of diverging lenses which refract the light to the desired beam form.

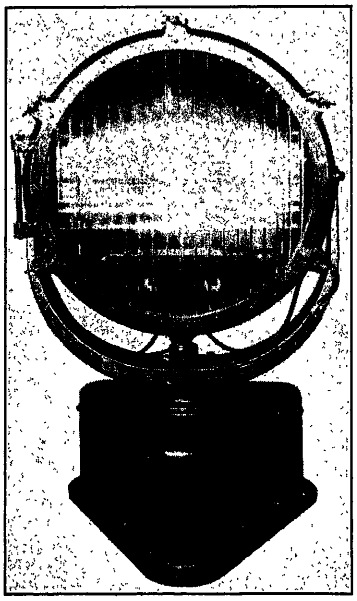

Fig. 3—Airport Beacon Conforming to Department of Commerce Specifications

Beacons at airports are generally located either on a steel tower similar in construction to the tower used on the airways or else on a platform on the roof of a hangar or other building. In any event, it is desirable to elevate the beacon to a height sufficient for the beam to clear surrounding obstacles and not become a nuisance to the neighborhood.

There are two types of lamps available for use in the beacon, differing in electrical and mechanical characteristics. Both lamps are of 1000-watt capacity, one operating at 115 volts, the other at 30 volts. The latter lamp having a more concentrated filament and operating at a higher temperature, obviously is more appropriate for use with a parabolic mirror and delivers a more definite and intense beam. Both filaments are in a T-20 bulb with a light center of 4¾ in. and a rated life of 500 hr. The beam candlepower with the 30-volt, high-current lamp is approximately 5,000,000; with the 115-volt lamp approximately 2,000,000. In order to carry out the policy of indicating the landing conditions at beacons along the National Airway System, it is recommended by the Department of Commerce that a green flashing auxiliary beacon be placed above the principal beacon to indicate that there is an airport at that location.

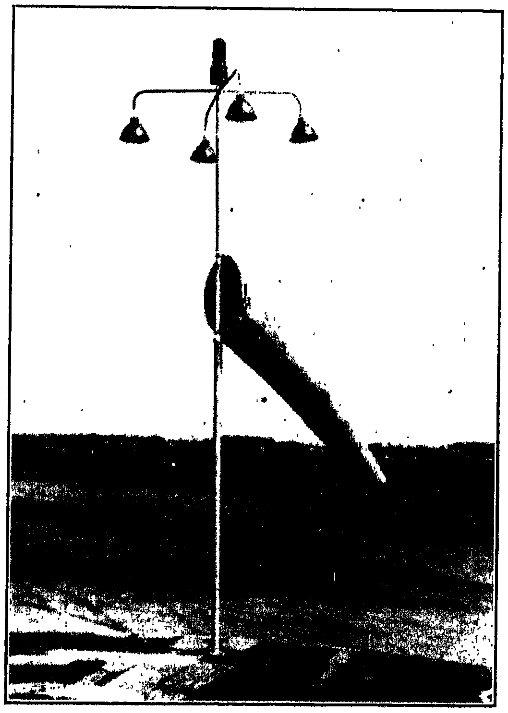

FIG. 4—WIND CONE MOUNTED ON ROOF OF HANGAR SHOWING METHOD OF LIGHTING

It is very essential that the pilot wishing to land be apprised of wind conditions on the field. The wind cone is perhaps the most generally used device for giving this information as it not only indicates the direction of the wind but also an approximate idea of its velocity. A method of lighting a wind cone (Fig. 4) consists of four 100-watt lamps in deep bowl porcelain enameled steel reflectors grouped at right angles to each other on two foot arms approximately 6 ft. above the wind cone. An obstruction light is placed on the top of the assembly. It is customary to operate these lights either from the commercial service in the building or from the boundary light circuit. Another type of wind indicator (Fig. 5) is known as a wind tee and consists of a structure simulating an airplane and pivoted to change position with change of wind direction. It is illuminated at night by means of lamps placed on top of the tee and provided with a reflector so as to throw the light on the structure and shield the direct light from the eyes of the pilot.

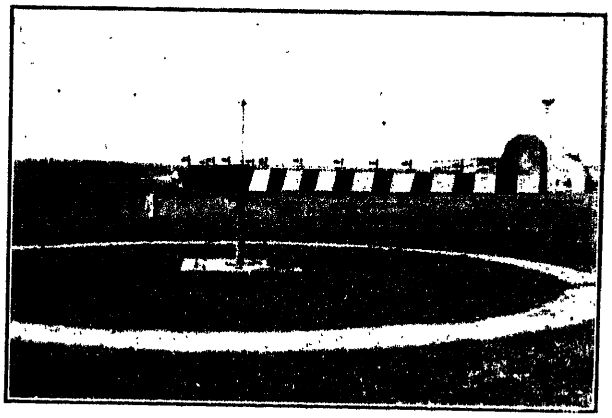

FIG. 5—WIND TEE

A wind direction indicating device illuminated by means of reflectored lamps on top of tee

The extent of the field is made known to the pilot by means of light sources placed around the border of the landing area on approximately 300-ft. centers. A type of unit used for this service (Fig. 6) consists of a weather-proof housing and etched clear globe enclosing a 600-lumen (60 cp.) 6.6-ampere series lamp or a 25-watt multiple lamp. Owing to the length of run necessary to mark the boundary of the average airport, it has been found in most cases more economical to adopt the series system of distribution. In this system a 6.6-ampere series circuit regulated by a constant current transformer is distributed by means of parkway cable buried in the earth. The boundary fitting is mounted on a pipe and planted in the ground, the parkway cable being brought up through the pipe to the lamp socket. A film or disk cutout is provided in the socket to maintain the continuity of the circuit in the event of lamp failure. A recent suggestion of the Department of Commerce recommends that boundary lights surmount a cone three feet in diameter and three feet high, painted in alternate bands of chrome yellow and white, separated by a narrow black band.

A green globe is substituted for the clear etched globe used on boundary fittings to serve as range lights for runways or as markers for the most advantageous landing directions. Obstructions on or in the vicinity of the airport are marked with red lights placed at the highest point of the structure. It is also the practise at some airports to make obstructions visible to the pilot by floodlighting them. This is found particularly advisable in connection with the hangars, for in addition to making the buildings visible, it makes evident their height and in this way gives the pilot a measure of his altitude. Such a system of lighting also provides illumination at the aprons of the hangar for the movement of planes and illuminates any signs or markings on the roof of the hangar. The illumination intensity recommended for hangar exteriors by the Department of Commerce is 2.5 foot-candles.

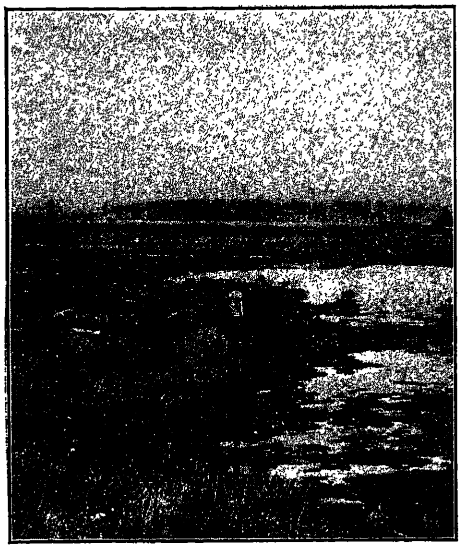

FIG. 6—A FORM OF BOUNDARY LIGHT EQUIPPED WITH 600-LUMEN 6.6-AMPERE SERIES INCANDESCENT LAMP

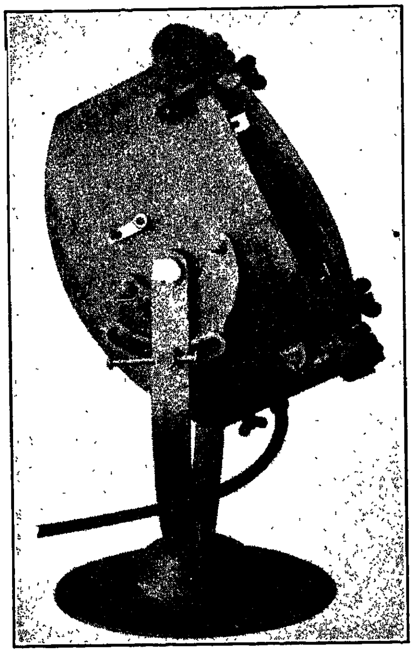

FIG. 7—A 14-IN. INCANDESCENT SEARCHLIGHT EQUIPPED WITH A 500-WATT T-20 MAZDA LAMP AND USED FOR DETERMINING CEILING HEIGHT

A beam of light has been found useful in determining the height of ceiling. An incandescent searchlight (Fig. 7) equipped with a 500-watt, 110-volt, T-20 projection lamp may be used for this purpose. The beam is directed on the clouds at a fixed angle and by sighting at the spot on the clouds from another point the height of ceiling is determined by triangulation.

Whereas the lighting facilities previously referred to follow rather definite regulations established by the Department of Commerce, a wider range of choice is left to the engineer in selecting a system for the actual illumination of the field. In studying this question, engineers very naturally turned to the pilots for information as to what sort of lighting they found most helpful to them in landing and taking off at night. Much information was obtained but many conflicting opinions were expressed. However, certain fundamental requirements were established and very generally agreed upon. The Department of Commerce requires for a field entitled to an "A" rating a minimum vertical illumination of not less than 0.15 foot-candles over the usable part of the field. This is the illumination measured on a plane at right angles to the field surface and is a measure of the value of illumination

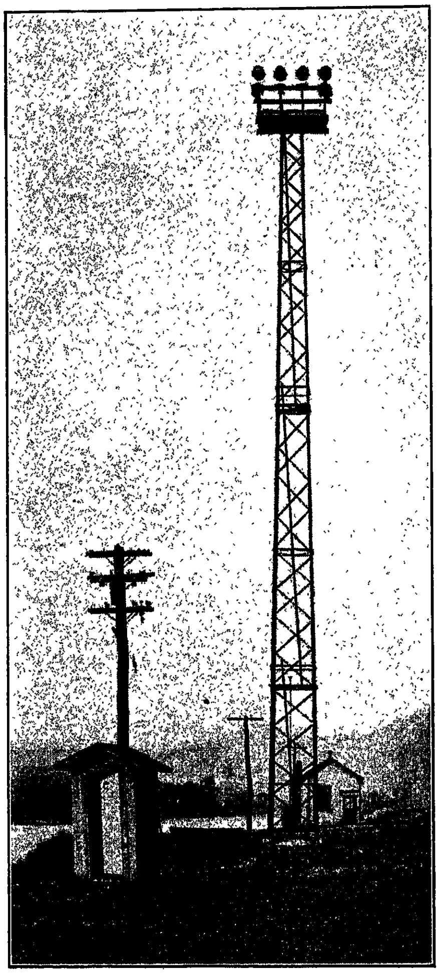

FIG. 8—THE TOWER SYSTEM OF LIGHTING LARGE AREAS

incident upon the vertical sides of obstacles, grass, etc., facing the source of light. This component is, therefore, the illumination which will enable a pilot to see obstacles ahead of him as he taxis along the ground or approaches very near to the ground provided he is moving in the direction of the projection of the light. The component of illumination at right angles to the vertical and called the horizontal illumination is the illumination which enables a pilot to see the ground from the air; it makes visible the area on which he is to land. If, therefore, a field is to be clearly visible from the air and at the same time objects and irregularities on the field are to be clearly discernible as the pilot approaches the field at low altitude for landing, the light must be distributed so as to provide adequate illumination in both the horizontal and vertical planes.

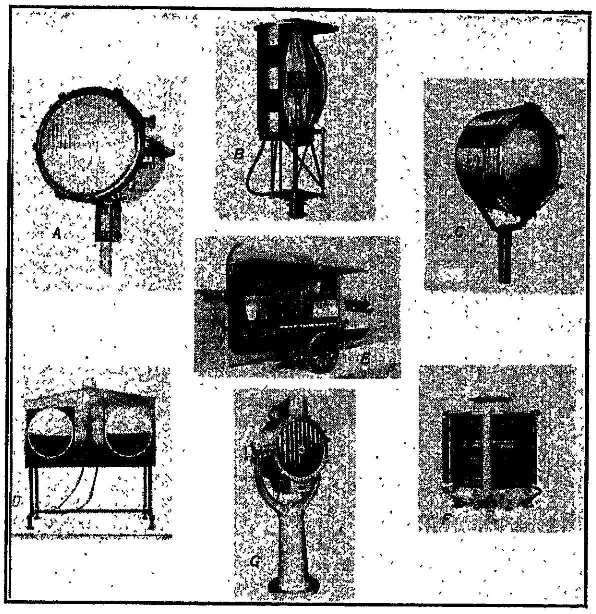

Fig. 9—Typical Forms of Airport Floodlighting Equipment

A. Incandescent Airport Floodlight. A 24-in. projector with chromium plated reflector and 45-deg. spread lens designed for use with 1500 watt and 3000 watt, 32-volt incandescent lamps.

B. Incandescent Airport Floodlight. Unit equipped with glass reflector designed to deliver a 180-deg. spread of light in the horizontal and limit the vertical spread to a few degrees. Lamp used may be either the 5-kw., 110-volt or 3000-watt, 32-volt incandescent lamp

C. Incandescent Airport Floodlight. Unit equipped with 25-in. mirrored glass reflector and spread convex heat-resisting lens of approximately 40-deg., 50-deg. or 80-deg. horizontal spread. Unit takes either the 1500-watt, 32-volt or 3000-watt, 32-volt incandescent lamp

D. Incandescent Airport Floodlight. Unit equipped with two 5-kw. or 10-kw. incandescent lamps, each in focus of 24 in. parabolic reflector. Beam spread in horizontal plane to about 80 deg. by diverging roundels.

E. Incandescent Airport Floodlight. Unit equipped with 14-3000-watt 32-volt incandescent lamps in the focal axis of a parabolic trough reflector The horizontal spread is 180 deg.

F. Airport Arc Floodlight. Unit equipped with a 150-ampere high intensity arc mechanism in the focus of a 21-element Fresnel lens. The horizontal spread is 180 deg. The 5-kw. or 10-kw. incandescent lamp may also be used with this unit.

G. Airport Arc Floodlight. Unit equipped with an 18-in. glass parabolic mirror and two front door glasses, one with clear glass and the other with 80-deg. spread lens. The spread lens may be swung back and the unit used as a high powered ceiling projector or as an auxiliary beacon.. The lamp mechanism is automatic, operating at 55 amperes on a line voltage from 80 to 125 volts direct current

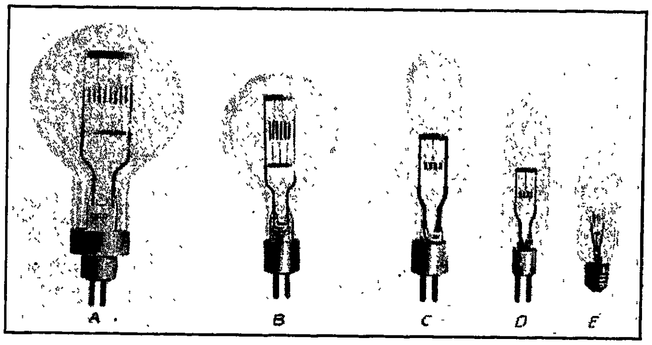

Fig. 10—Incandescent Lamps Used in Aviation Service

A. 10-kw., 115-volt, G-80 lamp for base down burning
B. 5-kw., 115-volt, G-64 lamp for base down burning
C. 3000-watt, 32-volt, GT-38 lamp for base down burning
D. 1500-watt, 32 volt, T-24 lamp for base down burning
E. 1000-watt, 115-volt airway and airport beacon lamp

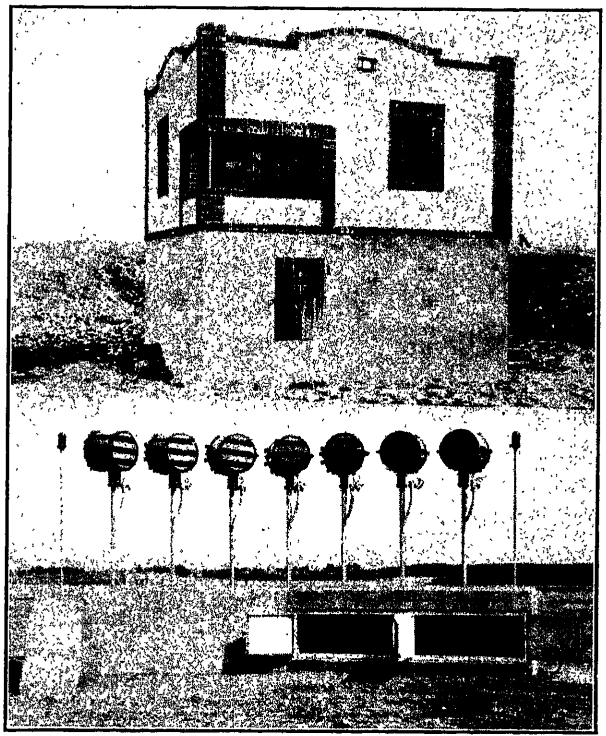

Fig. 11—Methods of Installing Field Floodlighting Equipment

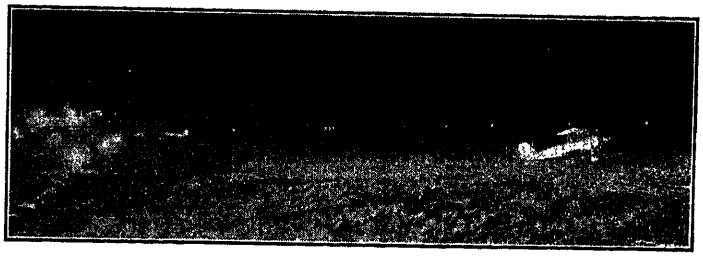

Fig. 12—Night View of Airport Floodlighted by Means of a Single Unit Equipped with Two 10-Kw. Incandescent Lamps

Furthermore, the light sources must be located so as to cause the least amount of glare in the eyes of the pilot for excessive glare is a potential cause of accidents. Needless to say, simplicity in operation and maintenance together with reliability are essential characteristics of the light sources. Light sources located close to the ground and delivering a fan of light at a grazing angle over the field surface give a maximum vertical illumination component and a minimum horizontal component. In order to increase the horizontal component of illumination it is necessary to elevate the light sources and deliver the light in a more downward direction. An example of this type of lighting is found in the systems in use for railroad yards or other large areas where floodlights are grouped on towers from 75 to 120 ft. high. Such a tower is shown in Fig. 8. This method of lighting has to date been frowned upon for airports largely on account of the hazard which such a tower presents in the vicinity of landing areas. The system employing light sources close to the ground has met with general favor and may be divided into two divisions; *viz.*, the centralized system and the distributed system. In the former system a single large light source or a group of smaller light sources concentrated at one point is employed for obtaining the field illumination; in the latter a number of relatively smaller units placed at two or more locations around the landing area are employed. The equipment (Fig. 9) used for field illumination employs both arc and incandescent light sources and a variety of optical systems for obtaining a fan distribution of light. In all cases the vertical spread of the beam is restricted to a few degrees and the horizontal spread expanded to 45 deg. to 180 deg. Incandescent lamps (Fig. 10) for this class of service are available up to 10 kw. capacity; in fact, lamps of even larger capacity are being built but at the present time there is no commercial equipment in which they are used.

It is interesting to note a trend toward the study of the character of the field surface with respect to night illumination. Obviously, the higher the coefficient of reflection of the surface, the greater the amount of light reaching the observer's eye. Just as our highway executives have found it desirable to mark road boundaries and obstacles with white paint to make them conspicuous at night, so our airport engineers and managers will find that the judicious use of light surfaces and white paint will greatly assist in the discrimination of essential objectives at night.

In locating the field floodlighting equipment, consideration should be given to prevailing winds and the site for the floodlights selected so that ships landing under prevailing winds may land in the same direction as the direction of projection of the light or at right angles to this direction in order that the pilot may not have to face the light source. The topography of the field is also an important consideration if shadowed areas are to be avoided. Modern practise (Fig. 11) provides substantial structures for the field floodlighting equipment with ample provision for housing transformers, control apparatus, etc. A night view of an airport illuminated by a single unit is shown in Fig. 12.

Other aids to the aviator in the form of advertising beacons, roof signs, etc., are daily being added by industry and municipalities. Under the guidance of the Department of Commerce these land marks become helpful to the aviator in following his course. There are many problems still confronting the engineer engaged in providing means for guiding the airman. The menace of fog still offers serious handicaps and attempts to counteract it by means of light have met with little success. The neon light, thought to possess fog penetrating properties superior to other forms of illuminants, appears from recent tests of the Bureau of Standards and others not to have any advantage for this purpose. Some of the largest research organizations in the country are devoting study to means of facilitating and increasing the safety of air travel and we may confidently look to the future for solutions to the problems which now appear most difficult.

Discussion

F. C. Caldwell: I should like to ask Mr. Mahan if he has the latest information with regard to the use of the neon lamps as beacons.

P. D. Manbeck: Is the incandescent lamp or is the carbon arc lamp the best suited for airport lighting? Experiments have been conducted, I believe, with both, and I should like to ask the question, has the carbon arc a field in this work?

A. W. Lee, Jr.: (by letter) I am very glad to see that Mr. Mahan questions seriously the penetrating properties heretofore ascribed to the neon light. I believe its advantages have been greatly exaggerated.

I believe that under the flood lighting of airports another apparently feasible scheme should be mentioned and studied, especially in such ports as have permanent runways, and that is the installation of translucent slabs in the runways themselves, these slabs to be lighted from below by ordinary incandescent lamps.

The majority of aviators consider outline lights on the field of far greater importance than the flood lights,—in fact, many of them make a practise of landing without any flood light whatever. It therefore seems very reasonable that if each permanent runway has scattered throughout its length these translucent rectangles, the pilot will more easily know not only where to set his ship down but when to set it down.

The main point which I should like to see discussed is the statement, "other aids to the aviator in the form of advertising beacons, roof signs, etc., are daily being added by industry and municipalities."

Roof signs with arrows pointing to the field, advertising signs, etc., are very helpful, but there is a growing tendency to establish special revolving beacons on high office buildings and I believe this is one of the most dangerous practises which we have to face.

Even though such a beacon flashes in code or stops once in each revolution and points toward the field, it is obvious that many conditions occur (such as fog, sleet, rain, etc.) which prevent the pilot from hanging around to spell out the beacon's message. If a revolving beacon is to be the standard indication throughout the country that a pilot is on a regular airway or is coming to a registered landing field, all other flashing beacons should be strongly discouraged.

Just two months ago a pilot on one of our contract mail lines missed an office building by a scant few feet. He had seen the flash of a beacon and had mistaken it for a beacon at the city airport, swooping downward after passing it in the same manner that he usually landed at the airport. Three other pilots of the same company have also been misled by this beacon, but have not had such narrow escapes.

If this practise is continued, I feel certain that sooner or later one of our night pilots will fly head on into some office building or smoke stack.

May I ask that this criticism be fully discussed and weighed and that the members present at this regional meeting do everything in their power to persuade the aeronautical branch of the Department of Commerce to disapprove any such future installation.

D. K. Wright: (communicated after adjournment) The writer wishes to take this opportunity to mention a recent development in lamp making which promises to make a big improvement in airport floodlighting. The various incandescent lamps used for this purpose have a rated life of 100 hr. at an efficiency of approximately 24 lumens per watt. The limiting factor which has prevented the operating of these lamps at a higher efficiency has been the blackening of the bulb. The new development which is to be put into effect on April 1 is the placing of a small quantity of specially prepared granular tungsten powder inside the lamp before it is exhausted and filled with nitrogen gas. When the bulb blackens from use the lamp is removed from its socket, inverted, and shaken gently with a circular motion for a few seconds. The abrasive action of the tungsten powder removes the black deposit from the walls of the bulb, leaving the glass bulb clear again. It even improves the lamp, because it will not blacken so readily the next time it is used.

The cleaning of the bulb makes it possible to operate the lamp at a substantial increase in efficiency. Higher efficiency in lamps of this type is of immense value to the floodlighting of landing fields, because the higher efficiency means a more than proportional increase in brilliancy of the filament and this greater brilliancy means, in turn, a beam of higher intensity and greater effectiveness.

The new development is to be incorporated in the 1500- and 3000-watt, 32-volt lamps, and the 5000- and 10,000-watt, 110-115-120-volt, airport floodlighting lamps.

W. T. Harding: (communicated after adjournment) Inasmuch as I have had charge of the development and standardization of night lighting equipment for the past three years for the U. S. Army Air Corps, I should like to bring out several points not clearly developed in the paper, *Lighting of Airways and Airports*.

The Air Corps is not charged with the development of airways equipment as that is a function of the Department of Commerce. Consequently, that part of the paper will be passed over at once, although, as a matter of historical interest, the Air Corps established the first lighted airways in this country.

The Air Corps, however, is vitally interested in the development of airdrome lighting equipment, and has done, and is still doing pioneer work along this line. Almost without modification the requirements for airport lighting as laid down by the Department of Commerce, were based upon those established and used by the Air Corps prior to the Civil Air Commerce Act of 1926. At the present time the major differences are merely in manner of installation as the Air Corps is interested in operation in time of war and is preparing to add auxiliary emergency equipment.

The Air Corps uses a beacon similar to that described merely because at the present time that beacon is the best available. These beacons, however, are essentially fair-weather aids. When needed most they are badly lacking in range and are hard to locate. Several other types have been developed, both incandescent and neon, but so far they have shown no better solution to the problem. The Air Corps and the Guggenheim Association are at present at work upon the problem of fog penetration at Wright Field and on completion of the experiments the problem of development of a beacon for fog will no doubt be in order.

With respect to wind indication, the Air Corps uses the wind tee. The indication of direction at night is made by the lamps themselves, covered with a green globe, instead of reflected light from the sides. From the air it appears as a green "T". Wind velocity is not indicated. The wind cone is not satisfactory even though illuminated internally or externally, as the three-dimensional position of an airplane at night coupled with lack of perspective, may give an incorrect indication.

The boundary light also is of value in landing or taking off, as a pilot may keep a straight course by using the light as a landmark and thus prevent ground looping. The Air Corps uses series circuits with individual lamp transformers for each light or group of lights. By this method the high voltage is kept underground.

With respect to floodlighting, a wide diversity of opinion prevails in the Air Corps. However, standardization has proceeded to a point where two types of installations are being made:

(a) *Primary Fields.* For airdromes where formation landings are made a (centralized) single source unit is used. This floodlight consists of a 180-deg. fresnal lens equipped with a 10-kw. lamp, spherical reflector, and automatic lamp changer and starter. The Air Corps has developed a special housing with automatically operating shutter. Two units are employed on a given field so placed that one or the other is used, depending upon wind direction.

(b) *Secondary Fields.* For airdromes where single ship landings are contemplated, the distributed type of lighting is used.

High mounting of floodlights is not recommended as the beam in misty or foggy weather curtains the ground and makes it hard to discern the exact location when landing. The floodlights should be mounted as low as the contour of the ground permits.

H. E. Mahan: Professor Caldwell has asked about the use of the neon tube for beacons. As far as can be determined, the neon light does not possess any fog-penetrating properties not possessed by the incandescent lamp. The Bureau of Standards has made rather extensive tests in this connection which are reported in the Bureau of Standards bulletin.[1] The form of the neon tube does not lend itself to projection equipment such as the standard form of airway beacon, and where used it generally takes the form of tubes built up to form a cylindrical or truncated conical structure. Its color is distinctive as contrasted with the ordinary lights of a city and to this extent is conspicuous, although similar results may be obtained by the use of color screens on incandescent lamps.

It is often stated that in clear weather the beacon is not needed and in foggy weather it cannot be seen. I think, however, that all agree that the beacon serves a real purpose, for in clear weather its distinctive flashes identify the airport and the airway and it is only in very heavy weather that it is obscured entirely. As far as fog is concerned, it is true that we have not made much progress in overcoming this handicap and there does not appear to be a definite solution in sight. Undoubtedly, the radio beacon is the most promising. A certain amount of success has attended attempts at fog dissipation, but I am afraid this would be very expensive and not altogether a practical solution.

Regarding the incandescent versus the arc lamp, the arc lamp is an excellent light source for any projection equipment and its color is in favor with many pilots. Its principal disadvantage has been that it requires direct current and an operator. The simplicity and flexibility of the incandescent lamp have brought this type of light source into very general favor although some of the larger airports are using arc equipments.

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[1] *Technical News Bulletin*, 1928, p. 156.

Iron Losses in Turbine Generators

BY C. M. LAFFOON[*] and J. F. CALVERT[*]
Associate, A. I. E. E. Associate, A. I. E. E.

Synopsis.—In large turbine generators there is no convenient method for determining the iron losses under load. It is the purpose of this paper to show, analytically, the factors influencing the change in iron loss with load. While very definite results are given for a simplified machine, it is not recommended at this time that any change be made in the A. I. E. E. Standards for computing load loss, because there is much to be done before the work can be considered complete.

This particular problem is typical of many which are entirely suitable for purely analytical study, and should furnish very desirable work for investigators who do not have access to large machines for extensive tests, but who desire to carry on studies related to this type of apparatus. This, as well as similar loss problems, can be studied much more readily by theoretical work and model tests, than from measuring the total losses of an entire machine, because the value sought is never a predominant proportion of the total value measured; and even if satisfactory results be obtained on a particular machine, they are not generally applicable to all such apparatus.

In this article, it is shown that on the basis of the flux distributions, the machines can be most conveniently divided into end zones, which present three dimensional problems and a large central zone, which presents only two dimensional problems. The losses in each part are discussed qualitatively, and the losses in the central zone are obtained quantitatively for an ideal machine in which only the fundamentals of the flux waves are present. It is shown that, contrary to the usual view, these losses do not vary as the square of the generated voltage, but are predominantly affected by the ratio of the slot to total leakage reactance. The results are given in curves. The complications in the problem introduced by the usual commercial designs by non-sinusoidal field forms, phase bands, etc., are discussed qualitatively.

* * * * *

THE losses which occur in the magnetic circuit of a synchronous machine depend upon the weight, magnetic properties, and physical shape of the iron and upon the arrangement and current values of the conductors. The total value of the no load iron losses can be measured with a satisfactory degree of accuracy, but no simple method is available for obtaining a direct measurement of these losses under load conditions. A method for obtaining the rate of loss at any point in the machine by means of temperature time curves was described by the writers in their paper on *Additional Losses of Synchronous Machines.*[1] Practical results can be obtained by this method, but it requires care and a large number of readings to permit the calculation of the total losses in the iron parts of the machine which is being tested.

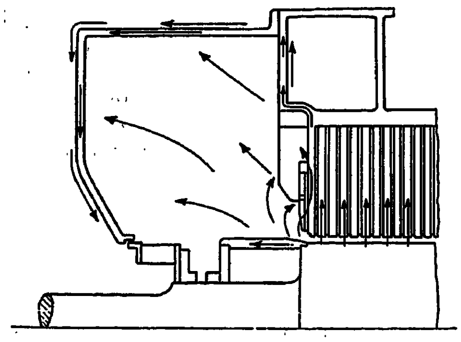

FIG. 1—FLUX DISTRIBUTION IN CENTER PLANE OF THE POLE—ROTOR EXCITED ALONE

The writers are not aware that anyone has presented an analytical study of the magnetic circuit losses which exist in synchronous machines under operating conditions. It is the purpose of this article to illustrate the flux distributions and iron loss conditions for certain principal parts of the commercial machines with the aid of diagrams and to determine analytically the ratio of the magnetic losses which exist under various load conditions to those which exist at no load for the case of an ideal, or simplified generator.

A better conception of the factors producing iron loss in turbine generators can be obtained by investigating the flux distributions in the different parts of the magnetic circuit. The end zones will be discussed briefly and then the central zone in greater detail. Figs. 1, 2, and 3 indicate diagramatically the direction of the flux in the plane passing through the center lines of the magnetic poles. There is a convenient division of the fields into end zones and central zone. In the end zone it is necessary to consider the flux distribution as a three-dimensional problem whereas in the central zone it can be considered as a two-dimensional problem.

It is necessary to decide between just what two radial planes two-dimensional conditions may be assumed. An estimate of this was made by assuming a simplified magnetic circuit as shown in Fig. 4. The magnetization curves for the paths were made as nearly equal to those existing in an actual machine as could be computed. The axial permeances were concentrated at the stator side of the air gap as shown, thus making the assumptions as pessimistic as possible. It was found that when the end leakage flux was something like fifty per cent of the radial air gap flux per packet, the influence of the end zone conditions became negligible four or five packets toward the center. This was thought to be worse than the actual case because effective stepping of the end packets reduces these inequalities to a minimum. The fact that the end packets and central packets do not vary greatly in temperature indicates that the penetration is not more than two or three packets under normal conditions. Fig. 5 has been drawn in an attempt to give a three-dimensional picture of these conditions. It is probably reasonable to assume that the end effects are confined to the first three packets on each end and that the large central zone presents only two-dimensional problems.

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[1] For references see Bibliography.

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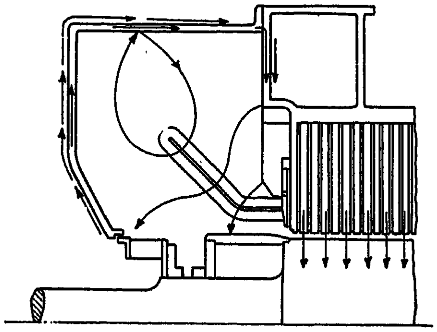


Fig. 2—Flux Distribution in the Center Plane of the Pole, Stator Excited Alone

The flux distribution in the end bell air space, due only to the rotor m. m. f., is influenced predominantly by the rotor retaining ring saturation and to a certain extent by the saturation in the stator iron. The latter causes a leakage flux from the frame to the bell. In this enclosed space there is a revolving field which produces losses in the metal coil braces and somewhat higher local losses in the finger plates. The iron end bells form parallel magnetic paths for the core and carry revolving fields of the distribution shown in Fig. 6. The losses in all of the solid parts are produced by currents which are near the surface of the iron, and consequently depend as much or more on the area than on the volume of material.

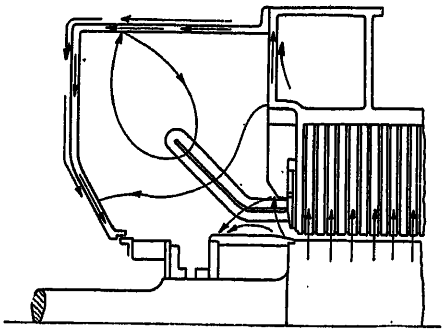


Fig. 3—Flux Distribution in the Center Plane of the Pole, Both Rotor and Stator Windings Excited at the Full-Load, Zero Power Factor, Over-Excited Condition

When the armature is excited alone there is a rotating flux in the end bell iron and in the enclosed air space. However, these flux distributions do not always maintain a constant shape, and therefore do not behave quite as simple revolving fields. This is due partly to the chording effect of diamond coil end windings, partly to the grouping of conductors in phase bands and to the normal chording of the machine, and partly to the influence of the phase connecting rings.

The flux studies indicate that the armature is probably a more important factor in causing local heating in the finger plate than is the field. The radial components of the slot leakage flux exist only at the end of phase bands. If the winding is full pitch (Fig. 7), or if the pitch is an even multiple of the reciprocal of the number of phases (as 1/3, 2/3, 3/3, 4/3, etc., for a three-phase winding) then this radial flux is concentrated in one tooth per phase per pole. This condition may produce high densities and high local temperatures

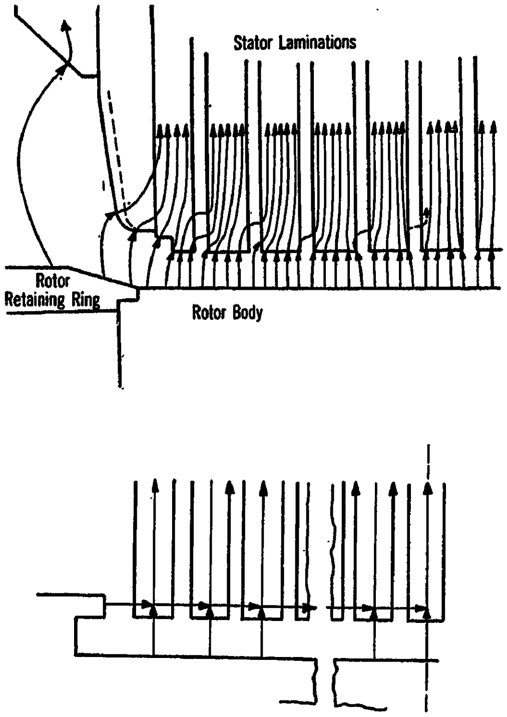


Fig. 4—Paths Assumed in Estimating the Axial Penetration of End Flux

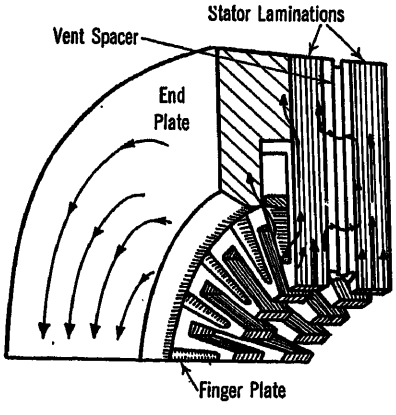


Fig. 5—Flux in the Stator End Parts

in some parts of the machine if proper precautions are not taken. For other coil pitches, the radial components of slot leakage flux occur in one tooth per phase per pole for each winding layer, and usually are not factors of importance.

At the zero per cent power-factor over-excited conditions, the armature predominates within the end bell enclosure, because the retaining ring acts as a partial magnetic shield for the rotor winding and thus renders the rotor field magnetomotive forces rather ineffective in this space. However, the rotor m. m. f. establishes the gap flux, the core flux, and hence the distribution in the end bell iron. The field in the end bell iron is about the same at this load condition as at no load normal voltage. The local effects of the phase connecting rings are present to some extent.

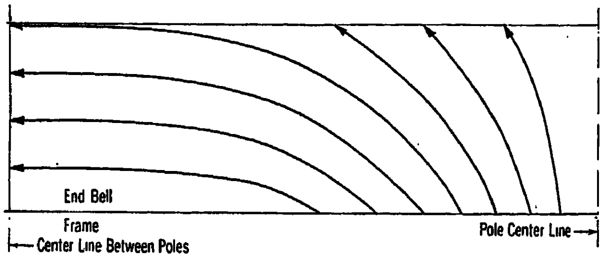


Fig. 6—Direction of Flux within the Stator End Bell Iron

Discussion of the Stator Iron Losses

It has been pointed out that the magnetic conditions in a turbine generator may be divided most conveniently into two parts. It is the large central part of the stator iron which will be discussed in this article. An ideal machine for calculation will be considered which has the following characteristics:

(1) A large number of teeth will be assumed so that practically no tooth taper is present;

(2) There will be as many phases as slots per pole so that the fundamental of the slot leakage flux may be considered alone; and

(3) A full pitch winding will be assumed.

In addition to the above assumptions for a special machine, the following general assumptions will be made:

(4) The rate of eddy-current loss in watt seconds per second per unit volume will be assumed proportional to the square of the instantaneous value of the resultant flux density in the laminated iron;

(5) The hysteresis loss per cycle will be assumed proportional to the square of the maximum flux density for the conditions encountered in commercial machines; and

(6) No allowance will be made for the redistribution of the flux in the main magnetic circuit due to changes in permeability between no-load normal voltage and full-load normal voltage conditions.

It is intended to consider the factors involved by the variations in the stator teeth and core loss on load as compared with the no-load conditions, due to the fundamental of the cross slot leakage flux and the increase or decrease in the air-gap flux. On all loads which require over excitation, the air-gap flux must be increased to produce a generated voltage equal to the vector sum of the terminal voltage and the armature leakage reactance voltage. Conversely, for practically all under excited conditions the air-gap flux is less at the load than at the no-load full voltage condition.

The armature leakage reactance voltage is usually divided into two main parts,—the voltage due to the end winding flux, and the voltage due to the slot leakage flux. (In this machine there will be no "tooth tip leakage flux," because no higher harmonics have been assumed.) The end winding reactance flux influences the densities in the main magnetic circuit, because it usually necessitates a change in the air-gap flux to maintain constant terminal voltage. It has been explained, previously, that the effect of the end winding flux, as such, can be neglected when considering densities in the central part of the stator iron. The reactance produced by the armature slot leakage flux also necessitates a change in the air-gap flux in order that constant terminal voltage shall be maintained. However, the paths of the latter flux (Fig. 8) are through the same stator iron parts as are occupied by the air-gap flux.

Loss in the Core

It is now possible to estimate the relative losses in the core back of the teeth on the basis of the foregoing assumptions. Both the air-gap flux, ${}_1\dot{\Phi}_a$, and the radial component of the slot leakage flux, $2\,\Phi_s$, have sinusoidal distributions at some circumferential line $d - d$ as shown in Figs. 8 and 9. These components of flux combine to give a resultant sinusoidal flux wave so that on the basis of assumptions 4, 5, and 6, it follows that the ratio of core loss on load to that on no load must be

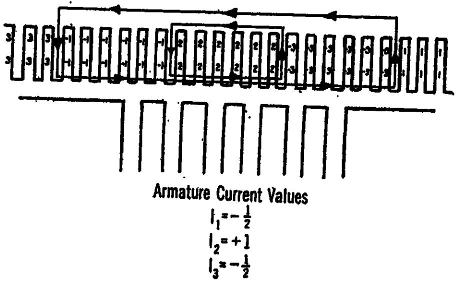


Fig. 7—Full Pitch Three-Phase Winding Showing the Concentration of the Radial Components of Slot Leakage Flux at the Ends of Phase Bands

$$\frac{{}_1W_c}{{}_0W_c} = \left\{ \frac{{}_1\dot{\Phi}_a + \overline{2\,\dot{\Phi}_s}}{{}_0\Phi_a} \right\}^2 \tag{1}$$

In this case the vector quantity $\overline{2\,\dot{\Phi}_s}$, is in phase with the maximum radial density due to the cross slot flux.

LOSS IN THE TEETH

The changes in tooth loss with load are much more difficult to describe than the core loss but can be illustrated with drawings. The difficulty is that the air-gap and slot leakage flux distributions do not have the same general shape in the teeth, whereas they did have in the core. In the teeth, the armature windings establish both tangential and radial densities while the air-gap field produces only radial densities. See Figs. 8 and 9. Hence the tangential density and the resultant radial density must be known at every point in the tooth throughout one cycle (see Fig. 10 for the component densities at one point in a tooth). Obviously, at every instant of time, the resultant density squared is equal to the sum of the squares of the tangential and the resultant radial densities.

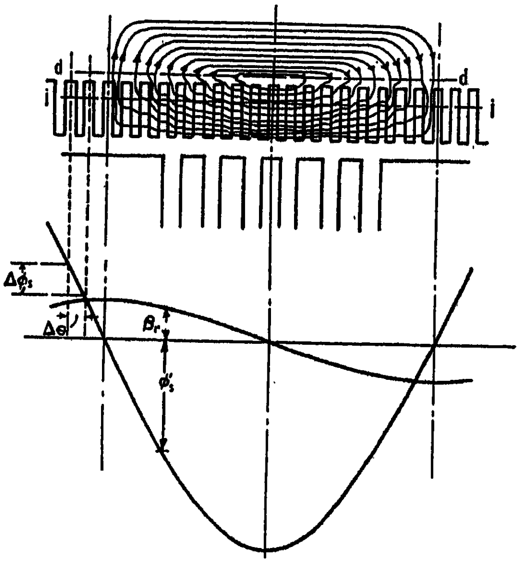


FIG. 8—ASSUMED DISTRIBUTION OF ARMATURE SLOT LEAKAGE FLUX

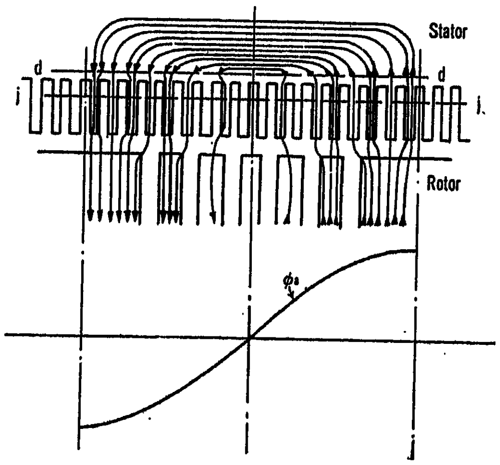


FIG. 9—ASSUMED DISTRIBUTION OF AIR-GAP FLUX

$$\beta_T^2 = \beta_R^2 + \beta_s^2 \qquad (2)$$

The rate of eddy current loss per unit volume per cycle is proportional to the mean square value of the resultant density, $\frac{1}{\pi}\int_{\theta=0}^{\theta=\pi} \beta_T^2\, d\,\theta$, but,

$$\frac{1}{\pi}\int_0^\pi \beta_T^2\, d\,\theta = \frac{1}{\pi}\int_0^\pi \beta_R^2\, d\,\theta + \frac{1}{\pi}\int_0^\pi \beta_s^2\, d\,\theta \qquad (3)$$

Hence the eddy current loss due to the density components displaced 90 deg. in space may be computed separately and added.

The hysteresis loss per cycle is proportional to the square of the maximum density, or β_{Tm}^2. The vectors β_R and β_s may have almost any particular time-phase relation, so that to determine the hysteresis loss a relation between the latter vectors is necessary to determine β_{Tm}^2.

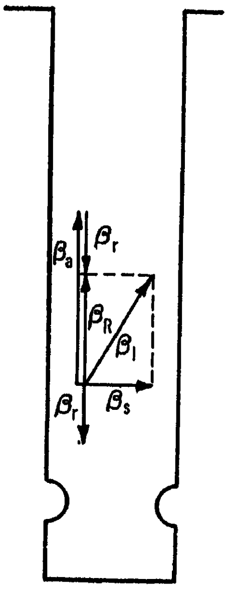


FIG. 10—TANGENTIAL, RADIAL, AND RESULTANT FLUX DENSITIES AT ONE POINT IN A STATOR TOOTH AT ONE INSTANT OF TIME

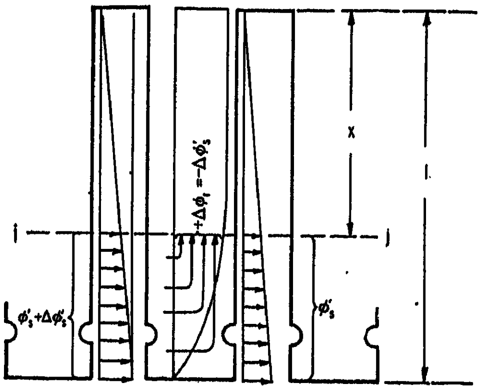


FIG. 11—TANGENTIAL AND RADIAL ARMATURE SLOT LEAKAGE FLUX

Before proceeding to a quantitative discussion of these densities and the resultant losses, it would be well to consider the angular or time relations between these values at any one point in a stator tooth. A machine will be assumed which has a counter clockwise rotation of the field. The simple Potier diagram for a non-salient pole generator carrying a reactive load, is shown in Fig. 12, with the density vectors under consideration superposed on the drawing. The radial density due to the air-gap flux, ${}_1\beta_a$, is in phase with this flux. The radial component of density due to the slot leakage flux, β_r, must be in phase with the armature demagnetizing flux, Φ_{dm}. The tangential density, β_s, due to the slot leakage flux is shown leading the radial value, β_r, by 90 electrical degrees; because it has been chosen to consider β_s positive in the direction opposite to that of rotation (see Figs. 8 and 12) and β_r positive in the outwardly radial direction.

Plainly, it would be desirable to relate the densities to voltages so that the loss equations may be expressed in terms of terminal voltage, reactance, and power factor. The first step is to relate the cross-slot flux and the air-gap flux to the voltages. The circumferential line at the base of the teeth will be used for reference. All of the air-gap flux and all of the slot leakage flux cross this line. They may be considered as two synchronously rotating fields, each of which establishes a voltage in the stator winding. Roughly, their distributions are as shown in Figs. 8 and 9. It can be seen that the slot leakage field is less effective in producing voltage than is the air-gap flux. The former enters at the base of the teeth, but does not link all of the conductors; whereas the air-gap flux has the same value at the gap as at the base of the teeth, and links more armature turns for a given number of flux lines. It may be written that,

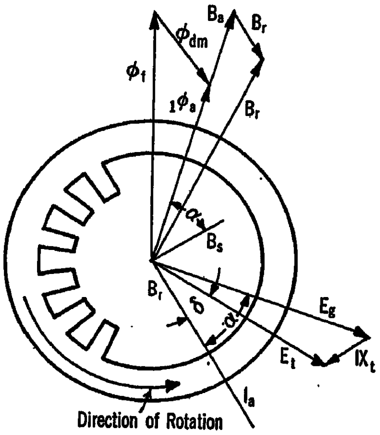


FIG. 12—VECTOR DIAGRAM FOR NON-SALIENT POLE A-C. GENERATOR

$$K_1\left(\frac{I\,X_s}{E_g}\right) = \frac{2\,\Phi_s}{{}_1\Phi_a} \tag{4}$$

where K_1 is a constant greater than one. Since a full pitch winding has been assumed, the density due to the cross slot flux varies directly with x (Fig. 11) in every slot. The constant, K_1, is the ratio of the flux turn interlinkages per slot ampere which would obtain for any one slot if all of the existing slot flux linked all of the turns in the slot to the flux turn interlinkages per slot ampere which actually exist. It can be shown (see Appendix) that $K_1 = \dfrac{3}{2}$.

On the basis of the foregoing analysis the loss equations to follow have been obtained for the central portion of the machine. The derivations of these equations are given in the Appendix.

I. Ratio of the Core Loss on Load to that on No Load.

$$\frac{{}_1W_c}{{}_0W_c} = 1 + 2\left(\frac{I\,X_t}{E_t}\right)\sin\gamma + \left(\frac{I\,X_t}{E_t}\right)^2 - 3\left(\frac{I\,X_s}{E_t}\right)\left(\frac{I\,X_t}{E_t} + \sin\gamma\right) + \frac{9}{4}\left(\frac{I\,X_s}{E_t}\right)^2. \tag{5}$$

This includes both hysteresis and eddy current loss.

II. Ratio of the Eddy Current Loss in the Teeth on Load to that on No Load.

$$\frac{{}_1W_{et}}{{}_0W_{et}} = 1 + 2\left(\frac{I}{E_t}\right)\sin\gamma + \left(\frac{I\,X_t}{E_t}\right)^2 - 2\left(\frac{I\,X_s}{E_t}\right)\left(\frac{I\,X_t}{E_t} + \sin\gamma\right) + \left(\frac{I\,X_s}{E_t}\right)^2\left(\frac{3}{K_3^2} + \frac{6}{5}\right) \tag{6}$$

III. Ratio of the Hysteresis Loss in the Teeth on Load to that On No Load.

$$\frac{{}_1W_{ht}}{{}_0W_{ht}} = \frac{1}{2}\,\frac{{}_1W_{et}}{{}_0W_{et}} + \frac{1}{2}\int_{\frac{x}{l}=0}^{\frac{x}{l}=1} \sqrt{\begin{aligned} &\left\{\cos^2\gamma - \left(\frac{I\,X_t}{E_t} + \sin\gamma\right)^2 + 3\left(\frac{I\,X_s}{E_t}\right)\left(\frac{I\,X_t}{E_t} + \sin\gamma\right)\left(1 - \frac{x^2}{l^2}\right) \right.\\ &\left. - \frac{9}{4}\left(\frac{I\,X_s}{E_t}\right)^2\left[\left(1 - \frac{x^2}{l^2}\right)^2 - \frac{4}{K_3^2}\,\frac{x^2}{l^2}\right]\right\}^2 \\ &+ \left\{3\left(\frac{I\,X_s}{E_t}\right)\left(1 - \frac{x^2}{l^2}\right) - 2\left[\frac{I\,X_t}{E_t} + \sin\gamma\right]\right\}^2\cos^2\gamma \end{aligned}}\; d\left(\frac{x}{l}\right) \tag{7}$$

It should be remembered that certain design constants must be involved in the numerical constants of these equations. For instance, the ratio of the iron area for the cross slot flux to that for the radial flux must be introduced, and it depends upon the physical proportions of the machine. These constants are expressed algebraically in the Appendix.

Curves based on the above equations are shown in Figs. 18 to 29 inclusively. Figs. 30 and 31 give the total variations in iron loss with load as functions of power factor. Various ratios of slot to total reactance and two values of total per cent reactance are chosen as shown. Certain ratios for the loss on no load were assumed, as follows:

$$\frac{\text{No-Load Core Loss}}{\text{Total No-Load Iron Loss in the Machine}} = \frac{{}_0W_c}{{}_0W_T} = 0.6$$

$$\frac{\text{No-Load Eddy Current Loss in the Teeth}}{\text{Total No-Load Iron Loss in the Machine}} = \frac{{}_0W_{et}}{{}_0W_T} = 0.1$$

$$\frac{\text{No-Load Hysteresis Loss in the Teeth}}{\text{Total No-Load Iron Loss in the Machine}} = \frac{{}_0W_{ht}}{{}_0W_T} = 0.3$$

All of these curves apply to the large central zone in the stator, and are for four-pole machines.

DISCUSSION OF RESULTS

To analyze the curves, it is convenient to consider the loss ratios in terms of the revolving fields which produce the losses.

The variations in core loss with flux were expressed by Equation (1). Consider an over-excited condition. The air-gap flux on load (and hence this component of the core flux on load) is increased over the no-load value, in the ratio of the generated to the terminal voltage. However, the core flux on load is decreased by the vector addition of the slot leakage flux. The slot leakage reactance may be considered $\frac{3}{2}$ times as effective in decreasing core densities, as the total reactance was in increasing these densities. Consequently, if $\frac{I\,X_s}{I\,X_t} < \frac{2}{3}$, the core flux and loss increase with load; if $\frac{I\,X_s}{I\,X_t} > \frac{2}{3}$, the core flux and loss decrease with load; and if $\frac{I\,X_s}{I\,X_t} = \frac{2}{3}$, the core flux and loss remain constant. The first two conditions are reversed if $E_g < E_t$, *i. e.*, at most under-excited conditions. The last condition is true at all power factors, namely, that the core loss does not vary with load when the ratio of slot to total reactance is two-thirds. The 100 per cent power factor case is somewhat unique. The reactance and terminal voltages add at right angles. Hence if the ratio $\frac{I\,X_s}{I\,X_t}$ is either greater or less than $\frac{2}{3}$, the core flux and loss must be greater on load than on no load.

In the teeth there are three components of flux which produce loss. The radial components of slot leakage flux combines with the air gap flux. They add in the radial direction, and give a resultant radial density producing loss at every point. The radial flux components corresponding to the reactance vectors $I\,X_s$ and $I\,X_t$ are about equally effective in producing eddy current loss. When $\frac{I\,X_s}{I\,X_t}$ approaches zero, the loss ratio for the teeth must increase with $I\,X_t$ just as it did for the core. When $\frac{I\,X_s}{I\,X_t}$ approaches unity, there is less variation in loss with load due to the radial density, but the tangential densities produce an appreciable value which must be added arithmetically to obtain the total loss. For any value of $\frac{I\,X_s}{I\,X_t}$, the change in loss on load over no load is a function of $I\,X_t$.

On the under excited conditions, it is well to consider the extreme ratios of $\frac{I\,X_s}{I\,X_t}$. As this ratio approaches zero, the eddy-current loss becomes predominantly influenced by the air-gap flux, and the loss ratio decreases with increase in $I\,X_t$ for fixed values of $\frac{I\,X_s}{I\,X_t}$. As $\frac{I\,X_s}{I\,X_t}$ approaches unity the tangential flux adds very considerably to the density, but the effects of $I\,X_s$ and $I\,X_t$ on the losses due to the radial densities more nearly nullify each other. The aggregate result is an increase in loss with $I\,X_t$.

In considering the variations in the hysteresis loss with load, it is interesting to note that the tangential densities are zero at the instant the radial densities are maximum, if the machine is operating at zero per cent power factor, either over or under excited. Consequently, the hysteresis loss at these operating conditions must be proportional only to the square of the radial densities. As $\frac{I\,X_s}{I\,X_t}$ approaches zero the hysteresis loss ratio must approach the corresponding ratios for the total loss in the core and for the eddy current loss in the teeth. When $\frac{I\,X_s}{I\,X_t}$ approaches unity, the ratio of the hysteresis loss in the teeth differs from the ratio of eddy current loss in the teeth, and from the ratio of total losses in the core. When the current is in phase with the voltage, the tangential density and the resultant radial density attain their maximum at almost the same instant of time for all points in the teeth. The maximum radial density squared is about equal to the sum of the squares of the maximum radial and tangential densities; and it is nearly proportional to the sum of the squares of the effective values of the latter densities. Thus the variation in the hysteresis loss in the teeth will be practically the same as the variation in the eddy current loss, when the current is in phase with the generated voltage. This is almost the case at the 100 per cent power-factor condition. (Compare Figs. 24 and 28.)

GENERAL CONCLUSIONS

In the central portion of the ideal generator (which has only fundamental flux waves), the ratios of iron loss on load to those of no load are not only functions of total reactance and power factor, but also of the ratio of slot to total reactance.

The actual loss in these parts is considerably less at operating conditions than is calculated by the usual method wherein the assumption is made that the iron loss varies as the square of the generated voltage. This is illustrated in Figs. 30 and 31. If a machine has a ratio of slot to total reactance of about 0.7, there is practically no variation in iron loss with load.

The quantitative analysis of the variations in iron loss for an actual commercial machine is vastly more complicated than for the ideal generator in the central portion of the stator, and some of the factors cannot be considered at all. (Some explanation of the probable difficulties to be met in the commercial case are outlined toward the end of the Appendix.)

It is possible to make analytical calculations for the loss in certain of the end zone parts. For instance, if the flux distribution in the end-bell iron can be established, a two dimensional problem may be assumed within the metal. However, the effect of the axial flux in the stator fingers, end plates, and first two or three packets, probably can only be properly investigated by tests which will permit the separation of the end zone from the central zone losses. Tests on machines of different lengths could be used if it could be established that the loss per inch axially in the central zone was substantially the same in each test; and not at variance due to building differences.

ACKNOWLEDGMENTS

The writers are indebted to Messrs. L. A. Kilgore, A. M. Harrison, and H. C. Myers of the Power Engineering Dept. for valuable theoretical assistance and for calculating data for the curves.

Appendix

Preliminary Derivations.

The constant K_1 was defined by Equation (4) and the accompanying descriptive matter. Referring to Fig. 11 it will be seen that

$$d\,\varphi_s = K_2\,L\,\beta_{sm}\,\frac{x}{l}\,d\,x \qquad (8)$$

$$n = N\,\frac{x}{l} \qquad (9)$$

$$K_1 = \frac{N\,\varphi_s}{K_2\,L\int_0^l \frac{N\,x}{l}\,\beta_{sm}\,\frac{x}{l}\,d\,x} \qquad (10)$$

$$K_1 = \frac{3}{2}\,\frac{N\,\varphi_s}{K_2\,L\,N\,\beta_{sm}\,\frac{l}{2}} \qquad (11)$$

but,

$$\varphi_s = K_2\,L\,\beta_{sm}\,\frac{l}{2} \qquad (12)$$

Hence,

$$K_1 = \frac{3}{2} \qquad (13)$$

And rewriting Equation (4)

$$\frac{3}{4}\,\frac{I\,X_s}{E_g} = \frac{\Phi_s}{{}_1\Phi_a} \qquad (14)$$

and since

$$\frac{{}_1\Phi_a}{{}_0\Phi_a} = \frac{E_g}{E_t} \qquad (15)$$

then

$$\frac{3}{4}\,\frac{I\,X_s}{E_t} = \frac{\Phi_s}{{}_0\Phi_a} \qquad (16)$$

From Fig. 12 it may be seen that

$$E_g = \sqrt{E_t^2\cos^2\gamma + (I\,X_t + E_t\sin\gamma)^2} \qquad (17)$$

then

$$E_g = \sqrt{E_t^2 + 2\,(I\,X_t)\,E_t\sin\gamma + (I\,X_t)^2} \qquad (18)$$

and

$$\sin\alpha = \frac{I\,X_t + E_t\sin\gamma}{E_g} \qquad (19)$$

$$\cos\alpha = \frac{E_t\cos\gamma}{E_g} \qquad (20)$$

$$\sin 2\,\alpha = \frac{2\,[E_t\,(I\,X_t)\cos\gamma + E_t^2\sin\gamma\cos\gamma]}{E_g^2} \qquad (21)$$

$$\cos 2\,\alpha = \frac{E_t^2\cos^2\gamma - (I\,X_t + E_t\sin\gamma)^2}{E_g^2} \qquad (22)$$

$$\sin\gamma = \sqrt{1-\cos^2\gamma} \qquad (23)$$

$$\cos\gamma = \text{power factor} \qquad (24)$$

In order to express the losses in terms of voltages it is necessary to relate the voltages and densities produced by the flux. In accordance with Equation (15)

$$\frac{{}_1B_a}{{}_0B_a} = \frac{E_g}{E_t} \qquad (25)$$

From Fig. 11

$$B_{sm} = \frac{2\,\Phi_s}{K_2\,L\,l} \tag{26}$$

$${}_0B_a = \frac{A_p}{A_i}\,\frac{2\,P}{.636\,\pi\,D\,L}\,{}_0\Phi_a = \frac{A_p\,P}{A_i\,D\,L}\,{}_0\Phi_a \tag{27}$$

Dividing Equation (26) by (27)

$$\frac{B_{sm}}{{}_0B_a} = \frac{A_i\,D}{A_p\,2\,K_2\,l\,P}\,3\left(\frac{I\,X_s}{E_t}\right) \tag{28}$$

Let

$$K_3 = \frac{A_p\,2\,K_2\,l\,P}{A_i\,D} \tag{29}$$

Then

$$\frac{B_{sm}}{{}_0B_a} = \frac{3}{K_3}\,\frac{I\,X_s}{E_t} \tag{30}$$

The constant K_3 depends upon the physical proportions of the machine. In the calculations it was found to be nearly constant for two-pole machines, and of another nearly constant value for four-pole turbine generators of modern design. Two average values were used accordingly—one for two poles and one for four-pole alternators.

QUANTITATIVE CONSIDERATION OF THE STATOR IRON LOSS

A. Core Loss

Equation (1) stated that the core loss ratio was

$$\frac{{}_1W_c}{{}_0W_c} = \left\{\frac{{}_1\dot{\Phi}_a + \overline{2\,\dot{\Phi}_s}}{{}_0\Phi_a}\right\}^2 \tag{31}$$

From Fig. 12 it can be written that

$$\{{}_1\dot{\Phi}_a + \overline{2\,\dot{\Phi}_3}\}^2 = [{}_1\Phi_a - 2\,\Phi_s \sin\alpha]^2 + [2\,\Phi_s \cos\alpha]^2 \tag{32}$$

(It should be remembered that α is positive when the current lags the generated voltage.)

From the preceding equations

$$\frac{{}_1W_c}{{}_0W_c} = \frac{{}_1\Phi_a^2 - 4\,{}_1\Phi_a\,\Phi_s \sin\alpha + 4\,\Phi_s^2}{{}_0\Phi_a^2} \tag{33}$$

In order to express this loss equation in terms of voltage and reactance it is only necessary to refer to Equations (15) and (16). Then

$$\frac{{}_1W_c}{{}_0W_c} = \frac{E_g^2 - 3\,E_g\,(I\,X_s)\sin\alpha + \frac{9}{4}(I\,X_s)^2}{E_t^2} \tag{34}$$

However, E_g and $\sin\alpha$ are cumbersome to use and by the application of Equations (18) and (19) to the last form it follows that,

$$\frac{{}_1W_c}{{}_0W_c} = 1 + 2\left(\frac{I\,X_t}{E_t}\right)\sin\gamma + \left(\frac{I\,X_t}{E_t}\right)^2$$

$$- 3\left(\frac{I\,X_s}{E_t}\right)\left(\frac{I\,X_t}{E_t} + \sin\gamma\right) + \frac{9}{4}\left(\frac{I\,X_s}{E_t}\right)^2 \tag{35}$$

If $\frac{I\,X_s}{E_t}$ and $\frac{I\,X_t}{E_t}$ are expressed as decimals, the formula is greatly simplified for calculation.

B. Eddy Current Loss in the Teeth

Equations (3) gave the general relation for the instantaneous densities and the resultant rate of eddy current loss at any one point in the teeth.

In Equation (2),

$$\beta_T^2 = {}_1\beta_R^2 + \beta_s^2 \tag{2}$$

By Fig. 10

$$\beta_T^2 = ({}_1\beta_a + \beta_r)^2 + \beta_s^2 \tag{36}$$

The values must be expressed as functions of θ and x before a quantitative solution can be obtained.

From Equation (8)

$$\frac{1}{K_2\,L}\,\frac{d\,\varphi_s}{d\,x} = \beta_s = B_{sm}\,\frac{x}{l}\cos\theta \tag{37}$$

Referring to Fig. 11 it may be seen that the decrease in cross slot flux below any circumferential line $j - j$ in the teeth must travel radially in the same distance. When the chosen positive directions of flux and rotation are taken into consideration also, it may be written that

$$\frac{d\,\varphi_s'}{d\,\theta} = -\frac{d\,\varphi_r}{d\,\theta} \tag{38}$$

From Equation (37)

$$\varphi_s' = K_2\,L\int_x^l B_{sm}\,\frac{x}{l}\cos\theta\,d\,x \tag{39}$$

$$\varphi_s' = K_2\,L\,B_{sm}\,\frac{(l^2 - x^2)}{2\,l}\cos\theta \tag{40}$$

$$\frac{d\,\varphi_r}{d\,\theta} = K_2\,L\,B_{sm}\,\frac{(l^2 - x^2)}{2\,l}\sin\theta \tag{41}$$

$$\beta_r = \frac{d\,\theta}{d\,A_i}\cdot\frac{d\,\varphi_r}{d\,\theta} \tag{42}$$

$$\frac{d\,\theta}{d\,A_i} = \frac{A_p}{A_i}\cdot\frac{2\,\pi\,P}{\pi\,D\,L} = \frac{K_3}{l\,K_2\,L} \tag{43}$$

Then,

$$\beta_r = K_3\,B_{sm}\,\frac{1}{2}\left(1 - \frac{x^2}{l^2}\right)\sin\theta \tag{44}$$

Since tooth taper is neglected, and in accordance with Fig. 12

$${}_1\beta_a = {}_1B_a\cos(\theta + \alpha) \tag{45}$$

Substituting Equations (37), (44), and (45) in (36),

$$\beta_T^2 = {}_1B_a^2\cos^2(\theta + \alpha)$$

$$+ K_3\,({}_1B_a)\,B_{sm}\left(1 - \frac{x^2}{l^2}\right)\cos(\theta + \alpha)\sin\theta$$

$$+ B_{sm}^2 K_3^2 \frac{1}{4} \left(1 - \frac{x^2}{l^2} \right)^2 \sin^2 \theta$$

$$+ B_{sm}^2 \frac{x^2}{l^2} \cos^2 \theta \qquad (46)$$

The eddy current loss in the teeth on load must be

$${}_1W_{et} = \frac{K_e V}{l} \int_{x=0}^{x=l} \left\{ \frac{1}{2\pi} \int_{\theta=0}^{\theta=2\pi} \beta_T^2 \, d\theta \right\} dx \qquad (47)$$

The eddy current loss in the teeth on no load must be

$${}_0W_{et} = \frac{K_e V}{2\pi} \int_{\theta=0}^{\theta=2\pi} {}_0B_a^2 \cos^2 (\theta + \alpha) \, d\theta = \frac{K_e V ({}_0B_a^2)}{2} \qquad (48)$$

The ratio of the eddy current loss on load to that on no load is

$$\frac{{}_1W_{et}}{{}_0W_{et}} = \frac{2}{l({}_0B_a^2)} \int_{x=0}^{x=l} \left\{ \frac{1}{2\pi} \int_{\theta=0}^{\theta=2\pi} B_T^2 \, d\theta \right\} dx \qquad (49)$$

Substituting Equation (46) in (49) and performing the double integration indicated by the latter, the following loss ratio results,

$$\frac{{}_1W_{et}}{{}_0W_{et}} = \left[\frac{{}_1B_a}{{}_0B_a} \right]^2 - \frac{2 K_3}{3} \frac{{}_1B_a}{{}_0B_a} \frac{B_{sm}}{{}_0B_a} \sin \alpha$$

$$+ \left[\frac{B_{sm}}{{}_0B_a} \right]^2 \left[\frac{2 K_3^2}{15} + \frac{1}{3} \right] \qquad (50)$$

Substituting Equations (25) and (30) in (50)

$$\frac{{}_1W_{et}}{{}_0W_{et}} =$$

$$\frac{E_g^2 - 2 E_g (I X_s) \sin \alpha + (I X_s)^2 \left(\frac{3}{K_3^2} + \frac{6}{5} \right)}{E_t^2} \qquad (51)$$

Also substituting Equations (18) and (19) in (51)

$$\frac{{}_1W_{et}}{{}_0W_{et}} = 1 + 2 \left(\frac{I X_t}{E_t} \right) \sin \gamma + \left(\frac{I X_t}{E_t} \right)^2$$

$$-2 \left(\frac{I X_s}{E_t} \right) \left(\frac{I X_t}{E_t} + \sin \gamma \right) + \left(\frac{I X_s}{E_t} \right)^2 \left(\frac{3}{K_3^2} + \frac{6}{5} \right) \qquad (52)$$

C. Hysteresis Loss in the Teeth

In the qualitative discussion of the losses it was explained that the rate of hysteresis loss per unit volume was proportional to the square of the maximum instantaneous density. Hence it may be written that this loss on load is

$${}_1W_{ht} = \frac{K_h V}{l} \int_{x=0}^{x=l} B_{Tm}^2 \, dx \qquad (53)$$

and on no load is

$${}_0W_{ht} = \frac{K_h V}{l} \int_{x=0}^{x=l} {}_0B_a^2 \, dx = K_h V ({}_0B_a^2) \qquad (54)$$

Then,

$$\frac{{}_1W_{ht}}{{}_0W_{ht}} = \frac{1}{l} \int_0^l \frac{\beta_{Tm}^2}{{}_0B_a^2} \, dx \qquad (55)$$

From Equation (46)

$$\frac{\beta_T^2}{{}_0B_a^2} = \frac{{}_1B_a^2}{{}_0B_a^2} \cos^2 (\theta + \alpha)$$

$$+ \frac{{}_1B_a}{{}_0B_a} \frac{B_{sm}}{{}_0B_a} K_3 \left(1 - \frac{x^2}{l^2} \right) \cos (\theta + \alpha) \sin \theta$$

$$+ \frac{B_{sm}^2}{{}_0B_a^2} K_3^2 \frac{1}{4} \left(1 - \frac{x^2}{l^2} \right)^2 \sin^2 \theta + \frac{B_{sm}^2}{{}_0B_a^2} \frac{x^2}{l^2} \cos^2 \theta \qquad (56)$$

Substituting Equations (25) and (30) in (56)

$$\frac{{}_1\beta_T^2}{{}_0B_a^2} = \frac{E_g^2}{E_t^2} \cos^2 (\theta + \alpha)$$

$$+ \frac{E_g}{E_t} \frac{I X_s}{E_t} 3 \left(1 - \frac{x^2}{l^2} \right) \cos (\theta + \alpha) \sin \theta$$

$$+ \left(\frac{I X_s}{E_t} \right)^2 \frac{9}{4} \left(1 - \frac{x^2}{l^2} \right)^2 \sin^2 \theta$$

$$+ \left(\frac{I X_s}{E_t} \right)^2 \frac{9}{K_3^2} \frac{x^2}{l^2} \cos^2 \theta \qquad (57)$$

Let,

$$M = \frac{E_g^2}{E_t^2} \qquad (58)$$

$$R = \frac{E_g}{E_t} \frac{I X_s}{E_t} 3 \left(1 - \frac{x^2}{l^2} \right) \qquad (59)$$

$$Q = \left(\frac{I X_s}{E_t} \right)^2 \frac{9}{4} \left(1 - \frac{x^2}{l^2} \right)^2 \qquad (60)$$

$$S = \left(\frac{I X_s}{E_t} \right)^2 \frac{9}{K_3^2} \frac{x^2}{l^2} \qquad (61)$$

Then expanding Equation (57) and substituting Equations (58), (59), (60), and (61),

$$\frac{\beta_T^2}{{}_0B_a^2} = \frac{M - R \sin \alpha + Q + S}{2}$$

$$+ \frac{(M \cos 2\alpha + R \sin \alpha - Q + S) \cos 2\theta}{2}$$

$$+ \frac{(R \cos \alpha - M \sin 2\alpha)}{2} \sin 2\theta \qquad (62)$$

The maximum value of $\beta_T = \beta_{Tm}$ will occur at the same value of θ as the maximum value of $\frac{\beta_T^2}{{}_0B_a^2}$. Equation (62) may be considered with θ as the only independent variable and rewritten

$$\frac{{}_1\beta_T^2}{{}_0B_a^2} = A_1 + A_2 \sin 2\,\theta + A_3 \cos 2\,\theta \qquad (63)$$

where A_1, A_2, and A_3 are constants corresponding identically to those in (62).

By inspection the maximum value of

$$\frac{\beta_T^2}{{}_0B_a^2} = \frac{\beta_{Tm}^2}{{}_0B_a^2} = A_1 + \sqrt{A_2^2 + A_3^2} \qquad (64)$$

Substituting these values in Equation (55),

$$\frac{{}_1W_{ht}}{{}_0W_{ht}} = \frac{1}{l}\int_{x=0}^{x=l} \frac{(M - R\sin\alpha + Q + S)}{2}\,d\,x$$

$$+ \frac{1}{l}\int_{x=0}^{x=l} \frac{1}{2}\sqrt{(M\cos 2\,\alpha + R\sin\alpha - Q + S)^2 + (R\cos\alpha - M\sin 2\,\alpha)^2}\,d\,x \qquad (65)$$

By substituting Equations (58), (59), (60), and (61) in the first integral of Equation (65), performing this integration, and comparing with Equation (51) it is found that Equation (65) may be written

$$\frac{{}_1W_{ht}}{{}_0W_{ht}} = \frac{1}{2}\,\frac{{}_1W_{et}}{{}_0W_{et}} + \frac{1}{l}\int_{x=0}^{x=l} \frac{1}{2}\sqrt{(M\cos 2\,\alpha + R\sin\alpha - Q + S)^2 + (R\cos\alpha - M\sin 2x)^2}\,dx \qquad (66)$$

Substituting Equations (19), (20), (21), (22), (58), (59), (60), and (61) in (66), then

$$\frac{{}_1W_{ht}}{{}_0W_{ht}} = \frac{1}{2}\,\frac{{}_1W_{et}}{{}_0W_{et}} + \frac{1}{2}\int_{\frac{x}{l}=0}^{\frac{x}{l}=1} \sqrt{\begin{aligned} &\left\{\cos^2\gamma - \left(\frac{I\,X_t}{E_t} + \sin\gamma\right)^2 + 3\left(\frac{I\,X_s}{E_t}\right)\left(\frac{I\,X_t}{E_t} + \sin\alpha\right)\left(1 - \frac{x^2}{l^2}\right)\right.\\ &\left.- \frac{9}{4}\left(\frac{I\,X_s}{E_t}\right)^2\left[\left(1 - \frac{x^2}{l^2}\right)^2 - \frac{4}{K_s^2}\,\frac{x^2}{l^2}\right]\right\}^2\\ &+ \left\{3\left(\frac{I\,X_s}{E_t}\right)\left(1 - \frac{x^2}{l^2}\right) - 2\left[\frac{I\,X_t}{E_t} + \sin\gamma\right]\right\}^2 \cos^2\gamma \end{aligned}}\; d\left(\frac{x}{l}\right) \qquad (67)$$

The solution of the latter equation is most readily obtained by choosing $E_t = 1$, and expressing $\frac{I\,X_s}{E_t}$ and $\frac{I\,X_t}{E_t}$ as decimals. A number of values of $\frac{x}{l}$ were assumed and the indicated integration performed graphically in obtaining the results shown in Figs. 18 to 29 inclusively.

Discussion of the Losses in the Central Zone in Commercial Machines

In the first portion of the article a general discussion was given of the losses in the various parts of a turbine generator, but it seems advisable to consider the losses for the central zone of commercial machines in somewhat greater detail. The alternators which now will be discussed have trapezoidal no-load field forms and balanced three-phase two-layer windings.

The higher harmonics of the no-load flux increase the eddy current loss. Neglecting their effect in changing the permeability, and consequently, the flux distribution, their eddy current losses could be computed entirely independently of the fundamental, and these component losses added. The hysteresis loss in the core will be almost proportional to the square of the total flux per pole, because in turbine generators the maximum densities are usually attained at the centerline between poles.

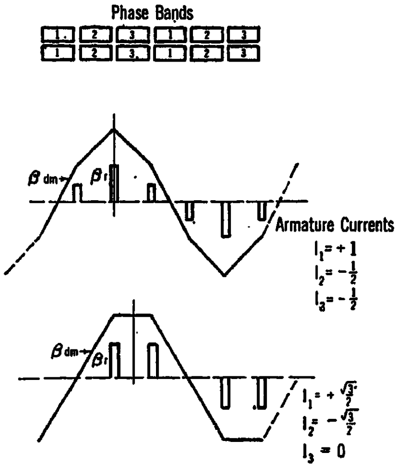


FIG. 13—ARMATURE MAGNETOMOTIVE FORCES AND DENSITIES DUE TO A BALANCED THREE-PHASE FULL PITCH WINDING

β_r = Radial density at the base of the teeth due to cross slot flux

β_{dm} = Radial density at the base of the teeth due to armature demagnetizing flux

If the no-load field form is a trapezoidal wave composed primarily of a first and a third harmonic, the third harmonic will increase the total eddy current loss in both the teeth and the core, above that which the fundamental alone would produce. This third harmonic will increase the hysteresis loss in the core, because it will increase the total flux; but it will decrease the hysteresis loss in the teeth, because the peak density is less than that which would be produced by the fundamental alone.

In discussing the effect of the armature magnetomotive forces, a full pitch balanced three-phase winding will be assumed first. Fig. 13 shows the winding and the density components at the base of the teeth produced by the armature demagnetizing flux and by the radial part of the cross slot leakage flux. It will be seen that at points between phase bands the actual radial flux density exceeds the fundamental of this wave by an amount which varies with time. The distribution with respect to the stator either in the gap or at the base of the teeth can be expressed very roughly by the equation

$$A = A_4 \cos(\lambda + \theta) + A_5 [\cos(\lambda + \theta)] \cos 6\lambda \qquad (68)$$

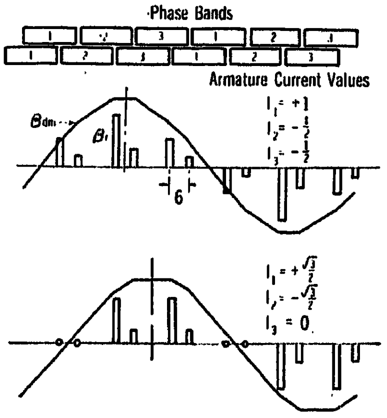


Fig. 14—Armature Magnetomotive Forces and Densities Due to a Balanced Three-Phase Winding of $\frac{8}{9}$ Full Pitch

β_r = Radial density at the base of the teeth due to cross slot flux

β_{dm} = Radial density at the base of teeth due to armature demagnetizing flux

where A, A_4, and A_5 are the amplitudes of the wave and its components.

λ = space angle in electrical radians.

θ = time angle in electrical radians.

The above equation can be rewritten

$$A = A_4 \cos(\lambda + \theta) + \frac{A_5}{2} \cos(5\lambda - \theta)$$

$$+ \frac{A_5}{2} \cos(7\lambda + \theta) \qquad (69)$$

These equations illustrate that both the fundamental and the higher harmonics produce fundamental frequency pulsations in the stator core and that their losses cannot be separated for that part of the machine. (Only the higher harmonics produce pole face losses which, of course, appear in the load loss measurements.) In this case the armature slot leakage flux simply increased A_5 at the base of the teeth above that value which existed at the gap.

It is now possible to consider a two layer chorded winding, because each layer will simply produce the same shaped flux waves, and these must be added with their proper displacement to obtain the total. The distribution at the base of the stator teeth is as shown in Fig. 14. The peculiar fact that the radial densities due to the slot flux produced by the two layers are not equal arises because the slot leakage flux due to the conductors in the base of the slots is greater than that due to those in the top, even though the top and bottom conductors are equally effective in producing gap flux. Hence, the slot leakage flux on entering the core is found to be out of phase or at least displaced with respect to the armature demagnetizing flux or ampere turns by some angle less than $\frac{\phi}{2}$, where ϕ is the angle by which the winding is chorded.

The flux distribution in the core which the slot flux alone would produce is shown in Figs. 15A and 15B. Fig. 16 shows the distribution in the core based on a sinusoidal distribution at the base of the teeth. These fields are plotted on the basis of uniform permeability.

There are six possible conditions which may establish

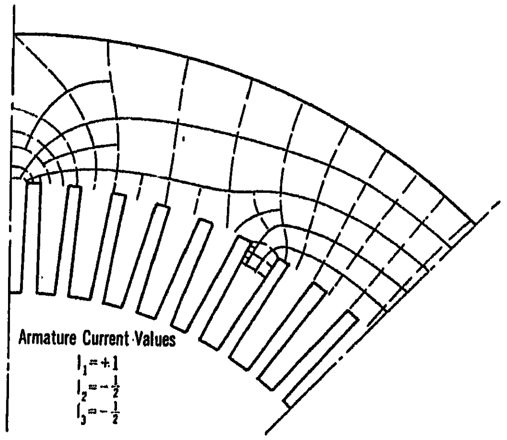


Fig. 15A—Flux in the Stator Core—See Corresponding Current Conditions in Fig. 13

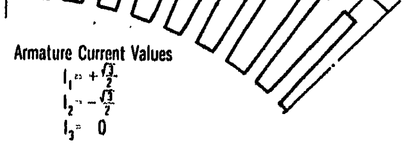


Fig. 15B—Flux in the Stator Core—See Corresponding Current Conditions in Fig. 13

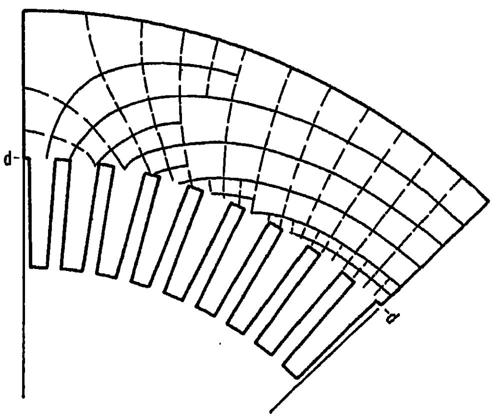
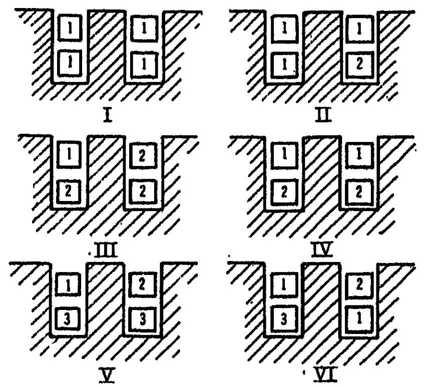


Fig. 16—Core Flux on Basis of Sinusoidal Distribution at d-d and Uniform Permeability in the Iron



The Numbers 1, 2, 3 Indicate Different Phase Currents

Fig. 17—Six Possible M. M. F. Conditions Affecting a Stator Tooth which can be Established by a Three-Phase Armature Winding

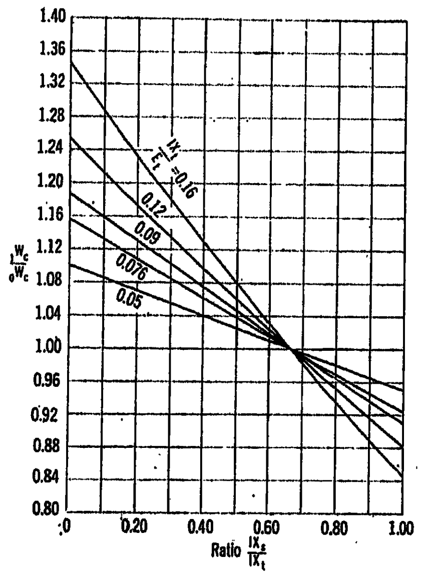


Fig. 18—Ratio of Full Load to No-Core Losses (Back of Teeth) *vs.* Ratio of Slot to Total Reactance, at 0.0 Power Factor Over Excited

the flux cycles for a given tooth (Fig. 17). That is, if the tooth is totally within one phase band, it has one set of flux boundary conditions due to the armature slot flux; if it is influenced by conductors of more than one phase, there are five other slot boundary combinations. In addition to these, the air-gap boundary condition will vary from tooth to tooth at any fixed load and power factor.

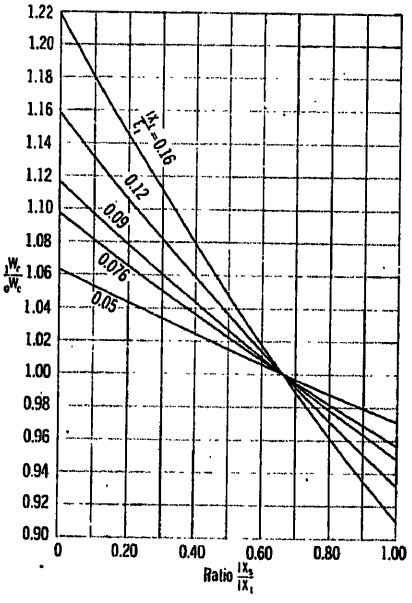


Fig. 19—Ratio of Full Load to No-Load Core Loss (Back of Teeth) *vs.* Ratio of Slot to Total Reactance, at 0.8 Power Factor Over Excited

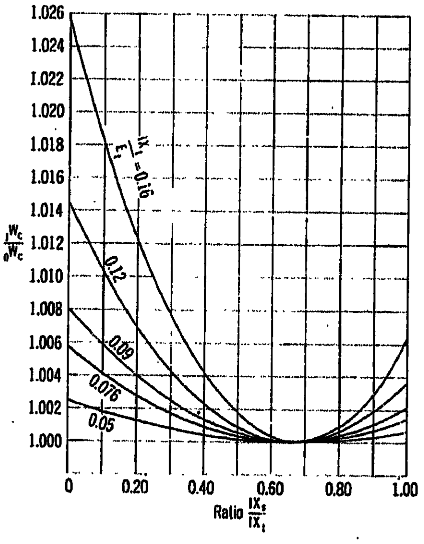


Fig. 20—Ratio of Full Load to No-Load Core Losses (Back of the Teeth) *vs.* Ratio of Slot to Total Reactance at 1.0 Power Factor

Neglecting tooth taper, these flux distributions and the resultant losses in the teeth may be related to per cent reactance, pitch, etc., but the solution becomes tremendously complicated. The distribution at the base of the teeth may be approximated by the addition of a few more harmonics to Equation (68). It will fit the distribution at a slight distance back of the base of the teeth and be effective for the flux entering about 96 per cent or 98 per cent of the core.

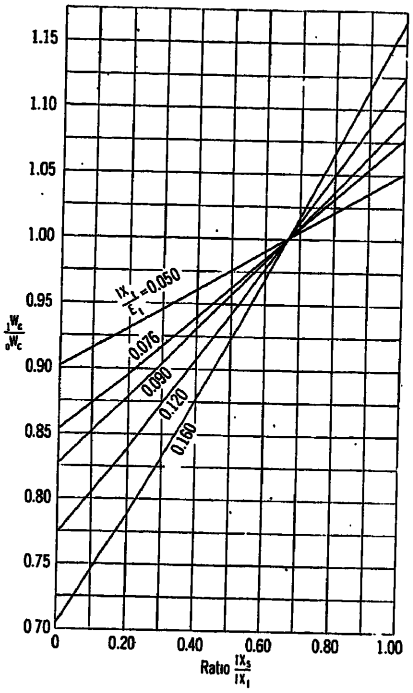


Fig. 21—Ratio of Full Load to No-Load Core Losses (Back of Teeth) *vs.* Ratio of Slot to Total Reactance, 0.0 Power Factor Under Excited

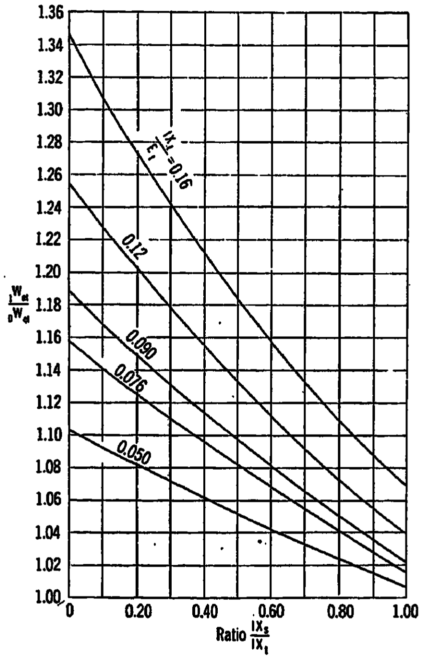


Fig. 22—Ratio of Full Load to No-Load Eddy Current Loss in the Teeth *vs.* Ratio of Slot to Total Reactance, at 0.0 Power Factor Over Excited

These possibilities have been investigated to a certain extent, but the factors which are neglected still are numerous, namely, tooth taper, variable permeability, shorted punchings, punching strains, and a number of sources of eddy currents which become more important as the main densities are increased. It was concluded that for the present, the refinements which have been discussed did not offer sufficient possibilities of improvement over the simplified, or ideal machine, to warrant the increased complexity of calculations which they involve. Nevertheless it is undoubtedly possible to extend this investigation if the proper amount of effort can be put on it.

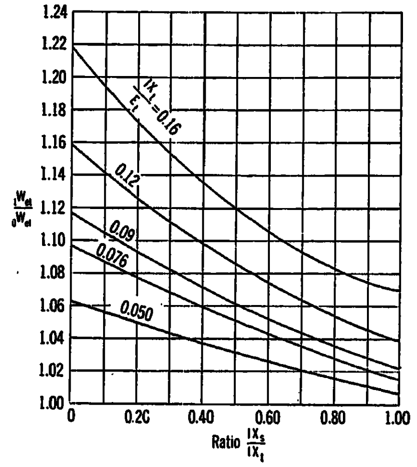


Fig. 23—Ratio of Full Load to No-Load Eddy Current Loss in the Teeth *vs.* Ratio of Slot to Total Reactance, at 0.80 Power Factor Over Excited

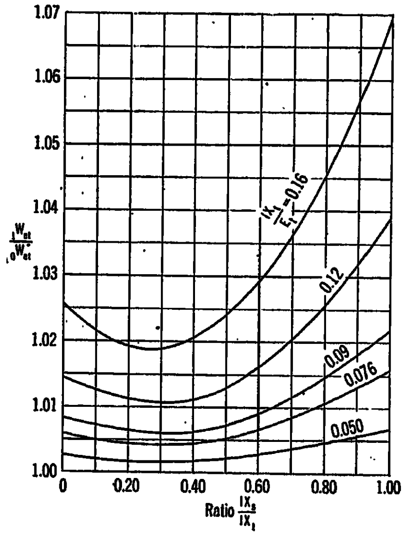


Fig. 24—Ratio of Full Load to No-Load Eddy Current Loss in the Teeth *vs.* Ratio of Slot to Total Reactance, at 1.0 Power Factor

Notation

Flux Symbols

Φ_f = air-gap flux per pole due to the field winding only.

Φ_{dm} = air-gap flux per pole due to the armature winding only.

$_1\Phi_a$ = resultant air-gap flux per pole on load at normal voltage.

$_0\Phi_a$ = air-gap flux per pole on no load at normal voltage.

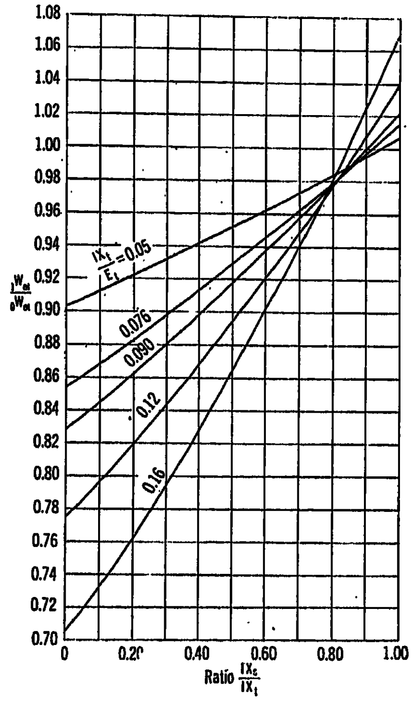


FIG. 25—RATIO OF FULL LOAD TO NO-LOAD EDDY CURRENT LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.0 POWER FACTOR UNDER EXCITED

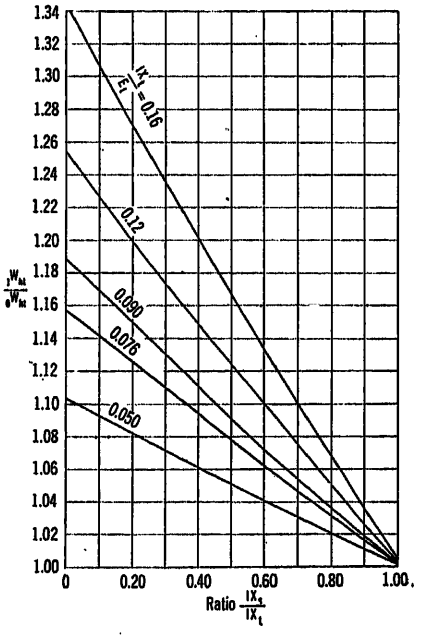


FIG. 26—RATIO OF FULL LOAD TO NO-LOAD HYSTERESIS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.0 POWER FACTOR OVER EXCITED

Φ_s = maximum flux across any one slot due to the armature windings.

φ_s = instantaneous value of Φ_s.

φ_s' = instantaneous value of cross slot leakage flux between any circumferential line $j - j$ in the teeth and the teeth tips. (Fig. 11.)

φ_r = one-half the armature slot leakage flux per pole across the line $j - j$.

$_1\beta_a$ = instantaneous radial density at any point in a tooth due to $_1\Phi_a$.

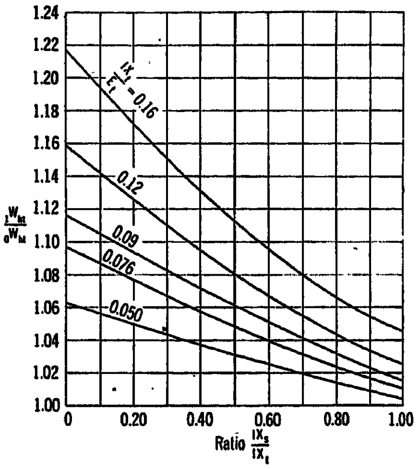


FIG. 27—RATIO OF FULL LOAD TO NO-LOAD HYSTERESIS LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.8 POWER FACTOR OVER EXCITED

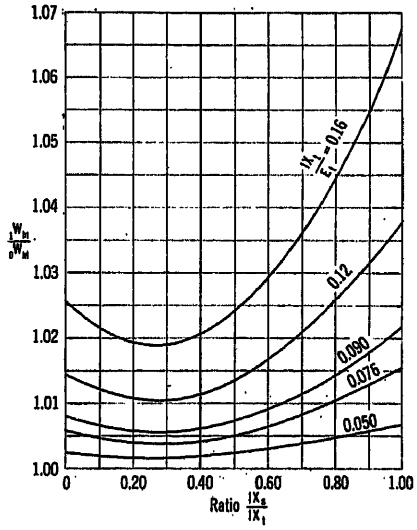


FIG. 28—RATIO OF FULL LOAD TO NO-LOAD HYSTERESIS LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 1.0 POWER FACTOR

$_0\beta_a$ = instantaneous radial density at any point in a tooth due to $_0\Phi_a$.

β_s = instantaneous tangential density at any point in a tooth due to the armature cross slot leakage flux.

β_{sm} = β_s at the tooth tip.

β_r = instantaneous radial density at any point in a tooth due to armature slot leakage flux.

β_R = instantaneous resultant radial density at any point in a tooth.

(β_R = vector sum of ${}_1\beta_a$ and β_r)

β_T = instantaneous resultant density at any point in a tooth.

($\beta_T^2 = \beta_R^2 + \beta_s^2$)

${}_1B_a$, ${}_0B_a$, B_{sm}, B_r, B_R, and B_T are the maximum values of ${}_1\beta_a$, ${}_0\beta_a$, β_{sm}, β_r, β_R, and β_T, respectively.

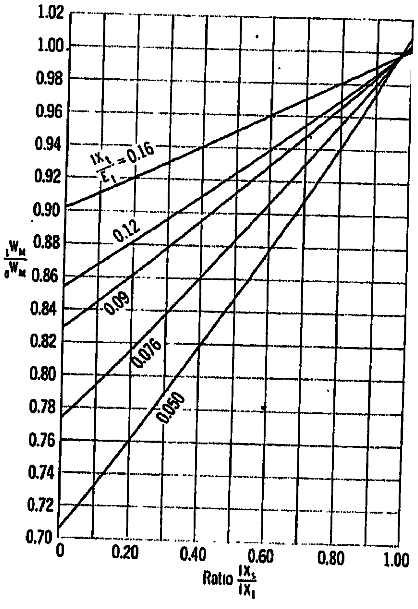


FIG. 29—RATIO OF FULL LOAD TO NO-LOAD HYSTERESIS LOSS IN THE TEETH *vs.* RATIO OF SLOT TO TOTAL REACTANCE, AT 0.0 POWER FACTOR UNDER EXCITED

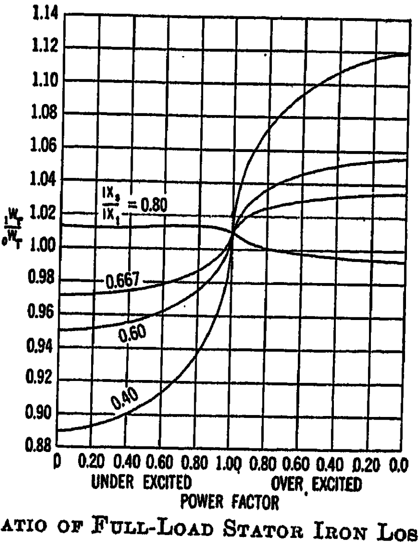


FIG. 30—RATIO OF FULL-LOAD STATOR IRON LOSS *vs.* POWER FACTOR FOR FOUR-POLE TURBINE GENERATOR WITH $\frac{I\,X_t}{E_t} = 0.12$

The radial values are considered positive when outward; and the tangential values positive in the clockwise direction (*i. e.*, opposite to the direction of rotation as later assumed).

Voltage and Current Symbols

(All values are per phase)

E_0 = generated voltage.

E_t = terminal voltage.

I_a = full load armature current.

$I\,X_t$ = total armature leakage reactance.

$I\,X_s$ = armature slot leakage reactance.

Loss Symbols

${}_1W_c$ = total loss in the stator core on load.

${}_0W_c$ = total loss in the stator core on no load.

${}_1W_{et}$ = eddy current loss in the teeth on load.

${}_0W_{et}$ = eddy current loss in the teeth on no load.

${}_1W_{ht}$ = hysteresis loss in the teeth on load.

${}_0W_{ht}$ = hysteresis loss in the teeth on no load.

${}_1W_T$, and ${}_0W_T$ = total iron loss in the machine on load and on no load, respectively.

K_e = rate of eddy current loss in watts per unit volume per unit of mean square density.

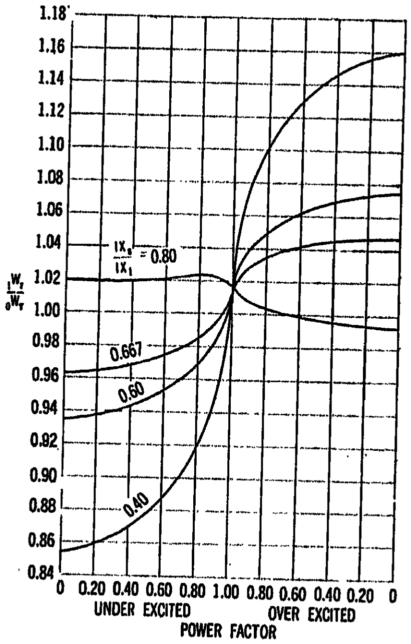


FIG. 31—RATIO OF FULL-LOAD TO NO-LOAD STATOR IRON LOSS *vs.* POWER FACTOR FOR FOUR-POLE TURBINE GENERATOR WITH $\frac{I\,X_t}{E_t} = 0.16$

where IX_s = slot leakage reactance
IX_t = total leakage reactance
E_t = terminal voltage

K_h = hysteresis loss per unit of volume per unit of maximum density squared.

Angular Symbols

θ = angular displacement with respect to some one point on the stator. Counterclockwise rotation is assumed.

(θ = 2 π × frequency × time)

α = angle between armature current and generated voltage, and is assumed positive when the current lags this voltage.

γ = power factor angle and is assumed positive when the current lags the terminal voltage.

λ = a constant angle as defined in Equation (68).

δ = angle by which the armature winding is chorded.

Dimension Symbols and Constants

x = radial distance from the base of the slot toward the tooth tip.

l = total depth of slot.

L = net length of stackings axially.

D = diameter at stator tooth tips.

P = number of pairs of poles.

A_p = total area of cylindrical surface at the tips of the teeth.

A_i = total area of magnetic material in the cylindrical surface at the tips of the teeth.

K_2 = stacking factor.

K_3 = constant for a given machine.

$$\left(K_3 = \frac{A_p}{A_i}\ \frac{2PK_2 l}{D}\right)$$

n = number of turns included in one slot in a distance x from the base.

N = total turns per slot.

K_1 = ratio of flux turn interlinkages per slot ampere which would obtain for any one slot if all the existing slot flux linked all of the turns in the slot, to the flux turn interlinkages per slot ampere which actually exist (on the basis of the current distribution assumed in the body of the article).

V = volume of iron in the teeth.

$M, R, Q, S, A, A_1, A_2, A_3, A_4$, and A_5 are constants which are defined where used.

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1. *Additional Losses in Synchronous Machines*, C. M. Laffoon and J. F. Calvert, A. I. E. E. Trans., Vol. XLV., 1927, p. 84.

2. "A Practical Application of Graphical Flux Mapping," J. F. Calvert, *Electric Jl.*, November, 1927.

3. "Graphical Flux Mapping II, III, IV," J. F. Calvert, and A. M. Harrison, *Electric Jl.*, April, May, and July, 1928.

4. *Induction Motor Core Losses*, P. L. Alger and R. Eksergian, A. I. E. E. Jl., Vol. XXXIX, October 1920, p. 906.

5. "Magnetic Properties of Silicon Steel in Large Transformers," G. H. Cole, *Electric Jl.*, January 1924.

6. "Principles of Alternating Current Machinery," Ralph R. Lawrence, (Second Edition), p. 100.

Discussion

L. A. Doggett: A number of very interesting points has been brought out in this paper. However, the paper does not present any experimental evidence. I realize that the more complicated the problem, the more difficult it is to get that evidence; but at the same time the more complicated the problem the more important it is to substantiate some of these assumptions that had to be made.

J. L. Beaver: I should like to ask the authors how the lines of flux shown in Figs. 2, 3, 4, and 5 were obtained. I am wondering just how accurate the authors feel those apparently assumed lines of flux distributions are. Has any model ever been used with currents? For instance, I was thinking of a closed circuit carrying currents in some sort of a liquid, in which the flow of amperes might be comparable to flux distribution.

L. A. Kilgore: (communicated after adjournment) In this paper, the authors have made a careful study of flux distribution in the machine under load, and of the resulting losses. They have obtained a definite relation between stator iron loss on load and no-load in relatively simple terms involving only slot reactance and total leakage reactance of the machine.

As they have stated, in order to obtain such relations a simplified machine must be assumed and certain other assumptions made in regard to the relation between losses and densities. The use of a simplified machine is justified only after a careful consideration of the factors neglected.

The chief difference in flux distribution from the ideal machine is the concentration of radial slot-leakage flux in one or two slots per phase per pole for a two-layer winding. These differences are illustrated in the paper. It appears that for the two-layer windings considered, the differences are not great. But for a one-conductor-per-slot machine near ⅔ pitch, as used in low-voltage two-pole turbo generators, the flux distribution in the teeth is very different. Due to interleaving of different phase belts in alternate slots, the densities of slot flux going radially are relatively high, and the resultant stator tooth loss is somewhat higher.

The assumption that the hysteresis loss is proportional to the square of the maximum instantaneous density is consistent with test results on rotating machines. This fact appeared to be in disagreement with the observation of physicists that at higher densities, the hysteresis in a rotating field approached zero.

Recently Dr. Rudenberg, in a series of lectures at the Massachusetts Institute of Technology,[1] and also before the Westinghouse engineers at East Pittsburgh, presented the theory of rotational hysteresis and gave a great many experimental data. Simple rotational hysteresis was found to increase more rapidly than alternating hysteresis at low densities, but to drop off rapidly above 12 kilogausses. In the core of a rotating machine, the tangential and radial components of density are unequal and out of phase, resulting in an elliptical field. The combination of rotating and alternating hysteresis gave an increasing loss at much higher densities than a simple rotating field.

The results of these tests seem to bear out quite well the assumption that the hysteresis varies as the square of the maximum density over the range encountered in rotating machines.[1]

J. F. Calvert: In reply to the questions by Professor L. A. Doggett: I do not believe that there is any possible way of directly measuring the variation in iron loss with load on commercially built machines. However, I should like to make a few observations along this line.

By means of a large number of thermocouples, the rate of change of temperature with time could be measured at various points in the iron when there was a sudden (theoretically instantaneous) change in load. From the measured rates of change of temperature with respect to time and the specific heat of the iron, the change of loss incident to the change of load could be computed at each of these points, for it can be shown that at the first instant of time, all the change in loss at a point must be stored. At one time, we seriously considered this method, but some experience with it showed that the time involved in such measurements would be tremendous. We outlined this method, and our efforts, in a paper on *Additional Losses of Synchronous Machines*,[2] presented before the Institute at New York in 1927. There is an article by M. C. Holmes on *Separation of Stray Losses in Turbine Generators* which appeared in the March issue of the A. I. E. E. Journal (p. 224), describing certain tests made by him. Incidentally, these readings of his seem to check our calculations of the axial penetration of flux from the end with satisfactory exactness. (Compare the discussions of Fig. 8 in Mr. Holmes' paper with that of Fig. 4 in the present paper).

Summing up the measurement by means of time-temperature curves, we may say that it was a possible means of check, but was too lengthy a process for us to use on this job.

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[1] The M. I. T. Lectures are to be published at a later date.

[2] A. I. E. E. Trans., Vol. XLVI, 1927, p. 84.

A number of calorimeter tests on turbine generators has been made by measuring the temperature rise and volume of air passing through a machine under steady-load conditions. Since about 98 per cent of the loss occurring within the frame is dissipated in this manner, it follows that very accurate loss measurements may be made. It is our experience to date, based on quite a number of these tests, some at zero power factor and some at 0.80 power factor, that the additional losses on short circuit are almost equal to those at normal voltage for equal values of armature current. This is what one would expect, if there were little or no change in iron loss with load when operating at normal voltage, and calculations by the method under question indicate just that.

However, the analysis given in this paper is one of simple character, which can be checked step by step. If there are no errors in the analysis, then the results must be as accurate as the assumptions. Assumptions 1, 2, and 3 involve only geometrical considerations and their import may be readily considered and checked. Assumptions 4 and 5 are in agreement with most Epstein tests on commercial iron at the densities used in turbine generators. Bibliography references 4 and 5 add some weight to the latter two points. Assumption 6 is unquestionably an approximation, but if there is little change in core flux between no-load normal voltage and full-load normal voltage, the change in distribution must be slight. This gives some fairly substantial evidence to validate our analysis. We recognize that it is a first step toward the solution of this problem and have called attention to this fact in the synopsis to the paper.

In reply to Professor J. L. Beaver, the basis for the flux distributions indicated in Figs. 1, 2, and 3 are described in detail in Bibliography references 1 and 3. These flux distributions can and are arrived at by purely analytical methods. However, in the case of the end-bell fields, they were first solved experimenally by means of magnetic compasses and iron filings. The use of electrical flow in metal templets to solve flux problems by analogy was described in 1915, in a paper by Professor John F. H. Douglas on *The Reluctance of Some Irregular Magnetic Fields*, A. I. E. E. TRANS., Vol. XXIV, Part 1, pp. 1067-1134.

Professor Douglas gives by far the most complete Bibliography on field mapping we know of for articles prior to 1915. We have attempted to add to this for the time since 1915 in the series of articles mentioned in our Bibliography, reference 3.

Figs. 4 and 5 are not intended to be exact, though very considerable efforts were made to achieve accuracy for the calculations, indicated by Fig. 4 and the discussion of that sketch.

In general, I may say that our Bibliography was intended to give the basis for our assumptions and to justify our flux analysis. We realize that we should have given specific bibliographical references in the body of the paper to cover any item which might be open to question.

The Fabrication of Large Rotating Machinery*

BY H. V. PUTMAN[1]

Associate, A. I. E. E.

THE manufacture of machinery is undergoing a change—probably the most significant in history. From very early times man has moulded metal into the things he desired to make. Once moulded, changes in surface or outline were difficult and expensive. And it was the need to simplify this part of the process that led to the invention and development of modern machine tools. Thus, the conventional manufacturing process of pattern making, moulding, and machining was established.

Now this fundamental method of manufacture is changing. Man has learned to handle steel much as a carpenter handles wood. With the gas torch he can take it apart and with the electric arc he can put it together in almost any shape or form desired, eliminating the pattern making and moulding entirely. That it can be done this way more economically than the old way, is due largely to the low cost of steel sheets, bars, and shapes which form the raw material. Strangely enough, it was the electrical industry which contributed most to the development of the steel industry which has made low cost steel possible. And now the low cost of steel is primarily responsible for the radical change taking place in the manufacture of electrical machinery.

Like other manufacturers, the Westinghouse Company has been using structural steel in parts of its produce for many years. In fact, the use of rolled steel in d-c. motor frames by this company, dates back sixteen years or more—antedating even the general use of electric arc welding with the metallic electrode. But the general application of fabricated steel in place of castings in the construction of our larger rotating machinery, is new within the last few years, and the change only now nearing completion.

Before adopting so radical a change as the substitution of fabricated steel in place of castings, it was necessary to make a careful study to determine what economic advantages would be likely to result. Such a change meant the complete redesign of all lines of large rotating apparatus and a complete reorganization of manufacturing facilities. Obviously, these changes involved large expenditures both in the engineering department and the factory, and the decision in favor of such a change was made only after a careful comparison of the cost with the resulting advantages.

In many cases it was found that the use of structural steel permitted little, if any, saving over a good cast design, made with good foundry equipment and machined on modern machine tools. This conclusion cannot, however, be applied generally. Some types of structures showed very appreciable savings while others, especially small ones having high activity, were found to be cheaper when cast.

Even at the same cost, however, the use of fabricated steel may be justified for the following reasons:

1. The possibilities of future cost reduction are greater because the fabrication of steel is a comparatively new art while the cost reduction possibilities of the cast construction have been largely exploited.

2. The elimination of patterns is a clear gain. This is of greatest value in large special machines where only a single unit is made. It applies also to large bedplates where there are few exact duplicates and where, with castings, continual pattern changing was necessary.

3. The fabricated steel design is flexible. A large number of different structures can be made from a limited number of parts by welding them together in different combinations. Minor changes are easily made. Greater strength may be had when required by adding additional structural members.

4. Shorter deliveries are possible with the fabricated steel construction. With the foundry and the pattern problem out of the picture, a shorter and more direct manufacturing sequency is possible. In fact, the whole process can be done in the same shop. Greater concentration of responsibility in the manufacturing organization can be had and this means greater efficiency and quicker production.

5. The saving in weight due to the use of fabricated steel is sometimes a very valuable feature. This is especially true in the case of turbo generators where the saving in weight is greatest. Previous to the use of fabricated steel, a 25,000-kv-a., 4-pole generator had to be shipped disassembled and the armature laminations and coils assembled in the customer's plant. In a recent case, it was possible to ship assembled a 100,000-kv-a. machine with the armature iron and coils in place. This also makes it possible to test these large machines for losses before being shipped, a highly desirable feature.

The saving in weight is also desirable for all machinery used on shipboard. Usually, there is a penalty and bonus on weight written into contracts for such equipment.

6. Fabricated steel structures are often better

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*This paper is not intended as a general treatment of the above subject. It presents some of the more interesting experiences of the company with which the author is connected, in changing their designs of rotating machinery to employ fabricated steel in place of castings.

[1] Section Engineer, Synchronous Motor Section, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

structures than were the cast. There are many sales arguments to be advanced in their favor and they are being demanded more and more by the public.

Most of these points were listed in the beginning as economic advantages of fabricated steel before the decision to adopt it was made. Since then, the truth of these statements has been demonstrated many times.

Already, developments in the arts of gas cutting and arc welding are taking place, which improve the quality and reduce the cost. New machines, comparable with our best machine tools, are being built to facilitate the practical application of these arts to quantity production.

The truth of the second and fourth points mentioned above was recently demonstrated in connection with an order for two 5000-hp., 82-rev. per min. horizontal synchronous motors for steel mill drive. These machines were urgently needed, so that short delivery was of utmost importance. Accordingly, a delivery of 90 days was guaranteed on the first machine and the second machine was to be delivered two weeks later. These motors are among the largest synchronous motors, in physical dimensions, ever built for an industrial drive. (See Fig. 1.) These were entirely new, new dies being required for both the armature and pole laminations. Steel had to be ordered special from the mills for the stator frame and for the spider. Incidentally, the slabs for the spider rim, because of their size and unusual length had to be rolled special. When these facts are considered, one may well wonder how such a short delivery is possible.

The first motor was turned over to the test department exactly 82 days after receipt of order, for complete engineering tests. On the 90th day five cars, bearing the first motor, left the works for the customer's plant. Eleven days later another five cars rolled out of the works with the second motor.

This production record was made possible through the use of fabricated steel. There were no patterns to make. The whole manufacturing process took place in one factory under the responsibility of one manufacturing department. A production schedule was formulated and carried out in an orderly fashion. The manufacture of other machines was not interrupted or delayed. A completely equipped fabrication shop was, of course, a necessity, and was available for handling this job.

Some of the special problems encountered by the engineering department in redesigning the various lines of rotating machines are of interest.

One of the greatest problems was to get designers to think clearly and with an open and unprejudiced mind about the fundamental design problems of fabricated steel. Incidentally, much new thinking was done about the old designs in cast iron, and it was discovered cast structures were designed the way they were designed, not because of the requirements of the structure or the machine, but because of the requirements of foundry practise or the machine tools. Such a conclusion seems so obvious as to be almost axiomatic, but a consideration of it helped greatly to establish a new point of view or basis of thinking for our new design work in fabricated steel.

It was determined that all knowledge of cast design should be put aside and even the appearance forgotten in an effort to make new designs with open, unbiased minds. The requirements of each machine or structure should be determined, then a study should be made of the structural material, plates, bars, and beams to see how these materials could be best employed to meet these requirements. Sometimes our knowledge of gas cutting or welded joints was found to be inadequate, and it was necessary to call upon the research laboratory for definite information backed up by actual experiments and tests.

There had to be a definite reason for every step taken in our design work. What was the result? We found our new designs radically different from cast structures and usually much simpler in construction. Any savings were found to depend but little on reductions in weight, but very materially on the simplicity of the design, and its general suitability to the use of structural materials. Structural materials are partially finished materials and a design to be suitable for their use should be so made as to use the rolled surfaces of the plates and bars as finished surfaces in the completed structure. This means that much of the machine work required on cast structures can be eliminated when the fabricated construction is used—provided the design is properly made.

The design of the different lines of fabricated machines is now completed, at least to the point of determining the types of the structures, and it is interesting to see how this point of view has been followed out in the different lines.

The frame construction of the smaller synchronous motors and generators having the core stacked on bolts between two frame rings gas cut from steel plate and left open in the back is now well known. It is a radical departure from the old cast frames in appearance, but it is adapted to the use of structural materials and it fulfills all the requirements of the electrical machine even better than did the cast construction. Fig. 2 shows a typical synchronous motor having this frame construction.

Even the use of the bolt for holding the laminations instead of the old fashioned dovetail was adopted because it was more suited to the fabricated construction. At first it was thought the use of the bolt for holding the core was adapted only to machines with narrow cores, but it was gradually extended to wider cores until now it is used exclusively. The 220-in. core of the 100,000-kv-a. Hell Gate turbo generator was stacked on bolts. Figs. 3 and 4 show pictures of this stator.

In the water-wheel generators the upper and lower brackets are of special interest because of their radical departure from similar cast structures and their suitability to structural materials. Figs. 5 and 6 show these brackets and Fig. 7 a complete vertical machine.

Another excellent example is the frame construction of the new line of 3600 rev. per min. turbo generators. (See Figs. 8 and 9.) It is built up of plates spaced periodically along the core. These plates support the frame bolts on which the punchings are stacked and at the same time provide the necessary air passages for the re-entrant ventilating system. Except for drilling the end plates, which receive the punching bolts, the whole structure is produced by the torch, arc welder, and the bending rolls and presses in the structural shop. A plate on the back serves as a cover to the air passages and extends over the armature winding on each end where it meets the pressed steel end bells that are bolted to it. This structure is peculiar in that, while it is especially adapted to fabricated steel construction, it can hardly be distinguished from the previous cast structures.

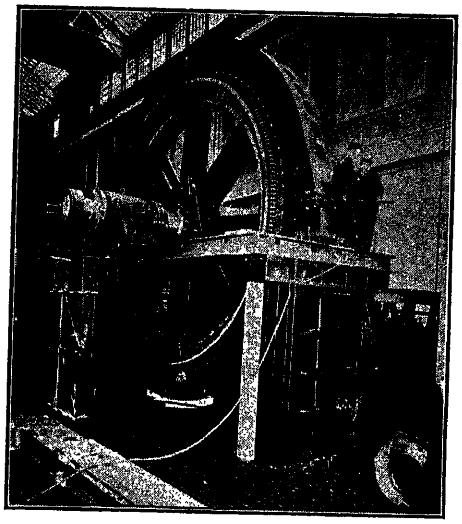

Fig. 1—5000-Hp., 82-Rev. Per Min. Synchronous Motor for Steel Mill Drive Assembled for Test

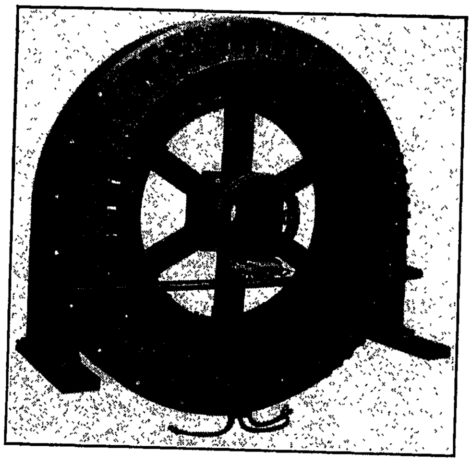

Fig. 2—Typical Low Speed Synchronous Motor Stator with Fabricated Rotor in Place

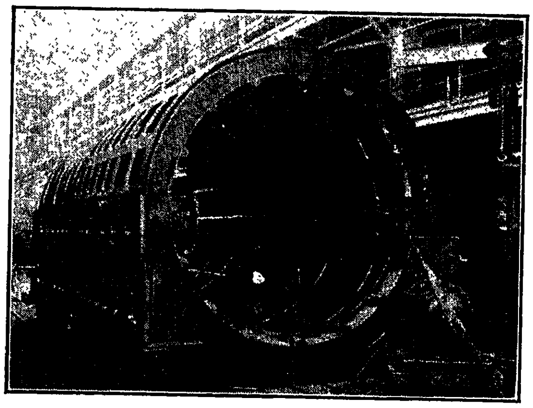

Fig. 3—100,000 Kv-a. Turbo Frame Showing Fabricated Skeleton Construction With Outside Lagging Removed

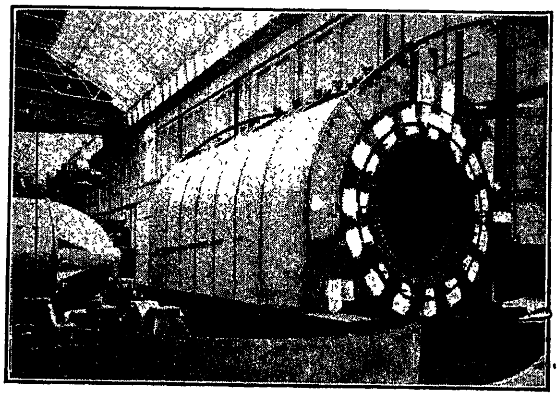

Fig. 4—100,000 Kv-a. Turbo Generator Frame with Core Assembled and Lagging Partially in Place

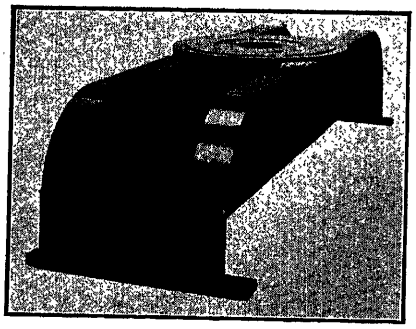

Fig. 5—Typical Fabricated Upper Bracket for Vertical Waterwheel Generators

Other examples might be mentioned but one of especial interest is the fabricated commutator spider for large high-speed commutators developed as a part of the general redesign of the d-c. machines. The special feature in this construction is the support of the front V ring which provides flexibility in the direction of the shaft. The space between the V rings can thus adjust itself to suit the expansion and contraction of the commutator bars. Tests on this construction have proved its superiority over the cast construction from the standpoint of performance. An idea of the construction can be gained from Fig. 10.

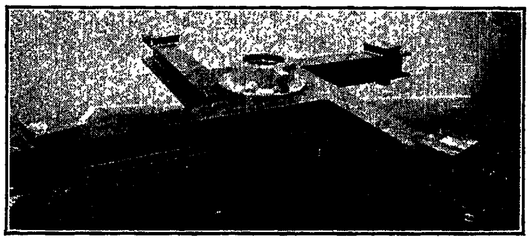

Fig. 6—Typical Lower Bracket for Vertical Water-Wheel Generator

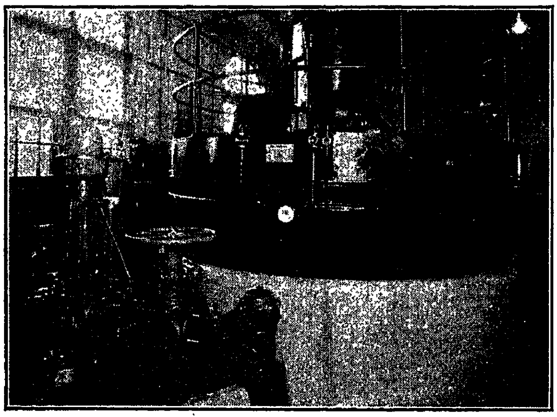

Fig. 7—Typical Vertical Waterwheel Generator, 625-Kv-a., 2400-Volt, Three-Phase, 60-Cycle 200-Rev. Per Min.

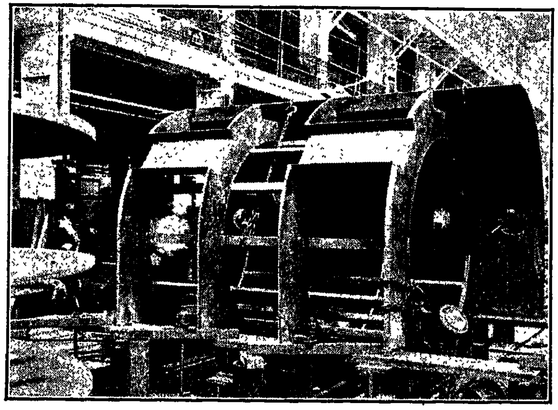

Fig. 8—9500-Kv-a., Turbo Generator Showing Fabricated Frame Construction

Another problem of the engineering department has been the "appearance problem." While some of the fabricated machines resemble closely their cast iron predecessors, many of them are radically different in appearance. It is a fact that what looks good to one is simply a matter of what he is accustomed to. If one is accustomed to looking at cast machines they look best, but as fabricated machines become more familiar they improve in appearance. What appears radical today seems commonplace tomorrow.

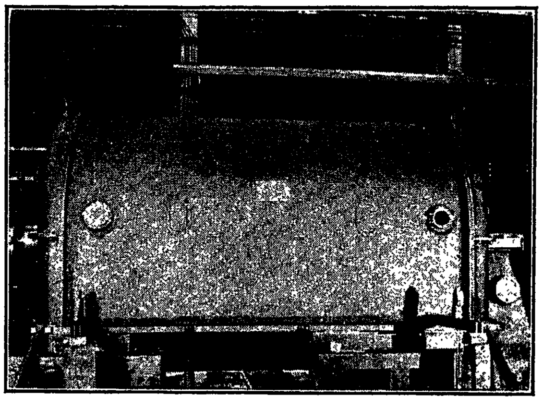

Fig. 9—9500-Kv-a. Turbo Generator Assemblor for Test

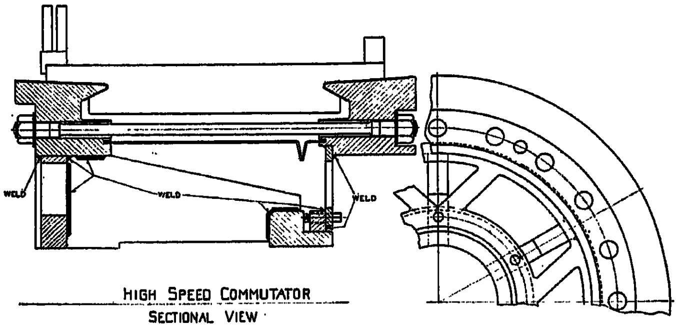

Fig. 10—Sketch Showing Construction of Fabricated Commutator Spider for Large High Speed Commutator

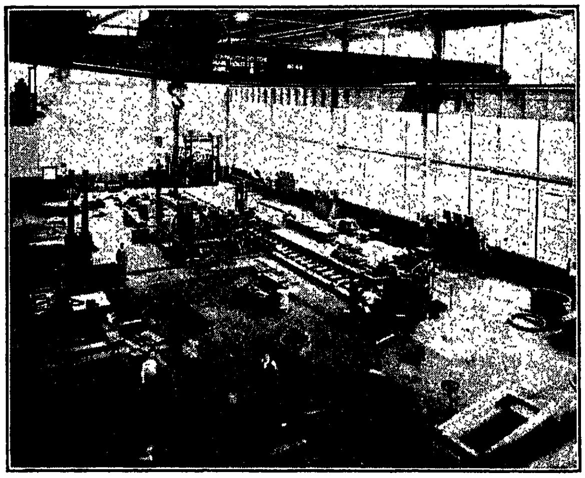

Fig. 11—General View of New Fabrication Shop

This was especially true in the case of fabricated H beam bed-plates. They produced considerable unfavorable comment at first, because of their appearance, and some customers actually refused to have them. But it is a fact that H beam bed-plates are really better bases than cast bed-plates. The U section of the cast bed-plates, with its smoothly rounded corners, was not dictated by any requirement of the structure, but they were made that way so the pattern could be pulled more easily from the sand. The idea of grouting bed-plates into the top originated with the fabricated bed-plate and when so grouted, its appearance is equally as good as that of the cast bed-plate. It is just as natural to grout a fabricated bed-plate into the top, as to set a cast base above the floor line. Architects are realizing this, and are designing the floors of power houses and motor rooms with recesses to receive the bed-plates, so that after the grouting is done, the floor covering can be placed flush with the top of the bed-plates. And today, no one hears a word of criticism regarding the appearance of fabricated bed-plates, and very little indeed, regarding the appearance of fabricated machines.

Round corners and sweeping curves are descriptive of cast structures and cast machines. Flat surfaces, square corners, and abrupt lines are equally descriptive of fabricated machines. It is only a matter of time when they will appear equally attractive. In fact, the straight line and the square corner have already come into prominence in the design of modernistic furniture, lighting fixtures, and decorations.

The major problem of the manufacturing department was the construction of a new fabrication shop, so that a brief description of the building and equipment is included. Fig. 11 gives a general view of the shop. The largest piece of equipment is a vertical bending roll for rolling into circles the frames of d-c. machines and spider rims of a-c. machines.

There is also a 1000-ton crimping press for putting an initial bend in the ends of the slabs to facilitate starting them in the rolls. A large furnace which will take slabs 37 ft. long, is provided for heating the slabs preliminary to rolling, and also, for annealing of completed structures. An approach table is provided for carrying the slabs from the furnace, through the crimper and to the bending rolls. The furnace, approach table, crimper, and bending rolls are all controlled from a single pulpit located near the rolling mill.

Another major piece of equipment is a flanging press used for forming the pressed steel end bells for turbo generators. This press has two vertical rams on top of 100 tons capacity each, a smaller horizontal ram, and a lower vertical stripper ram. The end bells shown on the turbo generator in Fig. 9 were made on this press.

One of the most interesting pieces of equipment is the shot blast room. This is a large room capable of receiving the largest completed fabricated structures. It is entirely closed and lined with steel plate. The operator is dressed in a suit and helmet which resembles that of a diver. The floor is a steel grating. The shot drops through the floor and into a hopper where it is picked up pneumatically and recirculated.

Here all fabricated structures are shot blasted before going to the assembly section, every bit of rust and scale is removed so that the surface resembles that of cast aluminum. When the paint is applied to this surface, there is no danger of its chipping, or scaling off, because it has been applied directly on the mother metal—to a surface 100 per cent clean.

Discussion

J. B. Silverman: With reference to these large machines, I wonder if a saving in weight is accomplished in the increased permeability of the material used, that is the structural steel instead of cast iron.

L. L. Bosch: I wonder whether or not pockets are formed in the spokes of the large fabricated machinery, which cause more loss due to air friction.

J. F. Lincoln: (communicated after adjournment) Mr. Putman makes the statement that the application of welding to small machinery does not show a saving, which I am sure he will find with further experience is not true. As a matter of fact the percentage of saving is probably greater when made in the proper way than the saving shown by Mr. Putman in his large application, the reason being that with the design shown by Mr. Putman much gas cutting is necessary, together with the resultant scrap, while in small machinery means have been devised to eliminate all scrap and all gas cutting.

The point that Mr. Putman makes that the hardest job that he had was to get his organization to think in new terms of arc welding, is undoubtedly the keynote to most of the hesitation in adopting this new method. It is necessary if we are to use this new tool, to do new thinking. This new thinking is not difficult, —it is only different.

H. V. Putman: The greater strength of fabricated steel, rather than its greater permeability, has been responsible for the decreases in weight which have been made in changing to fabricated structures. The frames of d-c. machines and the spiders of low-speed a-c. machines have always been cast steel which has a permeability fairly high compared with that of rolled steel. There has been no marked reduction in the weight of these parts.

The use of H-beam spokes in fabricated spiders does increase the windage loss slightly, but we have found it unnecessary to use blowers with such spiders to secure proper ventilation. This increase in windage is probably not as much as would be produced by the addition of blowers to an ordinary spider. Any construction which improves ventilation without increasing cost is, of course, a decided advantage from the design standpoint.

I hope Mr. Lincoln is right in his statement that small fabricated structures can be made to compete in cost with castings. Perhaps we shall find that to be true. It is difficult to make definite statements regarding comparative costs of structures which will hold true for more than one company. What is true in one company may be quite the opposite in another. We have small castings weighing only a pound or two which we produce at about the same price per pound as castings weighing several thousand pounds. We made fabricated steel brush holders for years and they were cheap; but today we produce die-cast holders at a much lower cost.

In conclusion, I should like to say just a word about reliability in connection with fabricated structures. Inspection of fabricated structures is probably not as easy as the inspection of cast structures, because of the difficulty of inspecting the welds. Much depends upon the welder. We take every precaution to make certain that our welders are able to do the work and to do it correctly. They have been trained in our own welding school, and are required to pass rigid tests before they are permitted to weld on our machinery. Each welder is required to stamp every weld he makes with his number so that it is possible to identify every weld on any machine in the shop. Should visual inspection reveal any defect in a weld, it can be traced back to the man who made it. We also occasionally inspect each operator's welding by chipping out a section for examination. This has a tendency to maintain a high quality of work on the part of each individual welder.

Electrical Equipment of Bar Plate
and Hot Strip Mills

BY J. B. INK[1]

Member, A. I. E. E.

THE purpose of this paper is to describe the electrical features of the continuous bar plate and hot strip mill recently placed in service by the American Rolling Mill Company at Middletown, Ohio.

Fig. 1 shows general arrangement of mill. Single line diagram, Fig. 2, shows the principal electrical connections.

The new mill produces ingot iron and various grades of steel strip in gages down to 0.093 in. and widths up to 48 in., also plates ¼ in. thick and 60 in. wide in 75 ft. lengths. These products are rolled from 6 in. thick slabs, 39 in. long, and of the width required in the finished product. For ease in handling and storing, the strip steel is coiled on leaving the last stand of the hot strip mill.

The mill consists of 11 stands. The first 7 constitute the "bar mill" and normally reduce the sheet to about ⅜ in. thickness. The last 4 stands are the "hot strip mill." Between stands 7 and 8 are a runout table, transfer, and bar piler for taking off sheet bar or other heavy gage product.

The initial length of slab is 39 in. when it enters No. 1 stand. This length of course increases as the piece progresses through the successive stands. When the length has increased to about 20 ft., as it does in No. 4 stand, it is about equal to the minimum spacing between stands possible with this type mill. Thus, the following stands, to be economically spaced both for conservation of floor area and heat, must be on centers less than the length of the piece, with the result that the piece, after leaving No. 4, is in two or more stands at the same time. This requires very close control of speed, as any change in speed or draft in one mill must be accomplished by a corresponding change in the other stands in the train. In rolling wide strip, wearing of rolls makes frequent changes in draft necessary.

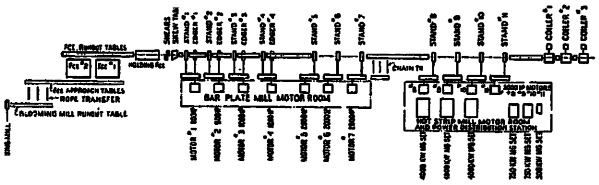

FIG. 1—GENERAL ARRANGEMENT OF BAR PLATE AND HOT STRIP MILLS

From the above it follows that stands No. 1 to No. 4 are independent as regards speed while No. 5 to No. 7 and No. 8 to No. 11 are inter-dependent.

The independent stands No. 1 to No. 4 are each driven by a wound secondary induction motor and are equipped with fly-wheels. The motor horsepower varies from ¼ to ½ the total rolling horsepower, the balance of the rolling energy being taken from the fly-wheel. The output of the motor is limited by automatic liquid slip regulators which introduce resistance in motor secondary as the motor primary current increases and so permits the fly-wheel to give up stored energy.

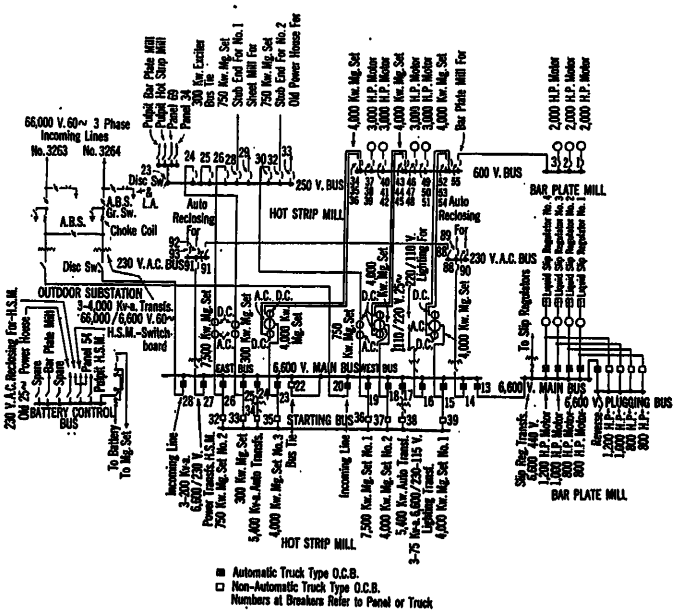

FIG. 2—DIAGRAM OF ELECTRICAL CIRCUITS IN ROLLING MILL

The characteristics of the first 4 stands are as follows:

Motor	Hp.	Speed Syn-chronism	Fly-wheel hp. sec.	Gear ratio	Mill rev. per min.	Roll diam-eter	Roll ft./min.
1	800	514	30,000	35:1	14.42	32 in.	121
2	800	514	30,000	35:1	14.42	32 in.	121
3	1000	600	40,000	29:1	20.18	32 in.	169
4	1200	720	40,000	29:1	24.3	32 in.	203.5

Motors driving stands 1 to 4 and their control are housed in a ventilated brick and steel motor room. They are equipped with thermocouples in primary windings for checking temperature, thermostatic relays in bearings, and space heaters to prevent "sweating" during shutdown. Primary winding is mica insulated and coil ends are braced to insulated steel ring supported from stator frame to resist shocks incident to the service.

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[1] American Rolling Mills Company, Middletown, Ohio.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

The rating and performance data of these motors follow:

All—6600-volt, 3-phase, 60-cycle, 40 deg. cent. rise.

Motor	Hp.	Poles	Efficiency			Power Factor			Per cent slip full load	Max. torque times full load
			½	¾	Full load	½	¾	Full load		
Nos. 1 and 2	800	14	90.8	92.8	93.25	67.8	78.4	82.9	1.13	2.9
No. 3	1000	12	92.4	93.6	94.	74.2	82.3	85.7	1.167	2.62
No. 4	1200	10	91.3	93.	93.6	78.6	85.8	88.9	1.	2.92

These motors are controlled from bar plate mill pulpit, which is similar to hot strip mill pulpit shown in Fig. 3.

Motor primary is connected to main bus by Westinghouse type OE6 oil circuit breaker. For plugging there is provided a Westinghouse type B2 breaker which connects the motor to reverse bus. The reverse bus is connected to main bus by an OE6 breaker. The forward breaker and reverse bus breaker are automatic on overload and undervoltage. Thus one automatic breaker gives undervoltage and overload reverse protection to the four motors.

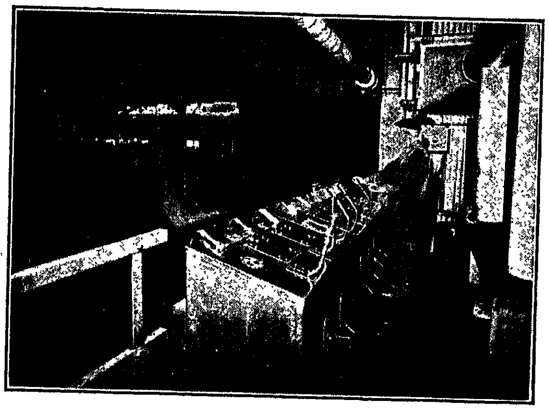

Fig. 3—Control Pulpit, Hot Strip Mill

This pulpit controls four 3000-hp. adjustable speed d-c. motors and approach and run-out tables for this mill.

Oil circuit breakers are truck type having all hot parts enclosed. Forward and reverse breakers are interlocked electrically and mechanically to prevent simultaneous closing.

The motor secondaries are connected to the bottom electrodes of liquid slip regulators, a form of water rheostat in which the electrolite is a solution of sodium carbonate. Three top movable electrodes are connected together and the distance between top and bottom electrodes is varied by the action of a torque motor energized from current transformers in the motor primary circuit. The torque of the torque motor is proportional to the square of the primary current so that it is very sensitive to change in load. When the load exceeds a pre-determined amount fixed by counterweights, the torque motor raises the electrodes, thus increasing the secondary resistance and reducing torque due to primary current to a point where the balance between weight of electrodes and motor torque is re-established. Fig. 4 shows a graphic wattmeter record of input to the a-c. motors. Superimposed upon these curves of motor input are shown the actual rolling hp. Note that the motor furnishes directly only a small part of the energy used in rolling, the fly-wheel supplying the major portion.

The interdependent stands 5 to 11 inclusive are driven by 600-volt d-c. adjustable speed motors. Due to the necessity for close speed regulation, and the fact that the piece is in the stand a considerable length of time, fly-wheels are impracticable on these stands. The motors, therefore, must provide maximum rolling torque.

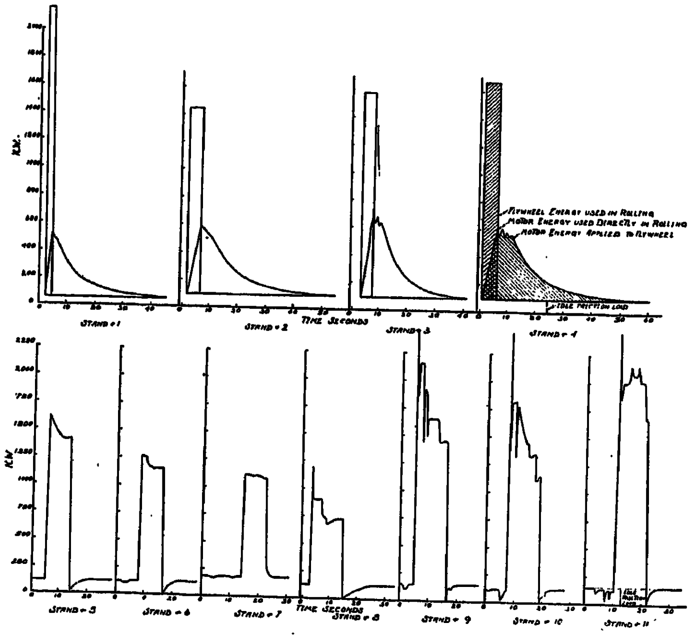

Fig. 4—Input to Mill Motors

Characteristics of stands 5 to 11 are as follows:

All 4-high 56 in., working rolls 18 in. diameter, backing up rolls 36 in. diameter, roller bearings.

Motor	Hp.	Speed	Gear ratio	Mill rev. per min.	Roll ft./min.
5	2000	300–500	8.1 :1	37.1/ 61.70	174.7–291.0
6	2000	300–500	6.00:1	50 / 83.3	235.4–392.0
7	2000	300–500	5.32:1	56.4/ 94.1	266.0–443.0
8	3000	180–360	3.02:1	59.6/119.2	280.5–561.0
9	3000	180–360	2.26:1	79.7/159.4	375 –750.0
10	3000	180–360	1.87:1	96.3/192.6	454. –908.0
11	3000	180–360	1.72:1	104.8/209.6	494 –988.0

The seven d-c. 600-volt motors are heavy duty type with low pedestal bearings and fabricated structural steel bases. The 3000-hp. motors, being very low speed, require forced air ventilation. Motors are equipped with bearing thermostatic relays, over speed devices, and heaters to prevent sweating during shut down. To each shaft is geared a speed indicating magneto. These motors are constant horsepower, speed adjustable by field control. Motors are compounded by use of series exciters. The series exciters consist of generators driven by 5-hp. induction motors. The generator field is excited by ¾ turn of the main motor armature circuit, so that the voltage generated is proportional to the input to the main motor. The generator circuit supplies a series field winding similar to the shunt winding on the main motors. Shunt field excitation is at 250 volts.

Compounding is adjusted to different motor speeds by a rheostat in the series exciter circuit. This rheostat is on the same shaft as the main shunt field rheostat so it is automatically adjusted with the main shunt field.

A speed regulation of ½ per cent was obtained in test of these motors.

To avoid readjustment of rheostat after a shut down a resetting face plate is provided on shunt field rheostat so that by its manipulation full field for starting may be secured and the main rheostat setting not disturbed.

In addition to the main rheostats which are motor operated, a hand operated fine adjustment rheostat is mounted in the operating pulpit.

The d-c. motors are entirely controlled from the operator's pulpit, except that emergency stop control switches are provided in motor rooms. They are started from 600-volt d-c. bus through series resistance and contactors using current limit accelerating relays. Dynamic braking is used in stopping mills.

Motors are protected against overload, low voltage, field failure, and over speed. Each motor control panel is connected to main bus through disconnects so that panels may be isolated for repair without killing main bus.

Rating and performance data on the d-c. motors follow:

All 600 volt—Separately excited at 250 volts, 40 deg. cent. rise.

Horse-power	Poles	Rev. per min.	Efficiency				Efficiency			
				½ load	¾ load	Full load		½ load	¾ load	Full load
2000	10	300–500	At 300 r. p m.	93.2	93.7	93.6	At 500 r. p m.	91.3	92.4	92.6
3000	14	180–360	At 180 r. p m.	92.5	93.85	93.8	At 360 r. p m.	91.9	92.9	93

The mill auxiliaries are with a few exceptions motor operated; 220-volt squirrel-cage induction type motors with magnetic cross line starters are used for all constant speed continuous duty applications such as fans, pumps, and chain tables between mills. These motors are in sizes from 2 to 75 hp. They are standard sleeve bearing motors with drip proof covers. The cross-line starters provide low-voltage protection and have thermal overload relays.

For intermittent duty, on loads having high starting torque and requiring quick acceleration, 250-volt d-c. mill type series or compound motors are used. This includes all cranes, transfer tables, roller tables, except hot strip run-out table; furnace pushers, pinch rolls, and screw downs. The controls for these motors are all of the magnetic definite time limit acceleration type. The control panels excepting for cranes are arranged in groups in dust tight brick rooms. With each group is a spare panel which may be substituted for any control panel in the group by manipulating transfer switches.

Power for operating the new mill is purchased from the local power company at 66,000 volts. Service is over about four miles of double circuit steel tower line from the power company's substation at Trenton, to a step-down transformer station at Middletown, built and owned by the American Rolling Mill Company. The step-down transformer substation consists of two banks of O. I. S. C. transformers 12,000 kv-a. each, connected delta-delta, stepping voltage down to 6600 volts. From the transformers the power is taken 1000 ft. underground on 3-conductor, lead covered, multiple cables to the main 6600-volt distribution bus.

Automatic oil circuit breakers are provided in the line at the 66,000-volt substation at Trenton and at the 6600-volt bus at the mill. Normally a 66,000-volt line, transformer bank, and 6600-volt line form an independent unit. No switching is done at 66,000 volts at the mill. However, disconnects are provided for isolating transformers and lightning arresters and provision is made for adding oil switches when required.

The transformers and lines are of such size that one bank will carry the mill load and the transformer capacity may be increased 50 per cent by adding radiators. Thus even with a large increase in power demand the duplicate sub-station and transmission equipment may be used in case of interrupted service on the line in use.

The main bus for distribution of power at 6600 volts is in two sections. Each section is supplied by one of the transmission units. Feeders are so arranged on the bus sections that one section may be taken out of service for additions or repairs without a complete shut down of the mill. The two bus sections are tied together by a non-automatic oil circuit breaker. The 6600-volt bus and oil circuit breaker structure is of the safety enclosed truck type. Westinghouse type OE6 breakers are used on all main line and feeder circuits. Westinghouse type B2 are used for non-automatic applications such as bus tie, and starting breakers for motor generator sets. A spare truck breaker of each type is provided and may be substituted for any other breaker.

All 6600-volt circuits are protected by induction type overload relays. The two incoming lines are equipped with balanced reverse power and overload relays. These lines are similarly equipped at the power company's substation and have, in addition, ground relays. By means of this system either transmission line is immediately switched off at both ends if it develops any trouble other than a single-phase ground on the 66,000-volt line. In the latter case the circuit is opened automatically at the power company's substation and an alarm is sounded at the mill to notify the operator to open his switch by hand. In the meantime no harm is done as the transformers are delta connected. To simplify the maintenance of relays a test panel is provided with load rheostat, ammeter, and cycle counter. This test panel is wired to a test bus and any relay on the system may be transferred to the test bus by means of a "push-pull" type test switch. The testing of relays is a part of the switchboard operator's routine.

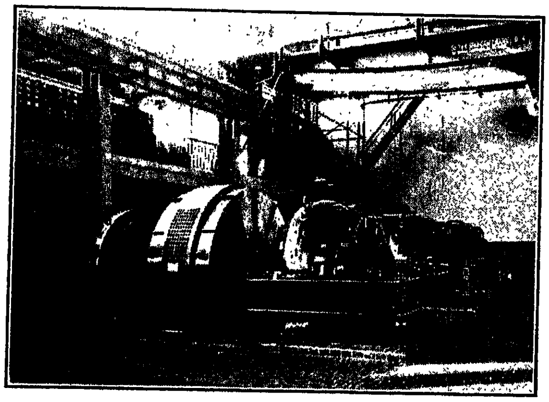

FIG. 5—4000-KW. 600-VOLT D-C. 6600-VOLT A-C. MOTOR GENERATORS

These machines supply d-c. power for the d-c. mill motors

Three 4000-kw. 3-unit synchronous motor driven MG sets supply 600-volt direct current for main mill motors; 250-volt direct current for auxiliary motor circuits is supplied by two 750-kw. MG sets, and 250-volt excitation by a 300-kw. MG set. The excitation bus may also be supplied from the 750-kw. MG sets.

Each 4000-kw., 600-volt motor generator set consists of two 2000-kw., 600-volt generators rigidly coupled to a 5800-hp., 85 per cent power factor, 6600-volt, 60-cycle, 14-pole synchronous motor. One of these units is shown in Fig. 5.

The field of each 2000-kw. generator is arranged for 125-volt excitation. The fields of the two generators are connected in series and excited from a 250-volt constant voltage excitation bus. These generators have straight differential and crossed cumulative series fields assuring equal division of the load between the two generators of the set. The three sets are operated in parallel by the use of double pole equalizers between sets.

The two 750-kw., 250-volt generators are driven by 1080-hp., 900 rev. per min., 80 per cent power factor synchronous motors, and the 300-kw., 250-volt exciter by 432-hp., 1200 rev. per min., 80 per cent power factor 6600-volt synchronous motor. The synchronous motor generator sets are started from reduced voltage supplied by either of two auto transformers each having capacity to start all the MG sets at intervals of 15 sec.

The starting auto transformers are oil insulated self cooled with outdoor tanks and bushings. They are located outdoors and protected top and sides with wire mesh.

Transfer of motors from starting bus to running bus and application of excitation is automatic.

Paralleling the 600-volt d-c. machines with wide differences of voltage is prevented by voltage balance relays.

Rating and performance of motor generators are as follows:

All motors are 6600-volt, 60-cycle, synchronous, 40 deg. cent rise.

	Generator			Motor		Set efficiency		
Kw.	Rev. per min.	Volts	Poles	Hp.	Power factor per cent	½ load	¾ load	Full load
4000	514	600	10	5800	85	86.	88.75	89.7
750	900	250	6	1072	80	83.1	86.4	87.7
300	1200	250	4	432	80	84.5	87	87.5

An unusual feature of the new mill is the equipment for handling the strip as it leaves the mill.

The metal is carried from last stand to coilers by a roller table in which each roll is direct coupled to an individual motor. These motors are 3-phase 88/220 volts, totally enclosed squirrel-cage type operating on variable frequency from 24 to 60 cycles, rated from 0.4 to 1 hp., speed range 190 to 480 rev. per min. The rolls are 10 in. diameter so that the speed of the table is from 500 to 1260 ft. per min. The motors are grouped on three circuits so that one circuit may be opened without interrupting operation of the table. When one circuit is opened every third motor stops, the other two being sufficient to handle the material.

The variable frequency current for runout table is furnished by a 24/60-kv-a., 24/60-cycle, 88/220-volt generator driven by an adjustable speed d-c. motor. The table is started from the 220-volt, 60-cycle mill circuit. With this arrangement it is only necessary to provide a variable frequency motor generator of sufficient capacity to run the table. Since the starting current is four or five times running current and has a low lagging power factor, a much larger generator would be required to maintain voltage in starting than is required for running only.

Three coilers take the strip steel from the last stand of the mill and form it into a coil about 30 in. in diameter. The sheet is at a red heat when it enters the coiler and is cooled by water before being discharged. The coiler is driven by a 75-hp. adjustable speed d-c. motor which drives the coiler rollers through multiple spindle gears. The rollers bear on the surface of the coil and as the coil grows are expanded by a cam driven by a 5-hp. adjustable speed motor. The strip steel is forced into the coiler by a pinch roll. The machine is started automatically when the strip completes a circuit from ground to the surface of an insulated roller.

About 30 sec. time is required to coil, cool, and discharge a strip of 12-gage steel 180 ft. long and 48 in. wide.

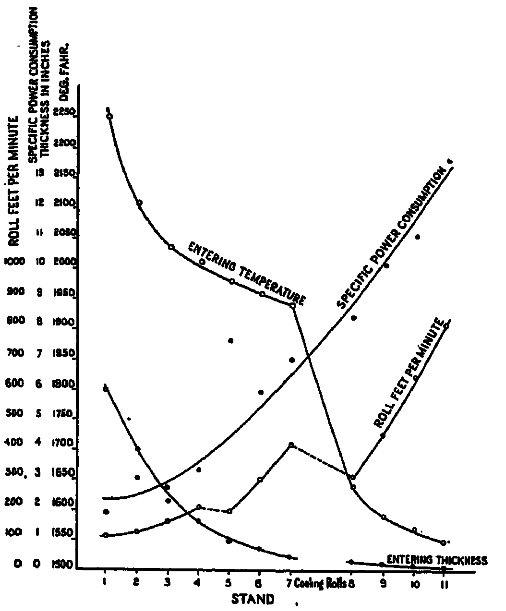

FIG. 6—RELATIVE VALUES AT EACH STAND OF ROLL SPEED, SPECIFIC POWER CONSUMPTION, THICKNESS OF METAL AND ENTERING TEMPERATURE

Specific power consumption equals hp.-sec. per cu. in. of metal displaced

Two hundred and twenty-volt, 3-phase power for auxiliary motors is from two automatic substations located near load centers. Each consists of a bank of three 200 kv-a., 6600/220-volt transformers and two automatic reclosing 1200-ampere, 220-volt, 3-phase feeder panels. A tie connection is run between substations to carry part of substation load in the event of transformer failure.

The 250-volt d-c. system is supplied through automatic reclosing feeders from the 250-volt bus in the power distribution station. This bus is connected by tie lines to two other 250-volt d-c. generating stations in the plant.

Lighting is provided by an overhead system of lights suspended from messenger wire. The messenger wire suspension prevents lamp breakage due to vibration of buildings, and permits arranging units for uniform light distribution.

Illumination values (foot-candles) are as follows:

Bar Plate Mill	3.5
Hot Strip Mill	3.5
Warehouses....	3.
Switchboard and Motor Rooms	6

Lighting service is supplied by a bank of three 75-kv-a. transformers, secondaries of which are connected to provide three 220/110 volt circuits. Wires are run open on specially designed supports mounted on lower chord angles of building roof trusses.

Provision is made for emergency lighting so that in case of failure of the 60-cycle service a system of emergency lights is automatically thrown on to storage battery service until the station operator can transfer the main lighting to a bank of transformers connected to the plant 25-cycle system.

Branch circuits are protected by circuit breakers with direct acting time limit feature.

Lighting fixtures are individually fused and are arranged with a disconnecting device for ease of replacement and maintenance.

Both motor rooms are ventilated with filtered air. In the bar mill motor room the air is delivered by a 57,000-cu. ft. per min., 1¼ in. s. p. fan to the basement and thence to the pits under the motors. The air is filtered by two 30,000 cu. ft. per min. rated, rotary type air filters. The system is so arranged that all or part of the air may be recirculated and also may be heated by passing through a group of unit type steam heaters.

The hot strip motor room is ventilated by two 70,000-cu. ft. per min., 1¼-in. s. p. fans which take care of the heat losses from motor generators, and two 65,000 cu. ft. per min., 3-in. s. p. fans which supply the forced air ventilation for the mill motors in this room. The air is cleaned by six 50,000 cu. ft. per min. rotary type air filters and may be recirculated and heated when necessary.

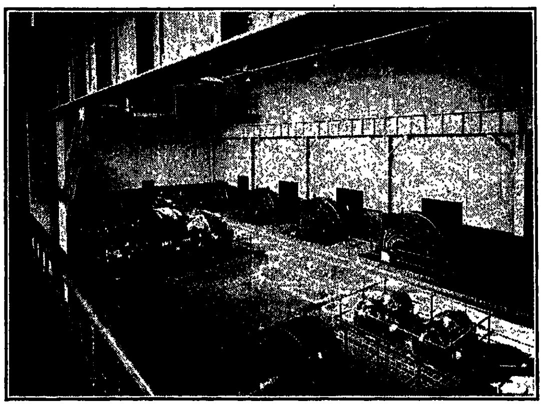

FIG. 7—HOT STRIP MILL MOTOR ROOM

This room also houses the motor-generator sets and main control switchboard

Safety features of the installation include the use of truck type oil circuit breakers in which all hot parts are entirely enclosed in steel housings making accidental contact impossible. The use of truck breakers avoids disconnecting switches with attendant hazards. Guards and barriers are placed over all bare conductors and moving parts of machines that would otherwise be accessible. High-voltage wiring and apparatus such as 66,000-volt substation buses, arresters, and disconnects are made inaccessible by elevation. Rooms and enclosures containing exposed electrical apparatus are kept locked when possible and arranged so that there is no occasion for unauthorized persons to enter.

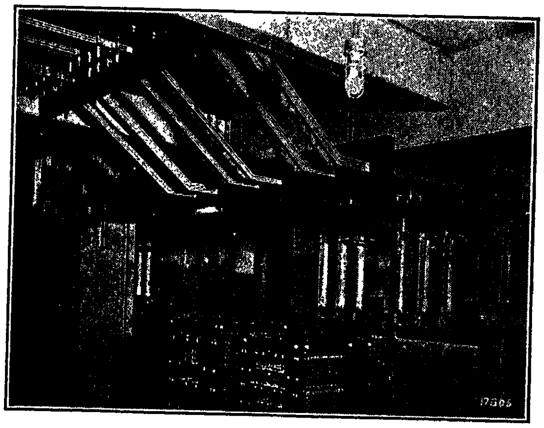

FIG. 8—600-VOLT D-C. ALUMINUM BUS STRUCTURE

In basement of hot strip mill. The supports are ebonized asbestos in welded steel frames

All switches, starters, panels, etc., are clearly marked with distinguishing names to avoid confusion in identification.

Conduit containing high-voltage conductors are painted a distinguishing color.

The grounding system is complete in all parts of the system.

Insulating mats are provided in front and rear of all switchboards.

A safety switch is installed in the leads of all motors, at the motors, so that when repairs or adjustments are made to the machines driven by such motors the switch may be opened and so make it impossible for others to start motor.

The power required to roll steel sheets varies with temperature, per cent reduction, speed of rolling, kind of material, and diameter of roll. It is probable that a large part of the input to the old type mill with brass bearings is consumed in friction in the bearings. This fact is demonstrated in comparative tests made on the four high stands of the mill herein described, which are equipped with roller bearings, and another mill making similar product and having brass bearings. In these tests it was found that the specific power consumption in the brass bearing mills was much higher than in the roller bearing mill. This difference increases as the temperature of the piece falls and bearing pressure increases.

Curves of Fig. 6 illustrate the relative value of different variables at each stand of the mill.

The drop in temperature between stands 7 and 8 represents the critical range of ingot iron, between 1922 deg. fahr. and 1562 deg. fahr., in which range the material may not be rolled.

From the accompanying test data the power consumption per ton of steel rolled is 32 kw-hr., to which must be added the idle friction of mills which make total 40.3 kw. hr. per ton, with a production of 150 tons per hr. To this must be added the power consumption of various mill auxiliaries amounting to about 800 kw-hr. per hour, making a total of 45.6 kw-hr. per ton from blooming mill runout to coil storage.

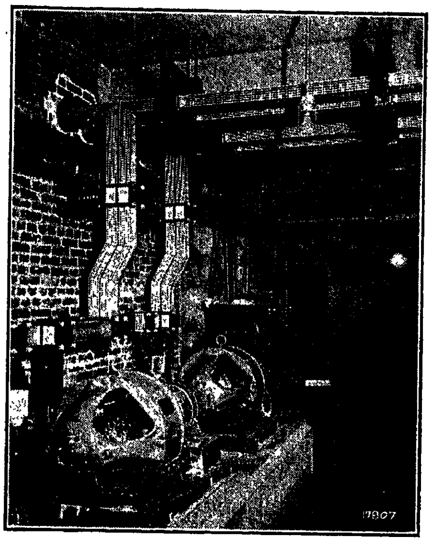

FIG. 9—SERIES EXCITER SET AND 600-VOLT D-C. ALUMINUM BUS STRUCTURE

Discussion

J. W. Bates: There is one question I should like to ask Mr. Gage. Referring to Fig. 6, in regard to the specific power consumption, the increase in the specific power consumption is evidently due to the combined effect of the lower metal temperature and the higher rolling speed. I should like to know if any data were obtained to enable us to know just how much of this increase in power consumption is due to the high rolling speed and how much of it is due to the lower metal temperature.

Referring further to the figure on power consumption of 45.6 kw-hr. per ton, I should like to know if this represents test or operating results and if it pertains to the material mentioned further in the paper, that is, 12-gage sheets, 48 in. wide.

J. B. Ink: Referring to Mr. Bates' question regarding the factors that determine specific power consumption. This power is consumed in fluid friction in the steel and bearing friction in the mill. In the modern mill with roller bearings such as the last seven stands of the mill under discussion, the losses in mill bearings is probably a small percentage of the total. The fluid friction, according to well known laws, varies with the viscosity and is proportioned to the square of the velocity.

In the range of temperature shown, the change in viscosity is not great so we may suspect, until test data upon which to base conclusions are available, that speed of rolling is the predominant factor in fixing the specific power and that this varies somewhere between the first and second power of the rolling speed.

A New Type of Hot Cathode Oscillograph and its Application to the Automatic Recording of Lightning and Switching Surges

BY R. H. GEORGE*

Associate, A. I. E. E.

Synopsis.—This paper presents a new general purpose type of hot cathode oscillograph, which employs a new electrostatic method of focusing the beam. This oscillograph will operate at any beam potential from 500 to 20,000 volts or more, and at any gas pressure below 30 microns. High photographic sensitivity at medium voltages is attained by the use of a high intensity beam.

A portable form of the oscillograph for recording lightning surges on transmission lines is described, together with circuits by means of which the lightning surge automatically starts the beam in from ½ to ¼ microseconds after the surge voltage begins to rise from zero. This oscillograph was put into operation on one of the 140-kv. transmission lines of the Consumers Power Company of Jackson, Michigan, on August 27, 1928.

* * * * *

INTRODUCTION

With the interconnection of large power systems, and with the rapid developments in the art of radio and carrier current communication, have come new problems many of which involve the study of high frequency phenomena beyond the range of the ordinary Duddell oscillograph. Consequently the importance of solving such problems has stimulated active interest in the development and application of the cathode ray oscillograph,† a device capable of recording extremely high frequency phenomena. As a result there have been developed, within the last decade, some three or possibly four general types of cathode ray oscillographs as follows: the low voltage hot cathode type of the Western Electric Company,[1] the medium voltage hot cathode type of Wood,[2] the high-voltage cold cathode type of Dufour,[3] Norinder,[4] the General Electric Company,[5] and more recently the high voltage hot cathode type of Rogowski.[6]

This paper presents a new general purpose type of hot cathode oscillograph, capable of operating at any potential from 500 volts to 20,000 volts or more, and a special portable form for lightning recording. The object in developing the general purpose oscillograph was to produce one having sufficient flexibility to combine, as far as possible, the desirable qualities of the previous types of cathode ray oscillograph, and yet be simple and reliable in operation.

SALIENT FEATURES OF THE NEW OSCILLOGRAPH

Some of the outstanding features of the new oscillograph are as follows:

1. The use of a special hot cathode electron gun which makes possible automatic starting and stopping of the beam for recording lightning surges on transmission lines.
2. The use of a new electrostatic method of focusing the beam, effective at any pressure from about 30 microns down to the lowest pressure obtainable with a mercury pump.
3. The entire beam passes through the high voltage anode, thus eliminating the problem of anode heating.
4. Bakelite is used for insulating the high voltage cathode and the deflecting plates, thus reducing the danger of breakage.
5. The deflecting plates are adjustable from outside the vacuum which makes it possible to vary the deflection sensitivity if desired.
6. All parts are readily accessible and easily replaced in case of damage.

General. The method of attack in the design and development of the general purpose oscillograph has been to gain the necessary photographic sensitivity through the use of a high intensity beam at a minimum beam voltage. The principal problem, then, has been one of devising satisfactory means for producing and focusing a high intensity beam over a sufficient range of beam voltages to insure the necessary photographic sensitivity. It is the solution of this problem which constitutes the major contribution of this paper.

Description of the Oscillograph. A general idea of the appearance of the oscillograph may be gained from Fig. 1, which shows a portable outfit designed especially for recording lightning surges. Fig. 2 illustrates the internal construction.

The beam is produced by an electron gun employing a plate voltage of 100 to 1500 volts, which drives a high intensity jet of electrons through a small hole in the plate into a cup-shaped higher voltage cathode where the jet is further accelerated and focused.

The principal object of the electron gun is to protect the filament, although it is made to perform other important functions as may be seen later.

Referring to Figs. 2 and 3, the electron gun consists

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†For a history of the development of the cathode ray oscillograph with complete bibliography see "Measurements in Electrical Engineering by Means of Cathode Rays," by J. T. MacGregor-Morris and R. Mines, *Journal of I. E. E.*, Nov., 1925, Vol. 63, p. 1056.

[1] See Bibliography.

Presented at the Regional Meeting of the Middle Eastern District, Cincinnati, Ohio, March 20-22, 1929.

of an oxide coated ribbon filament at (1), a filament shield at (2), and a positive electrode or plate at (3). The oxide coated filament provides ample electron emission without sufficient light emission to affect the photographic film. Films have been exposed as long as one hour without showing appreciable darkening from the light. The filament shield serves to limit the emitting area of the filament, and to protect it in case of flashover. With the proper spacing between filament and shield, and shield and plate, a converging field can be produced between the filament and plate, which concentrates the electrons from a relatively large area of filament and drives them through the small hole in the plate. As indicated in Fig. 3, the plate protects the filament from the high potential field, and by employing a small hole for the beam to pass through, largely protects it from bombardment by high velocity ions. Arc-over between the gun electrodes is prevented by making the spacings small, and providing for the rapid escape of gases emitted from the hot surfaces.

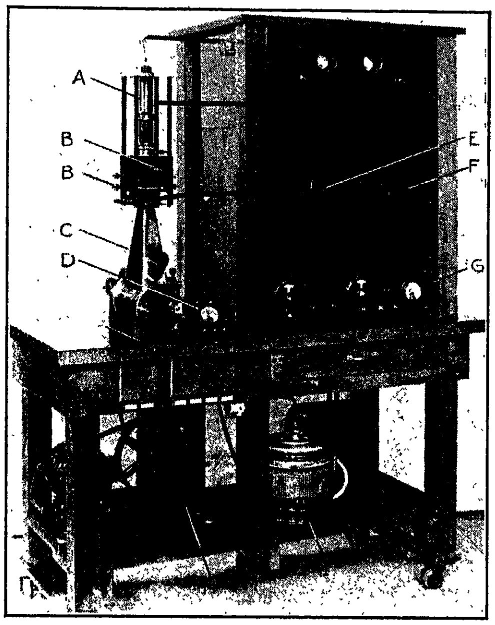

Fig. 1—Portable Hot Cathode Oscillograph for Surge Recording

A Electron gun housing.
B. Terminals of deflecting plates.
C. Oscillograph bell.
D. Vacuum gage.
E. Gun voltage control.
F. Cathode voltage control.
G. Timing oscillator.
H. Vacuum pump.
I. Drying chamber for films.

Aside from the protection of the filament, the use of the gun is an advantage for automatic recording of lightning surges, since the beam may readily be cut off by either removing the plate voltage or impressing a sufficient negative bias on the filament shield. By these means the filament may be held at the proper temperature for good emission, and full voltage maintained on the high voltage electrodes without producing a beam. Then by the proper circuits the transient can be made to apply the plate voltage automatically, or reduce the bias of the shield and permit the beam to form in time to record most of the wave front. As soon as the surge has passed the beam is automatically cut off again.

The high-voltage electrodes are of quite radical design as shown in Fig. 2. The cathode (4) is a cup-shaped piece of metal with the plate of the electron gun forming what would be the bottom of the cup. The cylindrical anode (5) extends inside the cathode, and can be raised or lowered by means of the knurled head at (6).

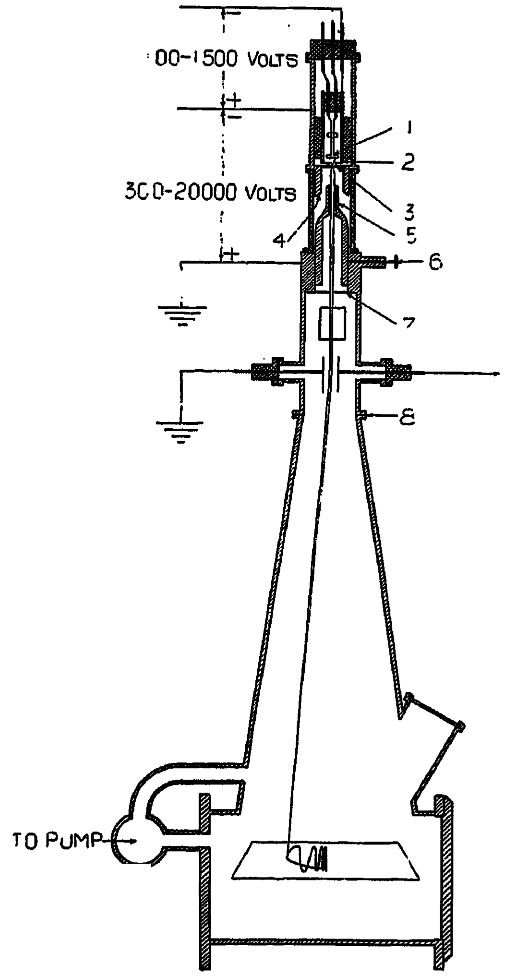

Fig. 2—Diagramatic Scheme of the Portable Oscillograph

(1) Ribbon filament.
(2) Filament shield.
(3) Positive plate.
(4) Cup-shaped cathode.
(5) Cylindrical anode.
(6) Screw for raising and lowering anode.

The entire beam is permitted to pass through the anode, thus eliminating the problem of anode heating, and at the same time increasing the beam intensity. The spacing between the high-voltage electrodes is small compared to the length of the mean free path of the electrons at pressures within the operating range, thus preventing a cold or gaseous discharge taking place. The anode extending into the cathode prevents electrons from bombarding the bakelite insulation, and also prevents flickering of the beam on continuous operation by shielding it from the field of surface charges on the insulation.

Provision is made for inserting a diaphram at (7), (Fig. 2), for cutting off the small amount of stray emission caused by a positive ion bombardment of the cathode. If a diaphram is used, the hole in it is made large enough to permit all of the main beam to pass through.

The two pairs of deflecting plates are insulated from the grounded metal housing by bakelite bushings. The plates can be adjusted without breaking the vacuum, by the use of a special packed joint in the bakelite.

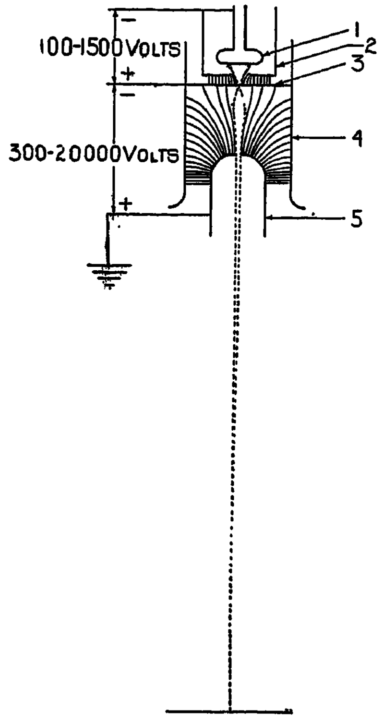

Fig. 3—Diagram Showing Method of Focusing the Beam

No provision is made on the portable type shown in Fig. 1 for magnetic deflection, but the laboratory type is provided with a section of bakelite at (8) (Fig. 2), to prevent eddy currents when using deflecting coils. A more recent design of portable oscillograph also provides for magnetic deflection.

The general purpose laboratory type of instrument is provided with both a rotating film drum and plate drum. The film drum takes film 5 in. by 30 in. with which the equivalent of a continuous record 140 in. in length can be obtained. The plate holder takes six 4- by 5-in. plates. The portable type shown in Figs. 1 and 2 is made much more compact to reduce volume, and takes 5-in. roll film. The oscillograms are 2½ in. by 5 in., although in a more recent design the oscillograms will be 3½ in. by 5 in. If special long rolls are used, as many as 100 exposures can be taken with one loading.

The Electrostatic Method of Focusing the Beam. The focusing of the electron jet has always been the most difficult problem in cathode ray oscillograph design, because of the natural tendency of the beam to diverge due to the mutual repulsion of the electrons. This is especially true of a general purpose oscillograph employing a wide range of beam potentials, since the positive ion method described by Johnson[1] does not function well at beam potentials much in excess of 2000 volts, and the magnetic method becomes unsatisfactory at potential much below 10,000 volts.

The most important feature of the oscillograph just described is the electrostatic method of concentrating and focusing the beam, which is effective at any beam potential from 500 volts to at least 20,000 volts.

Referring to Fig. 3, the beam from the filament (1) is concentrated and brought to a focus at the hole in the plate (3), by properly proportioning the spacing between the filament (1), the filament shield (2), and the plate (3), with respect to the size of the hole in (2), to produce a rapidly converging field between the filament and plate. After passing through the small hole in the plate into the high potential cathode (4) the beam diverges at first as indicated by the dotted lines, Fig. 3, due to the momentum received from the converging field in the gun.

The cathode (4) and the anode (5) are so shaped that the high potential field converges toward the anode. Therefore the electrons, in falling through this field, are not only accelerated in the direction of the anode, but are given a component of velocity toward the center of the beam. If this inward radial component of momentum of the electrons is great enough to overcome their force of repulsion the beam can be brought to a focus. It is therefore apparent that the distance from the cathode at which the beam will come to a focus depends upon the diameter of the beam when it starts to converge, hence the divergence of the beam on entering the cathode. This fact also necessitates the use of a large hole in the anode.

Since the time through which the mutual repulsion of the electrons in the beam acts, depends upon the velocity of the beam and consequently upon the beam potential, it is necessary to be able to vary the radial component of the high potential field in order to focus the beam. This is done by raising or lowering the anode by means of a rack and pinion operated by the knurled head (6), Fig. 2.

In practise, the anode is raised and the plate voltage of the gun increased to focus a high-voltage beam, and on the contrary the anode is lowered and the plate voltage reduced for a low voltage beam. The reason for raising the plate voltage when focusing a high voltage beam is to produce the same divergence of the beam on entering the cathode in spite of the higher potential gradient there. However, focusing is not critical to plate voltage and it is only with large changes in cathode voltage that it is necessary to change the plate voltage. The initial size of the beam is not so important for low voltage beams since focusing is assisted somewhat by the presence of positive ions.

Application of the Oscillograph

A. *With Continuous Beam.*

A steady beam can be produced and maintained for hours, if necessary, at any beam voltage from 500 volts to 20,000 volts, depending upon the deflectional and photographic sensitivities required.

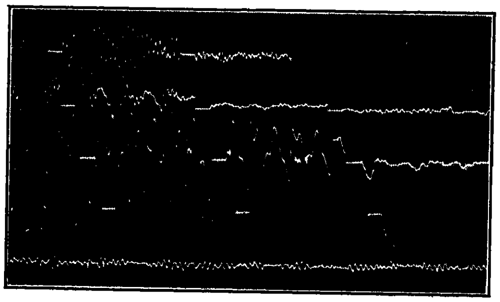

Fig. 4—Voice Record Taken with 2500-Volt Beam

The continuous beam permits study of the various forms of Lissajous figures, such as power loss loops including magnetization and dielectric loss curves, vacuum tube characteristics, frequency checking, etc. Another important use is in making long records of medium frequency phenomena where much greater photographic sensitivity is required than can be obtained with the ordinary oscillograph, and where it is necessary to obtain records free from distortion due to the mass of the recording element. A few examples are, studies of the operation of radio and carrier current apparatus, radio interference from corona, and testing voices for talking movies.

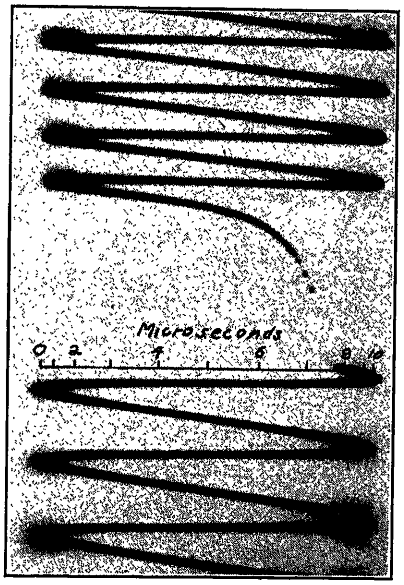

Fig. 5—High Voltage Impulse Superimposed on a 50,000-Cycle Timing Wave. 10,000-Volt Beam

Fig. 4 represents a section of a voice record taken with the laboratory oscillograph at a beam voltage of approximately 2500 volts. The timing is indicated by the dashes which occur at intervals of 1/60 of a second. The diameter of the beam was less than 1/2 millimeter.

Fig. 5 is an oscillogram of a high-voltage impulse super-imposed on a 50,000-cycle timing wave. Fig. 6 is a 500,000-cycle wave having an amplitude such that the beam was traveling at the rate of 100 kilometers (62 mi.) per second when crossing the zero axis. Both oscillograms were taken with a 10,000-volt beam.

B. *Recording Lightning Surges on Transmission Lines.*

It was at the request of the Consumers Power Company of Jackson, Michigan that the writer under the auspices of the Engineering Experiment Station of Purdue University undertook the design of a cathode ray oscillograph for recording lightning surges on transmission lines. This cooperative project resulted in the construction of the portable oscillograph illustrated by Fig. 1.

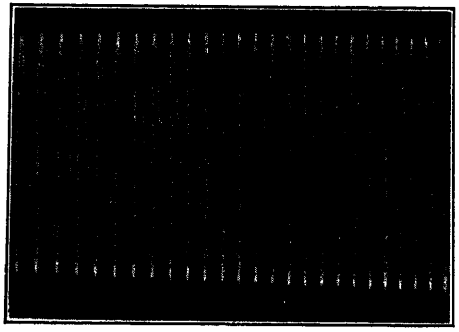

Fig. 6—500,000-Cycle Wave Taken with 10,000-Volt Beam

The recording of lightning surges presents a very difficult problem, in that the time at which a surge will occur cannot be predetermined. Therefore the oscillograph beam must either remain on continuously, or means must be provided whereby the transient, to be recorded, automatically starts the beam. The first method appears more logical since the time available to get the beam under way, by the latter method, is of the order of a fraction of a microsecond. However, there are practical difficulties encountered with the first method which may warrant the use of the second method. With the first method there is the difficulty when using a cold cathode of producing an intense beam which will hold its focus for any great length of time. And with either the hot or cold cathode oscillograph, the fogging of the film, due to bombardment by electrons scattered from the main beam by collision with gas molecules, presents a serious problem. By the proper regulation of vacuum and electrode temperature, Dr. Norinder[4] has solved both problems in a satisfactory way, but the equipment required is quite elaborate.

As a result of the difficulties encountered with fogging when employing a continuous beam, attention was turned to means by which the transient could be made to automatically start the beam in time to record the wave front. A method had already been devised in Germany, whereby the beam was initiated by the transient and the impulse to the deflecting plates delayed by passing it through a cable. However, with this method the important question arises as to whether the impulse has suffered distortion while passing through the cable. Since the beam of the oscillograph described in this paper can be cut off either by removing the plate voltage of the electron gun or by impressing a negative bias on the filament shield, it was evident that if a circuit could be devised for rapidly applying the plate voltage, or reducing the shield bias, the problem of automatically starting the beam might be solved.

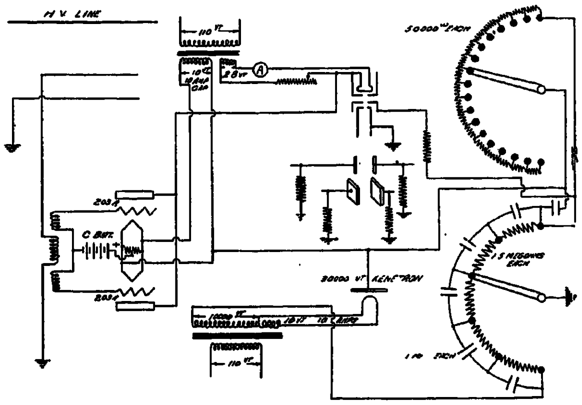

Fig. 7—Circuit for Automatically Starting Beam by Switching Plate Voltage of Electron Gun

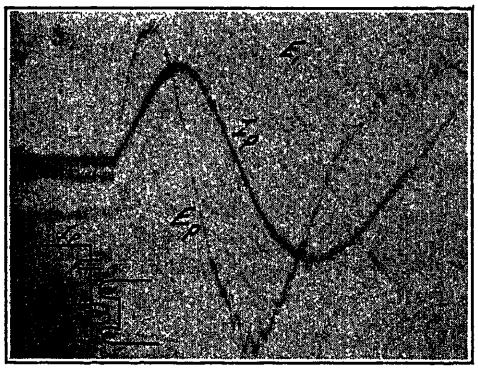

Fig. 8—Time Relation Between Applied Surge, Current, and Voltage Drop across Inductance in the Equivalent of Pick-up Circuit

Fig. 7 represents the circuit diagram for the portable oscillograph including a circuit for automatically initiating the beam by applying plate voltage to the electron gun. With this circuit the plate or beam current of the electron gun must pass through one of the two vacuum tubes inserted between the gun voltage supply and the filament. Therefore the beam can be stopped by applying a sufficient negative bias to the grid of the vacuum tubes to prevent the flow of plate current. Then in order to initiate the beam it is necessary to apply a small positive impulse to the grid of one or both tubes, after which the time required for the beam to build up depends upon the rate at which the electron gun can be charged as a condenser.

The circuit for producing the positive impulse on the grid of one of the vacuum tubes has some desirable characteristics which may not be evident. The first important requirement to be met is that the surge to be recorded must produce a positive impulse on the grid of one of the tubes regardless of the polarity of the surge.

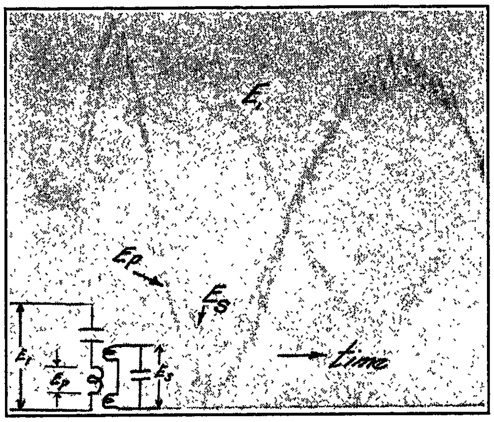

Fig. 9—Time Relation Between Applied Surge, Voltage Drop Across Inductance, and Terminal Voltage in Equivalent of Grid Circuit

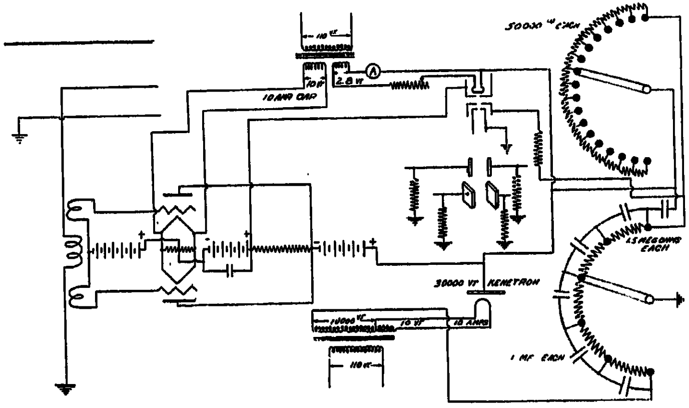

Fig. 10—Circuit for Automatically Starting Beam by Controlling Filament Shield Potential

This, of course, is most readily accomplished by an inductively coupled circuit which also has the advantage that the antenna of the dividing condenser can be insulated from the high-voltage direct current of the oscillograph. A rather unexpected advantage of the inductively coupled circuit comes from making the inductance such that the inductive reactance of the antenna circuit is small compared to the capacity reactance. This permits the charging current in the antenna circuit to reach its maximum well in advance of the maximum voltage of the surge, and since the voltage drop across the inductance is proportional to the rate of change of current, it therefore reaches its maximum ahead of the current. The inductance is, however, the primary of an air core transformer, therefore the voltage induced in the secondary or grid circuit also reaches its maximum ahead of the current in the primary. In order to check the theory and determine the time relation of the secondary terminal voltage, *i. e.*, the grid voltage, oscillograms were taken of the voltage and current relations in a similar low frequency circuit when the equivalent of a slow surge was applied. Fig. 8 shows the time relation between the applied voltage, current, and voltage drop in the inductance. Fig. 9 indicates the relation between applied voltage, the voltage drop across the inductance, and the voltage in the equivalent of the grid circuit. In studying the oscillograms it must be remembered that in actual magnitude the voltage drop across the primary, and the induced voltage of the secondary are small compared to the surge voltage, although they are shown to be of about equal magnitude by the oscillograms. Fig. 9 shows that the capacity load corresponding to the grid capacity causes quite an appreciable phase shift in the secondary terminal voltage with respect to the primary voltage but that the shift with respect to the surge voltage is small. In order to make sure that the phase relations were not being affected by the burden of the oscillograph elements, the burden was greatly increased but without apparent effect.

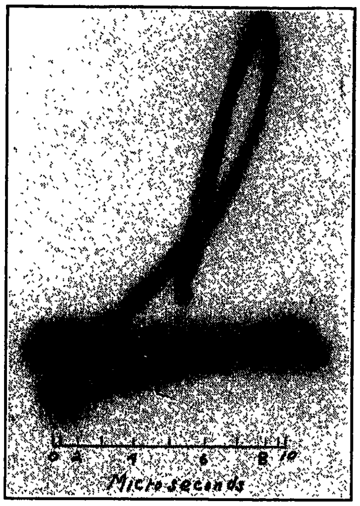

Fig. 11—Lightning Generator Impulse which Automatically Started Beam

An improved form of the circuit shown in Fig. 10 is employed when using a negative bias on the filament shield to stop the beam. Although this type of circuit is faster than the type shown in Fig. 7, it is liable to cut off too quickly on the tail of the wave unless special means are employed to hold the beam on. It has not been definitely decided which circuit is superior in practise, as either circuit will initiate the beam in half a microsecond or less from the time the surge voltage begins to rise from zero. Since klydonograph records indicate that lightning surges may reach a maximum in from one to ten microseconds, this circuit is capable of getting the beam under way in time to record practically the entire wave. The circuit may also be used for laboratory work to supplant a switch or sphere gap for turning on the beam if wave fronts of not less than one microsecond are employed.

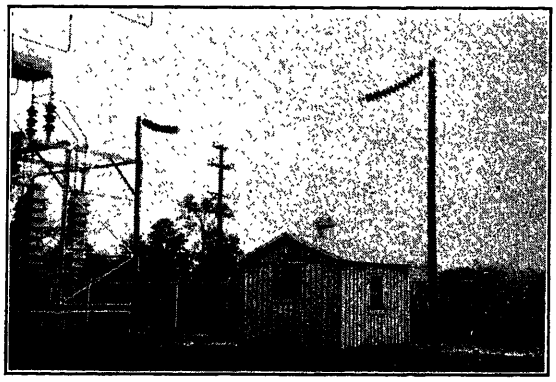

Fig. 12—Installation of Cathode Ray Oscillograph on 140-Kv. Line of the Consumers Power Company

Fig. 11 shows a record taken of a lightning generator wave in which the beam was automatically started by the surge. The surge was applied at right angles to a 50,000-cycle timing wave.

The portable hot cathode oscillograph, Fig. 1, employing the automatic beam starting feature, was put into operation on a 140-kv. line of the Consumers Power Company of Jackson, Michigan at their Blackstone Substation on August 27, 1928. Fig. 12 shows this installation.

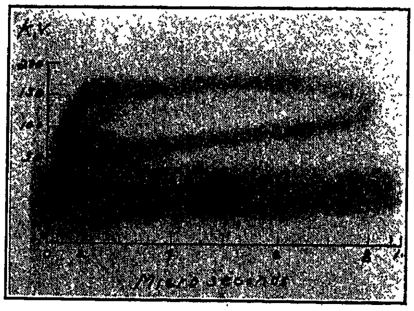

Fig. 13—Lightning Surge Recorded on 140-Kv. Line of the Consumers Power Company

Fig. 13 shows a 170-kv. surge which occurred during a distant lightning storm. The surge was hardly sufficient to fully turn on the beam and as a consequence it is very poorly focused.

A full description of the installation and results will probably be presented by the Consumers Power Company following the 1929 lightning season.

This oscillograph is entirely self-contained with the exception of the power supply which is taken from a 110-volt 60-cycle source. The power required is approximately 700 watts. The vacuum is produced by a rotary oil pump although a mercury pump will probably be added for use during the next season. The volume of the oscillograph has been reduced to a minimum so that with well dried film the oil pump alone will produce a workable vacuum in approximately three minutes. The film holder is daylight loading and takes a standard No. 104 film. A six exposure film is sufficient for eight oscillograms, although provision is made for sufficient film to take 100 exposures with one loading by the use of special film.

Development work is being continued along the following lines:

1. Further increasing the beam intensity.
2. Focusing still higher voltage beams.
3. The production of a linear time axis for lightning records.
4. Making the recording of lightning surges more nearly automatic.

CONCLUSION

As a result of a long series of investigations a general purpose type of cathode ray oscillograph has been developed which is applicable to a wide variety of uses in the fields of science and engineering. A special form of this oscillograph has been developed and put in use for recording lightning and switching surges on high-voltage transmission lines.

It is felt that this oscillograph will prove particularly useful in laboratories where a wide variety of work is handled as well as in field investigations.

The author wishes to express his sincere appreciation to Professor C. Francis Harding for his continued interest and encouragement as well as making the work possible, to Mr. J. W. Raleigh for valuable assistance in the early stages of the work, and to Mr. J. R. Eaton of the Consumers Power Company for many valuable suggestions.

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Discussion

W. L. Everitt: I wonder if this new type of electrostatic focusing increases the life of the filament, so that, if it is made up in a permanent tube such as the Western Electric Company's No. 224 A type, a considerably longer life would be obtained than is now possible with the present method of using a small filament worked at a high current.

Is the filament narrow as it is shown or is it fairly wide?

Is there any dispersion due to collision with the molecules of gas after it has passed through this second focusing slot No. 5?

R. H. George: As to the life of the filament, I believe that with an apparatus sealed off the life would be reasonably long, since the filament is well shielded from positive ion bombardment and the apparatus could be operated at a much harder vacuum than the Western Electric oscillograph.

The life of the filament, *i. e.*, the oxide coat, has not been particularly long with our oscillograph, the maximum being 30 to 40 hr. operation. However, the filament can be recoated a number of times. The principal difficulty with filaments has not been caused by positive ion bombardment, but by the action of oil vapors which lower the emission. The emission can be restored usually by heating the filament in the presence of oxygen, providing it has not previously been heated to excessive temperatures. This difficulty would not be encountered with filaments sealed off in glass apparatus.

The filament shown in the diagram is quite exaggerated in size. It is made in the form of a ribbon 1/16 in. wide and the emission taken from the flat surface, which more than covers the hole in the filament shield.

The film is turned up by means of a metal shaft which passes through a packed joint in the oscillograph housing. The vacuum-proof joint consists of a cotton packing on the high-pressure side of which is a cavity containing heavy grease. With this type of joint the shaft can either be rotated or moved endwise without affecting the vacuum. This same type of joint is employed for the shutter lever and the deflecting plates.

The film drum employed when recording low-frequency phenomena is driven through a conical joint with a special grease seal. Speeds as high as 500 to 600 rev. per min. can easily be obtained without affecting the vacuum.

There is some dispersion of the beam after it passes through the last plate, but it is not appreciable at normal operating pressures, *i. e.*, pressures below 10 microns.

It has recently been found that some of the apparent dispersion was due to the fact that gas liberated from the metal of the electron gun was not escaping with sufficient rapidity around the high-voltage anode, and as a result electrons were being liberated by bombardment of the gas, giving rise to electrons of varying velocity. The design has been altered to take care of this trouble.

Fused Arcing Horns and Grading Rings

Design, Construction, and Operating Experience on 66,000-Volt Transmission Lines of the Union Gas & Electric Company

BY PHILIP STEWART[1]

Associate, A. I. E. E.

Synopsis.—This paper considers the use of fuses on insulator strings of high-voltage overhead conductors, to interrupt the arc at times of flashover before the line relays operate to disconnect the circuit. Consideration is first given to the original development of this idea, in which a fuse was connected between the line conductor and an arcing ring, attached to the second insulator unit. When an excessive voltage occurs on the conductor to ground, there is a flash between a two-pronged horn on the top insulator unit and the ring. The circuit is completed through the fuse, which immediately opens, breaking the arc. Further consideration is given to a later development of the principle, in which two expulsion type fuses replace the two-pronged horn at the top of the insulator string and the arcing ring is placed at the conductor end of the string. Data are presented from tests and from experience on about 100 mi. of 66,000-volt circuit, of the Union Gas and Electric Company, Cincinnati, Ohio.

INTRODUCTION

The general practise up to this time is to isolate faults at the ends of feeders or transmission circuits in trouble. This is done by means of oil circuit breakers actuated by relays. Lines and circuits so equipped immediately go out of service in case of faults. The fused insulator string is a device for isolating faults right at the point where the fault occurs, interrupting fault current only, and not interrupting the useful service of the line or circuit.

It is now generally considered good practise to provide some arrangement of arcing or grading rings or horns, at all insulator strings of important high-voltage transmission circuits. Experience and tests point to this means of preventing cascading and possible shattering of insulators by flashovers caused by transient voltages on the line conductors to ground. The use of such devices reduces the time the line may be out of service, due to flashed insulators. The development of the fused device for insulator strings is an attempt to further reduce the interruptions, especially those of short duration caused by the line switch opening to clear the surge flashover and subsequent short circuit which may last for only a few seconds.

One of the two generating plants in the Cincinnati territory is located at Columbia Park, on the Ohio River, about 20 mi. west of Cincinnati. This plant is operated as a base load station at as nearly full load as possible at all times. The major part of the load from Columbia to Cincinnati is carried over four 66,000-volt transmission circuits carried on two double circuit tower lines. These circuits are connected to Terminal Switching Station, north of Cincinnati. From Terminal Station, two 66,000-volt circuits connect to each of three major distribution substations. Two circuits also connect to the system of the Dayton Power and Light Company.

It is very important for the operation of the entire system that these circuits be kept in service without interruptions. It is especially so on the circuits from Columbia to Terminal Station. When these lines were built, the insulator strings were equipped with flux controls. Six insulator units were used in each string. The first year's operation was good, but the disastrous results of insulator flashovers and line outages were evident.

ORIGINAL DEVELOPMENT

The first arrangement using fused arcing rings consisted of a string of seven insulator units and was provided with two horns at the top and a split oval ring below the fifth unit. Fuse clamps were mounted on the ring and conductor clamp, thus shunting the two lowest insulators with a fuse. With the fuse in, the line is insulated with five units between the line and ground. After the fuse has been blown, the insulator will consist of seven units. In this way the flashover distance will be increased and it was thought that in case a second surge originated near this point before the fuse was renewed, the flashover would occur at the next adjacent point of support.

Tests were made with this arrangement to observe the performance in clearing flashovers on the 66,000-volt system. All 66,000-volt lines were energized into Terminal. The tests were made on a line from the main bus. The insulator string was shunted by a piece of small copper wire in order to start the flashover. The line was closed in on the shorted insulator by means of the station oil circuit breaker. An oscillograph was used in order to obtain the short-circuit currents and voltages. An average short-circuit current of 2970 r. m. s. amperes was obtained, with 9000 volts, phase to neutral, during short circuit. The 10-ampere 37,000-volt carbon tetrachloride fuses cleared in one cycle, in each case clearing before the relays operated to open the oil circuit breaker.

Tests were made in the high-voltage laboratory of the General Electric Company. In these tests it was established that a power arc can be produced by fusing

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[1] Union Gas & Electric Company, Cincinnati, Ohio.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

a fine copper wire connected between the arcing rings of an insulator string. This method of starting an arc was used throughout.

Tests were also made to investigate the performance of the fused rings when subjected to artificial lightning; first with a ring made of 1½-in. pipe, then a strap iron ring, and in the third case the horns were supplied with hemispheres. These three tests gave approximately the same flashover voltages and arcing characteristics, thereby indicating that for this short string, grading is not so necessary. The average flashover voltage determined in these tests was 530 kv. maximum, which is the same as the flashover voltage between needle points placed the same distance apart. This indicates that the voltage distribution on the comparatively short string of insulator units does not affect the flashover of the parallel gap. However, on longer strings, grading is necessary, and the multi-gap effect of the units must be overcome.

Further tests determined the effect of artificial lightning discharge on the fuse alone. Ten discharges of approximately 6000 amperes (maximum) had no apparent effect upon the fuse, due to the exceedingly short time during which the current was flowing. This shows that the fuse is probably blown by the follow-up current and not by initial current due to the lightning discharge.

Operating Experience

In the spring of 1927, fused arcing rings were installed on two of the Columbia—Terminal circuits, and four of the shorter lines, leaving Terminal, totaling approximately 65 mi. of circuit so equipped. During an eight month period succeeding this installation, there were six cases of the fuses blowing and clearing without an interruption to service. There were three cases in which the fuses failed to clear and the circuit breaker opened, interrupting service on the line. During this period, ten fuses were blown. The three service interruptions were caused by the destruction of the fuses at the flashover points.

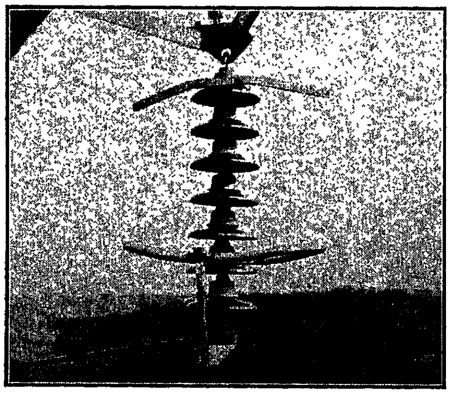

Fig. 1—Original Development of Fused Arcing Ring Using Carbon Tetrachloride Fuses

It is noted that the fuses functioned properly 77 per cent of the time and succeeded in preventing service interruptions 67 per cent of the time.

All failures of the device were due to failures of the fuse itself, caused by follow-up surge voltages. One case noted during the preliminary tests probably explains these failures. A blown fuse was inserted and an impact of very high voltage was applied. The arc followed a path from the conductor over the metallic part of the fuse within the glass tube, puncturing the glass, and passing over the outside to the ring and to the horn and ground. In actual practise, a dynamic arc would follow this first arc, causing destruction of the fuse.

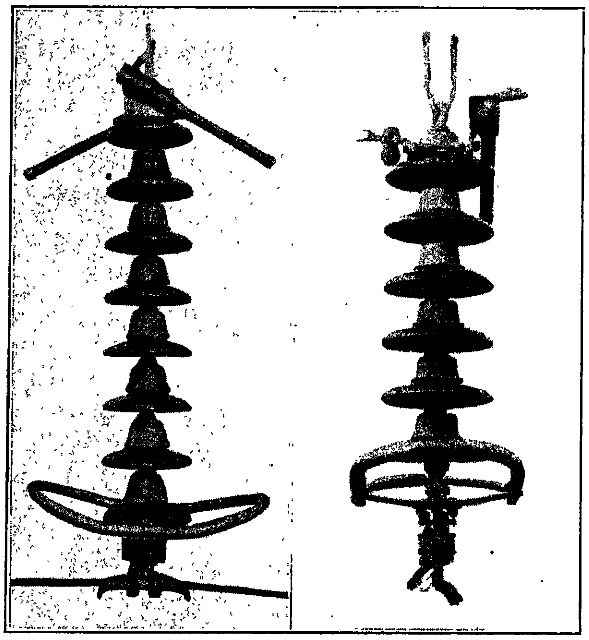

Fig. 2—Later Development of Fused Insulator String Using Expulsion Type Fuses as Arcing Horns

Development Using Expulsion Type Fuse

The operation of the fused arcing rings, using carbon tetrachloride fuses, (Fig. 1) was successful in preventing a number of service interruptions on the two Columbia—Terminal circuits on which they were installed. In considering the installation on the two remaining circuits, a more economical arrangement of the device was desired. First, the added cost for the extra insulator units shunted by the fuses did not seem to be justified by the first year's operation. Second, the carbon tetrachloride fuses were more expensive than expulsion type fuses. Therefore, the next development was an arrangement using two expulsion type fuses in place of the horn at the top of the insulator string, (Fig. 2) and a ring at the conductor end.

Tests were made on various makes of expulsion type fuses, some the standard products of different manufacturers and some specially designed by the Union

Gas and Electric Company. These tests were made at Terminal Station with the system set-up similar to that of the previous tests. In each case the fuse operated correctly to break the short circuit. The time required with a few exceptions was one-half cycle or less.

Further tests to determine the conductivity of the fuses after they had been blown established that there was practically no conducting material deposited in the fuse holder.

The diameter of the ring and the position of the fused horns were determined from tests made with the impulse generator of the General Electric Company so that flashovers on the assembled string occur from the end of the fuse to the ring without cascading.

In addition to more economical construction, the present assembly has an advantage over the original in that two fuses are employed. It was established by tests that both fuses will not blow at the same time, but one will remain in good condition. When one fuse is blown the arcing distance on that side is greater than on the side of the good fuse and second flashovers go to the tip of the good fuse. Therefore, the device will function for two consecutive surges originating on the same section of line before replacement of the fuses is necessary.

OPERATING RECORD WITH FUSED ARCING HORNS

Line	Year	Inter-ruptions	Flashed units	Type of fuse	Inter-ruptions prevented	Fuses blown
1761	1926	3	30	none		
	1927	2	6	S. & C.	4	9
	1928	0	0	S. & C.	1	1
1762	1926	4	14	none		
	1927	1	2	S. & C.	2	4
	1928	0	0	S. & C.	1	1
1763	1926	1	0	none		
	1927	4	18	none		
	1928	0	0	Expulsion	1	1
1764	1926	2	6	none		
	1927	3	24	none		
	1928	0	0	Expulsion	1	1
1261	1926	1	0	none		
	1927	0	0	S. & C.	0	0
	1928	1*	0	S. & C.	*	2
1262	1926	3	6	none		
	1927	0	0	S & C.	0	0
	1928	1*	6	S. & C.	*	1
861	1926	0	0	none		
	1927	0	0	S. & C.	0	0
	1928	0	0	S. & C.	2	3
862	1926	0	0	none		
	1927	0	0	S. & C.	0	0
	1928	2	0	S. & C.	0	3

*Flashover at substation.

Upon operation, a bright metal clamp at the out end of the fuse is blown off and the condition of the fuses can be determined by patrolmen's visual inspection from the ground.

OPERATION IN 1928

In the early part of 1928, the fused horns and grading rings were installed on the two Columbia—Terminal circuits not previously equipped with fused arcing rings. During this year there were no interruptions to service on any of these four circuits. Although there were comparatively few lightning storms, four fuses were found blown, indicating that they had been effective in preventing service interruptions.

For two years operation, there have been seventeen cases of flashovers on lines equipped with fused horns or arcing rings. Of these cases twelve interruptions have been prevented. This is an elimination of 70 per cent of service interruptions.

The accompanying table shows the operation record for the 66,000-volt tower lines during three years of service.

CONCLUSIONS

1. Arcing rings are essential equipment on important high-voltage transmission lines. Without arcing rings, many insulators are flashed and shattered due to cascading of the string. Shattered insulators mean a line outage for several hours caused by an abnormal condition existing only a fraction of a second.
2. Fuses in the flashover circuit interrupt and prevent the flow of follow-up current. Since the fuses function in approximately one-half cycle and protective relays are ordinarily set to operate in not less than 35 to 45 cycles, the short circuit will be cleared before the oil circuit breaker operates.
3. The fuse is not blown by the current of the initial surge, but by the current of the dynamic arc.
4. The use of fused insulator strings on the high-voltage circuits of the Union Gas and Electric Company has been a large factor in a considerable improvement in service and operating records.
5. Tests and operating experience, as well as economic considerations, show several advantages of the assembly using fused horns at the support end of the string, over the original assembly using a fused ring at the conductor end.
6. The adoption of fused horns and grading rings for important high-voltage lines has proved to be another means by which service and operation may be improved.

Discussion

R. L. McCoy: Mr. Stewart's very interesting paper on the development of the fused grading shield deals with its application to 66-kv. transmission line. The next thing which comes to mind, is how far can its application be extended to higher voltage lines. It is conceivable that its widest field of usefulness should be found on 110-kv. and 132-kv. lines. We have investigated this quite thoroughly, and there is now a fuse available which we believe will make this device practical for lines at these voltages. We have tested this very carefully in the laboratory and it worked perfectly. Of course, the final answer lies in field operation, and we are making arrangements for some small field installations during the coming lightning season which we hope will give some experience by the end of the season, so that there will be a definite basis for recommendations on 110- and 132-kv. lines by next Fall.

S. M. Hamill, Jr.: I think that Mr. Stewart has been a little modest in what he said about the fused grading rings. I notice the article states, "An average short-circuit current of 2970 r. m. s. amperes was obtained, with 9000 volts phase-to-neutral during short circuit." That, of course, is the drop from the point in the transmission line to what approximates the substation neutral, or rather what approximates a very good ground at the substation, and is not the actual phase-to-neutral voltage, open circuit, which was approximately 38,000 volts instead of 9000.

The second point I should like to bring up is that it is impossible to set the induction-type relays low enough so that they will clear with the fuses. In other words, we are not hindered in the relay settings by these fuses. We can set our relays at the minimum setting, namely, about 9 cycles, and still have the fuses clear ahead of them. As a matter of fact, all our low-setting relays are set at this valve on those transmission lines, which are protected by Mr. Stewart's fuses. While Mr. Stewart emphasized a very important item from the point of view of transmission-line maintenance, namely, the prevention of damage to insulators, I think that the most important item is the prevention of the line outage itself.

C. L. Fortescue: We at the Westinghouse Company have been working along similar lines, our idea being to incorporate into the insulating string the characteristics of a lightning arrester. This is really not in itself fundamentally a new idea. For instance, in wood poles we have an action due to the wood pole insulation, that is very similar to that in an arrester. If the lightning strikes the line and the wood pole functions as an insulator, so long as the pole is not ruptured the dynamic arc will not follow the flashover, and you have something very much like an arrrester action.

Of course, the trouble with a wood pole is that it is liable to rupture, due to a heavy lightning stroke, and of course in that case you get an outage. If we could incorporate in the string itself the properties of an arrester, we would have something that would be nearly ideal, because when the lightning stroke hit the line, the resulting voltage of the surge being too high for the insulator, it would function as an arrester and it would absorb the surge. The characteristic of an arrester would cut off any dynamic following current, and we would go right ahead without any interruption.

O. S. Clark: It should be of interest to know that, in connection with the statements of Mr. Fortescue regarding the performance of lightning on wood structures, one of our companies has a line in the construction of which is incorporated a large amount of wood insulation, and that in a number of cases in which the wood material has been ruptured by a lightning discharge, the service has not been interrupted.

In connection with the design of the fused horn and grading ring described by Mr. Stewart, the laboratory work that we have done, we believe, shows that it is possible to apply the device to any form of transmission-line insulation. The problem for any type of insulator and for any length of string is an individual one and it must be worked out, I should think, in an experimental laboratory before any conclusion can be reached. While the device is applied, at this time, to suspension strings only, there is no doubt that, if economically desirable, its application can be worked out for lines using insulators of the pin type.

There is one question that very likely occurs in the minds of many in connection with the double fused horn and grading ring assembly, and that is the possibility of flashes occurring at both fuse tips simultaneously. Laboratory tests have shown that in a very small percentage of the cases when the assembly was sparked over by the impulse generator, this did occur. The probability of such occurrences is too low to consider in its application.

H. C. Don Carlos: In discussing Mr. Stewart's paper, I wish to comment particularly on his conclusion No. 1 wherein he states that "arcing rings are essential equipment on important high-voltage transmission lines."

The Hydro Electric Power Commission has in operation approximately the following mileages of high-voltage lines which are equipped with suspension insulators.

220 kv.—25 cycles— 203 circuit miles
110 kv.—25 cycles—1125 circuit miles
110 kv.—60 cycles— 200 circuit miles
Less than 110 kv.—25 cycles— 127 circuit miles
Less than 110 kv.—60 cycles— 194 circuit miles

None of these lines is equipped with arcing rings and our flashovers on the 110-kv. 25-cycle system have been at the rate of 1.25 and 1.53 per 100 mi. of circuit during 1927 and 1928 respectively. I believe that there has been no case in which a string of insulators has been damaged to the extent that the line would not go back into service.

The 220-kv. transmission line was put in service October 1, 1928, and, of course, has not yet passed through a complete lightning season. We have therefore no data as regards the performance of this line throughout a complete season, though there was at least one rather severe lightning storm last fall in the district traversed by this line, and also a few this spring. While we have had four outages on this circuit, these have all been due to foreign interference and as yet there has been no evidence of a flashover occurring on this circuit. We therefore do not anticipate any serious trouble from lightning.

As regards our 60-cycle, 110-kv. lines at the head of the Great Lakes supplying Port Arthur and Fort William, about 70 circuit-miles have been in service eight years without a single flashover, and the remaining 90 circuit-miles, which have been in service 4½ years, have experienced a few flashovers, but in no case has an insulator string been damaged to such an extent that the line would not go back into service immediately.

While these lines can be returned to service immediately, there are in some cases one or two units in the string which are damaged, but these are located during line patrol and replaced at our convenience.

While there is no doubt that grading ring should, from a theoretical consideration, be of advantage in reducing the tendency of arcs cascading over the insulators, it would seem that if these are not properly mounted they may cause more trouble than they eliminate. The necessity, or desirability, of grading rings with present transmission voltages is a debatable question, but if the transmission voltage continues to increase it will undoubtedly be necessary to employ some means of improving the potential gradient over the insulator string so as to keep the voltage stress on the units next to the line below their flashover value. Practically, it is believed that wind conditions existing during flashovers are fairly effective in keeping the arc at a sufficient distance from the insulator strings so that in the majority of cases, no serious damage is done to the insulator string.

I am certainly interested to know that the Union Gas and Electric Company considered the expense of increasing their insulator string from six to seven units (one illustration shows eight units) and equipping them with this elaborate fuse protection as justified. It would seem that it would be very difficult to get a comparison of the performance of these circuits on a satisfactory basis to prove that these fuses were of value. For instance, would the service not have been improved to the same extent by adding the one or two units per string without equipping the strings with the grading rings and fuses? It must be expected that there will be many more flashovers on the reduced string than there would have been on the string without this equipment and such as might have occurred might not have resulted in service interruptions.

While no information is given regarding the four circuits between Columbia Park and Cincinnati Terminal (20 mi.) except that they were carried on two separate tower lines, it would seem quite reasonable to expect absolutely continuous service between these points if suitable relay protection were applied. However, if the relay equipment is opening the breakers in 35 cycles (0.58 sec.) it would seem that this is quite adequate protection and should be fast enough to ensure against permanent damage to the insulator in the event of a flashover. It is presumed that the relay protection has not been changed coincident with the change in insulator. No information is given about the grounded sky-wire protection applied to these circuits, nor is any mention made of the condition of the system neutral. We have found that a grounded sky wire properly applied has greatly improved conditions as regards flashovers on several of our lines and this is quite definitely standard practise with us now. Our 110-kv., 25-cycle system operates with a resistance-grounded neutral while the 60-cycle systems have the neutrals dead grounded. It might also be mentioned that we have not found a very heavy percentage of cases where trouble on one circuit of a tower line is communicated to the other circuit on the tower.

Philip Stewart: I might say briefly that the trouble we experienced was due to flashovers on two or three circuits at one time. In one case we had complete shutdown of all the circuits due to flashover on three circuits. I think there is considerable difficulty in comparing service records and requirements of one company with those of another company. You have to take into account the number of lightning storms, and perhaps there is quite a little difference in the various localities that have been mentioned.

Operating Experience with the Low-Voltage A-C. Network in Cincinnati

BY F. E. PINCKARD[1]

Associate, A. I. E. E.

Synopsis.*—This paper describes briefly the operating experience and problems which have resulted from the installation of a four-wire, 120/208-volt secondary network system with combined light and power mains. The problems with reference to network equipment, voltage regulation, and operation of consumers' equipment are stated with their subsequent solution. The results obtained from fifteen months operation of this system indicate that, with reasonable care in layout and selection of equipment, no operating problems of major importance will be met.*

* * * * *

INTRODUCTION

WHEN the a-c. low-voltage network system was inaugurated in Cincinnati, a comparatively slight amount of published data was available concerning actual network operating experience, and as might be expected, the operation of this system introduced a number of problems concerned with the design and operation of equipment, also with the success which could be attained with the application of the network system to various types of customers' equipment. During the period of fifteen months since the first network installations were placed in service, these problems have been treated in various manners and with varying degrees of success, and in this paper an attempt will be made to present the most interesting of these together with the steps taken for relief.

The a-c. network system here was designed ultimately to replace the existing three-wire d-c. Edison network in the congested business district of the city, a section approximately one mile square, with a peak Edison system load of 24,000 kilowatts. The ultimate layout of the system provides for the installation of eight 13,200-volt feeders with a transformer vault located at each intersection in the network district and between intersections where the load warrants, with an ultimate capacity of approximately 40,000 kv-a. Transformer banks of 300- and 450-kv-a. capacity with 10 per cent impedance and secondary mains of 400,000 cm. cross section were made standard, using a four-wire, 120/208-volt star connected secondary with combined light and power mains. Single line diagram showing schematic arrangement of the system is shown in Fig. 1.

The original installation consisted of two high-voltage cable feeders with transformer banks interconnected by secondary mains as required to serve certain new loads in the network district. As other new loads have developed and as a certain amount of changeover from direct current to alternating current has been effected, extensions have been made as necessary, at all times working toward the ultimate layout of the system. Fig. 2 shows the two-feeder network as operated at the present time. As the load increases and as additional vaults are installed, this system will be increased to a three-feeder, then a four-feeder, and ultimately to an eight-feeder layout. The four-feeder installation which completes the symmetrical layout of the network is shown in Fig. 3.

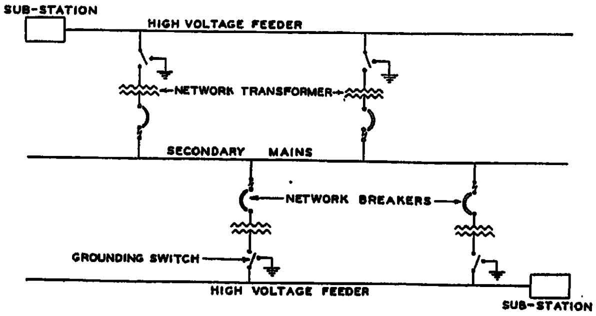

FIG. 1—SINGLE LINE DIAGRAM SHOWING SCHEMATIC ARRANGEMENT OF LOW-VOLTAGE NETWORK

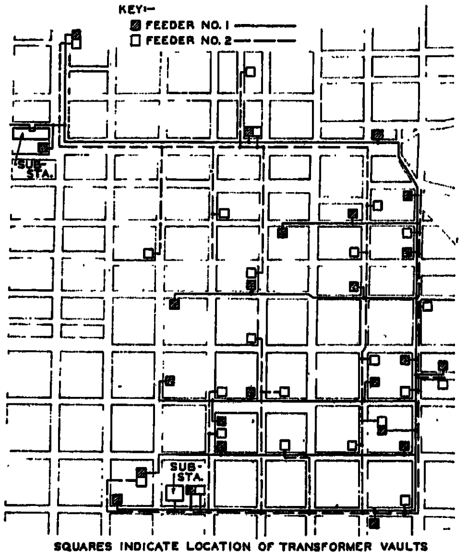

FIG. 2—DIAGRAM OF TWO-FEEDER NETWORK OPERATED AT PRESENT TIME

NETWORK EQUIPMENT PROBLEMS

With the slight amount of load served by the first network installations, it was natural that the first problems to be encountered were concerned with the network equipment and particularly with the automatic network circuit breakers. It should be mentioned here that a number of the problems encountered was the result of the extremely small load on the network at the start and is of importance, not so much from the standpoint of service, but rather from the experience gained in the operation of the equipment under this condition.

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[1] Electric Distribution Dept., The Union Gas and Electric Co., Cincinnati, Ohio.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

The first problem arose from the operation of various elevators connected to the network. In one new building where unit multi-voltage type elevators were installed and where the lighting load was at the time rather small, it was found that at certain times the network breakers would open due to a power reversal. Investigation of this condition revealed that this type of elevator, or any type of elevator using regenerative breaking, might feed a sufficient amount of power into the network to trip the breakers which were set for a reversal of approximately 5 amperes, which is considerably less than the energizing current of the transformers. Tests showed that the magnitude of the reversal at times reached a value of 15 kw., or more than one-third of the normal demand of the elevators. In order to correct this condition, heavier magnets were installed on the three-phase relay with some slight success and later the gradual increase in the lighting load served to absorb the reversal.

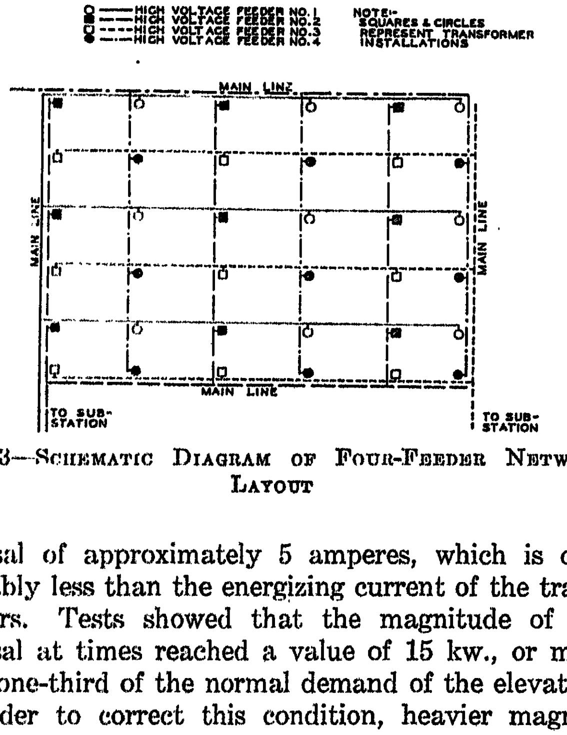

HIGH VOLTAGE FEEDER NO. 1
HIGH VOLTAGE FEEDER NO. 2
HIGH VOLTAGE FEEDER NO. 3
HIGH VOLTAGE FEEDER NO. 4
NOTE:- SQUARES & CIRCLES REPRESENT TRANSFORMER INSTALLATIONS
MAIN LINE
MAIN LINE
MAIN LINE
MAIN LINE
TO SUB-STATION
TO SUB-STATION

FIG. 3—SCHEMATIC DIAGRAM OF FOUR-FEEDER NETWORK LAYOUT

Later, additional work in connection with this same problem in other parts of the system led to further adjustment in the contact springs of this relay, so that the current required for tripping was just below the magnetizing current of the transformer. Although not working perfectly, this served to eliminate practically all complaints, which ordinarily would have continued throughout the time the transformer banks were lightly loaded.

After the previous problem had been investigated, a severe pumping of breakers under light load conditions was noted. This condition could not be attributed to power reversal, and a further investigation indicated that it was due to the slight phase angle difference between the two primary feeders which originated at different substations. This pumping effect should have been prevented by the operation of the single-phase breaker relay which is designed to control operation of the breaker on phase differences. A check into the operation of this relay indicated that the relay potential elements were reversed.

While the correction of the connections eliminated pumping, it resulted in a number of the breakers on one of the feeders remaining open a considerable part of the time.

When a load of even moderate size was thrown on and off intermittently in the vicinity of one of these breakers, severe voltage variations were noticed due to the breaker opening and closing. Since satisfactory service could not be rendered under those conditions, one of the feeders was extended to a generating station which reduced the difference in phase angle between the feeders from 2 deg. to practically zero, and eliminated this trouble.

The large number of breaker operations caused both by elevator reversals and by pumping due to phase angle difference, and later by variations in load, resulted in considerable attention to the mechanical features of the breakers themselves. A number of failures of breaker trip and closing solenoids was experienced, and in order to insure satisfactory operation, it was found necessary at this time to make a complete inspection of breakers as often as twice weekly. The number of breaker failures caused a great amount of consideration of breaker design and construction, particularly with respect to breaker life and to accessibility for repair.

The failures could be divided into two classes: failure to trip due to faulty trip mechanism, or to poor adjustment; and the burning out of closing coils on solenoid-operated breakers due principally to the tendency of the auxiliary contactor switches to weld closed, leaving the closing coil energized after the breaker had closed.

The failure to trip was solved primarily by the replacement of the old trip mechanisms with a new type developed by the manufacturer. In the case of closing coil failures, it was considered advisable to install some type of fusing in both the auxiliary and the main closing circuits, especially since it was so difficult to replace breaker parts due to the inaccessibility of the breaker when mounted in a submersible case. Tests were made on various types of fuses to secure the proper fusing characteristics, and finally link type thermal cutouts were installed so as to protect both the auxiliary and the main closing solenoids as shown in Fig. 4.

The idea of similar protection in the trip circuit was abandoned since it was considered undesirable to jeopardize the tripping function in any way. The difficulty of replacing parts on submersible breaker or of removing a submersible breaker from the case without cutting the secondary leads resulted in providing that the panel supporting the breaker parts should be divided into three parts, the upper and lower parts supporting the breaker terminals and the middle panel supporting the operating mechanism, this last so designed as to permit easy removal from the case for maintenance or repair.

As a check on the condition of the trip mechanism of all network breakers, each primary feeder was opened daily during light load period, which gave an immediate indication of the failure of any breaker to open. It probably served to keep the operating mechanism in a more flexible condition as well.

The behavior of some of the network breakers in service showed the necessity of more rigid laboratory tests before placing them in service than had been made before. With this in mind, additions were made to the acceptance test procedure which resulted in considerably less trouble in the field.

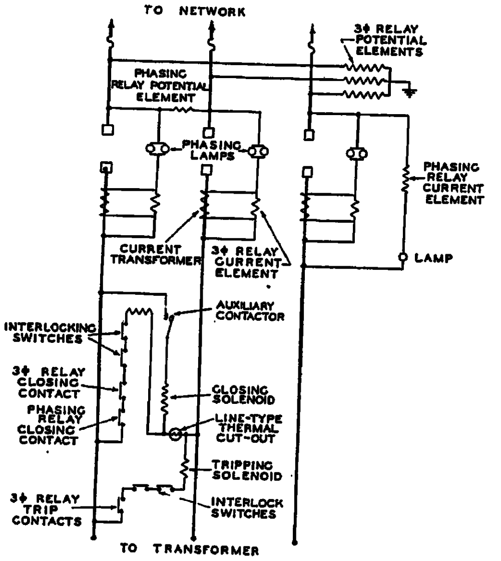

TO NETWORK
3ϕ RELAY POTENTIAL ELEMENTS
PHASING RELAY POTENTIAL ELEMENT
PHASING LAMPS
PHASING RELAY CURRENT ELEMENT
CURRENT TRANSFORMER
3ϕ RELAY CURRENT ELEMENT
LAMP
AUXILIARY CONTACTOR
INTERLOCKING SWITCHES
3ϕ RELAY CLOSING CONTACT
CLOSING SOLENOID
PHASING RELAY CLOSING CONTACT
LINE-TYPE THERMAL CUT-OUT
TRIPPING SOLENOID
3ϕ RELAY TRIP CONTACTS
INTERLOCK SWITCHES
TO TRANSFORMER

FIG. 4—SIMPLIFIED DIAGRAM OF SOLENOID-OPERATED NETWORK BREAKER

The laboratory tests are made primarily to reveal any mechanical or electrical defects due to manufacture or damage in shipment, and are used at the same time to check operating adjustments before the breakers are placed in service. These tests include inspection of electrical and mechanical parts, adjustment of settings of closing and tripping features, adjustment of contact pressure, and finally a detailed adjustment of three-phase and single-phase relays to operate correctly at the specified settings.

After the breakers are placed in service, certain routine tests are made in connection with the network inspections. These tests include checks on the operation of the tripping solenoid and of the closing mechanism, air pressure tests to assure the tightness of the submersible cases, and also tests of voltage drop across breaker contacts to indicate faulty contacts or breaker overload. No attempt is made to adjust network relays in service but a schedule is maintained whereby all relays on the system are returned to the laboratory for test at least once every three months. Better facilities for test are available in the laboratory than in the field, and it is considered advisable to make all tests on this important piece of equipment under the best possible conditions.

One function of the network which is of primary interest is its ability to burn clear any secondary faults. Due to the fact that the secondary mains installed here were composed entirely of new cable, the experience with secondary faults has been limited. In the few cases which have been observed, the faults have all cleared with no noticeable effect on network service. One of these faults occurred in a splice, a very severe test of the ability to clear, and even in this case the action was entirely successful, although the time of clearing was evidently prolonged much more than in the case of a fault in the cable itself.

Attention has also been devoted to assure in the case of a high-voltage feeder fault and the failure of a breaker to trip, that there will be a sufficient amount of secondary feed to blow the breaker fuses. In the case of breakers installed in connection with the 300 kv-a. transformer banks spaced farther than 450 ft. apart, it was considered advisable to replace the 2000 ampere fuses with 1500 ampere fuses which will blow more readily in case of a breaker failure and will still permit the feed of a sufficient amount of power into the network to clear any fault in a normal length of low-voltage cable.

Only one high-voltage fault has been experienced on the network, in this case the failure of a transformer bushing. The oil circuit breaker opened on current to ground and all breakers cleared successfully, but the fault is of some interest due to the difficulty in locating it.

The usual method of locating high-voltage cable faults has been to break down the insulation resistance at the fault, to circulate about 30 amperes direct current through the conductor and back through the lead sheath, then to locate the fault by means of a millivoltmeter used to indicate the direction of flow of current in the sheath. An attempt to apply this method to the location of the network fault proved unsuccessful, however, due to the large number of return paths for sheath current since the cable sheath is grounded to the transformer case and the case in turn is grounded to the neutral. In attempting to follow this method, a number of conflicting and misleading results was obtained.

Another method of locating primary network fault has since been devised, and tests on this method have been quite satisfactory. Essentially this procedure is to break down the fault and circulate about 30 amperes direct current through the faulted conductor with return path through the sheath as given in the previous method. The sheath current on the faulty primary cable is then neutralized in the various manholes by current supplied by a storage battery. The direction of the fault is shown by the effect of the fault current in the conductor on a compass needle.

Diagrams of these two methods are shown in Fig. 5.

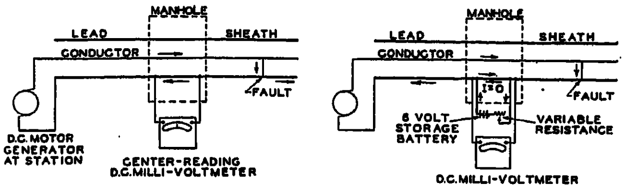


FIG. 5—(a) USUAL METHOD OF LOCATION OF HIGH-VOLTAGE CABLE FAULTS
(b) METHOD OF LOCATION OF HIGH-VOLTAGE NETWORK FEEDER FAULTS

REGULATION

Regulators were not installed on the primary feeders until some time after the network was in service. Two single-phase regulators, connected open-delta and mechanically interconnected, were installed on each feeder. A slight difference in secondary phase voltages noticed soon after they were in service was remedied by mechanical adjustments in the gearing between the two regulators on one of the feeders. Because of the possibility of causing pumping and also because of the various types of load connected, no attempt was made to regulate to any one point on the secondary, but rather to regulate to imaginary points approximately half way through the secondary of the transformers, using the same compensator setting on each feeder.

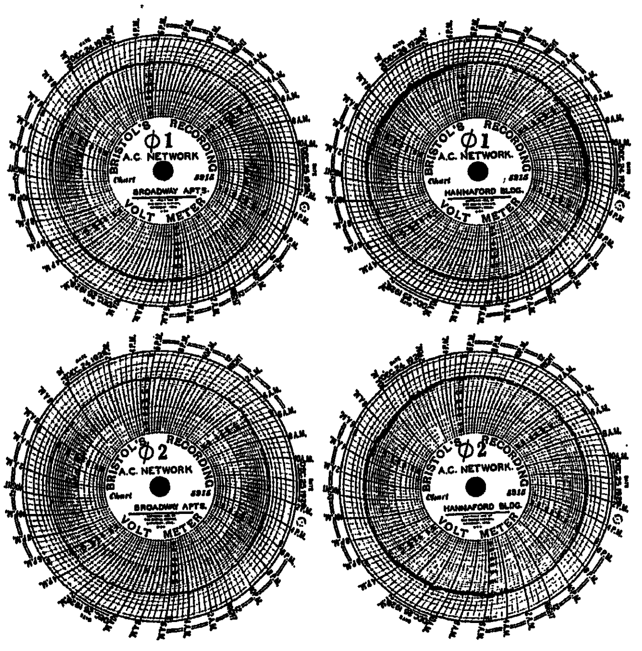


FIG. 6—VOLTAGE CHARTS FROM REPRESENTATIVE NETWORK SERVICES

Although the load both in kw. and reactive kv-a. had divided fairly evenly between the feeders with the regulators not in service, it was found that when in service, the kv-a. load on one was at times more than double that of the other. Tests showed that there was little difference in phase angle between the voltages impressed on the regulators, and also that changes in compensation failed to improve matters. It was finally discovered that one set of regulators was not operating properly due to an improper internal connection in the regulators.

When this was corrected, the compensation was arrived at by trial and during the comparatively short time since this correction has been made their operation has been satisfactory.

It may be of interest to note that there is no connection either mechanically or electrically between the two sets of regulators, one set of which is on a feeder from a generating station, the other on a feeder from a substation fed from this generating station and supplying both d-c. load and a-c. load through 4000-volt feeders.

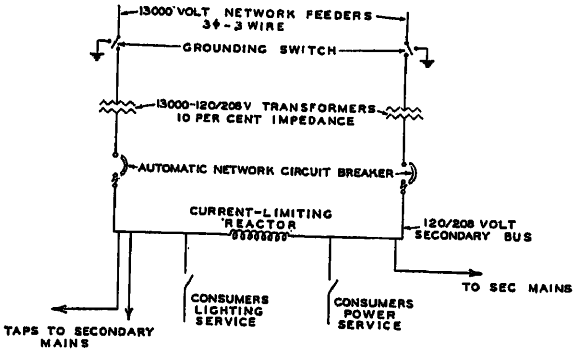

13000 VOLT NETWORK FEEDERS 3Φ-3 WIRE
GROUNDING SWITCH
13000-120/208V TRANSFORMERS 10 PER CENT IMPEDANCE
AUTOMATIC NETWORK CIRCUIT BREAKER
CURRENT-LIMITING REACTOR
120/208 VOLT SECONDARY BUS
TO SEC MAINS
CONSUMERS LIGHTING SERVICE
CONSUMERS POWER SERVICE
TAPS TO SECONDARY MAINS

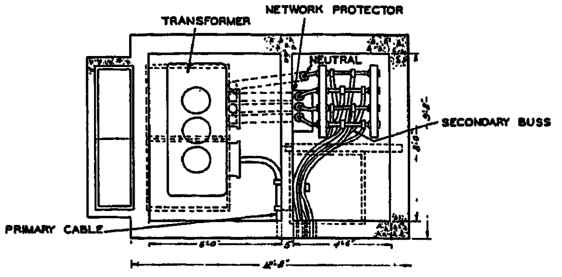
FIG. 7—LINE DIAGRAM OF DOUBLE TRANSFORMER VAULT INSTALLATION

Four general types of load are supplied by the network: manufacturing, hotel, office building, and theater. The charts shown in Fig. 6 are typical of voltage on consumers' premises, and indicate that regulation on the above basis gives very uniform voltage throughout the system in spite of the various types of load served.

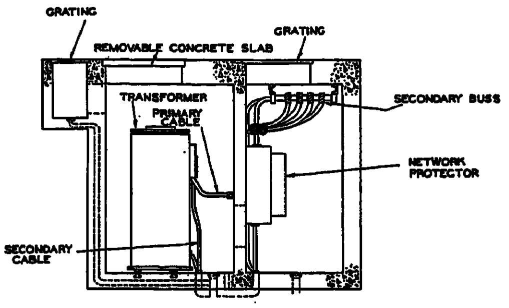
There have been no disturbances on the network since the regulators have been in operation, hence no data have been obtained as to their behavior under such conditions.

Another problem of regulation has been to prevent the starting current taken by large motors from affecting the lighting service. The results obtained by the use of reactors in the secondary bus in large buildings requiring more than one transformer bank indicate that it is practical to eliminate all effect whatever on the lighting voltage. A single line diagram of this plan is shown on Fig. 7.

This solution naturally could be used only where more than one transformer bank was installed. In other cases requiring only a single transformer, it was found necessary to take other steps to reduce the starting current of the motors. This has been accomplished by reconnection of the starters. It has resulted in requiring a longer starting time, but in no case have there been complaints on this account.



PLAN



ELEVATION

FIG. 8—STANDARD SIDEWALK VAULT FOR INSTALLATION OF ONE THREE-PHASE TRANSFORMER

It is felt that the requirements on starting current for motors connected to the network should be more stringent than for other parts of the system, and regulations are contemplated requiring the use of increment type of starters on motors on which the starting current is apt to cause light flicker.

The use of this type starter on 200-hp. motors connected to compressors has resulted in no noticeable variation in the lighting voltage.

OPERATING PROBLEMS ON CONSUMER'S EQUIPMENT

Since 120 volts for lighting had been standard on the distribution system in Cincinnati for a number of years, there was no reason for deviating from this for the network, especially since it offered more nearly rated three-phase voltage for motors.

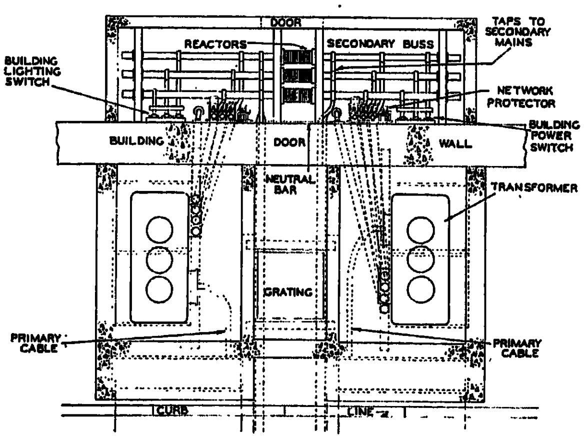
It is recognized that a motor operating with 75 per cent or less of rated load will give just as satisfactory service when operated on 208 volts as when operated on 220 volts and as had been expected, the greater part of the motor installations to be changed to the network service were found to fall in this class.

Auto-transformers have been installed for heating units rated at 220-240 volts and, in a few cases where contracts made with customers prior to the installation of the network contained clauses requiring 240 volts, three-phase service for power. However, it is no longer the practise to install them for new loads.

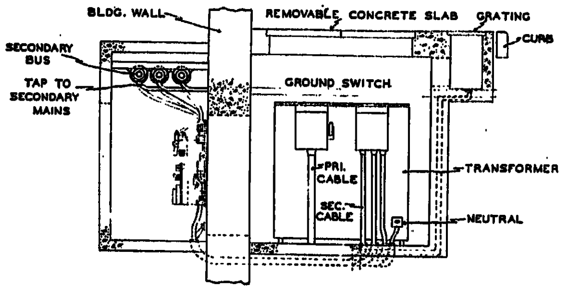
DESIGN AND CONSTRUCTION DETAILS

Figs. 8 and 9 show the construction details of sidewalk vault which are typical of a majority of the installations here. In the case of the double vault, the breaker compartment is located within the building wall, while the single vault is located entirely within the sidewalk. The transformers are of the type ordinarily used for low-voltage networks with grounding switch on the high side having line open, ground, and closed positions. A holding coil energized by the secondary of the transformer prevents the operation of the switch when the transformer is energized. Both open and submersible type network breakers are used, the former when installed in buildings, the latter in sidewalks. Transformers are of the submersible type in all cases.

Transformers first installed had the grounding switch in the transformer case. In those now used, it is contained in the high-voltage pothead. This change resulted in a considerable reduction in size and weight of the transformer, and in addition allows such work as change of oil to be done by throwing the grounding switch to the open position instead of keeping open the primary feeder as was necessary in the case of the original transformers.



PLAN



ELEVATION

FIG. 9—STANDARD SIDEWALK VAULT FOR INSTALLATION OF TWO THREE-PHASE TRANSFORMERS

While some single-phase transformer banks have been installed, the simplicity of installation of three-phase transformers has made them much more desirable.

From the standpoint of accessibility, both for installation and removal of equipment, and for operation, it has been the experience here that sidewalk vaults are preferable to those in buildings, especially where there is no direct connection between the building vault and the outside.

In order to prevent the necessity for phasing out from substations where additional transformer banks are being cut in, the practise of marking each phase in every splice by means of various colored tapes has been resorted to. This has resulted in a considerable saving of the time required for any such work on primary feeders. It also renders unnecessary any phasing whatever in replacing a faulty length of primary cable.

EQUIPMENT CHARACTERISTICS

At the time the network breakers were purchased, they were available in both the holding coil and shunt-trip type. The latter type was selected because of the possibility that voltage disturbances of a general nature on other parts of the system might cause unnecessary outages of the network due to the inability of the holding coil type breakers to remain closed on voltages lower than 35 per cent of normal. It was later found that this choice prevented a number of these outages, which while possibly not serious, were certainly not desirable from the standpoint of rendering first-class service to consumers.

In the selection of the amount of transformer impedance necessary, it was felt that this should be the maximum commensurate with satisfactory regulation. While the use of high impedance transformers reduces the size of secondary mains allowable due to limiting fault currents, this can be overcome by the installation of more than one set of mains where necessary, which is, in most cases, the most economical method to use in building up the secondary system. Transformer impedance of 10 per cent such as has been in use in Cincinnati has worked out very satisfactorily both from the standpoint of regulation and even distribution of load between transformer banks.

CONCLUSION

As has been stated before, the difficulties encountered were not of major importance and investigation as to their causes showed that the remedies to be applied were after all comparatively simple. They did serve, however, to bring out many of the main features to be considered in the layout of the system and in the selection and maintenance of the equipment.

It has been the aim here to furnish a-c. service to the downtown district comparable in reliability with that of the Edison system. Since there have been no interruptions in service other than local ones during the fifteen months that the low voltage network has been in operation, it is felt that this has been accomplished.

Discussion

L. L. Bosch: Mr. Pinckard's paper describes a two feeder a-c. network system which is probably the most difficult to operate. Should a fault occur on one feeder while the other is cut out of service, the system fails. Mr. Pinckard and his entire operating personnel have operated this type of system for fifteen months without an interruption. I think that they are to be highly complimented.

Since that time a third primary feeder has been added to the network, which materially reduced the operating hazards. During the fifteen-month period, the connected transformer capacity has been increased from an initial of 2000 kv-a. to a present of 15,000 kv-a.

The mechanical troubles with the a-c. network equipment which Mr. Pinckard described, I believe, show that the a-c. network is reliable rather than the contrary which might be believed at first. First, the mechanical troubles were numerous and difficult to overcome, but yet they did not cause an interruption to the system. In other words, the network is designed such that one, two, three, or any reasonable number of protectors may fail simultaneously without an interruption to the system's service.

It is believed that the question of starting currents of the motors on the a-c. network system should be given more consideration. In the d-c. system, it was necessary to have starting apparatus for the protection of the motor itself and of the connected mechanical apparatus. Many customers have come to the conclusion that since the a-c. motors are self-starting, nothing else need be considered, and that starting apparatus is unnecessary. They neglect entirely the question of line-voltage regulation. Motor starting currents on the network, as described by Mr. Pinckard, have been limited to increments of 100 amperes per ½ sec. This is calculated to give a voltage dip incident to starting the motor not greater than 2 per cent.

The quality of service delivered from this network indicates that this rule is justified.

H. C. Graves, Jr.: (communicated after adjournment) The difficulties Mr. Pinckard has experienced are typical and can be attributed to the breaker control equipment, and should not reflect on the a-c. low-voltage distributing system. Since the control and protective equipment is the weak link in the system, and since improvement here is most desirable and necessary, a brief recapitulation of the major faults emphasized by this article appears desirable.

1. Regeneration, or feed back through transformers causes incorrect breaker opening.
2. Small phase-angle differences between feeders, or regulation between feeders, will cause incorrect breaker opening.
3. System swings or synchronizing surges will cause incorrect breaker opening.
4. High-tension faults will frequently cause power flow through the network to cause incorrect breaker openings.
5. Regulators, both single-phase and three-phase, may cause incorrect openings. Regulator compensation must be carefully made and maintained. The hunting of regulators during system faults may frequently cause breaker operation.
6. Tripping relays must have characteristics suitable to trip on both lagging bank excitation current and leading cable charging current of very small values, and still give correct operation on current values of approximately one hundred times this value for both phase-to-phase and phase-to-ground faults on both high- and low-tension sides of the transformers. This is a severe design problem.

7. Tripping relays must maintain a delicate setting with little attention under very adverse operating conditions. Field tests are practically impossible. Necessarily delicate contacts fail because of the delicate settings and severe operating conditions.

That the equipment operates even as well as outlined by Mr. Pinckard reflects credit on the careful operation and maintenance of the equipment as well as on the ingenuity of the designers.

I should like to ask one question. Is it possible to select a secondary fuse for the network transformers that will blow under all conditions of transformer faults and yet will not operate under any condition of secondary faults such as the trouble in the splice mentioned in the paper? It would appear to me that numerous system conditions could exist which would prevent the proper discriminating action of a fuse.

F. E. Pinckard: Relative to the point brought up by Mr. Bosch on starting currents of motors connected to the a-c. network, while it is necessarily true in the case of a combined set of power and light mains that flicker is apt to occur, this matter has caused comparatively little trouble. It is also one that has a tendency to disappear as the system capacity increases.

As to the selectivity of the secondary fuse for the transformers and network breakers in Mr. Graves' question, this is accomplished by having three or four sets of mains feeding from each transformer bank. In case the network breaker failed to clear, the combined capacity of the mains should be sufficient to feed back enought current to blow this fuse. With the reverse condition, that is, a fault on the mains, it is assumed that this would occur on only one of the sets at a time, in which case the fuse capacity should be great enough to clear the fault.

Quick-Response Generator Voltage Regulator

Field Tests Made With Oscillograph

BY E. J. BURNHAM[1],
Associate, A. I. E. E.
J. R. NORTH[2],
Member, A. I. E. E,
and I. R. DOHR[3]
Non-member

Synopsis.—This paper describes elaborate and severe field tests made on high-speed excitation equipment used with a 5385-kv-a. turbo generator. Complete oscillograms and curves showing the performance of this equipment are included together with a discussion of the results obtained.

* * * * *

DURING 1928, high-speed excitation equipment was provided for a 5385 kv-a., house turbo generator which is used for supplying power to essential auxiliaries at the Saginaw River Steam Plant of the Consumers Power Company. Elaborate field tests were made on this regulating equipment, constituting the most severe tests made on a machine of such size, and the results are indicative of those which may be obtained on the larger size generators.

DESCRIPTION OF INSTALLATION

The Saginaw River Steam Plant, one of the largest steam generating stations on the system of the Consumers Power Company, will have, when completed in the near future, a total of five generators and two house generators, giving a generating capacity of approximately 140,000 kw. The No. 1 house turbo generator on which these tests were made is a General Electric unit rated 5385 kv-a., 3500 kw., 0.65 power factor, 2500 volts, 3600 rev. per min., 60 cycles and has a direct connected exciter rated 33.5 kw., 250 volts, four-pole, shunt wound. The house generators are connected to the 480-volt station auxiliary busses through 6000 kv-a., 2400-480-volt transformer banks. The 480-volt station auxiliary busses are connected to the main generating bus through 24,000-480-volt transformer banks.

The house turbo generators supply power to the more essential station auxiliaries consisting of boiler feed pumps, draft fans, circulating pumps, condensate pumps, etc. All motors used with these auxiliaries are started on full voltage and the larger ones, of 400 hp., are of the squirrel-cage type. The house generators may be operated in parallel with or isolated from the rest of the system.

A General Electric type FA-1 high-speed generator voltage regulator is provided to control the excitation of the No. 1 house generator. This regulator is equipped with a three-phase torque motor control and rheostatic follow-up features.

REQUIREMENTS

This high-speed excitation equipment was installed to accomplish the following:

(a) To accurately control the a-c. voltage under all conditions of picking up and dropping load without disturbing the operation of the other station auxiliaries operated from the same machine.

(b) To obtain a high speed of excitation response in connection with this generating unit in order that even the largest induction motors (400 hp.) used for station power service could be satisfactorily started at full voltage with minimum disturbance.

(c) To obtain first hand information concerning the operation of this type of quick response regulator, particularly regarding its speed of operation, precision, and stability of operation.

The range of excitation control on the generator is from 2300 volts, no-load and cold fields, to 2500 volts, full rated kw. load at rated power factor and hot fields. This range is necessary to provide adequate voltage on the 480-volt station auxiliary bus under various load conditions and allows for the regulation of the 2400-480-volt transformer bank. It corresponds to a range of 460 volts at no-load to 490 volts at full load.

DESIGN OF REGULATING EQUIPMENT

In order to meet the above requirements, a quick-response excitation system was provided consisting of an exciter with a high speed of voltage build-up and a high-speed regulator. The exciter is self-excited and has a speed of voltage build-up of approximately 1000 volts per second over the operating range.

Fig. 1 shows a simplified diagram of connections of the house generator excitation system and the FA-1 regulator. The regulator equipment consists of a main control element and an auxiliary control equipment. Figs. 2 and 3 show illustrations of this regulating equipment. The main control element consists of a d-c. system and an a-c. system and includes two lever arms, one controlled by a three-phase torque motor and the other by a d-c. magnet. A contact on the a-c. lever arm floats between two contacts on the d-c. lever arm. The torque motor is energized from two potential transformers which are connected to the three phases of the alternator, thus giving three-phase control. The d-c.

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1. General Electric Co., Schenectady, New York.
2. Stevens and Wood, Incorporated, Jackson, Michigan.
3. Consumers Power Company, Jackson, Michigan.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

magnet is connected across the armature of the exciter and provides the necessary anti-hunting feature and also the vibrating of main contacts, which will be described later.

The auxiliary control equipment includes two high-speed contactors, *L* and *R* (see Fig. 1), both of which are normally deenergized and arranged in such a manner that the contacts of *L* normally short circuit field reducing rheostat *l* in the exciter field circuit and the contacts of *R* are normally open but connected across the motor-operated field rheostat *f* and an additional field raising rheostat *r* in the exciter field circuit.

Quick response in the regulator is obtained by use of a spring dashpot connected to the a-c. lever arm of the main control element. The springs in the dashport allow either main contact to close without first moving the diaphram in the oil. With this arrangement, the torque motor may close the main contacts without waiting for the movement of the diaphram in the oil dashpot. This dashpot tends to stabilize the operation of the main contacts. This spring dashpot can also be applied to the vibrating type of regulator to obtain quick response.

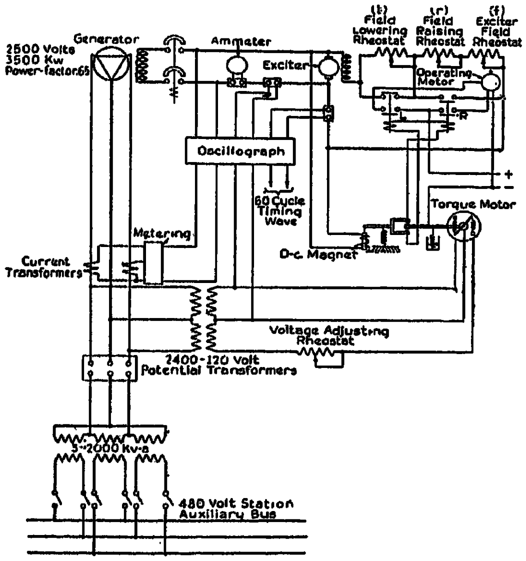

(l) Field Lowering Rheostat
(r) Field Raising Rheostat
(f) Exciter Field Rheostat
Generator
2500 Volts
3500 Kw
Power-factor.65
Ammeter
Exciter
Operating Motor
R
Oscillograph
+
−
60 Cycle Timing Wave
Torque Motor
Metering
D-c. Magnet
Current Transformers
Voltage Adjusting Rheostat
2400-120 Volt Potential Transformers
3-2000 Kv-a
480 Volt Station Auxiliary Bus

Fig. 1—Diagram of Regulator and Test Connections Saginaw River Steam Plant

The anti-hunting feature is provided by means of the d-c. coil on the main control element. This d-c. element, which is connected directly across the exciter armature, opens either the upper or lower main contact, whichever the case may be, after such contact is once made by the torque motor and the exciter voltage has changed the necessary amount.

Operation of Regulating Equipment

With the a-c. voltage of the alternator normal, the main control element is in equilibrium and the regulating equipment remains at rest. A decrease in a-c. voltage causes the torque motor to close the upper main contact, thus energizing contactor *R*, which in turn short-circuits rheostats *r* and *f* causing an increase in excitation. Upon an increase in a-c. voltage, the lower main contact closes, thus energizing contactor *L* which inserts rheostat *l* in the exciter field circuit to decrease the excitation.

For a decrease in a-c. voltage, the upper main contact and contactor *R* will open and close, thus functioning as a vibrating type regulator until the a-c. voltage returns to normal and the main element is in equilibrium. In the same manner, an increase in a-c. voltage will cause vibration of the lower main contact and contactor *L* until the a-c. voltage returns to normal.

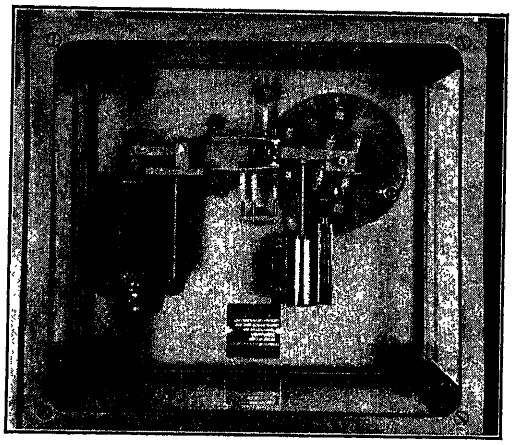


Fig. 2—Front View of Regulator Main Control Element

Continuous operation of the high speed contactors is kept to a minimum by means of the rheostatic follow-up feature which functions as follows: Each time that either high-speed contactor, *L* or *R*, is operated, an auxiliary contact energizes the motor of the motor-operated rheostat in the correct direction to bring the a-c. voltage back to normal. During a time when the

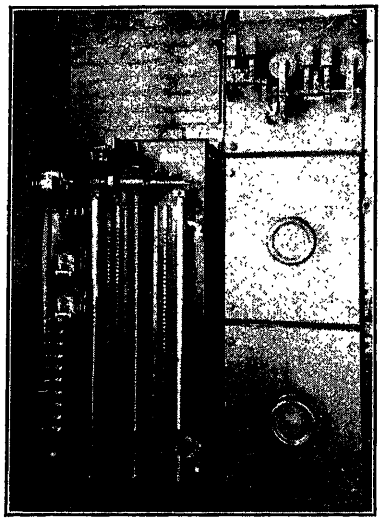


Fig. 3—Front View of Auxiliary Contactor Panel and Motor Operated Rheostat

a-c. voltage is low, the arm on the motor-operated rheostat is notched along to cut out resistance each time the upper main contact vibrates and the contactor *R* opens and closes. This follow-up action continues for a short time until the rheostat arm is in the correct position to take care of the new excitation requirements. When this correct position is reached and the a-c. voltage is back to normal, the main control element is in equilibrium and the regulating equipment is again at rest. During a time when the a-c. voltage is high, the equipment functions in a similar manner, opening and closing contactor *L* and notching the rheostat arm along to cut resistance in the motor operated rheostat.

FIELD TESTS ON REGULATING EQUIPMENT

An extensive series of field tests was conducted on this regulating equipment to enable it to be adjusted under actual service conditions and to obtain detail information regarding its operation. These tests consisted of suddenly picking up or dropping large amounts of load with the excitation of the generator under regulator control, and also with fixed excitation. Complete oscillographic records were obtained as well as graphic voltmeter charts and readings on indicating meters. The operation of the regulating equipment when the generator was paralleled with the rest of the system was also recorded.

Preliminary Adjustments. A series of preliminary tests was made for the purpose of observing the operation of the regulating equipment, making the necessary preliminary adjustments and checking its operation. Fig. 1 shows the connections used in making the test. The effect of the regulator on the generator voltage was recorded by means of an Esterline quick-trip graphic voltmeter and readings were taken on the switchboard indicating instruments. The sequence of operation of the various elements of the regulating equipment was noted and the field regulating rheostats were adjusted to the proper value of resistance. The effect of various adjustments was studied by suddenly applying load to the generator.

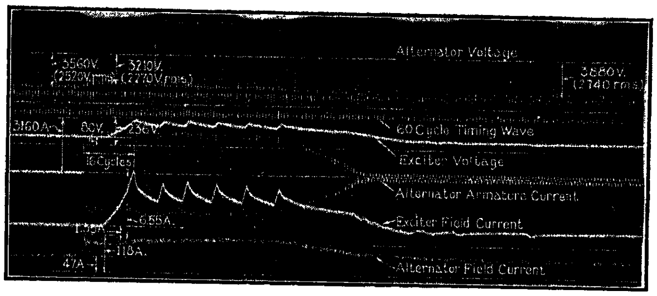
TABLE I
OUTLINE OF TESTS

Tests	Excitation	Generator		Test conditions
5,7	Regulated	Isolated	Carrying no load	Load applied (2 motors)
6,8	Fixed	Isolated	Carrying no load	Load applied (2 motors)
9	Regulated	Isolated	Carrying load	Load applied (2 motors)
10	Fixed	Isolated	Carrying load	Load applied (1 motor)
11	Fixed	Paralleled	Carrying load	Load dropped
12	Regulated	Paralleled	Carrying load	Load dropped

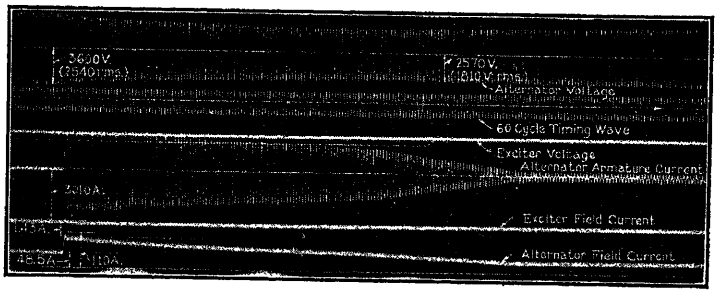
Final Tests. The final series of tests consisted of throwing a large amount of load on the generator with the regulating equipment in service and with fixed excitation, and under the conditions of load and operation shown on Table I.

Load was suddenly applied by starting one or two 400-hp. motors simultaneously at full voltage. Starting two motors simultaneously was equivalent to a momentary load of approximately 7500 kv-a. or approximately 130 per cent generator rating at approximately 35 per cent power factor lagging. The motors came up to speed in a very short time, approximately one second with the regulator in service and two seconds with fixed excitation, and their load demand dropped rapidly to a low value as soon as they came up to speed. Load was suddenly dropped by operating the generator paralleled with the system and carrying load, then suddenly opening the generator oil circuit breaker.

Oscillograph records were taken of generator voltage, generator armature current, generator field current, exciter voltage, exciter field current, and 60 cycle timing wave. Fig. 1 shows the connections used during the tests.



Test No. 7
Generator isolated and carrying no-load
Regulator in service
Started No. 3 and No. 6, 400-hp. motors



Test No. 8
Generator isolated and carrying no-load
Regulator not in service
Started No. 3 and No. 6, 400-hp. motors

FIG. 4—OSCILLOGRAMS OF TESTS NOS. 7 AND 8

Results of Tests. Figs. 4, 5, and 6 show representative oscillographic records obtained during the tests and Fig. 7 representative graphic voltmeter charts. The oscillograms have been carefully analyzed and the results are shown by curves in Figs. 8, 9, 10, and 11.

DISCUSSION OF RESULTS

From the curves, Figs. 8 to 11 inclusive, the results obtained with and without the regulator can be compared for the different test conditions.

As previously pointed out, the starting of two 400-hp. motors at full voltage constituted a load of approximately 7500 kv-a. at approximately 35 per cent power factor lagging. The inrush current to the motors upon starting is practically all wattless, therefore the duty upon the generator is severe as far as maintaining a-c. voltage is concerned because of the highly demagnetizing action of the armature current. The maximum momentary value of inrush current was the same with fixed excitation as with the regulator in service and was approximately 145 per cent of the generator current rating when starting two motors. It was approximately 100 per cent of the generator current rating when starting only one motor. With fixed excitation this current continually decreased in value after the first instant. However, with the regulator in service it was maintained near its maximum value until the motors approached full speed.

It will be noted that the instantaneous inrush of motor-starting current causes an initial increase in generator field current and an instantaneous drop of generator voltage, these values being the same whether the regulator was used or not. With fixed excitation the exciter voltage remained constant and the generator field current after its initial rise gradually decreases to its original value. With the regulator in service the exciter voltage rapidly increased to its maximum value and increased the generator field current to restore normal a-c. voltage.

When two motors were thrown on the line, the generator voltage dropped instantly approximately 8 per

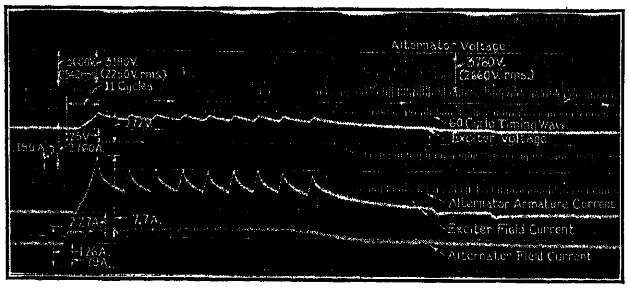


Fig. 5—Oscillograms of Test No. 9

Generator isolated and carrying load
Regulator in service
Started No. 3 and No. 6, 400-hp. motors

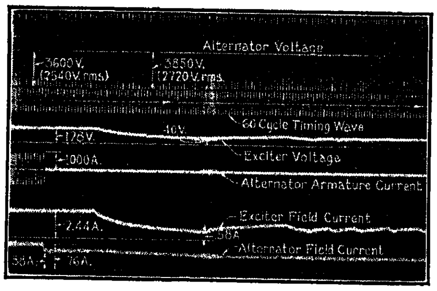


Fig. 6—Oscillograms of Test No. 12

Generator paralleled with system and carrying load
Regulator in service
Dropped load

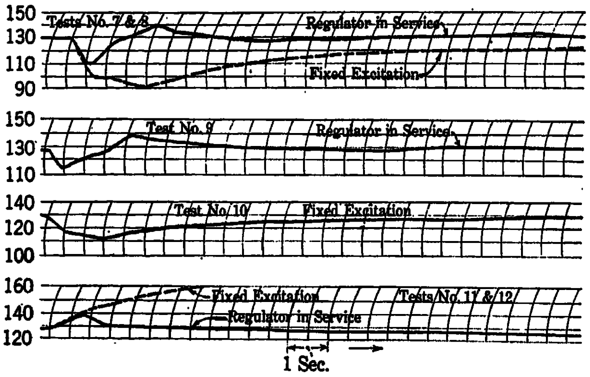


Fig. 7—Graphic Voltmeter Charts

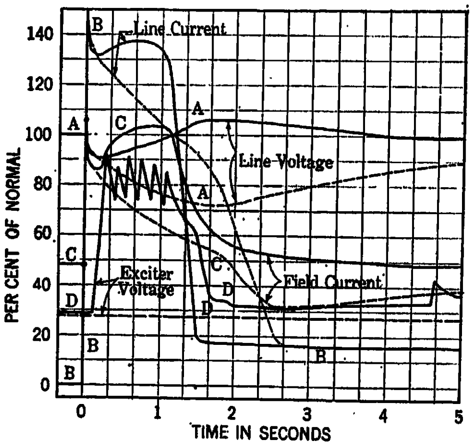


Fig. 8—Curves Showing Results of Tests Nos. 7 and 8

Generator isolated and carrying no load started two 400-hp. motors
——— Regulator in service
- - - - Fixed excitation

A.	Line voltage 2500 volt	= 100 per cent
B.	Line current 1245 amperes	= 100
C.	Field current 110 amperes	= 100
D.	Exciter voltage 250 volt	= 100

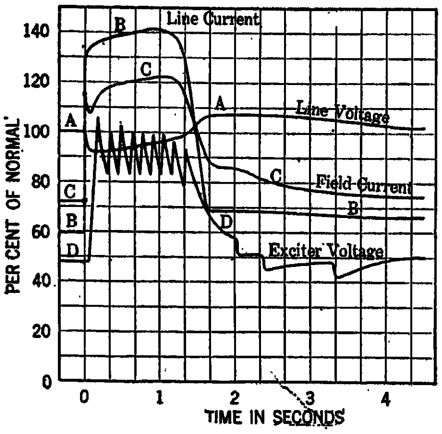


Fig. 9—Curves Showing Results of Test No. 9

Generator isolated and carrying load started two 400-hp. motors
Regulator in service

A.	Line voltage 2500 volt	= 100 per cent
B.	Line current 1245 amperes	= 100
C.	Field current 110 amperes	= 100
D.	Exciter voltage 250 volt	= 100

cent. After the initial dip of generator voltage there was a decided difference in the voltage curves obtained with the regulator in service and with fixed excitation. The maximum dip in generator voltage and the duration of the dip with fixed excitation was longer and much more severe than was the case under regulator control. Under regulator control the generator voltage was quickly raised to normal, but the sudden drop in line current when the motors come up to speed caused the voltage to rise above normal. The regulator was then called upon to adjust the excitation to reduce the voltage to normal. The vibrating action of the regulator is plainly shown by the rapid rise and fall of exciter voltage on the curves. At the time the regulator is vibrating, the frequency of the vibrations is about the same as would be expected with an ordinary vibrating type regulator.

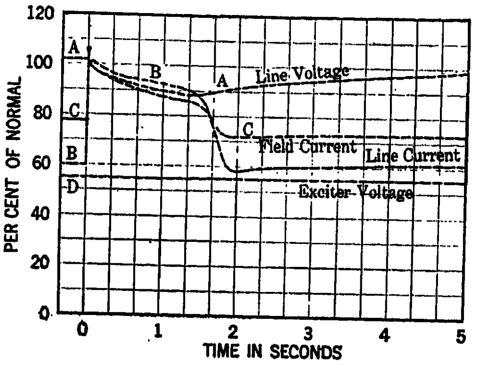


FIG. 10—CURVES SHOWING RESULTS OF TEST No. 10

Generator isolated and carrying load started two 400-hp. motors
Regulator not in service
A. Line voltage 2500 volt = 100 per cent
B. Line current 1245 amperes = 100
C. Field current 110 amperes = 100
D. Exciter voltage 250 volt = 100

Fig. 8 shows the results with and without the regulator in service under the condition of starting two motors at a time when the generator was carrying no load. The initial increase in generator armature current caused the generator field current to increase instantly from 48 per cent to approximately 100 per cent of normal full-load value. The generator voltage correspondingly decreased instantly to approximately 92 per cent of normal value. With fixed excitation, the generator voltage continued to decrease in value for about two seconds, at which time it reached a value 72 per cent of normal. From this point the generator voltage gradually returned to normal.

The effect of the generator carrying load when starting two motors can be seen by comparing Figs. 8 and 9. The line current did not momentarily increase to such a high value under load conditions as under no-load conditions. However, the duty was more severe under load conditions, as indicated by the duration of the disturbance and the fact that the regulator was not able to raise the a-c. voltage to normal as quickly, even though the speed of exciter voltage build-up was greater. The increased speed of build-up was due to the excitation being at a higher value initially.

Fig. 11 shows the effectiveness of the voltage regulator in limiting the rise of generator voltage upon the dropping of load.

Table II gives a summary of the test results and shows the successive operating intervals of the different portions of the excitation system from the instant the generator load changes until the generator voltage is restored to normal, the speed of voltage build-up of the exciter and the generator, and the limits of exciter and generator voltages reached during the tests.

The successive operating time intervals represent the time in cycles required for the individual portions of the excitation equipment to operate. It will be noted that the main contacts of the regulator closed in approximately 2.5 cycles after the load changed. The auxiliary raised contactors closed in approximately four cycles after the main contacts closed and the exciter voltage built up to its maximum value in an average time of approximately nine cycles after the auxiliary contactors had closed. The corresponding operating time intervals with the regulating equipment decreasing the excitation are somewhat longer. As shown by Test No. 12, the auxiliary lowering contactor closed in 14 cycles, and the exciter voltage decreased to its minimum value in 37 cycles after the auxiliary contactors had closed.

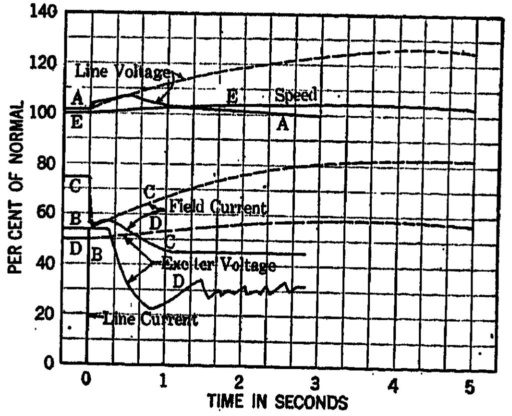


FIG. 11—CURVES SHOWING RESULTS OF TESTS NOS. 11 AND 12

Generator paralleled with system and carrying load
Dropped Load
——— Regulator in service
- - - - Fixed excitation
A. Line voltage 2500 volt = 100 per cent
B. Line current 1245 amperes = 100
C. Field current 110 amperes = 100
D. Exciter voltage 250 volt = 100

The above time intervals are of interest as regards the operation of the various component parts of the regulating equipment, but from an operating standpoint, the total time interval elapsing from the instant the load changes until the generator voltage is restored to normal is of prime importance. Referring again to Table II, it will be seen that with the regulator in

TABLE II
SUMMARY OF TEST RESULTS

Test No.	Test conditions			Successive operating time intervals (cycles)					Speed of build-up (Volts per sec.)		Exciter voltage		Generator voltage		
							Total‡								
	Excitation	Generator	Load	Main contacts	Aux. contactors	Exciter voltage	(A)	(B)	Exciter	Gen.†	Normal	Extreme	Normal	Min.	Max.
5	Regulated	Isolated No load	Picked up	2.0	3.0	9	65	660	930	300	80	236	2540	2240	2620
6	Fixed	Isolated No load	Picked up	....	....	....	1200	1200	....	−290*	76	76	2530	1870	2530
7	Regulated	Isolated No load	Picked up	2.0	3.0	11	70	720	1040	270	80	236	2520	2270	2740
8	Fixed	Isolated No load	Picked up	....	....	....	1200	1200	....	−500*	76	76	2540	1800	2540
9	Regulated	Isolated Carrying load	Picked up	2.0	2.0	7	83	600	1200	225	125	272	2540	2250	2660
10	Fixed	Isolated Carrying load	Picked up	....	....	....	720	720	....	− 170*	130	130	2630	2210	2630
11	Fixed	Paralleled Carrying load	Dropped	....	....	....	....	....	....	225	120	120	2660	....	3240
12	Regulated	Paralleled Carrying load	Dropped	3.0	14	37	110	720	− 140*	− 110*	128	40	2540	....	2720
13	Regulated	Isolated No load	Picked up	2.5	8.0	....	....	....	....	150	....	....	2540	2390	2600

NOTE: *Indicates rate of voltage decay.
†Approximate rate during time motors coming up to speed.
‡A Time interval refers to point when generator voltage first returned to normal.
‡B Time interval refers to point when generator voltage became stable at normal value.

service the generator voltage first returns to the normal value in an average time of approximately 81 cycles or about 1.5 seconds. As previously mentioned, the generator voltage overshot and finally became stable at the normal value after a total average time interval of 570 cycles or 9.5 seconds. With fixed excitation, the voltage returned to normal after an average time interval of approximately 20 seconds. It was rather hard to determine at just what point the generator voltage became stable at the normal value, but the above values are relative.

The speed of exciter build-up was very fast, considering that the exciter was self-excited, being of the order of 1000 volts per second, but it will be noted that the speed of exciter voltage build-down is very much slower. Additional resistance has been connected in series with the field lowering rheostat, since these tests were made, to increase the speed of exciter voltage decay. However, it should be pointed out that from an operating standpoint the speed of voltage build-up and decay of the exciter is of secondary importance to the speed of generator voltage response. The generator voltage increased, as shown by Table II, at an average rate of approximately 250 volts per second during the time the motor was coming up to speed.

CONCLUSIONS

The following conclusions may be drawn from the results of these tests.

(a) The quick response excitation equipment is effective in reducing the magnitude and duration of a-c. voltage surges due to sudden increase or decrease in load.

(b) From an operating standpoint, the over-all speed of a-c. generator voltage response is of prime importance rather than exciter speed of build-up. In other words, the total time interval from the instant of load change to the time when the a-c. voltage is restored to normal is most important.

Discussion

F. C. Hanker: When considering the selection of the proper size of auxiliary generator for a large station, the excitation scheme is of prime importance and the information presented by the authors is of considerable use to the application engineer. In the case of the auxiliary generator, continuity of service under varied conditions is very important.

Two years ago a similar set of tests was made by Messrs. Anderson and Monteith at the Richmond Station of the Philadelphia Electric Company, and the results were presented before the Institute. More information of this sort is needed to show the requirements of operation to be met by the apparatus.

The problem can be attacked either by having machines with good inherent regulation that will operate satisfactorily when heavy inductive loads, such as starting auxiliary motors on full voltage, are applied, or by the use of quick-response excitation which will tend to maintain the flux and thus allow only the armature reactance drop which is usually not serious. The degree to which the flux is maintained will depend on the layout of the excitation system.

The regulator which the authors describe is in reality what the Westinghouse Company term an "exciter rheostatic" regulator. This type of regulator has been used by the Westinghouse Company for the past several years. The first application was in the station of the Niagara Falls Power Company with normal speed of response of the exciter voltage. It has since been applied on a number of generators and condensers in connection with quick-reponse excitation, and in the last year or so also where the flux was built up at a very high rate.

The inherent speed of the regulator elements becomes a factor in the application where high-speed excitation is used. With the type of regulator under discussion, larger parts have to be moved than with a vibrating type of regulator; therefore greater forces must be applied. This brings up the question of whether contactors of the normal design will meet the operating conditions satisfactorily. We have found it necessary to use specially designed contactors for this service.

R. M. Carothers: The authors describe a certain and specific regulator problem, point out how that problem was attacked and the success with which it was met. The regulator they describe is not the vibrating type but is a regulator for customers who prefer this type of regulator, and also for places where the vibrating type of regulator is not required.

I might add that it is not the only type of regulator that can be termed "quick-response." The vibrating type of regulator with certain modification is also a quick-response voltage regulator.

H. L. Weinstein: All the test results shown in the paper were taken on the high side of the transformer bank, and the impedance of the bank was not given, so it is hard to say what effect the drop had on the low-voltage bus, but it is possible by maintaining voltage on the high side of the bus to have a varying low side due to the varying power factor of the motors as they come up to speed.

I should like to ask a question about this regulator. In the paper presented by Mr. Carothers and Mr. Nichols at the St. Louis meeting, the lever system contained in addition to the d-c. coil an a-c. coil. Is this a commercial form of regulator or has it been abandoned?

A. C. Monteith: (communicated after adjournment) This paper adds considerably to the meager amount of information available on the question of starting auxiliary motors, and it corroborates the work of Mr. Anderson and the writer which was presented in 1927 under the subject *Auxiliary Power at Richmond Station.* The conclusion drawn at that time was that a voltage regulator was very desirable.

In attempting to make a comparison between the so-called special regulator used by the authors, and the standard Westinghouse regulator used at Richmond Station, the following figures have been tabulated from the two papers on the subject. These are taken from Fig. 8 in the paper under discussion and Fig. 10 of the former paper:

Machine	Kv-a. rating	Reactance	Max. load	P. F.	Load per cent rating	Voltage	
						With reg.	Without
Saginaw.....	5385	6% App.	7500	35%	140	90	72
Richmond...	3333	5.6%	4200	22–35%	125	82	55

It is rather difficult to make a direct comparison on account of the factors involved, such as:

1. Regulation characteristics of the generator
2. Characteristics of the load
3. Speed of response of the exciter
4. Characteristics of the regulator

but nevertheless, the following conclusions are indicated:

(1) A detailed comparison of the regulation characteristics of the generators is difficult without complete details of the two ma-

chines. From the data available, it is believed that the inherent reactance is approximately the same in the two generators. The indications are that the Saginaw machine had somewhat better regulation since 140 per cent load dropped the voltage only 28 per cent without a regulator, while 125 per cent load on the Richmond machine dropped the voltage 45 per cent.

(2) In the case of the Saginaw machine, two 400-hp. motors were started, coming up to speed in one second with a regulator, and two seconds without a regulator. In the case of the Richmond tests, the motors consisted of one 500-hp., squirrel-cage motor driving a circulating water pump; two 150-hp., squirrel-cage motors driving ventilating fans; and two 125-hp., wound-rotor motors driving draft fans. The 500-hp. motor had approximately 22 per cent power factor starting current. The ventilating fans had in the neighborhood of 30 per cent and the draft fans were higher due to having a secondary resistance in the rotor. The last mentioned "CW" motor starting currents were small compared to the other three motors. Also the 500-hp. pump motor came up to speed in 1.4 sec. with a regulator, and in over 2 sec. without a regulator, while it took the four fans considerably longer to come up to speed. The tests indicate approximately 17 sec. with a regulator.

(3) The Saginaw exciter had a speed of response of 1000 volts per sec., while the Richmond machine had 100 volts per second speed of response of exciter terminal voltage.

(4) The characteristics of the regulator will have considerable bearing on the results obtained. The prime requisite is to put as much excitation into the machine as practical to offset the demagnetizing effect of the lagging current in the generator. The regulator used for the Richmond tests was a standard Westinghouse type vibrating regulator. When the load was picked up, the contacts closed and remained closed until normal voltage was restored, permitting the exciter to build up to its maximum or ceiling voltage. This is not the case for the regulator used for the Saginaw tests. Referring to Fig. 8 of the paper, it will be observed that after about 0.3 sec., the regulator contacts opened, arresting the rise of the exciter voltage in spite of the fact that the terminal voltage of the generator is very near its lowest value. Because of these inherent characteristics of the regulators, for a given exciter the former will supply more excitation than the latter.

It is seen from the above that in the case of the Richmond machine, the regulator was very effective, even with normal speed of response of exciter voltage as compared to the higher speed of response of exciter voltage on the Saginaw machine. For the tests tabulated, the Saginaw machine returned to normal in 70 cycles, while the Richmond machine voltage was normal in 96 cycles. The fact that it took the motors longer to come up to speed and that the voltage drop was greater would readily account for this difference in time.

Otto Naef: (communicated after adjournment) It is not often that actual data and test results on the operation of regulating devices are made public, and the authors are to be congratulated for the thoroughness with which they have analyzed and presented the test results obtained with the General Electric quick-response generator voltage regulator.

This regulator is of the combined rheostatic and vibrating type. The combination of these two elements in one regulator is indicative of the general trend in regulator design to get away from vibrating contacts with their attendant wear, and to replace them by something which gives more reliable operation. The question naturally arises, is it necessary to use vibrating contacts at all; it is possible to design an automatic regulator, which has quick response, without vibrating contacts?

I shall try to answer this question by showing a few test results obtained with a regulator built on the purely rheostatic principle, but distinguished from its slow-acting predecessor by its high speed of operation and powerful over-regulation.

The responsiveness of an automatic regulator may be expressed by the time, measured in cycles, which elapses from the moment a voltage change occurs until the regulator has adjusted itself to the position of maximum or minimum excitation, whichever may be required to bring the voltage back to normal. This time is generally a function of the magnitude of the voltage change acting on the regulator. That the regulator described in the paper has this same characteristic is indicated by the test figures presented. When switching on load, the instantaneous voltage drop was 8 per cent, and it required 6.5 cycles for the auxiliary contactor to close, of which 2.5 cycles were consumed in the main contacts. During this interval of 6.5 cycles, before the regulator would act, the generator voltage would drop another 3 to 4 per cent, the total drop being therefore 11 to 12 per cent. When dropping load, the paper states that it took 14 cycles for the auxiliary contactor to open, a considerable increase as compared to the first test. But the instantaneous voltage change acting on the regulator was also considerably smaller than before, as shown by the oscillograms. The influence of the magnitude of the voltage change on the responsiveness of the regulator is thus clearly demonstrated.

Before showing the test results obtained with the rheostatic type regulator it will be necessary that I explain briefly how it operates and in which way the tests were carried out.

Fig. 1 herewith shows a schematic diagram of the regulator, which is manufactured by Brown Boveri and is built in a large size, with four sectors, and in a small size, with two sectors. It is entirely self-contained, requiring no additional apparatus, relays, etc. The diagram explains the mechanism of the sectors which cut resistance in and out of the shunt field circuit of the exciter. Instead of gliding, the sectors roll over the contacts, thus doing away with gliding friction. The reactions from the two or four sectors are accurately balanced and in no position of the moving system do they exert a torque on the regulating spindle. The position of the regulating system is indicated by the position of the pointer. In position "Zero" all resistance is cut out. In position "4" all resistance is in. These two positions therefore correspond to maximum and minimum excitation, respectively. The diagram also shows how the regulator is connected to the generator whose voltage is to be controlled. The electric torque is provided by a powerful torque motor. The mechanical counter torque is obtained by spring tension. *o* and *p* is the anti-hunting device, consisting of an aluminum disk rotating between two permanent magnets. The disk is coupled to the regulating system by a soft spring, and therefore does not resist quick response of the regulator to sudden changes in voltage.

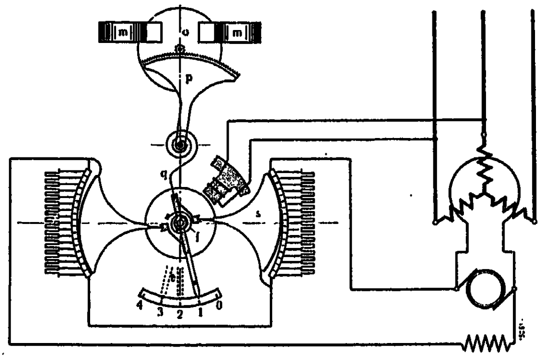


Fig. 1—Diagram Illustrating the Regulating Process.

The tests to determine the responsiveness of the regulator were made years ago on a standard four-sector regulator. For this purpose the voltage element was connected to a 60-cycle constant potential source through a resistance absorbing 11 per cent of the voltage. The voltage at the regulator terminals was 110 volts. By short circuiting the resistance by means of an auxiliary switch the voltage across the regulator terminals could be raised 11 per cent instantaneously. Different starting positions of the pointer were investigated, and the time required by the regulator to move from the starting position to position 4, corresponding to minimum excitation, was measured by connecting a battery and resistance across the regulator rheostat, and oscillographing the current flowing in the circuit.

The oscillogram represented in Fig. 2 herewith shows the regulator at rest in position 1.5, which generally corresponds to approximately full load on the generator. The test conditions were therefore similar to those that would exist if full load were thrown off a generator, resulting in a momentary voltage rise of 11 per cent. The curved line represents the variation of the current through the regulator resistances, the horizontal lines are calibration lines and show the current flowing through the sectors at intermediate positions. It will be noted that the time required by the regulator to begin motion is less than one cycle. After 6 cycles the regulator has completed half its course, and after 12 cycles it reaches its extreme position at 4, corresponding to minimum excitation. For starting positions below or above 1.5, the number of cycles required by the regulator to reach the end position 4 was found to be correspondingly larger or smaller.

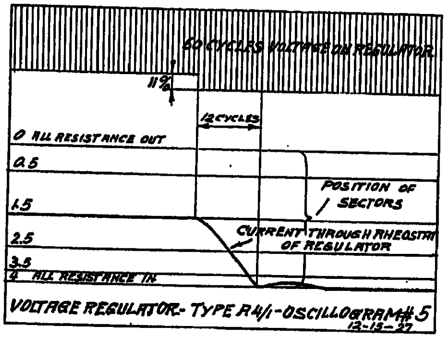


Fig. 2

It is to be noted that in its effect on the field current, the 12 cycles required by the rheostatic regulator to reach its end position is approximately equivalent to 6 cycles required by a vibrating type regulator to close the contacts of its auxiliary relay. This is due to the fact that the rheostatic regulator starts at once to cut the field resistance in or out, whereas the vibrating regulator has no effect on the field current during the first few cycles.

The above tests, which were made under conditions similar to those disclosed in the paper, as far as the regulator is concerned, indicate that the responsiveness of the rheostatic type of regulator compares very favorably with that of the vibrating type. There remains one difference, however, between the two types of regulators, which has some influence on the rate of generator field current build-up or decay. Whereas the regulator described in the paper begins to vibrate when a change in voltage occurs, and maintains the exciter voltage at an average value below the extreme minimum or maximum value, the rheostatic type of regulator keeps the exciter voltage at its very minimum or maximum until the generator voltage has been restored almost to normal. The rheostatic type of regulator therefore tends to produce a quicker response of the generator.

The purpose of this discussion was to show that the rheostatic principle is as applicable as the vibrating principle to quick-response automatic voltage regulators such as are demanded by the modern requirements of system operation.

E. J. Burnham: Mr. Hanker mentioned the contactors and inquired whether they were special. These contactors were special to the extent that they were made high-speed, that is, ordinary contactors were lightened so that they would vibrate much faster than the standard.

Regarding Mr. Weinstein's question I might say that the a-c. coil which is added to the left-hand member is replaced in this regulator by a spring dash-pot on the a-c. lever. In other words, the same thing is accomplished by this spring dash-pot, and is, you might say, a commercial form of the quick-response feature.

The tabulation cited by Mr. Monteith might indicate that the gain between 1000 and 100 volts per second is not nearly as great as the gain between 100 volts per second and constant excitation. This, of course, is natural. With a normal ceiling exciter it is questionable whether even an infinite volt-per-second rate would produce an appreciable gain over the 1000 volt-per-second arrangement. In other words the gain to be obtained is less and less pronounced as the volt-per-second build-up is increased.

Mr. Monteith also states that if the Westinghouse regulator were used in place of the one described, it would have supplied more excitation and implies that this is an advantage. Extensive tests such as those described in the two papers and discussion given at the St. Louis Regional Meeting, March 1928 (A. I. E. E. Trans., Vol. 47, pp. 944-974) have shown that in most cases it is not desirable to continue increasing the excitation until normal voltage is reached, when maximum benefit and stability are considered.

J. R. North: With regard to the question about the location of the potential transformer during the test, the idea was to get the actual results at the terminal of the generator. In this installation the load is served from the 189-volt bus and under normal conditions the regulator is controlled by potential transformers connected directly to the 480-volt bus.

Street Railway Power Economics
on the Cincinnati System

BY J. A. NOERTKER[1]
Associate, A. I. E. E.

Synopsis.—The Cincinnati Street Railway Company has, within the past few months, completed the rehabilitation of its entire power system. The system now consists of 19 full automatic synchronous converter substations upon which has been superimposed a complete system of supervisory control and remote metering. Papers have been presented by Frank W. Peters and Harley L. Swift covering the details of this installation. (See bibliography.) This paper discusses the economic factors involved in the selection of equipment and the design of the system. Part I points out that the most important economic consideration is service and presents a method for evaluating this factor. Part II discusses the design of feeder circuits with particular reference to the limitations of Kelvin's Law. Part III discusses system design with reference to the economics involved in the location of substations, and in the selection of control equipment. A general method for conducting extensive system studies is implied. Part IV discusses system load shifting characteristics and Part V points out the advantages of supervisory control.

I. INTRODUCTION

THE management of a modern street railway system, especially one operating under a "service at cost" franchise, should and does aim to provide a service that most nearly meets the requirements of the average car-rider which implies the fastest schedule speeds consistent with safety, and ultimate economy. As a matter of fact, ultimate economy as it concerns both car-rider and company is one of the most important considerations.

The car-rider is interested in continuity, cost, and quality of service. The primary factor affecting the quality of service is speed of car operation, and some idea of the magnitude of this factor may be had by considering that in a city of the size and character of Cincinnati the total annual time spent by the car-riding public in transportation is approximately 50,000,000 hr.

It is practically impossible to determine definitely a value for this time; however, assuming a rate of fifty cents per hour, the annual amount will be $25,000,000, which amount is of sufficient magnitude to urge a careful analysis of all factors affecting the speed of car operation, in the interest of both the car-rider and the management. Among the many factors affecting the speed of car operation the following are outstanding:

1. Topography
2. Traffic congestion and control
3. Number and duration of stops
4. Rates of acceleration and retardation
5. Motor characteristics
6. Trolley voltage.

Two of these factors, motor characteristics and trolley voltage, are directly under the control of the railway engineers. In selecting car equipments, it is necessary to assume average conditions of the above factors. The proper voltage has also usually been determined for average conditions and in the design of the power system the effort has been to furnish this average voltage over the entire system. It is not certain, however, that an average voltage will fulfill all conditions in the most effective manner. Since the primary function of the power system is to furnish the most economical and satisfactory trolley voltage for the operation of the cars, it is of the first importance to determine what voltage or voltages are the most economical and satisfactory for various operating conditions and then provide such voltage through the means of variable converter characteristics and transformer taps.

While poor voltage conditions may result in loss of time by passengers, aside from evaluation of this time, it can be shown that for a city the size of Cincinnati the annual variable cost of power supply subject to manipulation on which there is no practical check is approximately $180,000.00. This is made up of carrying charges on feeders, conversion loss, demand loss, energy loss, and carrying charges on that portion of the conversion equipment required to supply the feeder loss. It is evident from this fact alone that a thorough study of the power system is justified.

In an effort to determine the proper voltages for the operation of cars under various conditions, the following curves have been plotted from the data derived from speed-time and power-time curves of the latest type cars of the Cincinnati Street Railway Company.

Fig. 1 shows the minimum time required to make various runs at 600 volts and the per cent increase in time at lower voltages. It will be observed that for runs of less than 200 ft. the minimum time required to make the run is approximately the same. It should also be noted that for the 1200-ft. run on 6 per cent grade there is approximately a 6 per cent increase in the minimum time required when the trolley voltage is decreased from 600 to 500 volts.

Fig. 2 shows the energy consumption required to make various runs at several different voltages. The curves show that increasing the voltage on these runs materially increases the energy consumption. They

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Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

also show that the increased energy consumption corresponding to a definite increase in voltage is greater for the short runs.

Fig. 3 is derived from Figs. 1 and 2 and presents the same information in a more convenient form for a 300-ft. level run and a 1200-ft. run on 6 per cent grade. From a careful consideration of the data presented in Figs. 1, 2, and 3, it is obvious that the trolley voltage is an important factor in determining the speed of operation and energy consumption. Taking into consideration the evaluated time of the car-rider, the platform expense, and the cost of energy, it is possible, for any particular run, to determine the most economical operating voltage.

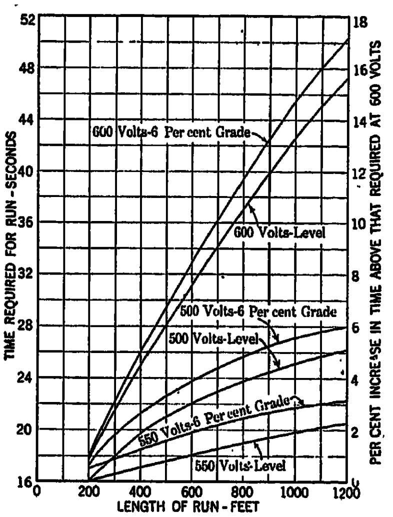

Fig. 1—Minimum Time Required to Make Various Runs (without Coast) at 600 Volts, and Percent Increase in Time at Lower Voltages

Car Data—Motors—4 W. E. No. 510 E. hp. 35
Gear Ratio—13.69
Size of Wheels—26 in.
Wt. of Car and Load—19.36 tons
Acceleration—1.5 mi. per hr. per sec.
Retardation—2.0 mi. per hr. per sec.

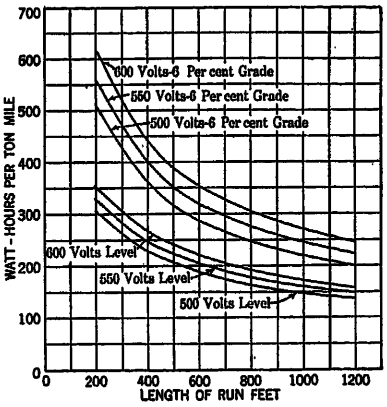

Fig. 2—Energy Required to Make Various Runs (without Coast) at Various Voltages

Car data—same as Fig. 1

Fig. 4 shows the most economical trolley voltages for a 300-ft. level run and a 1200-ft. run on 6 per cent grade, with platform expense and evaluated time of car-rider taken at $10.00 per car hr. and energy cost at the car taken as 1.5 cents per kw. hr. The curves show that for congested areas where runs are short the proper voltage may be considerably below 600 volts and that for long runs on heavy grades the proper voltage is above 600 volts.

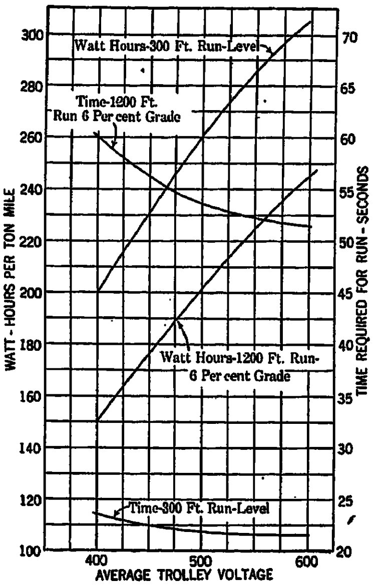

Fig. 3—Time and Energy Consumption Required (without Coast) for 300-ft. Level Run and 1200-ft. Run on 6 Per Cent Grade

Car data—same as Fig. 1

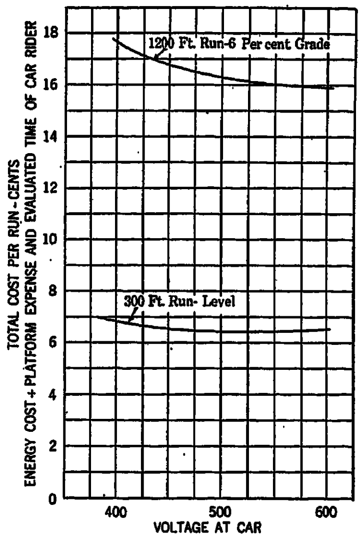

Fig. 4—Total Cost for Several Runs (without Coast) at Various Voltages

Car data—same as Fig. 1
Energy Cost—1.5 cents per kw. hr.
Platform expense and evaluated time of car-rider $10.00 per car-hr.

Inasmuch as increased car speed has been considered the desirable result of increasing trolley voltage, no consideration has been given to the effect of coasting. It can be shown, however, that if schedule speeds are kept the same, increase in voltage will usually result in decreased energy consumption at the car since higher speeds are reached more quickly with higher voltages and increased coasting is possible.

The above data will be considered in detail as they affect the distribution system, substation layout, and the selection of substation equipment.

II. DISTRIBUTION SYSTEM

The functions of a railway power distribution system are:

1. To sectionalize the system in such a manner that the effect of trolley faults on service will be as small as is consistent with the expenditure required to so sectionalize.
2. To provide the most economical trolley voltage, all factors considered, for the operation of the cars.
3. To provide facilities for shifting load in emergencies as when substations are overloaded or incapacitated.

Feeder Sections. A thorough study for the determination of feeder sections involves, in the first place, the setting up of every advantageous way of sectionalizing. Then, with a knowledge of the cost, together with a knowledge of the frequency, duration, and cost (both as to platform expense and evaluated time of carrier) of feeder faults, the distribution engineer can determine the most economical feeder sections. In this connection, the extensive use of outdoor sectionalizing circuit breakers should be considered.

Feeder Sizes. For the majority of cases, the application of Kelvin's Law offers a simple and effective solution to the problem of determining the most economical feeder size. It has the particular advantage that its conclusions may be presented in the form of curves which have a wide application. Further, inasmuch as the conclusions may be checked there is little danger of using them where they will not be applicable.

Kelvin's Law as modified by Kapp is as follows:

> "The most economical area of conductor is that for which the annual cost of energy wasted is equal to the annual interest on that portion of the capital outlay which can be considered proportional to the weight of metal used."

The application of this law to railway power circuits has been presented in a paper by Crecelius and Phillips. (Bibliography.) The following equations and curves present a convenient method for applying Kelvin's Law to extensive system studies. The most variable factor in such a study has been considered to be the cost and value of installed copper.

The total annual cost of a feeder circuit is equal to the annual investment charge plus the annual demand loss charge plus the annual energy loss charge. This may be expressed as a mathematical statement as follows:

$$C_p = (r_1\,C_u\,10^{-2})\,(3.67\,m\,D\,10^{-6}) + \left(\frac{M + r_2\,C_s\,10^{-2}}{\text{Eff. s. p.}}\right)\left(\frac{I^2\,11.0\,D\,10^{-3}}{m}\right) + \left(\frac{e}{\text{Eff. (av.)}}\right)\left(\frac{8760\,I^2\,11.0\,D\,L\,10^{-3}}{m}\right) \qquad (1)$$

The significance of the various parts of Equation (1) is as follows:

$(r_1\,C_u\,10^{-2})$—Annual charge on feeder, dollars per pound

$(3.67\,m\,D\,10^{-6})$—Pounds of feeder

$\left(\frac{M + r_2\,C_s\,10^{-2}}{\text{Eff. s. p.}}\right)$—Annual demand, dollars per d-c. kw.

$\left(\frac{I^2\,11.0\,D\,10^{-3}}{m}\right)$—One hour maximum kw. loss in feeder

$\left(\frac{e}{\text{Eff. (av.)}}\right)$—Energy charge, dollars per d-c. kw. hr.

$\left(\frac{8760\,I^2\,11.0\,D\,L\,10^{-3}}{m}\right)$—Annual kw. hr. loss in feeder

where

C_p —Total annual cost of positive feeder circuit (dollars)

r_1 —Rate of return on feeder investment (12 per cent) (Taxes 2.0 per cent; Depreciation 2.0 per cent; Interest 6.0 per cent; Reserve 2.0 per cent)

C_u —Cost of insulated feeder in place (dollars per lb.)

3.67—Pounds per ft. 1,000,000 cir. mil cable triple braid, weatherproof insulation

m —Cross-section of feeder at substation end (cir. mils)

D —Distance in feet to end of feeder or neutral point of feeder common to two or more stations

M —Annual maximum demand charge (dollars per a-c. kw. metered at 13,200 volts—\$12.00)

Eff. s. p.—Efficiency of substation during peak load (0.93)

r_2 —Rate of return on substation investment (15 per cent) (Taxes 2.0 per cent; Insurance 0.5 per cent; Depreciation 4.5 per cent; Interest 6.0 per cent; Reserve 2.0 per cent)

C_s —Cost of substation per d-c. kw. capacity (\$40.00)

I —One hour maximum demand (amperes)

11.0—Ohms per cir. mil ft. 30 deg. cent. 98 per cent conductivity

e —Energy charge on purchased power (dollars per a-c. kw. hr. \$0.004)

Eff. (av.)—Average efficiency of substation (0.90)

8760—Hr. in a year

L —Loss factor—the ratio of the average of the squared current demands to the squared maximum (1) hr. current demand. (0.25)

Substituting the above values in the equation for total annual cost and combining terms, the equation reduces to

$$C_p = 0.44 \times 10^{-6}\, C u\, m\, D + \frac{0.32\, I^2 D}{m} \qquad (2)$$

Equating first derivative to zero for minimum annual cost

$$o = 0.44 \times 10^{-6}\, C u\, D - \frac{0.32\, I^2 D}{m^2}$$

$$m = \frac{850\, I}{\sqrt{C\, u}} \qquad (3)$$

This is the proper size feeder for minimum annual cost for concentrated load at the end of a feeder of uniform cross-section.

Copper losses caused by a distributed load on a theoretically tapered feeder are equal to one-half those caused by a concentrated load on a feeder of uniform cross-section if this cross-section is the same as that at the station end of the tapered feeder. Since the volume of the tapered feeder is one-half that of the uniform feeder, the investment charge is one-half that shown by the equation for the uniform section with concentrated load.

The annual charge for this type of feeder is

$$C_p = \frac{0.44 \times 10^{-6}\, C\, u\, m\, D}{2} + \frac{0.32\, I^2 D}{2\, m} \qquad (4)$$

and the cross-section for minimum cost becomes

$$m = \frac{850\, I}{\sqrt{C\, u}} \qquad (5)$$

Also, since it can be shown mathematically that the copper losses caused by a distributed load on a feeder of uniform cross-section are equal to one-third those caused by a concentrated load, the annual charge for such a feeder is

$$C_p = 0.44 \times 10^{-6}\, C\, u\, m\, D + \frac{0.32\, I^2 D}{3\, m} \qquad (6)$$

and the cross-section for minimum annual cost becomes

$$m = \frac{850\, I}{\sqrt{3}\quad \sqrt{C\, u}} = \frac{490\, I}{\sqrt{C\, u}} \qquad (7)$$

If these values of m are now substituted in Equations (2), (4), and (6) for total annual cost, the following mathematical expressions will be obtained for minimum annual cost.

For concentrated load uniform cross-section

$$C_p\ (\text{min}) = 0.00075\, I\, D\, \sqrt{C\, u} \qquad (8)$$

For distributed load tapering cross-section

$$C_p\ (\text{min}) = 0.000375\, I\, D\, \sqrt{C\, u} \qquad (9)$$

For distributed load uniform cross-section

$$C_p\ (\text{min}) = 0.000434\, I\, D\, \sqrt{C\, u} \qquad (10)$$

The maximum voltage drop to the end of feeders and the average voltage drop at the load for minimum annual cost is

For concentrated load, uniform section

$$E_d\ (\text{max}) = \frac{I\, D\, 11.0}{m} = 0.013\, D\, \sqrt{C\, u}. \qquad (11)$$

$$E_d\ (\text{av.}) = \frac{I\, D\, 11.0}{m} = 0.013\, D\, \sqrt{C\, u}. \qquad (12)$$

For distributed load, tapering section

$$E_d\ (\text{max}) = \frac{I\, D\, 11.0}{m} = 0.013\, D\, \sqrt{C\, u}. \qquad (13)$$

$$E_d\ (\text{av.}) = \frac{I\, D\, 11.0}{2\, m} = 0.0065\, D\, \sqrt{C\, u}. \qquad (14)$$

For distributed load, uniform section

$$E_d\ (\text{max}) = \frac{I\, D\, 11.0}{2\, m} = 0.0112\, D\, \sqrt{C\, u}. \qquad (15)$$

$$E_d\ (\text{av.}) = \frac{I\, D\, 11.0}{3\, m} = 0.0075\, D\, \sqrt{C\, u}. \qquad (16)$$

It is of particular interest to note that the proper size feeder for minimum annual cost, Equation (3), is directly proportional to the load and does not depend on the length of the feeder. All of these equations and the curves presented below are applicable to the positive feeder circuit on both single and double trolley systems. Values for annual cost and voltage drops on the complete circuit of the double trolley system are twice that given for the positive feeder in the above equations. Some additional calculations are necessary to obtain these figures for the single trolley system.

The following curves have been drawn with the idea of expressing the results of the above equations in a more convenient form.

Fig. 5 illustrates how the factors of Equation (2) vary as the size of the feeder is increased. The curves have been plotted for 1000-ampere load concentrated at the end of a uniform cross-section feeder 1000 ft. long.

The curve "Per cent Return" has been plotted with the idea of showing the return on additional investment that would be required to build up an undersize feeder to the size corresponding to minimum annual cost as determined by Kelvin's Law.

Fig. 6 presents in a convenient form the results of applying Equations (3), (5), and (7) to a 1000-ampere load. From this figure the proper size feeder for minimum annual cost of any standard type railway feeder may be determined, provided the values of all factors used in solving Equation (1) are true for the particular feeder in question.

Fig. 7 shows the minimum annual cost of various standard types of railway feeders for 1000-ampere load, 1000-ft. length. Since the minimum annual cost is directly proportional to the ampere load and the length, the minimum annual cost for any other load and length can easily be determined.

Fig. 8 shows the maximum voltage drop to end of feeder and average voltage drop on feeder for the feeder size corresponding to minimum annual cost. These curves have been plotted for 1000-ft. length, and since the voltage drop is directly proportional to the length, the voltage drop for any other length can easily be determined. These curves are of particular value in checking the application of Kelvin's Law.

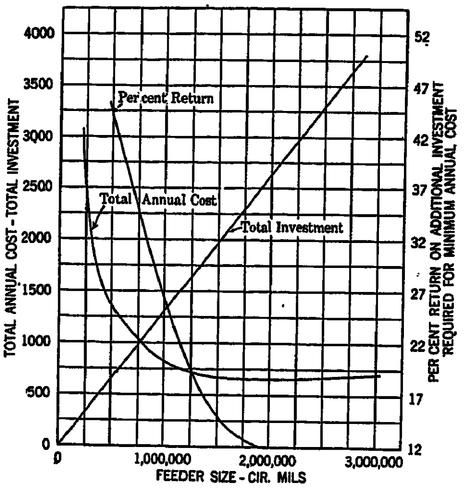

Fig. 5—Total Annual Cost and Investment for a Uniform Cross-Section Feeder 1000 ft. Long Carrying a 1000 Ampere Load Concentrated at the End

Copper at 18 cents per pound installed

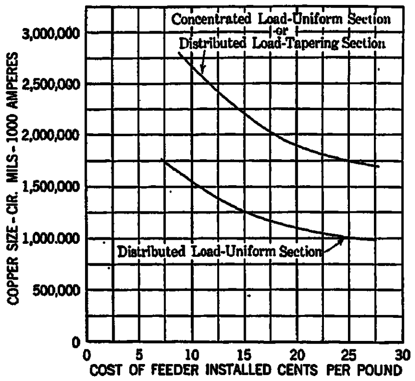

Fig. 6—Feeder Sizes for Minimum Annual Cost (Kelvin's Law)

Reconcilement to Kelvin's Law. After feeder sizes have been determined by Kelvin's Law, the resulting average feeder voltage should be checked against the most economical voltage required for the operation of the cars. If the average voltage at the load as determined by Kelvin's Law is considerably above or below that required for the most economical car operation as shown by curves similar to those of Fig. 4 for the average runs on the section in question, it will be necessary to plot graphs of Equation (2) (total annual cost against average voltage at the load). This graph should then be superimposed and added to the appropriate curve on Fig. 4. The resulting graph will indicate the point of overall minimum cost. The corresponding feeder size can then be read directly from the curves. The following examples illustrate the procedure:

Example 1. Consider a section 7200 ft. long having an average grade of 6 per cent and average length of runs of 1200 ft. The average one hour peak load of 1000 amperes is made up of 12 cars moving up-grade. Ninety-two cars carrying average loads of 60 persons per car and moving up-grade will traverse the section during the two-hour evening peak.

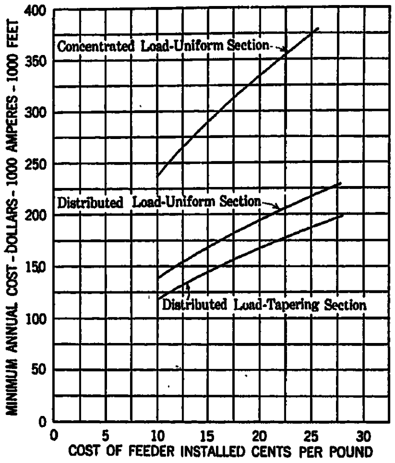

Fig. 7—Minimum Annual Costs (Kelvin's Law)

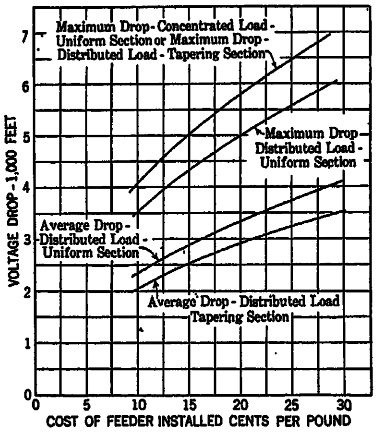

Fig. 8—Maximum and Average Voltage Drop on Feeders Designed for Minimum Annual Cost (Kelvin's Law)

To provide emergency tie-in facilities the feeder is of uniform section. Loss factor 0.25.

Substation voltage 600 volts.

From the standpoint of system stability, it has been found inadvisable to increase substation voltage above 600.

Cost of feeder installed \$0.18 per lb.

Value of car-time \$10.00 per car hour.

Cost of energy per d-c. kw. hr. \$0.015 (used for estimating cost of energy consumption at car).

Weight of car 19.36 tons.

Required to determine the proper size feeder.

From Figs. 6, 7, and 8, according to Kelvin's Law the proper size feeder is found to be 1,150,000 cir. mils corresponding to an annual minimum cost of \$2664.00, and to an average voltage drop of 46 volts; which corresponds to an average trolley voltage of 554. Referring to Fig. 4, it is found that the most economical voltage for the operation of cars is above 600 volts. Therefore, it is evident that Kelvin's Law is not applicable for this case. Fig. 9 illustrates the solution of this problem.

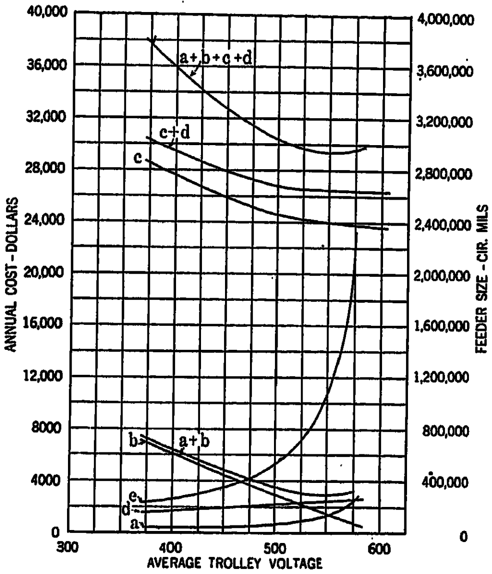

Fig. 9—Determination of Feeder Size—Example (1)

Curve *e* Feeder Size

Curve *a* Annual investment charges on feeder

Curve *b* Annual loss in feeder

Curve *c* Annual platform expense and evaluated time of car rider

Curve *d* Annual cost of energy consumption at cars

The data required for plotting curve *e* of Fig. 9 have been derived from Equation (**16**) for average voltage.

The data required for plotting curve *a* have been derived from the equation

$$C_a = 0.44 \times 10^{-6}\, C\, u\, m\, D$$

The data for plotting curve *b* have been derived from the equation

$$C_b = 0.32\, \frac{I^2 D}{3\, m}$$

The data for plotting curve *c* have been derived from the expression

$$C_c = (300 \times 92 \times 6 \times 10)\, \frac{t}{3600}$$

where *t* is the time in seconds required to make a 1200-ft. run as derived from Fig. 3.

The data for plotting curve *d* have been derived from the expression

$$C_d = \left(300 \times 92 \times 19.36 \times \frac{7200}{5280} \times \frac{W}{1000} \times 0.015 \right)$$

where *W* is watt-hr. per ton mile from Fig. 3.

The curve $(a + b + c + d)$ of Fig. 9 shows that the proper feeder size to secure minimum annual cost is 1,500,000 cir. mils.

Example 2. Consider the feed to a loop in the congested area. The cars operate over a level section 3000 ft. long. The lengths of runs average 300 ft.

The average one hour peak load of 1000 amperes is made up of 20 cars moving over the route.

Three hundred and twenty cars, carrying average loads of 60 persons per car, will traverse the section during the two-hour morning and evening peaks.

A feeder of uniform section feeds the loop from a substation approximately 4000 ft. from the center of the load; therefore, the load will be considered as concentrated at the end of a 4000-ft. uniform cross-section feeder.

Loss factor 0.25.

Substation voltage 560 volts.

From the standpoint of system stability, it has been found inadvisable to decrease full load substation voltage below 560 volts, this voltage being obtained through the use of shunt converters.

Cost of feeder installed \$0.18 per lb.

Value of car-time \$10.00 per car-hr.

Cost of energy per d-c. kw. hr. \$0.015 (used for estimating cost of energy consumption at the car).

Weight of car 19.36 tons.

Required to determine the proper size feeder.

From Figs. 6, 7, and 8, according to Kelvin's Law the proper size feeder is found to be 2,000,000 cir. mils, corresponding to an annual minimum cost of \$2720 and to an average voltage drop of 44 volts which corresponds to an average trolley voltage of 516 volts. Referring to Fig. 4, it is found that the most economical voltage for the operation of cars is 490 volts. Inasmuch as Kelvin's Law provides a higher voltage than can be economically utilized, it is apparent that it is not applicable for this case. Fig. 10 illustrates the proper solution of this problem.

The data required for plotting curve *e* of Fig. 10 have been derived from Equation (**12**).

The data required for plotting curve *a* have been derived from the equation

$$C_a = 0.44 \times 10^{-6}\, C\, u\, m\, D$$

The data for plotting curve *b* have been derived from the equation

$$C_b = 0.32\, \frac{I^2 D}{m}$$

The data for plotting curve *c* have been derived from the expression

$$C_c = (300 \times 320 \times 10 \times 10)\,\frac{t}{3600}$$

where t is the time in seconds required to make a typical run as derived from Fig. 3.

The data required for plotting curve d have been derived from the expression

$$C_d = \left(300 \times 320 \times 19.36 \times \frac{3000}{5280} \times \frac{W}{1000} \times 0.015\right)$$

where W is watt hr. per ton mile from Fig. 3.

The curve $(a + b + c + d)$ shows that the proper size feeder to secure minimum annual cost is 1,700,000 cir. mils.

The above examples have been worked out for a double trolley system. A somewhat similar procedure may be followed for a single trolley system.

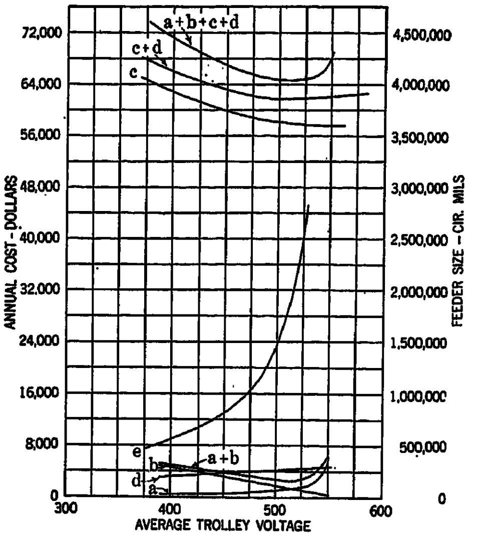

FIG. 10—DETERMINATION OF FEEDER SIZE—EXAMPLE (2)

Curve e Feeder size
Curve a Annual investment charges on feeder
Curve b Annual loss in feeder
Curve c Annual platform expense and evaluated time of car rider
Curve d Annual cost of energy consumption at cars

Tie Feeders. From the study of minimum annual cost of various types of feeders it is found that the most economical type of feeder for the usual distributed railway load is the uniformly tapering section. If the substation units and the high tension power lines connected to each unit were 100 per cent reliable, there would be no occasion to use any other type. However, since such reliability has not been obtained and since power outages represent such a tremendous loss to the public, the Cincinnati power system has been laid out with the idea of carrying the peak load service at reduced voltage with any one substation out of commission. This can only be done through the extensive use of tie feeders, which in many cases has lead to the installing of uniform cross-section feeders. A comparison of Equations (9) and (10) shows that the total annual cost of the uniform cross-section feeder is 15.7 per cent higher than that of the tapered section. In actual practise, due to the impossibility of obtaining a theoretically tapered section, this percentage is somewhat less. It is considered, however, that the advantages of the uniform cross-section feeder are worth the increased cost. This fact can be proved conclusively through the evaluation of power outages mitigated by the use of tie feeders.

Feeder Control Equipment. For systems similar to the Cincinnati power system where the ill-effects of substation outages have been lessened by the use of a large number of small stations and extensive tie feeders, the circuit breaker and its control circuit become the most important element in the system. Some conception of the expenditure justified to secure effective feeder control equipments may be had by considering that a single outage, during the peak, of 30 minutes' duration on the feeder used in Example 2 would result in a time loss of 2400 passenger-hours.

The feeder control equipments on the Cincinnati system were designed to:

1. Trip on rate of rise of current when such rate of rise corresponds to a short circuit. This feature is adjustable so that the rate of rise at which the breaker will open can be regulated.
2. Permit the breaker to remain closed when subject to any legitimate load or when subject to any decrease in load.
3. Reclose, following a suitable time delay, which time delay is adjustable when the resistance of feeder and load reach a predetermined value. This value is adjustable down to 0.3 ohm.
4. Reclose and stay closed on multiple feeders when feeder is being energized from another source.
5. Positively stay out on 0.4 ohm load with bus voltage at 620; and with no change in adjustment, will positively reclose on 1.0 ohm with bus or feeder voltage at 350 volts.
6. Completely isolate the feeder when the power dispatcher locks it out.

In addition, the supervisory equipment superimposed on the full automatic feeder equipment has been designed so that:

1. The power dispatcher will have indication as to whether the breaker is open or closed.
2. The power dispatcher will have indication as to whether the breaker is under automatic or supervisory control.
3. The power dispatcher can reclose the breaker on a short circuit, but if short circuit conditions exist, the breaker will re-open. In connection with this feature an anti-pumping circuit is used.
4. Failure of supervisory cable will cause feeder equipment to revert to automatic control except when a feeder circuit is locked out.

The circuit breaker is of the latched-in type and both closing and tripping coils are operated from the station control battery. It incorporates in its design a magnetic blow-out for extinguishing the arc.

III. CONVERSION SYSTEM

The study of railway power system design, with particular reference to the economics involved in the use of automatic substations, has been covered in considerable detail by committees of the A. E. R. A. These reports present both analytical and practical methods of considerable interest and value. In particular, they emphasize the importance of a thorough economic study of the distribution system and point out the desirable effects of decreasing voltage in congested areas. However, no definite methods are presented for determining the economic effect of the distribution system and trolley voltage on power system design.

The foregoing parts of this paper represent an effort to evaluate these factors so that they can be considered in relation to the other parts of the power system. The following discussion is an attempt to show how these and other factors have affected the location and number of substations on the Cincinnati system.

The immediate cause of the general rehabilitation of the system was the economy that the Street Railway Company could effect by purchasing power instead of operating its own generating plant. Inasmuch as this meant the replacement of all 25-cycle conversion equipment, amounting to one-half of the total installed capacity, an extensive study embracing the entire system was decided upon. The company had had in operation, for approximately six years, three full automatic single unit, 1500-kw. substations. The fine performance records of these equipments were a large factor in determining the selection of automatic control for the entire system.

Several topographical peculiarities of Cincinnati have considerably affected the power system design. (1) There are no large areas of uniform load density; (2) numerous heavy grades exist on which it is particularly advantageous to maintain the highest possible voltages; and (3) after leaving the congested area, the car lines extend radially out from the center of the city an average distance of ten miles with only one intersecting cross-town line. These factors tend to make any application of analytical methods very difficult. On the other hand, it is apparent that such a lay-out will immediately suggest a number of probable substation locations, especially after a careful study of voltage, load, and sectionalizing requirements has been made.

The general procedure for making the study of the system is given below:

Data. Of fundamental importance in making this study was the preparation of maps drawn to scale showing in detail the following data.

1. Location and routing of all cars operating during peak periods.
2. Length and location of grades.
3. Most desirable trolley sectionalizing, together with corresponding load in amperes on the various sections, and the most suitable average trolley voltage for peak periods.
4. Location of existing feeder copper.

Size of Units. Inasmuch as a large number of 60-cycle 1500-kw. units was already in use on the system, it was decided to extend the use of this size in so far as was feasible. However, after a few preliminary calculations it was obvious that a smaller size unit would be desirable for certain outlying sections, and a tentative set-up was made using 750-kw. units for these sections.

Reliability. Since the new 60-cycle conversion equipment would be more susceptible to system disturbance than the old 25-cycle equipment, and since it was decided to provide a service superior to that which had been provided by the old equipment, the principle was tentatively adopted that the loss of any single-unit station or any one unit in a double-unit station should not seriously affect transportation service. It afterwards developed that with a few minor changes in the preliminary system design this principle could be economically extended so as to include the loss of any complete double unit station.

Capacity for Expansion and Reserve. In general the expected one-hour integrated load was taken as approximately 80 per cent of the nominal substation capacity. This allowed about 20 per cent for expansion and unforeseen increase in load and figured on using the 50 per cent overload capacity of the converters to furnish the reserve required to meet the principle outlined above. In connection with expansion, the possibility of operating a proposed Rapid Transit System had a definite bearing. This resulted in some minor adjustments in location and in the providing of buildings large enough to house additional substation capacity.

Possible Locations. After the required data had been assimilated and fundamental principles decided upon, all possible substation locations were determined by inspection and a few preliminary calculations. The use of the system of moments was of particular value in the selection of tentative locations.

Zoning Restrictions and Available Sites. In Cincinnati the existence of some rigid zoning ordinances had considerable effect on the locations of substations. A survey was made as to the availability of sites in the permitted building zones.

Existing Feeder. It was evident that the considerable expense involved in re-arranging the feeder system would have a definite bearing upon substation location, and that as much advantage as possible should be taken of the feeder already installed.

Electrolysis. In the rehabilitation of its power system the railway company had to consider the possibility of changing to single trolley operation. From the first, it was realized that the extensive use of small single unit substations would be a definite factor in the mitigation of electrolysis when single trolley operation should be adopted. Discussing the development of automatic substations, the 1921 Report of the American

Committee on Electrolysis (p. 46) calls attention to the above fact in the following statement.

> "The increased savings attending this development will undoubtedly increase the number of substations which can economically be installed on both interurban and city systems, and if full advantage is taken of these economies, the feeding distances will be reduced to such an extent as to greatly reduce stray currents generally."

A specific instance supporting this statement was presented in a paper before the A. E. R. A. (Bibliography.)

Comparison. A general consideration of the data, principles, and limitations outlined above resulted in materially reducing the number of possible substation locations. As a matter of fact, few alternate schemes remained to be considered in detail. The total annual cost of the following factors, together with a consideration of system reliability, flexibility, and electrolysis conditions, were compared for each of the remaining schemes:

1. Service conditions
2. Distribution investment
3. Distribution loss
4. Equipment investment
5. Realty investment
6. Conversion loss
7. Maintenance and operating cost.

High-Tension Lines. No consideration was given to the high-tension system until after the substation lay-out had been determined. The high-tension system is usually one of the first considerations, but since the Cincinnati power system contemplated the purchase of power delivered at its substations and since the local power company had a number of strategically located high-tension distribution centers, it was thought that the high-tension system would not materially affect the ideal substation lay-out. Except in a few instances this proved to be the case. The negotiations with the power company developed the fact that considerable economy could be affected in the high-tension distribution system if the railway company would agree to the joint use of high-tension lines. In the case of a number of single unit stations this was agreed to, with the understanding that these lines would be protected with the most effective relay equipment that could be obtained. In the majority of cases these lines are parts of loop systems which are sectionalized at the railway substations. Inasmuch as the railway power system was designed so as to permit the loss of any one substation without seriously impairing the service, the above agreement was regarded as sound.

Final Determinations. The most economical substation lay-out as determined by the consideration of the foregoing factors involved the use of one 300-kw. single-unit automatic substation, five 750-kw. single-unit automatic substations, eleven 1500-kw. single-unit automatic substations, one 3000-kw. two-unit automatic substation, and one 4500-kw. three-unit manual station. In the final determination, it was decided that for the service under consideration, a slower speed machine would be more desirable than the 1200-rev. per min. standard 750-kw. units. Inasmuch as there was little difference in the cost and efficiency between the slower speed 750-kw. machines and the standard 1000-kw. machines and since the outlying territory in which these machines were to be located might be subject to considerable expansion, it was decided to use the standard 1000-kw. units.

The final calculations showed that it would be impossible to maintain reasonable service in the congested areas if the three-unit station should be entirely lost. It was, therefore, decided to tranfer one of these units to an adjacent single-unit station. From the standpoint of system stability, since the surrounding units would all be protected by the use of load shifting equipment, it seemed absolutely essential that the central station should also be protected. It was, therefore, decided to use full automatic control over the entire system.

Converter and Transformer Characteristics. The engineers in selecting the car equipments must necessarily base the application upon an average trolley voltage and average operating conditions throughout the system. If an average trolley voltage is furnished, the car equipments will perform in a more or less satisfactory manner. However, the operating conditions for any street railway car vary between wide limits. Fig. 4 shows the most economical voltages for a 300-ft. level run and a 1200-ft. run on 6 per cent grade. From this it will be seen that trolley voltages for cars operating under normal service conditions vary over a wide range.

It is evident that the railway power system should, in so far as possible, provide the most satisfactory and economical trolley voltage. Since adjustment of voltage through the manipulation of the distribution system does not provide an economical method for varying voltages, it is up to the substation engineers to provide for this either by making use of adjustable machine characteristics, suitable transformer taps, or both.

There is no argument as to the superior performance of shunt converters from the standpoint of substation operation and system stability, but it must be recognized that shunt converters cannot be used in outlying territories, or on heavy grades, except at the sacrifice of car speed. In congested areas, advantage can be taken of the superior operating characteristics of shunt converters with the additional advantage of power saving brought about through the furnishing of more economical trolley voltages. Inasmuch as the advantageous load shifting characteristics of the shunt converter may become a matter of importance on any part of the system, it will be a definite advantage to retain the possibility of shunt operation even on those parts of the system where compound operation is normally considered most effective. For the same reasons, it will be desirable to provide transformer taps with the idea of varying d-c. bus voltage. Two distinct advantages resulting from this practise are (1) the providing of a more satisfactory trolley voltage, and (2) the providing for emergency load shifting.

It may be of interest to note that all of the new converters on the Cincinnati power system may be operated full shunt, full compound, or at any intermediate degree of compounding. All new transformers are provided with tap-changing devices in the high-tension windings, so that the d-c. bus voltage may be varied in steps of 20 volts from 620 to 540 volts. It is expected that the high-tension power furnished by the power company will vary little from the 13,200-volt standard.

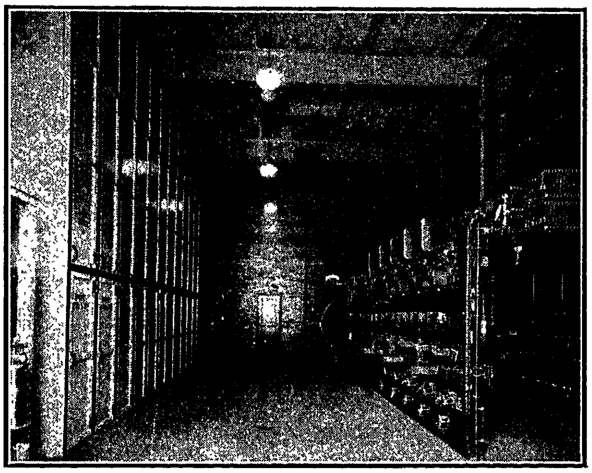

FIG. 11—INTERIOR OF SINGLE UNIT AUTOMATIC SUBSTATION

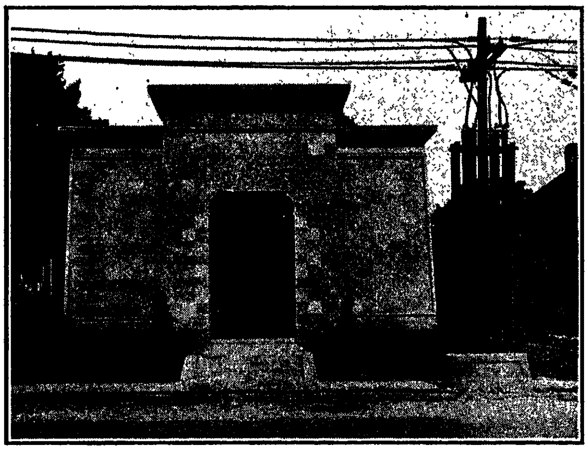

FIG. 12—EXTERIOR OF AUTOMATIC SUBSTATION

Manual vs. Automatic Control. Among the important factors that determined the type of control selected for the Cincinnati power system were maintenance and operating costs, cost of equipment, reserve capacity, efficiency, reliability, improved service, protection of equipment, and load shifting characteristics. Based upon six years' operating experience with three 1500-kw. single unit automatic substations, it was determined that a fair annual charge for maintenance and operating expenses on the automatically controlled equipment would be $1100 for a single-unit station and $1800 for a double-unit station. The corresponding charge for manual equipment was determined to be $3400 for a single-unit station and $5200 for a double-unit station. Annual savings creditable to automatic control, therefore, are $2300 for a single-unit station and $3200 for a double-unit station.

In the consideration of new single-unit substations, the investment chargeable against the use of automatic control was taken as the difference in cost between automatic control and the corresponding manual equipment. In the case of the rehabilitated station, however, the investment charge against automatic control was taken as the total cost of the new automatic control equipment less the scrap value of the superseded manual control equipment.

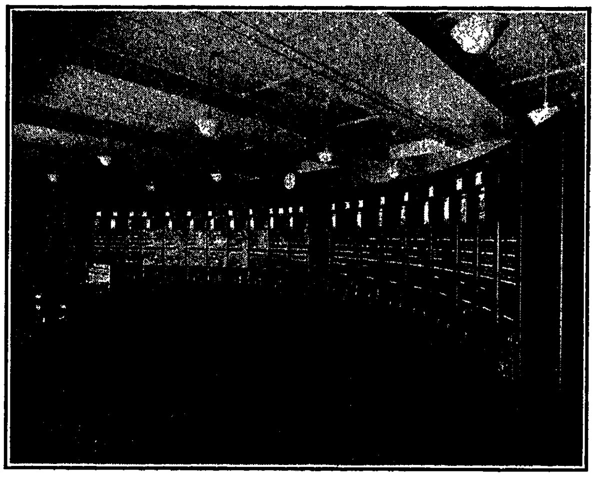

FIG. 13—SUPERVISORY CONTROL ROOM

For new single-unit stations, the difference in cost between automatic and manual equipment was found to be $13,000.00, which at 15 per cent amounted to an annual carrying charge of $1950.00. For rehabilitated single-unit stations the annual carrying charge at this per cent amounted to $2700.00; for new double-unit stations the annual carrying charge amounted to $3900.00, and for rehabilitated double-unit stations to $5400.00. From these figures, it is apparent that automatic control has a decided economic advantage only in the case of new single-unit stations.

Due to load shifting characteristics and thermal protective devices, automatic equipment can be safely subjected to heavier loads than could the corresponding manual equipment. Hence, it can be seen that less reserve capacity is required and that efficiency may be increased. The reserve capacity required for double-unit stations is particularly affected.

The greatly increased number of electrical devices required for automatic control makes this control equipment more susceptible to faults and failures. The only mitigating circumstance is the fact that this disadvantage is well recognized and as a consequence the automatic equipment is usually more liberally designed and is more carefully installed, inspected, and maintained. Since the functioning of the automatic control equipments represents the best possible procedure in the handling of all normal and emergency operating conditions and since the sequence is arranged so that the control will perform as intended or not at all, it is evident that a more reliable operation will result than could be expected from a manually operated equipment. Thus, the possibility of material damage due to an incorrect manual operation is eliminated.

A definite improvement to service, due to automatic control, is the increased speed with which service is restored after having been interrupted by the failure of the high-tension power supply or by faults on the d-c. feeder system.

Since the bearing, thermal, and ground devices on the automatic control equipment provide protection superior to that possible with manual control, this must be considered an advantage.

The advantage of the load shifting equipment provided as part of the automatic control is one of the most outstanding benefits and should be given considerable weight in selecting control equipment for any railway application.

Buildings. Inasmuch as the general policy of the Cincinnati Street Railway Company has been to provide substation buildings that would be a credit to the community in which they were located, the full economic benefits usually considered possible through the use of automatic control have not been realized. This policy, however, has been instrumental in developing a considerable feeling of goodwill towards the company.

A-C. vs. D-C. Control. The selection of the type of control (alternating or direct current) is an important factor in the design of the railway power system, and should be given considerable attention.

IV. Load Shifting Characteristics

One of the most interesting problems connected with the design of the railway power system is that of determining the proper load shifting characteristics of the various substations. This became a particularly important factor on the Cincinnati system when the principle was established that the loss of any one substation should not seriously affect service conditions. The extensive use of tie feeders and high capacity load shifting resistance was instrumental in materially reducing the reserve capacity that would ordinarily be required to meet this condition. The first step in the determination of the ohmic values of load shifting resistances was to lay out the distribution system and determine the equivalent resistances between each station and its adjacent stations.

The shifting in load occasioned by a drop in voltage can be expressed by the following formula:

$$I = \frac{E}{R} \qquad (17)$$

This equation may be proved mathematically correct for substation loads distributed in any way.

The ohmic values of the load shifting resistance were calculated from the following:

$$R_r = \frac{E}{1.5\, I_c} = \frac{R\, I.}{1.5\, I_c} = \frac{R\,(1.9\, I_c - 1.5\, I_c)}{1.5\, I_c} = 0.266\, R \qquad (18)$$

Where

I —Ampere decrease in load

E —Decrease in substation voltage or voltage drop in resistance

R —Equivalent resistance of tie feeders to adjacent stations

R_r—Ohmic resistance of one step of load shifting resistance

I_c —Full load current of converter

The above formula assumes that the proper ohmic value of a single step of resistance is that necessary to reduce the load on the converter from 190 per cent to 150 per cent. It is obvious that this assumption may not be generally applicable.

On the Cincinnati power system two equal steps of load shifting resistance were used. These were of sufficient thermal capacity to carry a 150 per cent load on the converter for 20 min.

A consideration of Equation (**17**) in connection with the use of shunt converters indicates the feasibility of eliminating some or all of the load shifting resistance in those stations where a low value of equivalent resistance can be obtained.

Load Shifting Resistor Relay Equipment. Since the cutting-in of load shifting resistors always results in a considerable loss of power, in impaired service, and in the possibility of shutting down the equipment on account of overheated resistors, it is of vital importance that this equipment does not cut-in unless this operation is absolutely essential to prevent overheating of the converters. In this connection, the proper application of relay equipment associated with the cutting-in of load shifting resistance is of particular importance.

V. Supervisory Control

Full Automatic System without Supervisory Control. There are conditions such as those brought about by storms and fires that cannot possibly be met by automatic control. These usually consist in the necessity for deenergizing trolley lines so as to eliminate the possibility of accidents.

In addition to these limitations, there appears to be a number of situations which are not met by the present automatic equipment. Such a situation is the inability of the low voltage time delay starting equipment to meet all required conditions. In order to promptly meet emergencies such as the loss of an adjacent substation, it is evident that the time delay starting relay must be set for short time. If this adjustment is made, because of fluctuating railway loads, it will lead to much unnecessary starting and stopping of the conversion equipment resulting in higher maintenance cost and decreased efficiency. If the time is lengthened so as to correct this condition, prompt operation in case of trouble is sacrificed. It may be possible to meet this situation by a more effective control, but so far as is known, this has not been done. A situation similar to the above is the proper operation of equipment for the providing of night owl service. Another case in which it is difficult to secure the proper functioning of automatic control is one where trolley voltage is adjusted by varying the machine operating characteristics and substation voltage, in which case the load on the converter may not determine the proper point at which the converter should be shut down.

Functioning of Supervisory Control. The primary functions of supervisory control are (1) to provide necessary checks on operation of substations, (2) to operate the system at the point of maximum efficiency, and (3) to change the normal automatic functions so as to better meet emergencies.

Inasmuch as the entire 600-volt feeder system is tied together so that the loss of any one substation during the light load periods has little effect on the movement of cars, it is apparent that without some sort of indicating system, there is no check on substation functioning at this time. As most faults occurring in automatic substations develop or become apparent during the starting operation, it is of particular advantage to have a check at this time so that the faults may be remedied before the peak period.

Since the load demands on a street railway system are approximately the same from day to day, it is possible to calculate the most efficient way in which to operate the system. This will usually call for a definite schedule for the various units. The maximum efficiency and the minimum maintenance will result from keeping the number of starting and stopping operations as low as possible. This may be accomplished most effectively by the use of supervisory control.

In addition to providing the means for meeting fire and storm emergencies, the supervisory system can often be used to meet emergency overload conditions. These are usually taken care of by automatic equipment through the cutting-in of the load shifting resistance which, as before stated, results in power loss, impaired service, and the possibility of overheated resistors. Improved service will frequently result from dropping tie-feeder loads since this may permit the reclosing of the resistance shunting contactors.

Remote Metering. As applied on the Cincinnati power system, the remote ammeters provide the dispatcher with a continuous check on the operating conditions of the converters both when starting and running. The ammeter, although calibrated to read in d-c. amperes, really measures the current in the a-c. side of the converter. The load readings are essential if the dispatcher is to shift loads during emergency overload conditions. The recording volt meters provide a continuous check on that most important operating factor, substation bus voltage.

CONCLUSIONS

Inasmuch as topography, arrangement of the city, density of population, and traffic conditions have such a definite bearing on the distribution of load and car operation, it is impossible to draw conclusions that are generally applicable to all railway systems. In any city, however, the comprehensive design of a power system must consider as one problem the effect of trolley voltage on car speed, the distribution system, the location of substations, and the selection of conversion equipment.

(a) On any railway system the effect of trolley voltage on service is a most important consideration and merits a more extensive study than has yet been given to it.

(b) Kelvin's Law with certain limitations is generally applicable to the design of railway feeders and offers a relatively simple solution to the problems of the determination of proper feeder sizes.

(c) Converters and transformers designed to give the greatest possible flexibility as to operating voltage should be selected. Compound wound converters capable of being operated as shunt machines are most suitable for the railway power system, the flat voltage characteristics being desirable for grades and outlying territories and the shunt characteristics being more desirable for congested areas.

(d) The use of automatic control results in the most economical and reliable system.

(e) With a complete automatically controlled system, it is necessary to have some sort of check on substation operation. Supervisory control, in addition to providing such a check, furnishes a method for more efficient normal and more effective emergency operation than is possible with full automatic equipment acting alone.

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Discussion

T. H. Schoepf: There is one point in the assumptions of fixed values which have been established by the author that I question the correctness of, and that is the fixed charge of 12 per cent in the one instance and 15 per cent in the other, wherein 2 per cent has been included for taxes and 2 per cent for a reserve fund. As a matter of fact the higher you select the total of that percentage, the smaller your cable becomes.

If the size of the cable is figured by the formula as established by the author and established by others on the Kelvin law, the cable is a little larger than common sense dictates, and usually it is trimmed down to some extent.

Those who have had experience with tax commissions I think know how they make up taxes. A budget is submitted to the tax commission of how much the state requires. They look over the companies they think can afford it, and they say, "This is your tax; this is your valuation based on the tax," and that is what you pay. So computing the property by the amount of this cable doesn't make any difference in the tax.

I think the whole percentage should be lower than 12 and 15 respectively. However, as a matter of fact the property here was figured out by the method Mr. Noertker has exposed. It was modified in some places where judgment dictated modification was required, but we were particularly concerned with the tie-feeders, and it is very gratifying to every one to see how completely the scheme laid out by this system works out in practise to accomplish the result aimed at.

J. C. Bailey: In comparing the installation of 1000-kw. capacity converters with 750-kw. capacity, and basing the capacity of these machines on approximately 80 per cent of the required normal substation capacity, it is of interest to note that the 740-kw. machines will operate at approximately 75 per cent of its rated load, and the 1000-kw. machine operates at approximately 50 per cent of its rated load.

The result is a loss in the over-all efficiency of 0.5 to 1 per cent in the operating expense per kilowatt hour. On this basis it is a question, due to the increased operating cost and carrying charges of the larger sized machines, whether there is an apparent saving in the distribution and general operating cost of the system.

An interesting feature on the rehabilitation of the Cincinnati Street Railway system was the adoption by the power company of the total synchronized metering scheme. The instruments on the market and available at the time were subject to variations in supply voltage and frequency, and in order to keep 19 instruments an instrument had to be selected in which the operation would not be affected by variations in supply of voltage. The totalized metering scheme was discussed and the cost was considered prohibitive, and so it was finally decided to adopt the use of the Landis and Garr mexagraph recording demand and kilowatt hour meter.

This instrument is built of two sections consisting of the standard watt hour meter element and recording power demand mechanism.

Briefly the outstanding features which may be mentioned here are: it will operate in synchronism regardless of whether the power is on or off as there is a 50-hr. reserve drive in the main spring for operating the mechanism with the potential removed from the motor. The power demand is recorded by means of a silver-tipped stylus moving over a specially prepared paper, and third, the minute adjustments provided for resetting the charts on demand periods within 15 sec.

In regard to the question of a-c. against d-c. control, I should like to state that our company has done considerable investigation work in regard to the merits of both types, and finds that the d-c. type of control for automatic substations is superior to the a-c. system. The maximum continuity of service is assured, as the control bus is not subject to the usual variation in the supply of voltage, while the station is carrying a load or coming into service with d-c. voltage condition, such as may occur under a general system disturbance.

J. A. Noertker: Colonel Schoepf questioned the allowance of 2 per cent for reserve. I have never found any place where such an item has been included, but in Cincinnati the street-railway properties are limited to a 6 per cent return on our investment because of the franchise, and I would hesitate to recommend any increase in feeder or any other scheme that would possibly show only a 6 per cent return on the investment; therefore, I included 2 per cent additional just to be sure that installing this feeder was a good investment. I did that rather than consider the thing only from the common sense point of view and put in a little smaller or a little larger feeder. It is at least definite.

Mr. Bailey raised the question of 1000-kw. *versus* 750-kw. units. A 750-kw. converter is a 1200-rev. per min. machine, and there is no question but that it is subject to higher maintenance than the 1000-kw. machine. In addition to that the places in which we installed the 1000-kw. machines were outlying territories, and although we couldn't at this time really justify more than a 750-kw., we felt that there may be some possibilities of future growth in those locations; so we put in the 1000-kw. machines. At the present time we are undoubtedly standing a little greater loss than if we had applied the correct capacity, but this is probably offset by reduced maintenance.

Automatic Mercury Rectifier Substations in Chicago

BY A. M. GARRETT[1]

Associate, A. I. E. E.

Synopsis.—Two automatic mercury-vapor rectifier substations having 3000-kw. 5000-ampere, 600-volt rectifiers are described in this paper. These rectifiers are the largest installed in this country. The reasons for the selection of this type of converting equipment instead of the rotary converter are enumerated. Information is given on the auxiliary apparatus and the control arrangement.

THE placing in service recently of two mercury-vapor rectifier substations in Chicago, automatic in operation, marks the fourth of a series of rectifier installations in this vicinity. The application of approximately 10 rectifiers to steam road electrification service, as well as city traction supply, has perhaps made the Chicago District the representative locality for this new type of converter.

The decision to make the latest units automatic in operation was based on two fundamental reasons, first, to determine whether the so called adaptability of the rectifier to automatic operation would be borne out by practical experience, and second, to demonstrate comparative performance with the automatic operation of the synchronous or rotating converter. Both types serve the same class of load supplying 600-volt energy to the elevated and surface lines systems of Chicago.

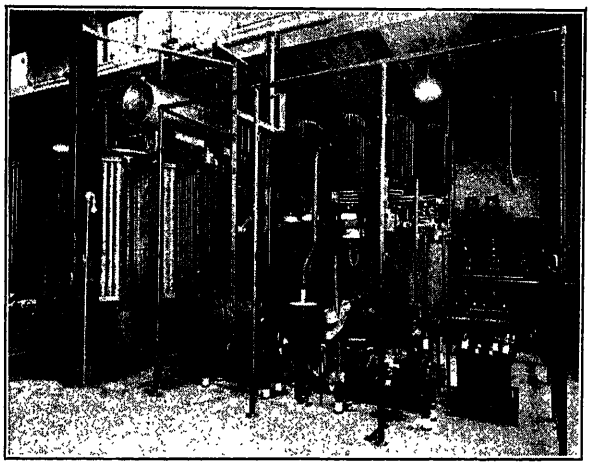

FIG. 1—3000-KW. MERCURY ARC RECTIFIER AT MAYPOLE AVENUE SUBSTATION

Because of the number of rectifiers of the iron tank type originally installed in Chicago or vicinity, the subsequent wide-spread interest in their performance, and the changes and additions in the latest units, making them more applicable to American practise, a word in explanation of the decision to use this type of unit for railway service is necessary.

Although from an operating point of view this decision is based upon a performance not equal as yet to that of the synchronous converter, improvements recently made in rectifiers, together with removal of handicap due to size, are some of the factors which lead us to believe the difference in reliability has practically disappeared.

The usual advantages generally understood to be characteristic of the rectifier are: a converting device simple in design and construction; practically no moving or wearing parts; low maintenance and inspection costs; absence of renewal expense and higher efficiencies. In addition, the advantages which are outstanding for rectifier installations when located in metropolitan areas are the use of water for cooling purposes instead of air, absence of noise and vibration, and the elimination of massive foundations and ventilating duct construction.

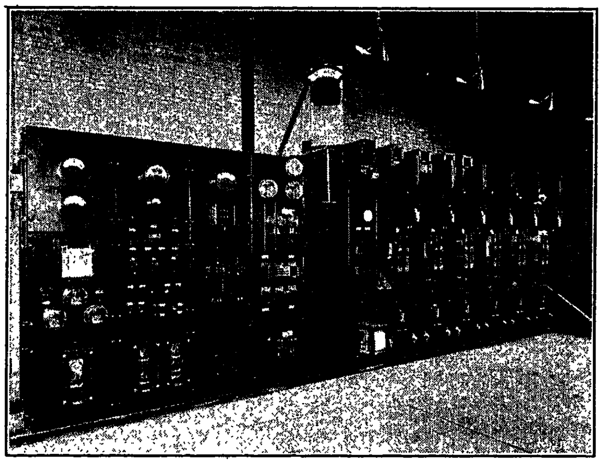

FIG. 2—CONTROL AND FEEDER SWITCHBOARD, MAYPOLE AVENUE SUBSTATION

The increased number of facilities required with the installation of the modern converter of the rotating type has become a problem and a matter of concern from an operating point of view, as well as that of investment. Whether located in leased space below the street level in a commercial building, or in a substation building of the conventional type generally found in the outlying territory, the present synchronous converter must be provided with enclosures to which are

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connected extensive space consuming air intake and exhaust systems, air cleaning and blower equipment, part of which is fitted in and around massive foundation work. These facilities or a modification of them must be provided for ventilation of the unit or for noise mitigation, or for both. The use of the mercury rectifier appears to be the answer to these problems which cannot be solved satisfactorily when the older type of converter is used.

The two rectifiers which are of the largest ampere size installed to date in this country, were manufactured by the American Brown-Boveri Electric Corporation. This capacity is obtained from a single cylinder arranged with 12 water cooled anodes. The rating of unit is 3000 kw. 5000 amperes, 600 volts, with 50 per cent overload capacity for two hours and 200 per cent load for one minute. Although nominally known as a 600-volt outfit, a no-load tap arrangement permits a range of voltage on the output side of the rectifier of 575 to 650 volts in steps of six full capacity taps. The rectifier which receives its a-c. energy from oil insulated self-cooled transformer is operated from the three-phase 12,000-volt, 60-cycle grounded neutral system of the Commonwealth Edison Company. The rectifier unit together with the 600-volt feeders are fully automatic; the 12,000-volt switching equipment is remote controlled from the nearest attended substation in the territory. The 12,000-volt supply to the rectifier substation is furnished in two lines arranged in a loop formation from the same attended substation.

The pump equipment for exhausting the air and gases from the cylinder is different from previous units in that two sets of pumps are used instead of one, and the mercury vapor pumps are separated from the rotary vacuum pumps, and are mounted upon the rectifier cylinder making a shorter and more effective piping arrangement with the condensing chamber.

Because the mercury rectifier is essentially a static conversion unit, devices necessary to prepare it for automatic operation are relatively few in number.

The starting of the unit is controlled by a time clock which can be set at any predetermined time to place in motion the customary control equipment commencing with the master control relay which functions in sequence to close the a-c. oil breaker, strike the ignition and excitation arcs within the cylinder, start the cooling water, and close the d-c. breaker connecting the unit to the 600-volt bus and the pick-up load that is available. Ordinarily the total time for this operation is less than one minute. To shut down the unit the master relay trips the a-c. oil breaker which through auxiliary contacts cuts off the excitation and trips the d-c. breaker. To the a-c. breaker operation is added the function of reclosing the three trials at reclosing before locking out after the third attempt.

The pumping equipment, which is used to maintain vacuum within the cylinder, operates independent of any of the other automatic functions of the unit and the control for this equipment is in operation at all times whether the rectifier is connected to the system or not, except at such times that the unit may be taken out of service for repairs or other reasons. This control consists essentially of a hot wire gage operating on the Wheatstone Bridge principle, arranged so that two of the arms of the bridge are subject to the gas pressure within the cylinder, while the other two are exposed to the pressure of the atmosphere. The difference in resistance due to difference in pressure causes a current to flow through a vacuum meter which is calibrated to indicate the vacuum directly in millimeters of mercury. This meter also carries contacts which control the action of the mercury and rotary pumps placing in operation the mercury pump only when there is a high vacuum, both mercury and rotary pumps when a medium vacuum exists, and operates the lockout relay taking the rectifier off the system when low vacuum is indicated.

The amount of water supplied to the rectifier is regulated in accordance with the load demands through the registering of the temperature of the discharge water upon a thermostatic device which in turn controls the solenoid operated water valves.

Briefly the entire installation is protected against overloads, short circuits, overheating of the rectifier cylinder, failure of the water supply, failure of the auxiliary power supply, and high-voltage surges as follows:

Overload relays of the induction type located on the high-voltage side of the transformer protect against overload and short circuit conditions. This action is selective with the d-c. breaker.

A series of continuous overloads which may occur below the setting of the overload relays and cause undue heating of the rectifier, if persisted in long enough, is guarded against by means of a thermal relay. The rectifier is further protected against overheating by a temperature relay which gives a visible warning when the temperature of the cylinder reaches 60 deg. cent. and at 65 deg. cent. locks out the substation. Protection is afforded to the equipment in case the cooling water to the vacuum pumps fails, fuses in the supply circuit to the pumps blow, or the service of the supply circuit is interrupted or fails.

Protective resistance and spark-gaps connected to the anode leads mitigate the effect of high-voltage surges.

An 8000-ampere Westinghouse carbon breaker provides the switching device for the output side of the unit, as well as protecting the rectifier against short circuit, overload, and reverse current conditions.

Further protection as to overload and short circuit conditions on the d-c. side of the rectifier including selectivity with the Westinghouse breaker is provided by the General Electric moderate speed breakers with which each feeder is equipped. The breakers are automatic and reclosing in action.

To reduce the amount of scale forming matter which may deposit in the water passages around the cylinder due to roiled or muddy water, the cooling water is first passed through sand filters. It is expected that the amount of cleaning of the water chamber will be materially reduced.

The interior of the substation building is designed to meet the requirements of this type of conversion unit, its auxiliaries, and switching equipment. Generally the arrangement is similar to that of a rotating unit except for the absence of those features mentioned before. The exterior architecture of the building presents a structure pleasing in outline and creditable in appearance to the neighborhood.

The length of time the two units have been in service is much too short to gain any information as to operating performance. It may be said, however, that the two rectifiers were placed in service just prior to the time the heavy winter load occurs on the traction systems, and although subject to only a short seasoning period, the units have shown good performance up to the first of January, carrying at times integrated loads almost equal to the rated capacity.

Discussion

Otto Naef: About half of the rectifier substation equipments furnished by the company with which I am associated are designed for automatic operation. The combined output of these automatic stations, however, is smaller than that of the manually controlled stations, which indicates the general trend of using automatic control for a large number of small, decentralized substations and reserving manual control for the larger, more centrally located substations. Among the automatic rectifier substations I wish to mention especially one which is designed as a portable unit and has all the equipment mounted on a standard railroad car.

The two new 3000-kw. rectifier substations in Chicago are outstanding by the size of rectifier used, each tank having a nominal rating of 3000 kw. at 600 volts. This is considerably more than has been thought possible until quite recently. The Chicago rectifiers are the first ones of this large size which were put in operation in this country and I think the operating company which installed them has rendered a very important service to the art by trying them out on their important system and obtaining valuable operating data that will be of interest to other users of converting equipment.

That the engineers have confidence in these large rectifiers is indicated by the fact that up to the present time more than 45 rectifier cylinders of this size have been ordered here and abroad, of which nearly 30 are already in operation. Among the outstanding installations in this country and Canada I wish to mention a repeat order of 4 more rectifiers for the Chicago District and an electrochemical application in a large zinc-smelting plant in Western Canada, where 6 rectifiers will be installed, having an aggregate capacity of 16,800 kw. These latter rectifiers, while having equal output as the rectifiers now in operation in Chicago, have considerably less weight and a smaller diameter, a development which has been made possible by the good results obtained with those already in service.

J. A. Noertker: I should like to ask Mr. Garrett whether rectifiers can be designed so as to have a flat voltage characteristic, and if they are so designed, whether this characteristic can be changed so as to have a shunt characteristic.

On our Cincinnati system we have found it to be advantageous to have a flat voltage characteristic in our outlying territories so as to maintain a scheduled speed. At the same time there is the possibility of these stations becoming permanently overloaded, and this could be rectified to a certain extent if we could later change this to a shunt characteristic, which we intend to do.

T. H. Schoepf: I should like to know the size of the sand filter mentioned in the paper. Probably the water in Chicago has no mineral deposit in it that would cause deposit in the pipes that would go through a sand filter bed.

Also, in an outlying district where the rectifier may be out of service as much as 12 to 24 hours, what provisions do you make to prevent the water from freezing, and is the filtered water used over again?

George Swallow: I think it is remarkable that the total installed capacity of the mercury arc rectifier for railway service has increased practically 100 per cent in the last two years, and I believe it is the consensus of opinion of the operating companies that the rectifier is now well on its way toward replacing the rotating type of equipment, due mainly to the better efficiency.

I don't think Mr. Garrett mentioned in his paper just what the efficiency is, and he might be able to give us some idea of the actual gain made under regular operating conditions at 600 volts. It is evident that at 1500 volts the efficiency is so much greater that rotating equipment cannot compete with the rectifier on the efficiency basis.

There have been figures made and tests carried out on the Illinois Central system on various rectifier substations, and it has been shown that the actual gain in efficiency has been about 17 per cent on a capacity factor of about 12 per cent with 3000-kw. units.

A. M. Garrett: Mr. Naef's contribution has brought out the point that the problems in Chicago may be a little different from those in other places. We have had problems where the space has been so valuable that we could not include in some installations the ventilating devices I mentioned in the paper. We had for years the problem of commutation on the existing converters of the rotating type. Most of them were brought to a satisfactory condition. We even went so far as to look into the question of the design of the rotating unit to determine whether we couldn't thereby improve the commutation. With the advent of the first 60-cycle converters in 1916 we experienced trouble with unstable machines. They have since been improved so that the later types are better and not so susceptible to the swinging loads that accompany heavy drafts of power serving the traction systems. We have also had trouble with flashing of converter commutators, which in some instances has put the substation out of service for some considerable time.

Flashovers are reproduced in the rectifier in the form of arc backs, but in many cases the unit can be switched right back into service. After a flashover no inspection is necessary unless the vacuum has dropped, and that doesn't happen very often. The rectifier has this advantage, it can be switched back immediately but this cannot be done with the rotary converter. Generally, at least, an inspection must be made and if there is a flashed commutator the unit is sometimes out for hours.

Mr. Naef, I believe, spoke of four units now on order. These together with others will total some 18,000 kw. for service in Chicago, and will replace existing 25-cycle converters so that the supply to the load will be changed over from 25-cycle generators to 60-cycle generators, thus working toward a unified frequency of 60 cycles, standard in the Chicago district. We find that it is a rather happy combination in an old 25-cycle converter substation, to install a 60-cycle rectifier in it, using the rectifier as a base-load machine and the converters for peak purposes only.

The efficiency is not so much greater than it is with the 60-

cycle converter as far as the actual plant efficiency of the substation is concerned, but we do have a machine that operates very nicely as a base-load unit. We gain a little in efficiency and saving in energy cost.

Mr. Noertker wanted to know about the voltage characteristics. I can't say exactly as to that. I would rather leave this to some representative of the manufacturers, but I don't believe you can get an absolutely flat characteristic unless you install load-ratio control equipment connected to the transformer winding on the high side.

Regarding the matter of vacuum, if it cannot be restored and the unit must be opened there will be a considerable loss of time. In some severe cases of trouble such openings may run as high as 75 to 100 hr., especially if it is necessary to re-form. If the unit is opened only for inspection of certain parts or possibly to clean it in case the mercury has become dirty, the time out of service is not so long. It then becomes only a matter of the time actually taken to disassemble and reassemble, followed by a short period thereafter to bring the vacuum back by reforming. This is because all materials have been thoroughly seasoned. If a unit needs a new anode or a new insulator, or any other new iron material, considerable time is necessary to lay the machine out of service, take it apart, and put in the new parts, clean it thoroughly internally, and reform until the new material has seasoned properly.

Colonel Schoepf mentioned the size of the water filter. It is about 8 ft. in diameter and stands about 7 ft. high. If it is in some outlying district where the water pressure may be low, possibly a pump arrangement may be needed. In the last installation we put in a pump, but we have not used it as yet.

The possibility of water freezing in cooling passages has to be guarded against. One manufacturer, who has a unit in a substation with low load, during the cold period in the morning in the winter months provides heaters in the water passages to keep the temperature up to a safe limit. Otherwise there will be trouble in starting the arc. We experienced such trouble with one unit during the past winter.

We don't use the water over again in Chicago, but waste it. In the Public Service Company territory, at the Vollmer Road Substation, there is no city water system, and water is drawn from a well and recirculated through surface coolers cooled in turn by a fan.

Mr. Swallow mentioned gain in efficiency. We did gain some efficiency in the case of the rectifiers on the Illinois Central System. That load, however, is a different one from city traction loads, as it is highly fluctuating, and the efficiency of the rectifier is much higher at quarter load or around quarter load, so there is a gain when light loads of considerable duration are placed on the rectifier. Where there are many units in the single substation, however, approximately this same gain may be obtained with rotating converters by cutting them in and out in accordance with the load demands.

Arc Welding of Steel Buildings and Bridges

BY FRANK P. McKIBBEN[1]

Non-member

THE art of welding structural steel by electricity is developing rapidly and this year witnesses many applications of this process to construction and to reinforcement of bridges and buildings. The electric arc furnished designing engineers with a new tool which has been widely adopted not only in the fabricating of bridges and buildings, but also in many factories for welding parts of machinery and for assembling structural steel sections as substitutes for castings.

With proper supervision of design and workmanship one can secure safe construction as is evidenced by the existence of over sixty welded buildings in which no failures have been recorded. For a new type of construction this is truly a remarkable record, but in this connection, we must not forget that welded construction in its early stages has received far more experimentation, more careful supervision in execution, than did concrete or riveted work at the corresponding stages of their development.

Among the most important matters now receiving attention are: revision of building codes, preparation of specifications for welded buildings, accumulation of cost data, training of designers, the qualification of welders and inspectors, and additional tests of welded joints.

BUILDING CODES

The recent movement to change fiber stresses in steel construction from the 16,000- to the 18,000-pound basis has resulted in the revision of various municipal building codes. Among political divisions now actively engaged in revising their codes are New York, Philadelphia, Pittsburgh, Chicago, Lansing, Schenectady, several cities in California, and the State of California, and it is hoped that in these revisions due consideration will be given to the advantages of welding and that enabling provisions will be incorporated therein.

The American Welding Society's Committee on Building Codes is now drafting a code for welding, and until its report is published, cities may either adopt an enabling ordinance without any attempt to specify details of welded steel construction, or on the other hand may draft codes as complete as is now possible to obtain, giving the technique of welding and its application to steel buildings.

An example of an enabling article of general nature is as follows:—"Welding may be substituted for or used in connection with those methods already prescribed in the building code of the City of .... for assembling the component parts of and connecting together various beams, girders, lintels, trusses, columns, or other structural steel used in building construction; provided, that welding be done by competent workmen in accordance with designs and under supervision of a competent architect or engineer skilled in this work; and under regulations or specifications recommended by the Bureau of Building Inspection of the City of...... and adopted by the Common Council of the City of ...."

In addition to an enabling article similar to that outlined above, a specific code should include articles pertaining to definitions, materials, apparatus, permissible unit stresses, workmanship, qualification of welders, proportion of parts, and the protection of steel.

SPECIFICATIONS FOR ARC WELDING STEEL BUILDINGS

Architects and engineers are finding that specifications concerning arc welding are not easy to secure, and so many are the inquiries regarding this subject that there has been included in the Appendix of this paper a proposed set of specifications based on experiences gained with four welded steel building frames, and prepared after conferences with several prominent structural engineers and welding specialists. These specifications contain nine sections under the following headings: 1–General Application, 2–Definitions, 3–Materials, 4–Apparatus, 5–Permissible Unit Stresses, 6–Workmanship, 7–Qualification of Welders, 8–Proportion of Parts, and 9–Protection of Steel.

COST OF WELDING

Judging from the welded trusses of Building No. 1 of the General Electric Company's West Philadelphia plant the cost of welded trusses should be below that of any other method of connecting truss members. In that case a very great reduction was found in weight of welded trusses as compared with riveted trusses designed to perform the same function. A 58-ft., 6-in. welded truss weighed 5000 lb. as compared with 6800 lb. for a riveted truss, a saving of 1800 lb. or 36 per cent of the welded truss. A 78-ft. welded truss weighed 9200 lb. versus 13,200 lb. for a riveted truss, a saving of 4000 lb. or 43 per cent of the welded truss. While in buildings with a large number of trusses a saving is effected by welding, I doubt if very great reductions in costs result at present from use of welding in buildings consisting entirely of beams and columns. But where a large number of plate girders is used in a structure the probabilities are that welding will materially lessen costs, especially where automatic machine welding is applicable.

However, the architect or engineer has a very good way at his command for ascertaining whether he can reduce costs with welding. All he need do is to ask for

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alternate bids between riveted steel and welded steel; a method recently adopted in several instances.

QUALIFICATIONS OF WELDERS

On important building or bridge work it is not sufficient to employ welders upon their claims of experience in welding. While their claims may be perfectly correct and made in good faith, nevertheless it is true that a welder may be capable of securing excellent results on one class of work and yet be unable to perform well the work at hand. Each man should be tested by at least one of the three methods mentioned in the specifications in the Appendix.

A tendency exists to exaggerate this matter of tests for qualification of welders. It is certainly desirable to have good welders, but over zeal in this test business is not wise because it tends to create scepticism regarding safety of welds, to unnecessarily increase the cost, to restrict welders with rules not applied to other workmen, such as riveters and mixers of concrete, and to create unnecessary municipal regulatory bureaus.

Recently, it has been suggested that all welders in a certain city be licensed by the municipal Bureau of Building Inspection. I hope this method will not be adopted in any city. All that is needed is that the engineer or architect in charge of a welded steel building frame have on the work a competent inspector who qualifies the welders by any or all of the methods set forth above. If some municipal regulation is necessary it would seem better to have the contractor for the steel submit evidence that the welders he proposes to use on the building in question meet the qualification tests. But from my experience in this matter, I think no municipal regulation should be adopted other than those commonly applied by codes to other types of construction.

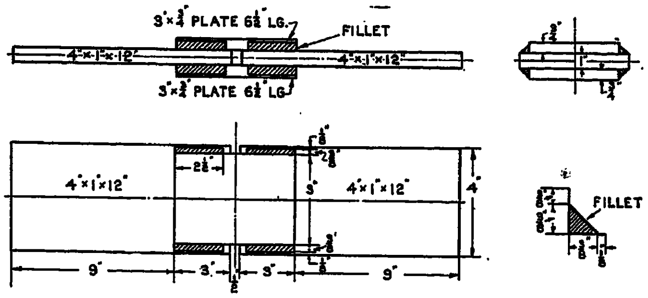

FIG. 1—TEST SPECIMENS FOR FILLET WELDS

Of the test methods, the tests to determine the longitudinal shearing value, Fig. 1 (Section 7, Article 3 of the specifications), is the best but most expensive for ordinary building construction involving trusses, beams, girders, and columns. The test specimen shown in Fig. 1 is designed in such a manner that when the 10 in. of fillets reach their ultimate shearing strength the unit tension in main and splice plates will not materially exceed the elastic limit. In fairness to the welders they should be given an opportunity before making the test specimens of familiarizing themselves with the type of electrode and current density to be employed.

It is desirable to test the welders in this manner from time to time during the progress of the work. The accompanying Table I gives the results of such tests for five welders at the Trenton welding shop of the American Bridge Company during the progress of welding the trusses for West Philadelphia Building No. 1 of the General Electric Co.

Table I shows a maximum ultimate longitudinal shearing stress per linear inch of fillet of 13,267 lb., and a minimum of 10,137 lb., with an average of 11,705 lb. As the minimum shearing section of a $\frac{3}{8}$-in. fillet one in. long is 0.266 sq. in., the above figures correspond to ultimate shearing stresses per square inch of minimum section as follows: maximum 49,900, minimum 37,100, average 44,000 lb. per sq. in. respectively. Moreover, when the fillets were stressed to the breaking loads the average unit tension in the two 5-in. by $\frac{1}{2}$-in. splice plates as well as the main 6-in. by $\frac{7}{8}$-in. plates were materially less than the elastic limit in each specimen. It will be noticed that the test specimens used in these tests were slightly different from those given under qualifications of welders, Fig. 1, Section 7, Article 3, of the specifications given in this paper.

TABLE I

TESTS ON TEN SPECIMENS TO DETERMINE VARIATIONS IN WORKMANSHIP OF FIVE SHOP WELDERS. LONGITUDINAL SHEARING VALUES ARE A MEASURE OF COMPETENCY

Number of specimen	Welder's number	Length of fillets total inches	Ultimate load pounds	Ultimate per linear inch pound	Average per linear inch pounds
3 A	3	10	132,670	13,267	
3 B	3	10	120,700	12,070	12,669
1 A	1	10	127,850	12,785	
1 B	1	10	120,630	12,063	12,424
2 A	2	10	120,570	12,057	
2 B	2	10	113,930	11,393	11,725
4 A	4	10	115,510	11,551	
4 B	4	10	111,580	11,158	11,355
5 A	5	10	105,660	10,566	
5 B	5	10	101,370	10,137	10,352
			Average	11,705	

EXPERIENCE OF WELDERS TESTED IN TABLE I

Welder Number	Experience
1.	Welding school but no previous experience.
2.	No welding school but previous experience.
3.	Welding school and previous experience.
4.	No welding school but previous experience.
5.	Welding school but no previous experience.

Table I reveals two interesting facts, though of course it must be remembered that only ten specimens are included. First, the highest result 13,276 lb. per linear in. of the best welder is 3130 lb. or 30.8 per cent above the lowest value of the least efficient welder. The maximum is about 14 per cent above and the minimum 14 per cent below the average value. Second, although a considerable variation exists between values of the best and poorest welders, a conspicuous similarity is shown between the two test results of each individual welder.

INSPECTION

As in riveted steel and in reinforced concrete construction, the conservative engineer provides inspection of welded work both in shop and field. It is the duty of the inspector to see that sizes and lengths of steel sections are correct, that welders are qualified, that the proper current and electrodes are used, that the current values are changed to suit the various sizes of fillets and thicknesses of steel, that the sizes and lengths of fillets are deposited to conform with those specified on drawings, that all parts conform to drawings as to size and position. The inspector should be required to make daily reports as to quality and quantity of work performed.

A good inspector can learn by visual inspection a good deal about a fillet. Rounded edges denote lack of penetration of fillet into parent metal, *i. e.*, into parts being welded; one short and one long side of a triangular fillet indicates that the wire electrode has been held at an incorrect angle of welding; a crater at any point in a fillet other than the end is evidence that the arc has been broken and the fillet not continuously laid; numerous gas holes on the surface of a fillet indicate too long an arc and a lack of penetration; a current, shown by the ammeter on the welding machine, too great for thin plates or too light for thick ones, is undesirable and the inspector should see that the welders adjust their machines to obtain the current suitable for the thickness of the material being welded.

In comparing the lengths of fillets actually deposited in the shop with those designated on the drawings it has been my experience that generally the total measured deposits exceed the total compiled from drawings, but now and then an individual fillet may be either greater or less than designated. For example, at the welding shop of the American Bridge Company at Trenton the total fillets on one welded truss aggregated 633⅛ linear in. as compared with 598 in. specified. And while the maximum excess of actual length of an individual deposit over that specified reached as much as 52 per cent, in two other fillets deficits of 8.8 per cent and 10 per cent respectively occurred. These deficits are not due exclusively to the welder, but sometimes may be attributed to inaccuracies on drawings where a specified length of fillet is impossible to execute.

TESTS ON WELDED JOINTS

Although sufficient test data pertaining to the strength of welded joints are available to enable engineers to produce safe designs for ordinary joints there is still need for additional tests covering several types of joints which will be used more freely in buildings or bridges when sufficient design data are obtainable.

Tests may be classified under three divisions:

1. Longitudinal shear on fillets.
2. Transverse shear on fillets, involving also simultaneous tensile or compressive stresses on some planes within the fillets.
3. Tensile and compressive strengths of welds in butt joints.

1. Longitudinal shearing values of fillets have been sufficiently well established for safe design of ordinary connections, but several important problems in connection with this shear have not been studied sufficiently. These are:

a. Relation between longitudinal shearing strength and the length of fillets. Our knowledge of longitudinal fillets is as complete relatively as it is of the shearing strength of a group of fillets. But the exact distribution of the shear along the fillet has not been determined. Several investigations indicate that the strength per linear inch lessens as the fillet becomes longer. We do not know the laws of distribution of stresses over a group of rivets; *e. g.*, given a lap riveted joint with five rivets in a row parallel to the line of stress; are the stresses on the rivets equal, or does each end rivet in the row carry more stress than the central rivet, if so, how much more? And so it is with a fillet; the end inch has a different stress from the central inch but how this intensity varies we know not. If we can learn this we will know more about welds than we know about rivets.

b. Relation between longitudinal shearing strength and the size of the fillet. A square inch of weld metal in a ¼-in. fillet may have a given longitudinal shearing strength. Will the strength per square inch of a ½-in. fillet be the same? Several series of tests indicate that the smaller the fillet the greater the strength per unit of area, while others do not show this characteristic.

c. Relation between longitudinal shearing strength and the number of layers used in depositing the fillet. Here again there is conflicting evidence, because some tests seem to indicate that the unit strength for a fillet of given cross-section increases if the fillet be deposited in two layers instead of one, while other tests fail to substantiate this.

2. Transverse shear, involving also simultaneous tensile or compressive stresses on some planes, exists in a fillet such as that across the top flange of an I beam welded to the side of a column. If the top of the beam is in tension the fillet is subjected to a transverse stress which apparently varies from pure shear on the horizontal side of the fillet to pure tension on the vertical side thereof. I say apparently, because the exact location of planes of principal stresses needs study.

3. Tensile and compressive strengths of welds in

butt joints have been given a great deal of attention. The former is useful in designing joints in pipes and cylindrical tanks, while the latter is of value in planning joints such as butt splices in some compression members such as at top chord splices. Many tests are added daily to the very considerable amount of available data of this nature.

A Series of Tests to Determine the Longitudinal Shearing Strength of Fillets

In May 1927, the General Electric Company made and tested at Rensselaer Polytechnic Institute a series of 48 welded specimens to ascertain the longitudinal shearing strength of fillet welds; results are given herewith in Tables II and III, and summarized in Table IV.

These two series of tests were designed to give ultimate longitudinal shearing values of ¼-in. and ⅜-in. fillets with triangular cross-sections without stressing the main and splice plates to the elastic limit. One-half the specimens were tested in tension and one-half in compression, the latter specimens being shorter than the former. All welding was performed by one operator, who did the work by hand, with current of 200 amperes, 3/16-in. electrode, arc voltage 20.

TABLE II
LONGITUDINAL SHEARING STRENGTH OF FILLETS
Specimens Tested in Tension

Specimen number	Fillets			Ultimate loads pounds	
	Size inch	Length each fillet inches	Number of linear inches each side	Total	Per linear inch of fillet
1 T—1	3/8	3	12	173,300	14,442
—2	3/8	3	12	164,100	13,675
—3	3/8	3	12	154,400	12,867
Average				163,933	13,661
2 T—1	3/8	4	16	212,000	13,255
—2	3/8	4	16	216,100	13,506
—3	3/8	4	16	191,600	11,975
Average				206,593	12,912
3 T—1	3/8	5	20	276,580	13,829
—2	3/8	5	20	268,300	13,415
—3	3/8	5	20	262,100	13,105
Average				268,993	13,450
4 T—1	3/8	6	24	317,000	13,208
—2	3/8	6	24	316,900	13,204
—3	3/8	6	24	330,300	13,763
Average				321,400	13,392
5 T—1	1/4	3	12	139,500	11,625
—2	1/4	3	12	131,200	10,933
—3	1/4	3	12	122,300	10,192
Average				131,000	10,917
6 T—1	1/4	4	16	176,740	11,046
—2	1/4	4	16	163,420	10,214
—3	1/4	4	16	164,860	10,304
Average				168,340	10,521
7 T—1	1/4	5	20	232,300	11,615
—2	1/4	5	20	227,620	11,381
—3	1/4	5	20	226,560	11,328
Average				228,827	11,441
8 T—1	1/4	6	24	257,720	10,738
—2	1/4	6	24	262,100	10,921
—3	1/4	6	24	251,800	10,492
Average				257,207	10,717

Examination of Table IV indicates that shearing values as determined on the tensile pieces are lower than those given by the compression pieces:—for example, 10,899 and 13,681 lb. per linear inch respectively for ¼-in. fillets, the difference being 2782 lb. or 26 per cent of the tensile average; while for ⅜-in. fillets the corresponding values are 13,354 and 16,126, a difference of 2772 lb. or 21 per cent of the tensile. The results in general seem to establish no relation between unit shearing strengths and lengths of the fillets,

TABLE III
LONGITUDINAL SHEARING STRENGTH OF FILLETS
Specimens Tested in Compression

Specimen number	Fillets			Ultimate loads pounds	
	Size inch	Length each fillet inches	Number of linear inches each side	Total	Per linear inch of fillet
1 C—1	3/8	3	12	185,000	15,417
—2	3/8	3	12	190,100	15,842
—3	3/8	3	12	212,300	17,692
Average				195,800	16,317
2 C—1	3/8	4	16	268,700	16,793
—2	3/8	4	16	259,600	16,225
—3	3/8	4	16	263,100	16,444
Average				263,800	16,487
3 C—1	3/8	5	20	318,300	15,915
—2	3/8	5	20	319,500	15,975
—3	3/8	5	20	310,200	15,510
Average				316,000	15,797
4 C—1	3/8	6	24	391,700	16,321
—2	3/8	6	24	370,400	15,433
—3	3/8	6	24	383,000	15,958
Average				381,700	15,904
5 C—1	1/4	3	12	180,000	15,000
—2	1/4	3	12	200,000	16,667
—3	1/4	3	12	182,000	15,167
Average				187,333	15,611
6 C—1	1/4	4	16	217,600	13,600
—2	1/4	4	16	225,700	14,106
—3	1/4	4	16	226,200	14,138
Average				223,167	13,948
7 C—1	1/4	5	20	251,700	12,585
—2	1/4	5	20	231,000	11,550
—3	1/4	5	20	250,600	12,530
Average				244,433	12,222
8 C—1	1/4	6	24	302,700	12,613
—2	1/4	6	24	332,900	13,871
—3	1/4	6	24	296,300	12,346
Average				310,633	12,943

TABLE IV
SUMMARY OF TABLES II AND III GIVING ULTIMATE LONGITUDINAL SHEARING STRENGTH OF FILLETS

	Length of fillet inches	Average ultimate longitudinal shearing strength per linear inch of fillet pounds	
		Size of fillet	
		1/4 in.	3/8 in.
For specimens Tested in Tension	3	10,917	13,661
	4	10,521	12,912
	5	11,441	13,450
	6	10,717	13,392
	Average	10,899	13,354
For Specimens Tested in Compression	3	15,611	16,317
	4	13,948	16,487
	5	12,222	15,797
	6	12,943	15,904
	Average	13,681	16,126

although the unit values of ¼-in. fillets tested in compressive specimens do decrease as the lengths of fillets increase, while others fail to show this trend.

Consider now the ultimate shearing strength per square inch of the ⅜-in. fillets as determined by the tensile specimens. For a ⅜-in. triangular fillet one inch in length, the net shearing area is 0.266 sq. in., which divided into 13,354 lb. (Table IV) gives a unit shearing stress of 50,200 lb. per sq. in. on the minimum section, commonly called the throat, *i. e.*, the section passing through the apex of the fillet's cross-section and perpendicular to its hypothenuse.

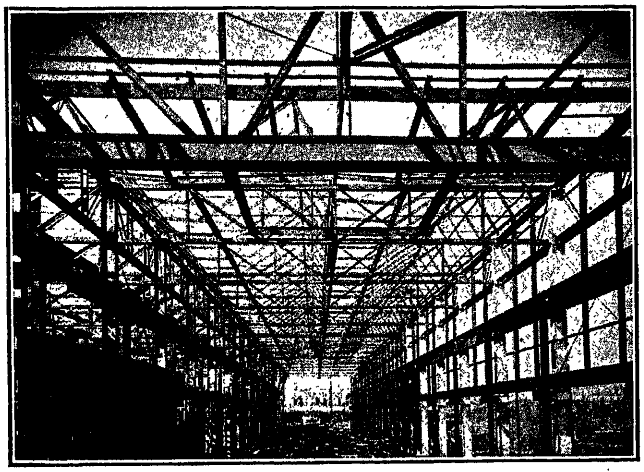

FIG. 2—SHOWING BUILDING WITH 56-ARC-WELDED ROOF TRUSSES OF GENERAL ELECTRIC CO. AT WEST PHILADELPHIA, PA.

The longitudinal shearing value of 3000 lb. per linear inch of a ⅜-in. fillet has been quite commonly used in designing welded joints of buildings. In the light of the tests herewith presented let us examine this working unit. Considering for a moment the ultimate values for ⅜-in. fillets as determined by tensile specimens it is seen that the lowest in Table II is 11,975 lb. per linear inch for specimen 2 T—3 and that is the only value below 12,000 lb. per linear inch. The unit working stress of 3000 lb. per linear in. has, therefore, a factor of safety of four. If we consider the average of all the tests on tensile specimens, 13,354 lb., Table IV, the 3000 gives a factor of safety of 4.4. Obviously, the factor of safety for the shearing values based on compression tests is much greater than that just determined.

This commonly accepted safe permissible value of 3000 lb. longitudinal shearing strength per linear inch for ⅜-in. fillets, divided by 0.266 (the throat distance) gives 11,300 shearing stress in pounds per square inch.

THREE BUILDINGS IN WHICH ARC WELDING WAS USED

During the past year the General Electric Company has completed a building in West Philadelphia, Pa., one in Bridgeport, Conn., and one in Pittsfield, Mass. in all of which the steel is connected principally by arc welding.

WELDED TRUSSES IN WEST PHILADELPHIA BUILDING

The West Philadelphia building, Fig. 2, consists of a head house 78 ft. wide by 171 ft. long with vertical clearance of 43 ft. below bottom chords of trusses, of one aisle 59 ft. by 474 ft., and of a second aisle 79 ft. by 474 ft. These aisles and the head house comprise a building approximately 138 ft. by 552 ft. Supporting the roof over these various parts are trusses of the Pratt type with horizontal chords in the two main aisles, and with a sloping top chord in head house. Each chord is an 8-in. H section with web horizontal, while each diagonal consists generally of two small channels with their backs lying in contact with, and on the outside of the vertical flanges of the H chords to which the channels are welded by fillet welds. Ordinarily each vertical member of each truss is a 7-in. I beam which fits into the trough of top and of bottom chords, to the vertical flanges of which it is welded. These direct connections of web members to chords obviated the use of gusset plates at truss joints.

In any building consisting of a main aisle and a transept, especially where there is a traveling crane in the latter, the truss at the intersection of transept and head house is somewhat difficult to arrange. In this building a truss 17½ ft. deep spans the main aisle, supports a portion of the roof load, the clearstory, a roof truss in the head house, and a head house runway girder spanning the 59-ft. aisle. On account of its depth this truss was shipped in parts, assembled lying flat on the ground, welded and erected. All other trusses were completely welded in the shop, and after erection with sufficient bolts to hold them in position were welded to the supporting columns.

FIG. 3—DETAILS OF COLUMN ANCHORS

This building has in addition to ten-ton bridge cranes in head house and in the 59-ft. main aisle, several smaller bridge and wall cranes.

WELDED TRUSSES AT BRIDGEPORT, CONN.

The Bridgeport building code is a flexible one in that it authorizes the Building Commission to permit the use of new materials if after investigation the Commission is satisfied that the new type is safe and satisfactory. After a study of tests data of welded joints the Commission granted a permit for the General Electric Company's welded steel building which connects two existing buildings.

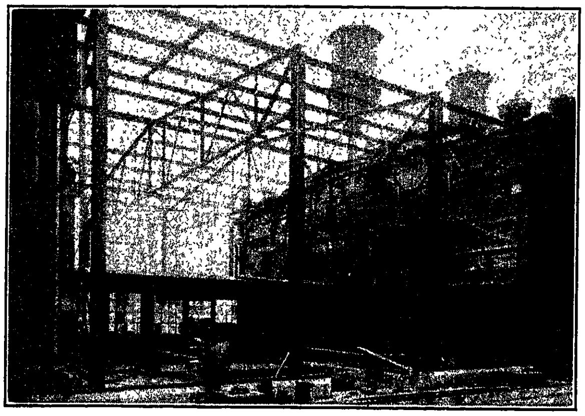

Fig. 4—Welded Trusses Erected in Position between other Buildings

This is a one story structure with roof trusses of 62 ft., span supported on side wall columns. The interesting details here are the use of T-shaped top and bottom chords, the truss end bearings at the tops of columns and the splice at center of top chord. By cutting the web of an H section along its center thus forming two T's for chords to which web members are welded directly, all gusset plates were eliminated. Beams can be cut in this manner either by cutting with the oxy-acetylene flame or by punching. At the bearing of vertical web of this top chord T on the base plate which in turn rests on the column cap, stiffener plates are shop welded to the T-web.

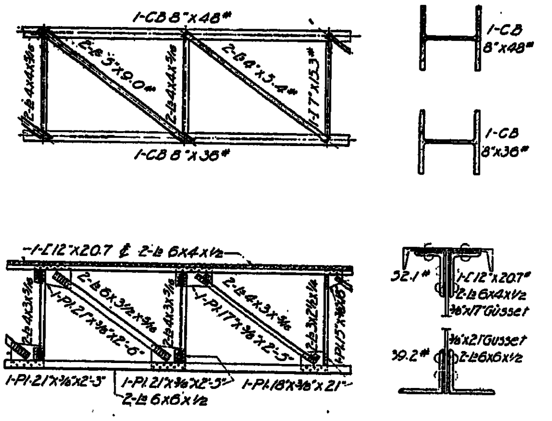

Fig. 5—Welded and Riveted Typical Panels and Chord Sections Pratt Type Truss

As the roof of this building slopes downward from the center line towards the ends of the truss the top chord is given a similar contour, thus necessitating a splice in the top chord at center. This is accomplished by a butt splice with a single V in the horizontal flanges of the T, while the vertical web is spliced on each side by a leg of a web member connected by fillet welds. As the roof trusses were fabricated completely in the shop by welding, the field welding consisted principally of connecting the top and bottom chords to the columns.

WELDED FACTORY BUILDING AT PITTSFIELD, MASS.

This structure is two stories high in part and three stories high elsewhere; length 280 ft.; width overall 60 ft., with approximately 40 ft. between centers of outside wall columns. The interest here lies entirely in the structural details, among which may be included the following:

a. Connection of a 27-in. main floor girder to the side wall column by a shop welded angle bracket upon which the girder rests and by field welding the girder web to face of the column as well as field welding the top and bottom flanges to the face of the column. The ends of these 27-in. girders were milled.

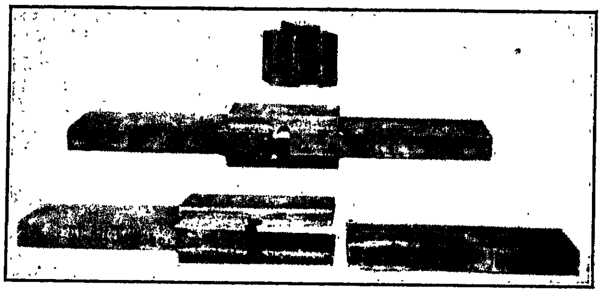

Fig. 6—Tests on Welded Structural Steel Showing Compression and Tension Specimens after Failure

b. Connection of main 18-in. floor beams to web of 27-in. main floor girders by resting the beams on heavy angle brackets shop welded to girder webs. These seat angles were designed to take the loads to which they were subjected.

c. The use of flat bars as stiffeners welded to the vertical webs at the center of 27-in. main floor girders where these rest upon short columns extending from foundation to second floor level only.

All shop and field welding on the above buildings was performed by single operator motor driven generator sets. For depositing 3/8-in. fillets on metal 3/8 in. or more in thickness, the current was about 175 amperes; 3/16-in. electrodes; arc voltage 20; with smaller current values for smaller fillets on thinner steel.

Appendix

PROPOSED SPECIFICATIONS FOR ARC WELDING STEEL BUILDINGS

SECTION 1. GENERAL APPLICATION

1–1. Arc welding may be substituted for or used in connection with rivets or bolts, or other methods prescribed in the Standard Specifications for Structural Steel for Buildings adopted by the American Institute of Steel Construction as amended to date, for connecting together, or assembling the component parts of beams, girders, lintels, trusses, columns, or other structural steel used in building construction.

SECTION 2. DEFINITIONS

2–1. *Arc Welding.* The process of joining steel parts in the molten, or molten and vapor states by the electric arc without the application of mechanical pressure or blows.

2–2. *Arc Weld.* A weld in which welding heat is generated by an electric arc formed between the base metal and the electrode, or between two electrodes, with or without the use of hydrogen or other equivalent gas.

2–3. *Electrode.* A specially prepared metal wire or rod used as a terminal in an electric circuit to produce an intense heat by means of the arc, and which may or may not supply the molten steel required for joining the parts to be welded.

2–4. *Filler Metal.* Metal which is added to a weld by the melting of an electrode or a welding rod.

2–5. *Weld Metal.* Material composing a weld.

2–6. *Base Metal.* Material to be welded.

2–7. *Fillet Weld.* A weld, at the intersection of two surfaces approximately at right angles to each other in which the weld metal has a triangular cross-section, its two sides being fused with the original surfaces of the base metals, and its third side being exposed. The size of a fillet weld is expressed in terms of its length and width, the length being the effective length exclusive of crater, the width being the length of the side of the largest isosceles triangle contained in the cross-section of the weld metal.

2–8. *Butt Weld.* A weld joining the edges of a butt joint in which two members are brought together edge to edge but not necessarily in contact, one surface of each member being in the same or parallel planes.

2–9. *Continuous Weld.* A weld of unbroken continuity used for strength or tightness or both.

2–10. *Intermittent Weld.* A strength weld consisting of a series of short welds in line, alternating with unwelded spaces.

2–11. *Tack Weld.* A short weld used not for strength, but for purposes of assembling only.

SECTION 3. QUALITY OF MATERIALS

3–1. *Structural Steel.* Steel used in structures covered by these specifications shall be of the grade designed as "Structural Steel for Buildings" as specified by American Society for Testing Materials, Serial No. A9 as amended to date.

3–2. *Electrodes and Welding Rods.* Wire used for electrodes or welding rods shall be commercial mild steel wire made for this purpose, of uniform homogeneous physical structure; free from irregularities in surface, from hardness, segregation, foreign matter, oxides, pipes, seams, or other defects.

3–3. Diameter shall not vary more than 0.003 in. above or below nominal values.

3–4. In the hands of an experienced welder, electrodes and welding rods shall show good welding qualities in flat, vertical, and overhead positions, and shall pass through the welding process without any unusual characteristics.

3–5. Chemical compositions shall conform to the following limits for uses specified:

	Wire grade A	Wire grade B
Carbon	0.13 to 0.18%	Not over 0.10%
Manganese	0.40 to 0.60%	0.25 to 0.45%
Phosphorus	Not over 0.045%	Not over 0.045%
Sulphur	Not over 0.045%	Not over 0.045%
Silicon	Not over 0.06%	Trace

Wire of Grade A shall be used for hand arc welding, and grade B for automatic machine arc welding.

SECTION 4. APPARATUS

4–1. Arc welding apparatus shall be of standard designs and products of reputable manufacturers.

4–2. Welding current shall preferably be direct current supplied by generators driven by electric motors or engines having suitable automatic speed regulation. Whether for use in hand or machine welding, generators shall be adjustable for different current values and shall deliver constant current at suitable voltage for each setting of the regulator.

SECTION 5. PERMISSIBLE UNIT STRESSES

5–1. Welded joints shall be proportional so that combined dead and live loads, and impact, if any, shall not cause the stresses therein to exceed the following amounts in *pounds per square inch*:

Shear on minimum section of weld metal	11,300
Tension on minimum section of weld metal	13,000
Compression on minimum section of weld metal	15,000

5–2. Maximum fiber stresses due to bending shall not exceed the values prescribed above for tension and compression respectively.

5–3. Stresses in welded joints due to wind only, and combined stresses due to wind and other loadings, may exceed by 33⅓ per cent the values prescribed above; provided, the section thus obtained is not less than that obtained if the wind force be neglected.

5–4. In designing welded connections allowance shall be made for bending stresses due to eccentricity, if any, except that in designing fillet welds no allowance need be made for the bending couple formed by resultant stresses on the two fused sides of the weld.

Section 6. Workmanship

6–1. Surfaces to be welded shall be reasonably cleaned, by wire brushing, chipping or hammering, of loose scale, rust, paint, or other foreign matter; except that a thin coat of linseed oil need not be removed before welding.

6–2. Welding operators shall submit evidence of experience in welding structural steel with the kind of welding to be used on the proposed work, and show ability to make flat, vertical, and overhead welds of uniform quality conforming to requirements of these specifications.

6–3. After being deposited welds shall be brushed with wire brushes, and shall show uniform sections, smoothness of weld metal, feather edges without overlaps, freedom from porosity and clinkers. Visual inspection at edges and ends of fillets and butt joint welds shall indicate good fusion with and penetration into base metals.

6–4. Arcs for electric welding shall be of proper lengths to produce good welds.

6–5. *Current Values.* The following figures are approximations only, intended as a guide to the operator who shall select the size of electrode and adjust the electric current to produce good homogeneous welds of above characteristics:

Electrode diameter in inches	Amperes hand welding	Corresponding plate thickness in inches
1/16	50–75	Up to 3/16
3/32	75–100	Up to 1/4
1/8	100–150	1/8 and over
5/32	150–200	1/4 and over
3/16	175–225	3/8 and over
1/4	200–250	3/8 and over

The same size electrode may be used with various thicknesses of plate, but thicker plates require heavier currents.

6–6. Current values for machine welding may exceed those specified above for hand welding but machine welding shall if required meet qualifications for welders hereinafter specified under Section 7.

6–7. In assembling and during welding, the component parts of a built-up member shall be held by sufficient clamps or other adequate means to keep parts straight and in close contact.

6–8. In welding, precautions shall be taken to minimize "locked-up" stresses, and distortion due to heat.

6–9. Before welding new steel to old, both shall be free from paint and other foreign matter, except that a thin coat of linseed oil need not be removed.

Section 7. Qualifications of Welders

7–1. *Butt Welds.* Welders may be required to weld four sample butt joints each made of two pieces ½ in. by 9 in. by 12 in. to form a piece about 12 in. by 18 in., prepared with double V described hereinafter; of these, two sample plates to be welded in the flat position and two in the vertical position with the joint located vertically. Each sample plate shall be machined to reduce the joint to the thickness of the base metal. From each sample plate, standard two in. tensile test specimens shall be made and tested in tension. The average tensile strength of each group of two sample plates shall not be less than 45,000 lb. per sq. in., and the tensile strength of the lowest in each group shall not be less than 40,000 lb. per sq. in.

7–2. *Lap Welds.* Welders may be required to weld four sample lap joints each made of ½-in. by 6-in. by 8-in. plates clamped one on the other with the 8 in. edges lined up but with the 6-in. edges offset ½ in. A full ½-in. fillet is then to be made along one 6-in. edge in accordance with specifications hereinafter described for lap joints. Upon completion of the welds and after cooling, the specimens shall be torn apart and broken by wedging at the unwelded 6-in. edges; and the fractured metal of specimen broken through the weld shall reasonably show:—bright, dense, even textured, crystalline or fibrous weld metal, irregularly torn and void of irredescent colors; good fusion of weld and base metals; good penetration into right angle corner of the fillet.

7–3. *Fillet Welds.* Welders may be required to weld three sample test specimens of size shown herewith by Fig. 1; each specimen consisting of two main plates 4 in. by 1 in. by 12 in. placed in line with their inner ends separated by ½ in., making the over-all length 24½ in. These places are to be connected by two splice plates, each 3 in. by ¾ in. by 6½ in. long, symmetrically clamped to the main plates, one above and the other below the two main plates, and each welded in a flat position to each main plate by two 2½ in. by ⅜ in. triangular fillets with their craters filled; each of these fillets is to be started at outer ends of the 3 in. by ¾ in. by 6½ in. splice plate and continue 2½ in. along the 6½ in. dimension towards the center, thus leaving a 1½ in. space between the inner ends of the fillets; the specimen is then to be inverted and the four remaining fillets deposited also in the flat position. In depositing the weld metal, craters shall be filled to full cross-section of fillets. Electrodes of 3/16 in. diameter shall be used. These specimens are to be tested in tension to ascertain the longitudinal shearing value of the ⅜-in. fillet welds; and the fractured specimens shall be examined for penetration into and fusion with base metals, freedom from gas holes or slag inclusions in fillets. The three specimens shall show at least an average ultimate longitudinal stress of 42,000 lb. per sq. in. and the lowest in the group shall show at least 38,000 lb. per sq. in. of minimum section of fillet.

Section 8. Proportion of Parts

8–1. *Plate Girders.* Girders shall be proportioned preferably by their moments of inertia but where the approximate method of flange design is used the web equivalent may be taken as 1/6 the web area, provided no holes exist in the web.

8–2. Web splices shall consist of a plate welded on each side of the web.

8–3. Stiffeners may be angles or flat bars; welded to the top and bottom flanges and also to the web by continuous or intermittent welds designed to transmit the stresses. Stiffeners at bearings or other points of concentrated loading shall be fitted to the flanges.

8–4. Connection of component parts of girder flanges to each other and flanges to webs shall be by continuous or intermittent fillet welds designed to transmit the stresses. Intermittent welds shall be not less than one and one-half inches long exclusive of crater, spaced in the clear not exceeding 16 times the thickness of thinnest piece connected, nor more than four inches in the clear.

8–5. Plug or slot welds may be used to fasten cover plates to each other and to other members.

8–6. *Beams.* Continuous beams and girders designed in accordance with generally accepted engineering principles may be used, provided that welded connections be properly proportioned to resist bending moments to which they are subjected.

8–7. Welded joints supporting non-continuous beams shall be designed with due regard for secondary bending stresses induced by deflection of the beams.

8–8. When the toe of a seat angle supporting a beam is to be welded and is not truly formed, but is rounded due to worn rolls or other causes, either a preliminary fillet shall be deposited to fill the open space, after which the full fillet required in the design shall be desposited, or the rounded edge of the toe shall be sheared square to receive the full fillet required.

8–9. *Trusses.* In tension members connected by fillet welds the gross cross-sectional area shall be considered as resisting the stresses.

8–10. For tension or compression members having symmetrical cross-sections the fillets shall either be arranged symmetrically about the axis of the member, or else proper allowance shall be made for unsymmetrical distribution.

8–11. For members of unsymmetrical cross-section, like angles for example, lengths of fillets shall be determined by taking moments about the gravity axis of the member.

8–12. *Columns.* Built-up or rolled steel columns shall contain no material, except filler plates, whether in the body of the column or used as lattice bars or stay plates, less than ¼ in. in thickness.

8–13. Ends of columns shall be faced to plane surfaces at right angles to their axes. Column splices, whenever practicable, shall consist of splice plates connected to the columns with proper amounts of fillets, and if filler plates are required they shall be connected with proper amounts of fillets. When sections of columns to be spliced are such that splice plates cannot be used, connections may be formed of plates and angles or other shapes designed to distribute the stresses properly. In all cases, column splices shall be so arranged that by bolts or otherwise the various tiers of a column can be accurately alined before welding, and shall be of sufficient strength to carry erection stresses.

8–14. Fillets connecting component parts of a built-up column shall be either continuous throughout the length of the column, or intermittent arranged as follows: Continuous fillets extending at each end of the column for a distance equal to the least width of column; then intermittent fillets one and one-half inches long spaced not exceeding four inches in the clear for the remainder of the column's length.

8–15. Lattice bars and tie plates, where used, shall be welded with fillets to secure strengths equal to those of the rivets in lattice bars and tie plates connected by rivets as specified in Standard Specifications for Structural Steel for Buildings adopted by the American Institute of Steel Construction and as amended to date.

8–16. *Welding and Bolting.* For welded structures, connections of main members carrying live loads producing impact, and connections of members subject to alternate stresses, shall be welded.

8–17. For welded structures, component parts of all columns, girders, and trusses shall be welded with fillet welds.

8–18. For welded buildings over four stories high all column splices, and all connections of beams and girders to columns, or within three feet thereof, shall be welded.

8–19. Unfinished bolts may be used in the shop or field for connections of small structures used for shelters and for connections of secondary members of all structures, such as purlins, girts, door and window framing, lining up bracing, and filling-in beams in floors. In buildings two or more stories high, sufficient bolts shall be used to connect properly all members during erection.

8–20. So far as possible all joints shall be designed to eliminate overhead welding; and arranged so that the welds will be in shear or direct compression.

8–21. Tension butt welds shall not be used in main truss members, but may be used in secondary members, wind bracing connections, beam and girder connections.

8–22. To the calculated lengths of each weld ½ in. shall be added to allow for the crater.

8–23. Fillet welds of lengths less than four times their width shall not be figured as part of any connection.

8–24. *Butt Joints.* The edges of base metal of members ¼ in. or more in thickness, transmitting stress by means of butt joints, shall be beveled with either single V or double V joints, or their equivalents.

8–25. Steel for single V shall be beveled not less than 30 deg. on each edge to form an open angle of not less than 60 deg., and the joint shall be backed-up in an approved manner.

8–26. Steel for double V joints shall be beveled not less than 37½ deg. on each side of each piece to form open angles of not less than 75 deg.

8–27. In all single V and double V butt joints the beveled edges shall have initial free distances of ⅛ in. to 3/16 in., and steel ¼ in. to 7/16 in. thick shall be welded in one layer, steel ½ in. to 11/16 in. in two layers, and ¾ in. to 1 in. in three layers.

8-28. All above joints shall be somewhat reinforced but not to exceed 20 per cent for single V nor 15 per cent for each side of double V joints.

8-29. *Lap Joints.* All lap joints in ¼ in. to 7/16 in. steel shall be fillet welded in one layer, ½ in. to 11/16 in. in two layers, and ¾ in. to 1 in. in three layers. The contour of the cross-section of fillet shall be as near as practicable to a triangle with equal sides, and with minimum side not less than the prescribed dimension of cross-section of fillet.

8-30. Weld metal deposited in two or more layers shall have each layer brushed with a wire brush or otherwise cleaned before subsequent layers are deposited.

8-31. *Drawings.* Drawings shall show clearly all welds, including the length and width of all fillets both for shop and field weldings.

SECTION 9. PROTECTION OF STEEL

9-1. Structural steel before being welded shall not be painted. If it is to be welded only in the fabricating shop and subsequently erected by bolts or rivets it shall receive in the shop one coat of acceptable metal protection after shop welding is finished, and one coat after erection. Steel to be field welded shall receive one coat of linseed oil after shop welding is completed; and after being erected and welded it shall receive two coats of acceptable metal protection.

9-2. No paint shall be applied to steel surfaces which are to be encased in concrete.

Discussion

S. M. Hamill: I should like to know how the welder tells when he has reached the proper depth of the fillet.

F. C. Caldwell: I should like to bring up the question of reliability of the electric weld and also the question as to whether the welder can be depended upon to produce as reliable work as the old fashioned riveter.

I. S. Campbell: What difference exists in the strength of weld made by the electric arc, the oxy-acetylene, or atomic hydrogen?

R. M. Easton: I should like to ask Professor McKibben for the requirements for inspectors on structural welding. It has been the practise in most structural work that is fabricated by means of rivets, to require that certain standards, which have been developed for the testing and inspection of rivets, be observed. I don't know of any means by which we could look at a weld or test a weld and tell positively whether it is a good or a bad weld.

M. M. MacDonald: I should like to ask Professor McKibben if he could enlighten us any in reference to the welding of the bearings on worn shafting, such as armatures for instance, or on other equipment and working parts of that sort, as to just what results have been obtained with electric arc welding on cast iron as applied to perhaps motor frames or parts of that nature.

J. F. Lincoln: (communicated after adjournment) Mr. McKibben in his Appendix makes certain recommendations which are somewhat open to question.

His specification on electrodes draws rather closely the chemical content of the steel which seems rather unnecessary. Why should it be so necessary to hold the content of the rod closely when the steel that is to be welded may vary in these respects very widely? Why should carbon and manganese be held to a comparatively low point in the welding rod when the amount of each one that is in the weld will be very greatly reduced by passing through the arc? If it is necessary for the carbon in the weld to be held to less than ten points, it would seem to me to be equally necessary for the carbon in the structural steel to be held to the same point, which would mean that at least half of all structural steel would be rejected.

Concerning Mr. McKibben's recommendations as to electrode diameters, currents to be used in them, and thicknesses of plate on which they are to be applied, some question may be properly asked. It is difficult to understand why a 1/16-in., 3/32-in., or 1/8-in. electrode can find any economic use in the welding of structural steel. The 5/32-in. and 3/16-in. electrodes may find a rather limited use in overhead welding, but 1/4-in. and 5/16-in. electrode should be used in doing at least 95 per cent of all structural-steel welding as the art is now known. The currents which should be used, should be in the case of 1/4-in. electrodes from 250 to 350 amperes, and on 5/16-in. electrodes from 300 to 400 amperes, that is for the usual electrode. Special coatings have been devised which will materially increase these currents. In this way the cost of welding and the ease of welding can both be materially bettered.

F. P. McKibben: Replying to Mr. Hamill's question, the welder can secure proper penetration (depth of weld) by watching the molten crater during welding. After completion of the weld the degree of penetration is determined by observing the depth of crater at the end of the weld; the thoroughness of penetration, by inspecting the edges of the weld. A sharp feather-like edge indicates good, and a rounded edge poor penetration. If a poor fillet is found it should be cut out and rewelded, just as a poor rivet must be replaced by a good one.

Replying to Professor Caldwell's question, by qualifying welders and by controlling the technique of welding, that is, the electrodes and current density, arc welding of structural steel is as reliable as other operations in building construction and I have long since ceased to worry about the results. The absence of any failure in the 60 or 70 welded buildings which have been constructed indicates sufficient reliability of control.

Replying to Professor Campbell's question, I would use in designing a welded structure the same allowable unit stresses for oxy-acetylene as for arc welding or atomic-hydrogen welding, namely, for shear on fillets 11,300 lb. per sq. in. of minimum section; for tension, 13,000 lb. per sq. in., and for compression 15,000 lb. per sq. in., both applied to minimum sections of the weld.

Replying to Mr. Easton's question, in the shop fabrication and field erection of welded buildings with which I have been connected, inspectors have been capable welders who first qualify the welders in accordance with Section 7 of the specifications given in the paper. They then inspect the work in progress and after completion as outlined in the paper under the heading Inspection.

Arc welding has been used on nearly all the welded buildings thus far constructed.

Replying to Mr. MacDonald's questions, it is common practise throughout the country to repair worn shafting and other parts by depositing welded metal using either oxy-acetylene, arc, or atomic-hydrogen processes. The most interesting case of this I have seen recently was in an iron-ore mining region where worn flanges on heavy ore cars were being replaced by automatic arc-welding machines which deposited welded metal to replace the original flange contour. Flanges, shafting, and other worn parts are frequently repaired in this manner; often a very considerable economy results. Arc welds in cast iron may or may not be successful. Frequently the results are excellent, then again are disappointing; by preheating or by use of special electrodes success is often secured when by ordinary direct welding failures occur.

Oscillographs for Recording Transient Phenomena

BY W. A. MARRISON[1]

Associate, A. I. E. E.

Synopsis.—Oscillographs developed for recording transient phenomena are described which obtain automatically records of amplitude, wave form, frequency, duration, and time of any electrical disturbance for which they are adapted. Two instruments are described for recording very short or very long transients; these may be used in combination. At power frequencies satisfactory records may be made on film or sensitized paper with a two-watt lamp. The instruments and their performance are illustrated by photographs and oscillograms.

* * * * *

IN this paper oscillographs are described which were developed primarily for recording transient phenomena of which the time of occurrence is neither known nor subject to control. The specific apparatus described was designed primarily for recording transient inductive disturbances in communication lines from neighboring power circuits. When the design of this apparatus was begun, there was no satisfactory way for determining the duration, frequency, or wave form of such disturbances, although apparatus was available by means of which the approximate magnitude of such transients could be determined, and, by constant supervision, the time of their occurrence. It was with the idea of determining part or all of these factors automatically in a single record that the oscillographs to be described were developed.

Transients in general may be of various types. They may have components in a large range of frequencies; they may occur in a large range of amplitudes and may be very long or very short or intermittent. Attention was directed toward recording devices which would obtain records of any disturbances in excess of a predetermined magnitude regardless of the time of occurrence. For practical reasons it was necessary also to give attention to the cost of operation, the power consumed, and the amount of servicing in operation.

To meet these requirements two somewhat different types of oscillograph were developed. One is capable of making records of short duration having uniform resolution throughout. By its use the wave shape of the first half cycle of a transient is recorded as clearly as that of any subsequent wave. The other instrument makes long continuous records and may be arranged to record a disturbance of any reasonable duration. The former instrument makes records in polar coordinates on a sheet of film rotating in its plane and will be called a "polar oscillograph." The latter records in rectangular coordinates on long strips such as motion picture film and will be called a "continuous-film oscillograph."[2]

FEATURES COMMON TO BOTH OSCILLOGRAPHS

Since these oscillographs were designed for recording the same sort of phenomena and for operating under somewhat similar conditions they have a number of features in common.

As in most oscillographs the optical system consists of a light source, a mirror capable of being vibrated about an axis in its plane with an amplitude proportional to the signal to be recorded, and a lens system, including the mirror, to form an image of the light source on light sensitive film moved in a direction perpendicular to the plane of vibration of the mirror.

The light source consists of a concentrated filament flashlight bulb placed as close as possible to a pinhole aperture in such a way that the aperture, as viewed from the vibrator side, appears to be completely filled by the lighted filament. No condensing lens is used because of the small size of the bulb which permits the filament to be brought close to the aperture. The filament is brighter than its image and the use of a condensing lens in this instance would waste light unnecessarily by reflection and absorption, and would make the optical system larger.

The vibrator is of the moving-iron balanced armature type similar to a driving element frequently used in loud speakers. The armature is attached by means of a stiff rod to a mirror free to vibrate about an axis in its plane in such a way that, as the armature of the element vibrates, the mirror vibrates at a relatively large angular amplitude. With this type of vibrator it has been possible to employ a mirror half an inch in diameter and still retain a satisfactory frequency range and sensitivity. The large mirror makes it possible to use either less sensitive film or a less intense light source than ordinarily would be required for a given recording frequency and film speed. With a half inch mirror it has been found practicable to use a lamp requiring only about two watts for recording on par-speed film.

A plane mirror is used on the vibrator, and a single meniscus lens mounted on front of it serves, in virtue of the reflection, as a symmetrical lens in forming an image of the pinhole on the film. When greater resolution along the time axis is required than can be obtained with this simple system, a cylindrical lens with short focus is placed in the light path just in front of the film.

Each oscillograph is equipped with a camera for the purpose of photographing a clock on the oscillogram to indicate the exact time of occurrence of the disturbance recorded. Any other information it is desired to associate with the records made by a particular oscillograph may be recorded photographically along with the clock. In several cases calendar clocks have been employed indicating the day and the month, and indicating whether the time is a. m. or p. m.

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[1] Bell Telephone Laboratories, Inc., New York, N. Y.

[2] This instrument is also known as the "movie oscillograph."

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

A schematic diagram of the optical system of the recorder and of the camera, as used in the polar oscillograph, is shown in Fig. 1. Some of the mechanism essential for operation is omitted for the sake of clearness.

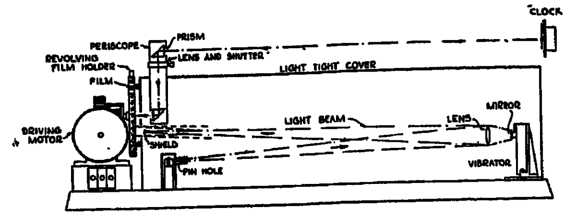


FIG. 1—ESSENTIAL ELEMENTS OF POLAR OSCILLOGRAPHS

One of the calendar clocks used in conjunction with these oscillographs is shown with cover open in Fig. 2. A space is left below the clock face for a card on which may be written identifying or other information relative to records that may be obtained. Lamps are mounted within the cover to illuminate the clock when necessary.

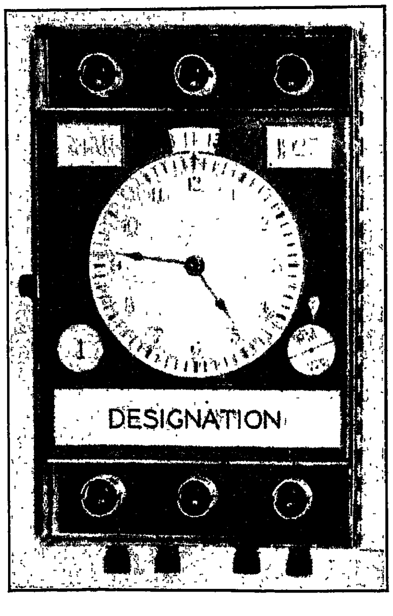


FIG. 2—CALENDAR CLOCK USED WITH OSCILLOGRAPHS, DOOR IS OPEN TO SHOW LAMP SOCKETS

Both oscillographs are equipped with automatic devices which enable them to make records of transients for which they are intended without the attention of an operator. These automatic features will be described in some detail in the following discussion of the individual oscillographs.

One part, however, a high-speed "line-relay," is common to both. It consists of a pair of high-speed polar relays, the windings of which may be connected into a line in such a way that, depending on the polarity, one or the other will be operated by any pulse of sufficient magnitude. The relays may be biased so that they operate only on pulses in excess of any given magnitude. They are connected so that when they do operate they remain operated until reset by some external means. Contacts on both relays are connected in parallel to the apparatus to be controlled so that a single positive or negative pulse will put that apparatus in operation. The time elapsed between the arrival of a pulse and the closing of the operating contact is less than 0.01 second. A schematic diagram of the line relay is shown in Fig. 3.

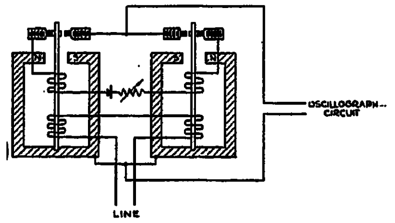


FIG. 3—CIRCUIT OF HIGH SPEED LINE RELAY

MAIN FEATURES OF POLAR OSCILLOGRAPH

A polar oscillograph is shown in Fig. 4 with the light-tight cover removed to show the optical system. The film is held in a standard film holder in a rotating member at the extreme right of the picture. The use of standard film holders facilitates loading in daylight as in an ordinary camera. The film is rotated by a small motor geared to the rotating member. The rotating member is separated from the remainder of the oscillograph by a circular light trap which permits free rotation while shielding the film from external light. The circular light trap used is illustrated in Fig. 1 and in Fig. 5. With this arrangement films may be exposed for days at a time under ordinary light conditions without appreciable fogging.

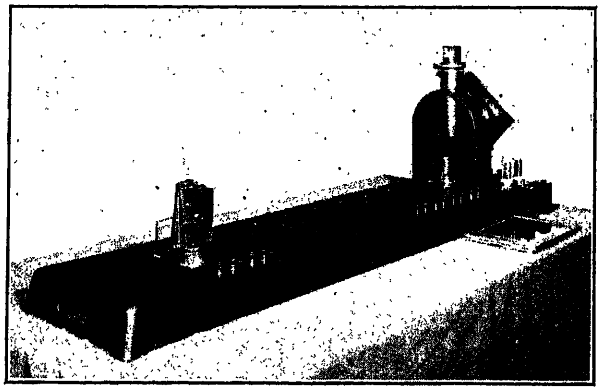


FIG. 4—POLAR OSCILLOGRAPH, SHOWING FILM ROTOR, PERISCOPE, LAMP HOUSING, AND VIBRATOR

The flashlight lamp is housed in the small light-tight box on the base just to the left of the film rotor. Excessive scattering of the light is prevented by a small tube, with a diaphram near the end, directed toward the vibrator mirror. The vibrator and mirror and the lens of the optical system are mounted on the base near the end opposite from the film mounting.

The chief value of this oscillograph lies in its ability to record with good resolution from the very beginning of a transient, regardless of the time at which it occurs, and regardless of the angular position of the film at which it begins. To accomplish this, the lamp is lighted continuously during the time a transient is expected and a narrow shield is placed in the light path of just sufficient width to prevent light from reaching the film when the vibrator is at rest. In this way fogging of the film is prevented during the time when no current is flowing into the vibrator but a record is made of any disturbance of sufficient magnitude to move the spot off the shield. A record made in this way appears like an ordinary oscillogram except that a narrow, clear space is left where a zero line is usually obtained. This is illustrated in Fig. 6A.

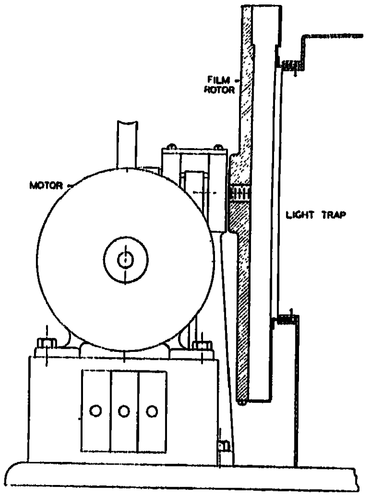
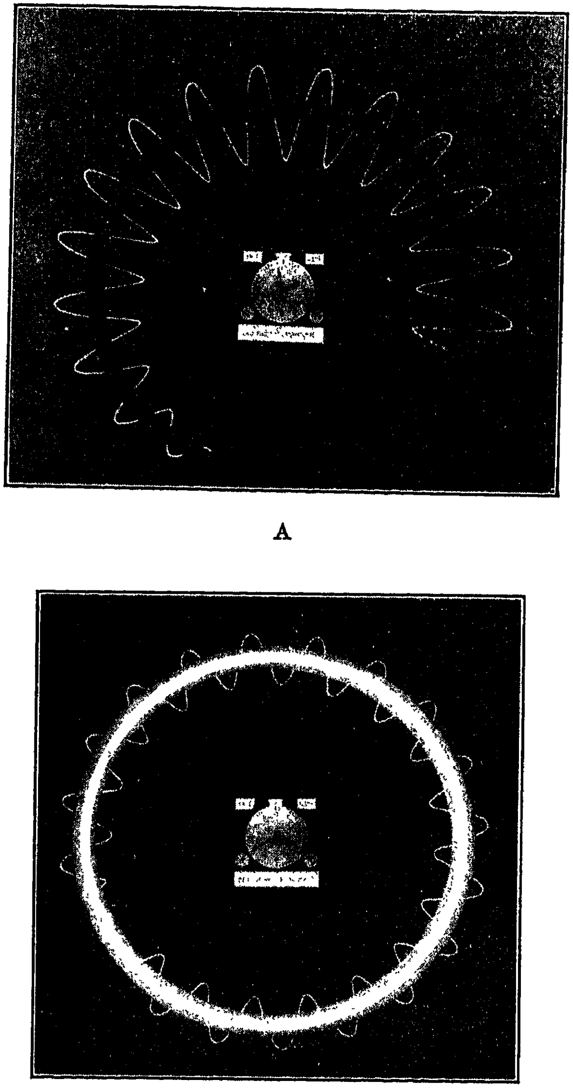


Fig. 5—Scale Drawing of Rotating Light Trap

In the absence of a shield the film soon becomes so badly fogged that an oscillogram made upon it is useless. Fig. 6B shows the fogging obtained with an exposure of one minute on the zero line without a shield.

If it is desired to record only disturbances in excess of a certain magnitude, the width of the shield may be increased so that no exposure occurs until the vibrator moves at more than a predetermined amplitude. In Fig. 7 a shield for this purpose is shown which may be adjusted to cover a portion in the center of the record from the width of the spot to about half an inch, or removed from the field entirely. As may be seen from the illustration this is accomplished by moving the shield in guides about the axis of film rotation.

There is a disadvantage in using a very wide shield of the type described. If, for example, a disturbance occurs which is just great enough to be recorded, the major portion of the wave is hidden by the shield and all that can be deduced from the record are the peak amplitude, frequency, and time. This difficulty can be avoided readily, however, by attaching a shield to the armature of an electromagnet so that it can be removed from the light path when the magnet is energized. The magnet may be operated by the high speed line relay, which is adjusted to operate when the disturbance exceeds a certain amount, in this case the same amount that moves the light spot at an amplitude greater than the width of the shield. The oscillogram shown in Fig. 8 was made with a removable shield of this type. The shadow of the shield is indicated in the first four cycles but does not appear during the remainder of the oscillogram. For convenience in interpreting the results a zero line is automatically recorded immediately after the recording of the oscillogram.



A

B

Fig. 6—Oscillograms Illustrating the Use and Omission of a Light Shield over the Zero Line

In order to reduce fogging due to stray light a large shield is placed between the film and the light source, having a vertical slit just large enough to allow the vibrating beam of light from the mirror to pass through to the film. This device reduces the fogging due to stray light to a small fraction of what otherwise would be obtained. The vertical slit can be seen in Fig. 7 behind the variable shield. A slight corona-like fogging on either side of the circular shadow of the shield, as shown in Fig. 8, is obtained after a few hours of exposure. In practise a film may be exposed for twenty-four hours or more without the fogging becoming so serious as to obscure a record.

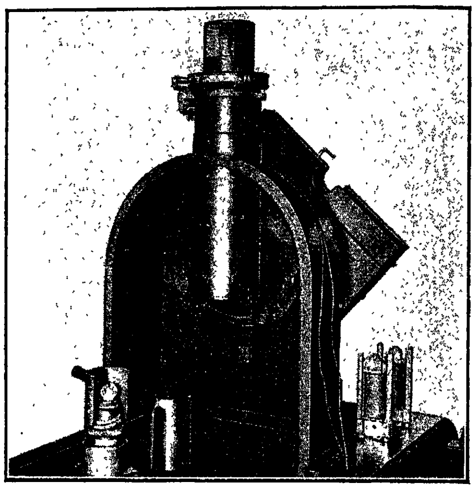


Fig. 7—Shields to Prevent Fogging on Polar Oscillograph

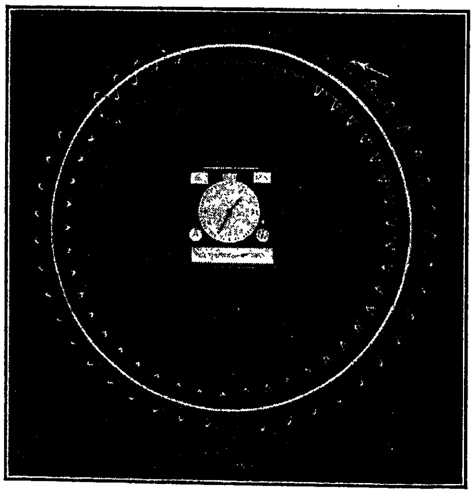


Fig. 8—Oscillogram Illustrating the Use of Removable Shield

To avoid confusion due to overlapping records a device is used to stop recording after one complete revolution of the film. It is put in operation at the beginning of a transient by the high-speed line-relay and allows recording to continue for one complete revolution, regardless of the angular position of the film at which it begins.

This device is shown on the oscillograph in Fig. 9. A vertical shaft driven at half the speed of the film carries a magnetic clutch fastened rigidly to it and a commutator which idles on the shaft except when engaged by the clutch. This commutator has one insulating segment and one conducting segment, each of angle about 180 deg. Contacts, controlling the current to a relay winding, are normally on the insulating segment of this commutator, but when the high-speed line-relay operates, the magnetic clutch is energized and the commutator is rotated until the contacts touch the conducting segment, thus operating the relay. One contact on the relay automatically releases the magnetic clutch, preventing further rotation of the commutator. Another contact interrupts the current going to the light source. Since the filament of the lamp is small the light is extinguished in a very short time and, of course, recording is stopped immediately.

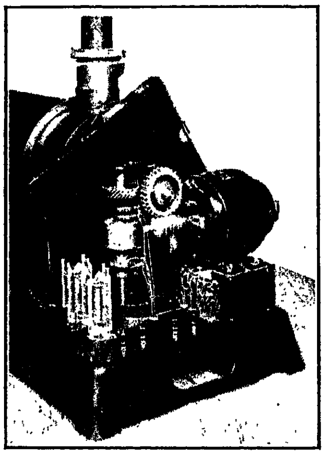


Fig. 9—Device on Polar Oscillograph to Limit Recording Time

After the exposed film has been replaced, and it is desired to put the oscillograph in operation again, the clutch and commutator are restored to their original condition by operating a key which energizes the clutch magnet and releases it again after one-half revolution of the vertical shaft.

If it is desired to make a record covering more or less than one revolution it can be arranged simply by changing the gear ratio between the film shaft and the vertical commutator shaft. The film speed may be changed either by changing the gear ratio between the motor and the film driving shaft or by varying the speed of the motor.

The camera for photographing a clock is shown in Fig. 4. It consists of a lens and shutter, shown at the top, and a periscope consisting of two right angled glass prisms mounted in the vertical tube. The periscope places the image in the center of the film and since there are two reflections the image will be the same as if none were used. The shutter is equipped with an automatic release which is operated a definite time interval after the beginning of an oscillogram, the time being determined by means of a slow acting relay or by a sequence switch. It is desirable to stop the film from rotating before photographing the clock which may be done by means of the same relay that turns off the lamp, or by means of a sequence switch.

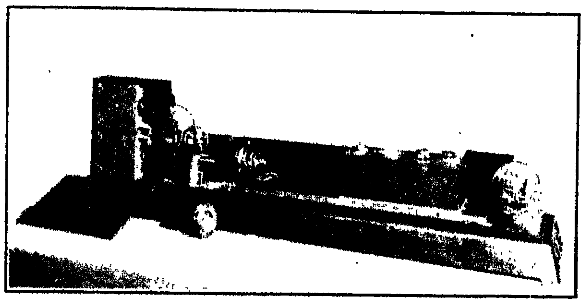


Fig. 10—Continuous-Film Oscillograph With Covers Removed

MAIN FEATURES OF CONTINUOUS-FILM OSCILLOGRAPH

A picture of the continuous-film oscillograph is shown in Fig. 10. It differs from the polar oscillograph mainly in the form in which records are obtained. As previously stated, records are made in rectangular coordinates on a strip of film and may, therefore, be of any length depending only on the length of film available and on the size of the storage magazines. The oscillograph shown makes records on motion picture film or sensitized paper of the same width. The film is stored in standard magazines for motion picture film holding up to 200 ft. It is advanced by means of a motion picture sprocket driven through gears and a magnetic clutch from a variable speed motor. The optical system is practically identical with that used on the polar oscillograph.

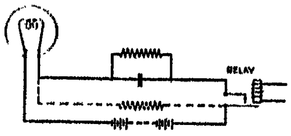


Fig. 11—Circuit for Lighting Lamp Quickly

With an oscillograph of this type it is not practicable to allow the film to be moving all the time on account of waste of film and the maintenance difficulties. In order to avoid this and still make it possible to begin recording soon after the beginning of a disturbance, the motor and associated gears are left running the whole time during which a transient may be expected, and, when a disturbance occurs, a quick acting magnetic clutch engages the film driving shaft with the motor which puts the film in motion very quickly. The line relay lights the oscillograph lamp at the same time. The whole recording mechanism may be put in operation within 0.02 sec., thus insuring a good record of any but a very short transient.

Normally the lamp would require several hundredths of a second to become lighted to full brilliancy if operated at normal voltage. However, with a voltage several times normal and by the use of the circuit shown in Fig. 11, it is possible to bring it to full brilliancy within 0.01 of a second without danger to the lamp. When the circuit is closed by the relay the condenser is charged suddenly to the applied voltage, the charging current passing through the lamp filament. This current, at the outset, is several times the normal current for the lamp and brings it to full brilliancy quickly. The resistance shunting the condenser has such a value that normal current flows through the lamp filament in the steady state, so, once lighted, the lamp remains at normal brilliancy as long as the circuit is closed. The lamp may be lighted in even less time if a small current is left flowing through the filament continuously in order to keep it hot but not hot enough to be luminous. A resistance of suitable value connected as indicated by dotted lines in Fig. 11 will accomplish this result.

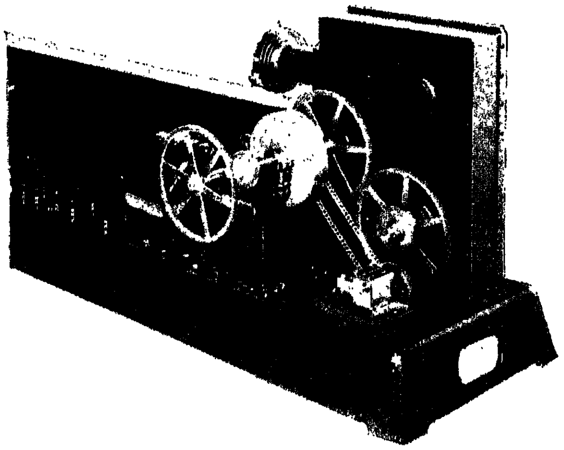


Fig. 12—Quick Acting Magnetic Clutch on Continuous-Film Oscillograph

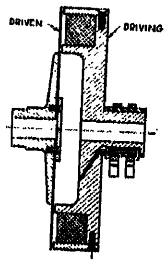


Fig. 13—Sectional Scale Drawing of Magnetic Clutch

The magnetic clutch, while especially designed to operate quickly, accelerates the sprocket and film without shock in order to avoid danger of tearing the film and to reduce wear and tear on the mechanism.

The clutch is shown mounted on the oscillograph in Fig. 12. It is also shown diagrammatically in Fig. 13 to illustrate its construction and operation. The annular coil in the driving member is connected, through slip-rings and contacts on the high-speed relay, to a battery. When current flows in this coil a steel diaphram on the driven member is drawn against the annular electromagnet, traction being obtained at the outer edge of the diaphram. Due to the small clearance between the diaphram and the electromagnet the diaphram is drawn into contact very quickly, and due to the small moment of inertia of the driven member it is rapidly accelerated to maximum speed. Since this is a friction type of clutch the acceleration is gradual and does not submit the sprocket and film to shock as would a toothed clutch.

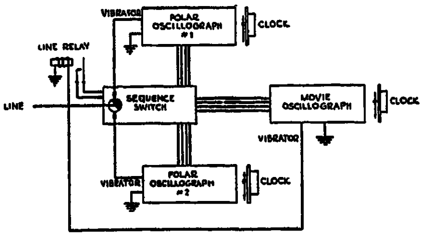


Fig. 14—Arrangement of Oscillographs for Recording any Transient in a Line

Two polar oscillographs and one continuous-film with control equipment and clocks are arranged so that a transient of any duration occurring at any time will be recorded with a record of the time of occurrence.

The delay in recording after the beginning of a disturbance depends on the time of operation of the high-speed relay plus that of either the clutch or the lamp, whichever is the longer. The relay requires only a few thousandths of a second to operate and both the clutch and the lamp may be adjusted to operate in less than one hundredth of a second. With this apparatus, therefore, it is possible to record all of a disturbance except that part which occurs during about the first 0.02 of a second. If desired, the lamp can be arranged to operate in considerably less time than the clutch, in which case the first part of the record will not be resolved but will indicate the amplitude of the disturbance which is frequently the most desired information. In the case of 25-cycle or 60-cycle disturbances the maximum of even the first half cycle may be recorded in this manner.

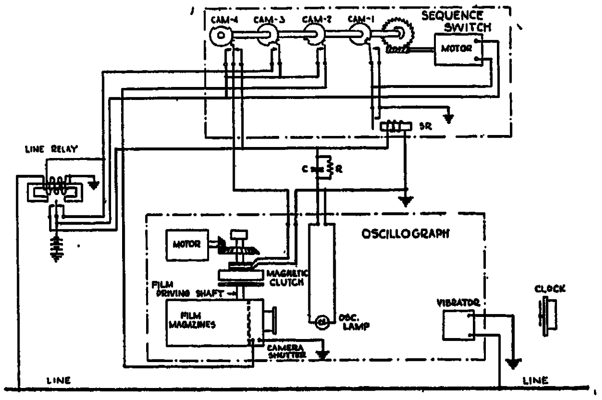


Fig. 15—Schematic Arrangement of Continuous-Film Oscillograph with Sequence Switch, Line Relay, and Clock, Arranged for Automatic Operation

The film driving mechanism is arranged so that the film may be advanced at any speed in a wide range, from a few inches per minute to about a foot per second. This is accomplished by means of a set of change gears and by changing the speed of the driving motor, or by both in combination.

As with the polar oscillograph, a camera is included for the purpose of recording the time automatically on the oscillograph film. This camera may be seen in Fig. 10. It is the same as that on the polar oscillograph with the exception that only one prism is used. This has the advantage, when recording is done on paper, that the image obtained through a lens and a single reflection is not reversed. The shutter is equipped with an automatic release that can be associated with slow acting relays or a sequence switch to take care of photographing the clock on the right portion of the film.

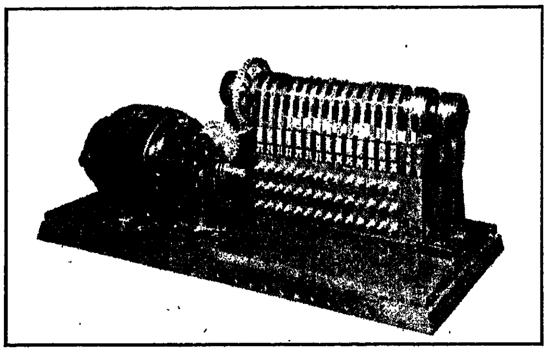


Fig. 16—Sequence Switch

Operation

There are a great many ways in which the oscillographs described above may be used. An arrangement is described in which two polar oscillographs and one continuous-film oscillograph have been used in conjunction for studying transients which are likely to occur at any time during long continuous periods. It was desired to determine the magnitude with considerable accuracy and at the same time to determine the time of occurrence, duration, frequency, and wave form.

The arrangement of oscillographs is shown diagrammatically in Fig. 14. One of two polar oscillographs is connected in the circuit being investigated so that it is in condition to record the first part of any transient that should occur. A high-speed line relay associated with it is arranged to put in motion the sequence switch which takes care of a number of operations consisting chiefly in starting the continuous-film oscillograph, in substituting the second polar oscillograph for the first after a certain small time interval, and in operating the camera shutters at the proper times.

Used in this way the polar oscillograph obtains a record having considerable resolution of the first part of a transient while the other oscillograph obtains a record of the complete transient with the exception of the first few cycles which are, however, obtained on the polar machine. The sequence switch is adjusted so that the continuous-film oscillograph is put in operation before the polar machine stops recording so that, with the two records, complete information of the disturbance may be obtained. Two polar oscillographs are used in this way so that, in case two transients occur close together, a record of one of them will not be lost during the time required for reloading. The sequence switch connects the line to the spare polar oscillograph automatically and gives the operator ample time to make any necessary adjustments on the remaining machine. The arrangement is perfectly symmetrical and either polar oscillograph may become the spare.

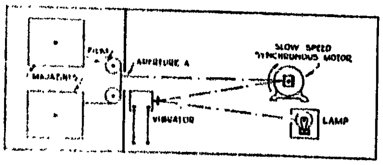


Fig. 17—Schematic Drawing of Sampling Oscillograph

This oscillograph records one wave out of a number at regular intervals, say one cycle in sixty, with considerable resolution in order to record slow variations in wave form.

An example of the record of a transient obtained with the two types of oscillograph, used in conjunction, is shown in Fig. 18. The polar oscillogram is similar to that shown in Fig. 8. It is obvious that the continuous oscillograph began recording about five cycles after the beginning of the transient while, of course, the polar oscillograph began recording immediately. The long record, however, continues 25 or 30 cycles beyond the end of the polar record and shows the manner in which the transient ended. Space does not permit of showing the clock that was photographed on this record.

When certain factors are known about the disturbances to be recorded the arrangement may be somewhat simplified. If, for example, it is known that any transient to be recorded will be of very short duration, the continuous-film oscillograph need not be used. If, on the other hand, it is known that the first cycle or two of the disturbance will be of no importance in the record, the polar oscillograph may be dispensed with.

The continuous film type of oscillograph offers some decided advantages over the polar type in that a large number of records can be made at one loading. Largely because of this it is possible to make the oscillograph entirely automatic in operation, causing it to record, without any attention whatever, all the transients in a circuit as they occur, until the supply of film is exhausted. Such an oscillograph may be left permanently connected into a circuit in which transients are expected, and at the end of any period the film that has been advanced into the "exposed" magazine will show on development records of the magnitude, frequency, and wave form of the disturbances and of the time of occurrence of each.

For automatic operation of the oscillograph a sequence switch of some sort must be used to insure that the various automatic operations are performed in the proper sequence. In Fig. 15 a schematic drawing illustrates the essential elements of such an arrangement. When standing by, ready to record a transient, the motor on the continuous-film oscillograph is running but does not engage the film advancing mechanism because the magnetic clutch is normally deenergized. On the arrival of a transient the sequence of operations is as follows:

The line relay operates and locks in operated position in virtue of the bias current through the outer winding being removed by the opening of the back contact. The lamp is lighted through the resistance and condenser combination *R C*, and at the same time, the motor is engaged with the film driving mechanism through the magnetic clutch. Since the vibrator is connected continuously to the line, recording begins immediately. Relay *S R* operates at the same time that the clutch is energized and starts the motor of the sequence switch which rotates the cams in the direction of the arrow. After a predetermined amount of film has been advanced, cam 3 rebiases the line relay and restores it to the original non-operated condition, (unless the disturbances on the line continue beyond this time). This releases relay *S R* but the cams con-

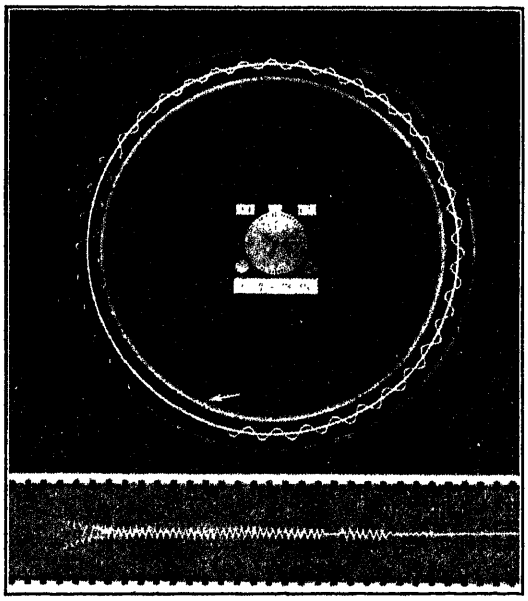


Fig. 18—Sample of Records Made by Polar and Continuous-Film Oscillographs Used Together

tinue to turn since the contact operated by cam 1 is in parallel with that on the relay, which allows the motor to run during one complete revolution of that cam. When the line relay is restored the film stops and the lamp is disconnected. After cam 3 stops the recording and resets the line relay, cam 2 operates the camera shutter which photographs the clock on the film at the end of the oscillogram. As the cam shaft continues to revolve, cam 4 operates the magnetic clutch for a short time without lighting the lamp. This advances the film far enough so that the beginning of the next record will not be superposed on the image of the clock. When the cam shaft has completed one revolution, cam 1 stops the sequence switch motor by opening the contact associated with it.

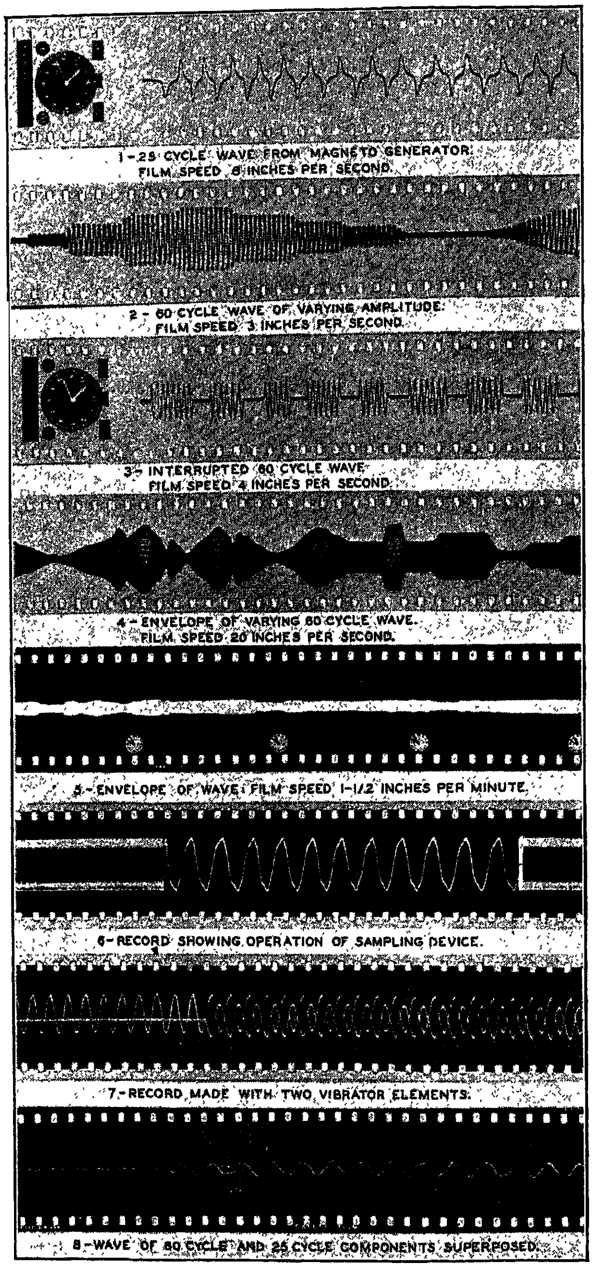


Fig. 19—Samples of Continuous-Film Oscillograms

After this sequence of operations everything is exactly the same as before except that a certain length of film has been advanced into the used-film magazine and a complete record made on it. The length of the record may be adjusted by an adjustment of cam 3.

In case the disturbance lasts until after the amount of film allotted to each record has been used, the line relay will remain operated and the film will continue to be exposed during one whole cycle of the sequence switch. Thus a continuous record can be made of a disturbance, however long, provided there is sufficient film.

Fig. 16 shows a sequence switch that has been used in the arrangements of both Fig. 14 and Fig. 15. With it a great many arrangements of automatically controlled equipment may be set up besides those described, permitting the oscillographs to be used in a great number of ways.

A modification of the continuous-film oscillograph which appears to have some novel and useful features is shown in Fig. 17. It is adapted especially for sampling a wave at regular short intervals instead of making a continuous record or merely a record of unusual disturbances. It involves, in addition to the usual optical system and means for advancing the film, an additional mirror in the light path between the vibrator and the film, rotating synchronously with the current or voltage to be recorded, about an axis perpendicular to both the direction of motion of the film and the axis of the vibrator mirror. The function of the rotating mirror is to sweep the light beam along the oscillograph film past an aperture shown at *A* in such a way that the effective film speed during exposures is many times the actual film speed, and to permit of exposure during only a small part of the total time.

As an example, suppose that the mirror makes one revolution in two seconds and that the wave to be recorded has a frequency of 60 cycles per second. If the distance of the rotating mirror from the film is approximately 8.5 in., one cycle of the wave recorded will be spread over one inch of film. If a rotating mirror with a single facet is used, and if the aperture is just one inch wide, the actual film speed should be one inch in two seconds and every one hundred and twentieth wave will be recorded. If two facets 180 deg. apart are used on the rotating mirror, and if the film speed is doubled, one wave in every 60 will be recorded.

Individual facets on the revolving mirror may be inclined to the axis of rotation in order that successive exposures may be made from different vibrator elements. For example, three facets suitably mounted could be employed to show in succession sample waves of current from the three circuits in a three-phase line, the motor driving the mirror being operated synchronously in phase with the three-phase voltage.

The advantage in this recording method lies in the ability to obtain a good record of slow changes with good resolution and without the use of a large amount of film.

Another method of sampling which gives a somewhat different kind of information may be used with a continuous-film oscillograph with the usual form of optical system. The film is run at very slow speed in order to obtain normally an envelope of the wave. At intervals the speed of the film is increased to a value sufficient to resolve the wave and show the actual wave shape. A record of the time may be made on the film by photographing a clock at regular intervals, say once a minute. A record made in this way is shown in No. 6, Fig. 19.

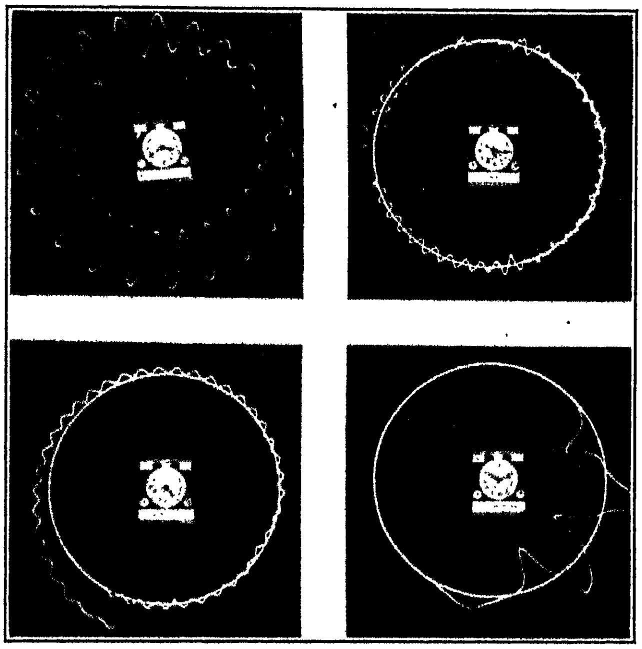


FIG. 10—SAMPLES OF POLAR OSCILLOGRAMS

Frequently there is an advantage in recording two or more variables simultaneously. A record obtained with two vibrators is shown in Fig. 19, No. 7.

PERFORMANCE

The limitations of an oscillograph lie mostly in the vibrator and, to a smaller degree, in the optical system and photographic emulsion used. The frequency characteristic of the vibrator up to 800 cycles is quite uniform, permitting records of disturbances having components in this range to be made with little distortion. This range includes the first 13 harmonics of 60 cycles and the first 32 harmonics of 25 cycles.

The vibrator may be wound to have any impedance in a wide range. If wound to have a high impedance it is especially suited for recording voltage waves, and if wound to have low impedance it is better suited for recording current waves. The sensitivity, as usually expressed, in millimeters deflection per milliampere of input, varies approximately as the square root of the impedance of the winding. The ratio of deflection to input is constant over a considerable range due to the balanced structure of the motor element.

As noted previously the oscillographs described are intended for recording in a comparatively low frequency range. In the range given there has been no difficulty in obtaining good records with a two-cp. flashlight bulb. This, of course, is largely due to the large size of the mirror on the vibrator.

Samples of records made with the oscillographs described are shown in Figs. 6, 8, 18, 19, 20, and 21. Those in Figs. 6, 8, and 18 have already been mentioned. Fig. 19 shows eight strip oscillograms and Fig. 20 shows four polar oscillograms illustrating some of the possibilities of these instruments. Fig. 21 is a response calibration, at constant frequency, of a polar oscillograph.

A number of field applications of oscillographs of both types has been made with satisfactory results. In some cases where cooperative studies were being made, the oscillographs have been used for recording transient neutral currents in power systems as well as to record voltages induced in telephone circuits by power system transients. Experience with the oscillographs in these field installations has suggested a few improvements of a mechanical nature and certain rearrangements of parts to increase the convenience of operation. These changes are now being embodied in a new design. It is hoped that it will be possible in a later paper to describe these features and to give the results of field experience more fully than can be done at this time.

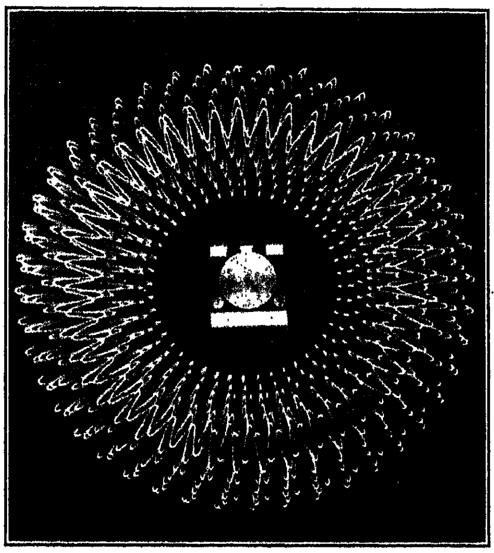


FIG. 21—AMPLITUDE CALIBRATION OF POLAR OSCILLOGRAPH

Discussion

Morton Sultzer: Oscillographs of the general type described by Mr. Marrison have been in field use for over two years. I wish briefly to discuss some of the results obtained. I hope at a later date to present to the Institute a more elaborate review of these results.

As stated in the paper, this apparatus was developed for recording transient inductive disturbances in communication lines from neighboring power circuits. In our observations of this kind of disturbance, there have always been cases where transient disturbances were manifest in communication circuits, for which there was no record of disturbances in the power system to which the communication circuits were exposed, and there have been records of disturbances in power circuits of sufficient magnitude to cause inductive disturbances in neighboring communication circuits, for which there are no records of disturbances in the communication system. An automatic recording oscillograph, equipped with a camera for photographing a clock to indicate the exact time of occurrence of the disturbance, should be a valuable tool in determining what disturbances correspond in the two systems.

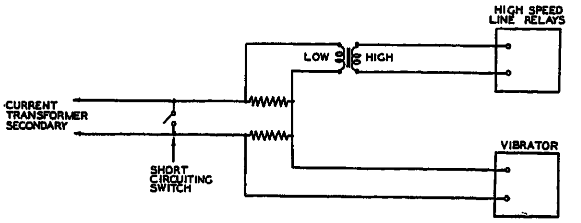


Fig. 1

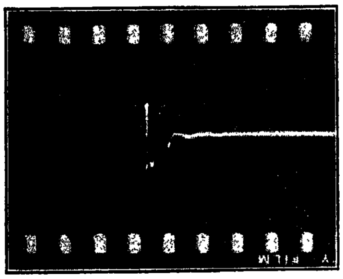


Fig. 2

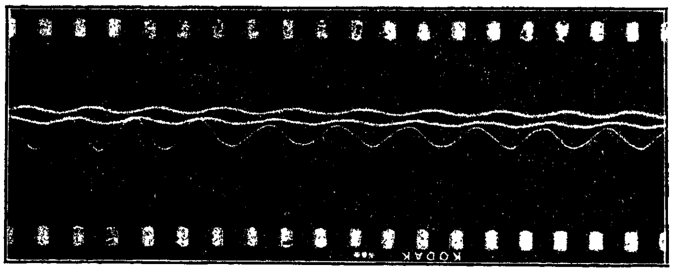


Fig. 3

In certain cases where co-operative studies are being made, oscillographs have been used to record simultaneously the disturbing abnormal in the power system, as well as the disturbance in the communication circuit. In addition to recording transient disturbances caused by abnormal conditions on power systems, this oscillograph has been found valuable in staged tests where the faults were applied to the power system at a predetermined time, and for measuring normal load induction.

Fig. 1 herewith is a diagram showing the connections of the continuous film oscillograph into a power system's neutral ground. In the secondary of the current transformer are inserted two small resistances. The drop across one resistance is supplied to the low side of a transformer, the high side of which feeds the tripping relay circuit. Bridged across the other resistance is the vibrator element of the oscillograph wound for low impedance. By this connection, it is possible to use a small radio-type transformer for operating the tripping relay without causing wave-shape distortion of the resultant oscillograms due to overloading.

Fig. 2 is an illustration of the rapidity of action of the oscillograph, for a surge of this type would probably not result in any breaker operation and therefore would not be called to the attention of the power company. The surge lasted slightly longer than one cycle on a 60-cycle system.

Fig. 3 shows 25-cycle induction from an electrified railroad, illustrating the use of the three-element oscillograph.

Fig. 4 shows oscillograms from stages tests showing the current in the neutral ground circuit of a 60-cycle power system and the resulting induction on communication circuits. These two oscillographs were situated approximately 60 mi. apart at the time of the tests. It will be noticed that communication circuit induction is recorded on two exposed lines by use of a two-element oscillograph, and an inspection of the two strips will illustrate how closely the induction followed the variations in the fault current. After the first six cycles, the second phase was faulted, causing an increase in the fault current, and after 31 more cycles, both phases had cleared. The current and the resulting induction after this point are due to the fact that the fault was placed on one of two transmission lines, working in parallel. After the fault had cleared on the side toward the generating station, it was fed over the non-faulted line until the breakers cleared at the far end.

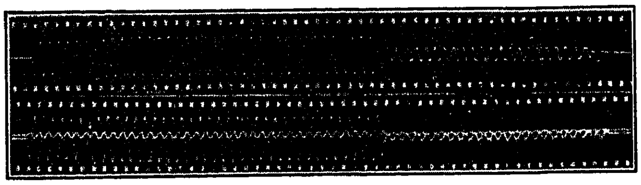


Fig. 4

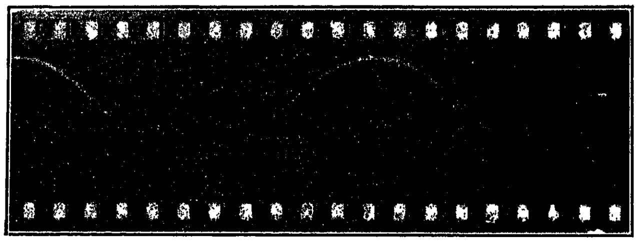


Fig. 5

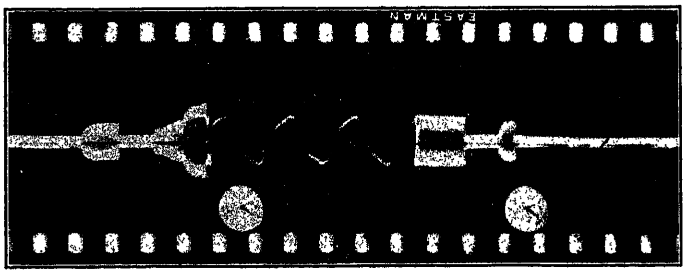


Fig. 6

Fig. 5 shows oscillograms of a 25-cycle calibrating wave. It will be noted that the film in this case is run at a much higher speed than on most of the other oscillograms, the speed in this case being approximately 5 ft. per sec., and in most of the other oscillograms 1 ft. per sec. This speed has been successfully used in staged field tests.

Fig. 6 shows the induction under normal load conditions from a 25-cycle electrified railroad system. This oscillogram was taken at a film speed of about 1½ in. per min., and at 5 min. intervals a short sample of the wave shape was taken by speeding up the film to approximately 1 ft. per sec.

Bushing-Type Current Transformers for Metering

BY A. BOYAJIAN[1]
Fellow, A. I. E. E.

and

W. F. SKEATS[2]
Associate, A. I. E. E.

Synopsis.—This paper describes a new development in connection with bushing type current-transformer metering circuits. While this development utilizes the two-stage principle and has the same order of accuracy as that of two-stage current transformers, it is different from the conventional two-stage current transformers in that it does not require two-stage wattmeters and watthour meters, but may be used with any wattmeter or watthour meter. The principle and connections of the transformer are explained, and performance curves of a typical unit are given.

* * * * *

INTRODUCTION

THE great simplicity, reliability, and low cost of the bushing-type current transformer have led engineers to an effort to improve its accuracy to such a point as will be satisfactory for metering purposes. The introduction of the two-stage principle[3] by Brooks and Holtz, six years ago, marked a satisfactory improvement in accuracy, but the two-stage principle as applied so far has required special two-stage wattmeters and two-stage watthour meters. There has still remained, therefore, the need for a bushing-type current transformer that would have metering accuracy and could be used with any wattmeter or watthour meter. In the development of the arrangement described below, which satisfies this requirement, the two-stage principle has been retained and such additional features have been incorporated as would make it adaptable to single-stage wattmeters and watthour meters.

CIRCUIT OF THE NEW CURRENT TRANSFORMER AND THEORY OF ITS OPERATION

Fig. 1 illustrates diagrammatically the conventional two-stage current transformer and a two-stage wattmeter or watthour meter. $P_1 S_1$ is the first stage of the current transformer feeding the first-stage current-coil C_1 of the meter W. The current in C_1 differs (vectorially) from the correct secondary current by the exciting current in P_1 divided by the turn ratio of the transformer. The second-stage transformation aims to put into C_2 a current equal to this difference. In the second stage of this current-transformer, P_2' and P_2'' act jointly as the primary; that is, the net ampere turns of P_2' and P_2'' act as the primary ampere turns, inducing a corresponding secondary current in S_2, which flows into the second-stage current-coil C_2 of the meter. The net ampere turns of P_2' and P_2'' are the exciting ampere turns of the first stage and thus the current delivered to S_2 and C_2 represents and makes up for the exciting current error of the first stage. In transforming the exciting current of the first stage, the second-stage current transformer requires an exciting current, so that the exciting current correction by the second stage is not 100 per cent exact. However, since the residual error is the exciting current of the exciting current, it is of second order of magnitude as compared with the error of the first stage or as compared with the error of an ordinary current transformer of the same proportions.

In order that the two stages may perform without interference from each other, it is necessary that the two circuits have no appreciable mutual impedance.[4] One obvious method of accomplishing this is to separate the current coils C_1 and C_2 from each other and provide a separate potential coil for each one to react with.

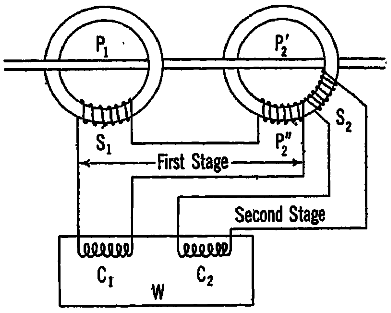


FIG. 1—TWO-STAGE CURRENT TRANSFORMER WITH SPECIAL WATTMETER HAVING TWO CURRENT COILS

This is the practise followed in most of the two-stage current transformer applications, requiring special wattmeters and watthour meters. This disadvantage has been obviated in the present arrangement as follows:

For the proper performance of the two stages, it is not necessary that there should be no mutual impedance between the two circuits at any point, but that there should be no *net* mutual impedance. Accordingly, C_1 and C_2 may have any mutual impedance at any point provided that it is neutralized by an equal and opposite mutual impedance at another point. If C_1 and C_2

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[1] General Electric Company, Pittsfield, Mass.

[2] General Electric Company, Schenectady, N. Y.

[3] See *The Two-Stage Current Transformer*, by H. B. Brooks and F. C. Holtz, A. I. E. E. TRANS., 1922, p. 382.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

[4] By the mutual impedance between two circuits is understood the ratio (vectorial) of the voltage in one circuit produced by current in the other to the current flowing in the other circuit. As far as the authors know, this conception has to date been limited to mutual inductance or mutual reactance, but if the two circuits have a part in common it is obvious that current flowing in the one will give rise to an in-phase voltage in the other due to the resistance of the common part.

are combined into a single current coil C (Fig. 2) as in any conventional wattmeter or watthour meter, the mutual impedance between the first and second stage circuits is the impedance of the coil C, and therefore it is necessary and sufficient to provide another impedance common to the two circuits and equal and opposite to C. This is accomplished by the auxiliary impedance Z and the transformer T. The impedance Z is substantially equal to the impedance C times the ratio of transformer T, and the transformer T accomplishes the desired reversal of sign so that one can neutralize the other. To make this clearer, assume that the switch A in the second stage is opened. The first stage current flowing through C and the primary T, will naturally circulate a corresponding current in the secondary of T through the impedance Z. If now we trace the second-stage circuit beginning at a and ending at b, through Z and C, we find that there are two impedance drops in it, one at Z, the other at C, and that these two are opposite to each other so that no net voltage appears from a to b induced from the first stage current. The equality of the two drops is accomplished by the equality of the two impedances, while the opposition of the drops is accomplished by the fact that the secondary induced current of T must be opposite to its primary current. Thus, the net mutual impedance between the first-stage and second-stage circuits is rendered zero; and, therefore, if the switch A is closed, the second-stage current transformer delivers its current into the current coil C of the meter unaffected by the first-stage current C. Thus, both the first and the second stage currents flow simultaneously in one and the same coil C without any interference.

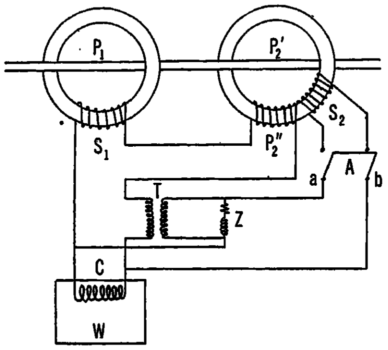

Fig. 2—Two Stage Current Transformer with Standard Wattmeter

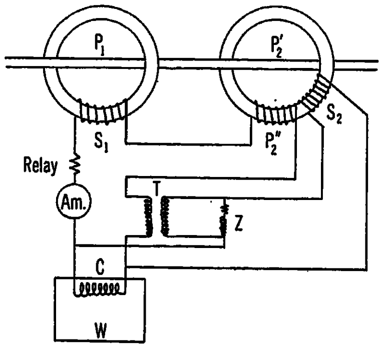

Fig. 3—Two Stage Current Transformer with Standard Wattmeter, Ammeter, and Relay

If Z is vectorially identical with the impedance of C, obviously the exciting current of T will cause the drop across Z to be slightly (vectorially) different from that across C. This error may, if desired, be compensated by making Z proportionately (vectorially) different from C. The magnitude of this error, however, is very small compared with the general improvement accomplished, especially in the bushing-type current transformers; because, first, the auxiliary transformer T is a very low-voltage transformer and can therefore be made far more accurate than the high voltage units P_1, P_2; and second, the auxiliary transformer T need not be bushing type, but of any high accuracy type, and thus again its error will be almost negligible for the circuit conditions. T is not to be mounted on the bushing but at any convenient point on the switchboard near the meters.

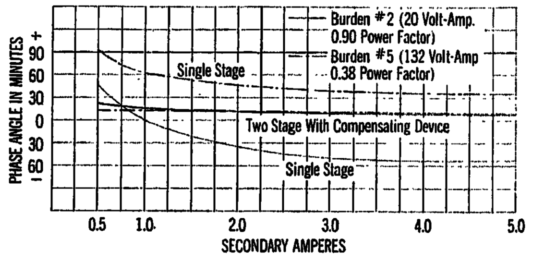

Fig. 4a—Performance Curve

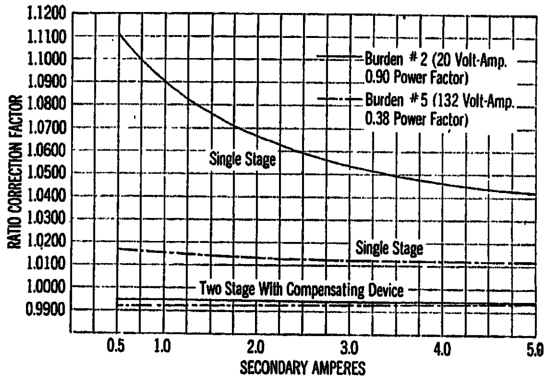

Fig. 4b—Performance Curve

Fig. 3 illustrates the connections of such a current transformer arrangement with a number of instruments. Since great precision is necessary only in the meters and not in the indicating instruments or relay coils, only the watthour meter is shown included in the second stage. This leads to greater accuracy by reducing the burden and exciting-current of the second stage. In such applications, the auxiliary impedance Z must obviously represent the impedance of only that part of the burden which is to be "two-staged." The auxiliary impedance naturally adds to the burden of both stages, but since it duplicates only that part of the burden which is to be "two-staged," it adds only a small amount to the total.

Performance Curves

Figs. 4A and 4B show performance curves of a 300-ampere turn, 60-cycle unit designed for a 132-kv. bushing. Single stage curves taken on the same unit, and using both first stage and second stage iron are also shown for comparison. The curves were taken for standard burdens No. 2 (20 volt-amperes, at 0.90 power factor) and No. 5 (132 volt-amperes, at 0.38 power factor). It will be noted that errors in phase angle of + 36 min. and − 52 min. at full load have been reduced to + 10 min. and + 8 min. respectively, whereas errors in ratio of 4.2 per cent and 1.2 per cent have been reduced to 0.6 per cent in each case. Moreover, whereas the uncompensated transformer showed much larger errors at low values of current than at high values, the two stage transformer with the compensating device is very nearly as accurate with 0.5 secondary amperes as with 5 secondary amperes.

Table I shows the final correction factor for each burden at 0.5 secondary amperes and at 5.0 secondary amperes and at various power factors.

The table shows that the combined ratio and phase angle error changes from 1.1 per cent at 10 per cent of rated secondary amperes to 0.8 per cent at full rated secondary amperes at 80 per cent line power factor, with additional improvement as the power factor is increased.

TABLE I
FINAL CORRECTION FACTOR

Line power factor	0.5 secondary amperes		5.0 secondary amperes	
	Burden No. 2	Burden No. 5	Burden No. 2	Burden No. 5
0.90 lead	0.994	0.997	0.995	0.995
0.95 lead	0.993	0.996	0.995	0.995
1.00	0.992	0.994	0.994	0.994
0.95 lag	0.991	0.992	0.993	0.993
0.90 lag	0.990	0.991	0.993	0.993
0.80 lag	0.989	0.989	0.992	0.992
0.70 lag	0.988	0.987	0.991	0.992
0.60 lag	0.987	0.985	0.990	0.991
0.50 lag	0.985	0.983	0.989	0.990

Discussion

C. T. Weller: The comparison between single- and two-stage operation suggests a slightly different view of the theory of two-stage operation, which may be of interest:

Referring to the authors' Fig. 1, the first stage of current transformers P_1 and P_2' may be thought of as a frequently used combination of bushing-type current transformers, in which the secondaries of two such transformers are connected in series to obtain better performance. The secondary "ampere-turns" (this term makes the discussion independent of the actual transformer ratio) are less (vectorially) than the primary ampere-turns by the amount of the exciting ampere-turns, which are necessary to force the secondary current through the windings and burden. In general, the secondary ampere-turns differ both in magnitude and in phase angle from the primary ampere-turns, and thus produce corresponding errors in the meter registration.

If, therefore, another set of ampere-turns, which is approximately equal to the exciting ampere-turns, could be superimposed on the secondary ampere-turns, the error would be considerably reduced. This is accomplished by the two-stage transformer in which a tertiary winding (S_2) on part of the core (P_2') is energized by the comparatively small exciting ampere-turns. This winding must, therefore, supply a small part only of the secondary burden. The exciting ampere-turns of the tertiary winding are comparatively large (although of a second order) even when supplying a comparatively small burden; a large burden might render the output negligible.

A comparatively small burden is obtained in the standard two-stage watthour meter by providing a duplicate current coil. The modification described by the authors obviates, as stated, the necessity for the separate current coil. Perhaps the best summary of the results obtained is given in their Table I, in which the "overall" correction factors differ from unity by nominal amounts only for a wide range of line power factors. It can be shown, however, that the accuracy obtainable with the modified method is inferior to that with the standard two-stage method, due principally to the addition of the burden of the auxiliary current transformer T.

P. O. Langguth: The fine performance shown in Figs. 4A and 4B, of the paper, I assume are on the basis of two-staging the watt-hour meter burden only, and that the 20 volt-ampere burden is carried on single stage only.

It would be interesting to know what performances can be expected when applying this transformer to conditions of lower ratio, and higher secondary burden where the complete burden is two-staged, and also if it is necessary to have a special impedance Z and transformer T when it is desired to change meters.

It seems that the space requirements of this type of transformer when used on circuit breakers may be a deciding factor in its application. At first hand it appears that the transformer would be of relatively large proportions when attempt is made to apply it to the lower voltage class of breakers, and probably very little gained over the conventional bushing transformer using a low-loss iron core.

A. M. Wiggins: (by letter) Several years ago the writer tried out a scheme with bushing-type current transformers very similar to that described in the paper by Messrs. Boyajian and Skeats. This scheme consisted of a main transformer and an auxiliary transformer having their secondaries connected in parallel, with an auxiliary burden connected in the circuit of the auxiliary transformer. This scheme was tried out on 50/5-ampere and 200/5-ampere ratio transformers. Very good performance resulted when a proper adjustment of the values of resistance and reactance of the auxiliary burden was obtained. The scheme was objectionable, however, because it was necessary to adjust its auxiliary burden on test for each unit and also because this adjustment was correct for one certain burden only. Apparently the same objections apply to the scheme described in this paper.

About this time the nickel-iron alloy, known as hipernik, had been developed to the point where it could be used in low-ratio designs to give a performance on a par with that of the compensated scheme for small and medium-size ring punchings and for ratios down to 150/5 amperes. However, no reliable tests for metering accuracy have been made on large rings, or for very large secondary burdens. Further experiments with the compensated scheme were abandoned in favor of the much simpler designs using hipernik. Some examples of the performance obtainable with hipernik designs are given in Figs. 1 and 2 shown herewith. Further work is yet to be done with designs employing very large ring punchings.

A. Boyajian: The modified two-stage current-transformer described in this paper is as yet in its developmental stage, and therefore some of the questions asked cannot be answered with certainty. The device is as yet unavailable in the market.

Referring to Mr. Weller's discussion, his point that the "modified" outfit will be somewhat less accurate than the straight two-stage outfit, may be true, if the compensating impedance is made an exact duplicate of the meter burden, but this auxiliary impedance need not be an exact duplicate of the meter burden.

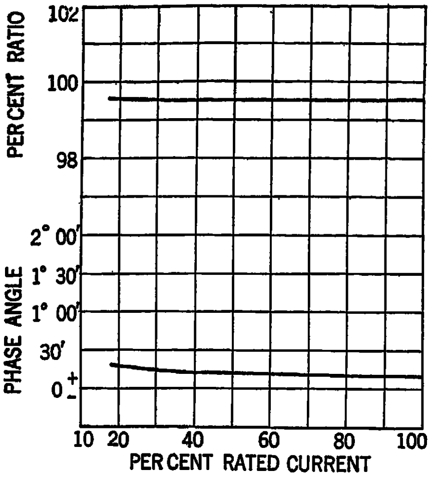

Fig. 1—Ratio and Phase-Angle Curves

150/5 A. ring-type current transformer

Answering Mr. Langguth's question, if the meter is changed, the auxiliary transformer T need not be changed, but the impedance Z may well be changed (or adjusted) if the impedances of the two meters are very different.

Referring to Mr. Wiggin's discussion, I may say that there is a world of difference between, (a), an attempt to feed back a variable magnetizing current from an auxiliary fixed impedance, and (b), a scheme to neutralize a fixed mutual impedance by another fixed mutual impedance. When an auxiliary impedance is used to feed back exciting current, the accuracy is seriously affected not only by changes in meter burden but also by changes in load current. It can be adjusted to excellent accuracy at one value of load current (no matter what the primary ampere-turns may be), but when the load current is increased or decreased, large errors may begin to show. The two-stage principle differs from this fundamentally, in that the variable exciting current is transformed in a second stage and fed to the meter burden. There is very little difference in the manner of functioning of the straight two-stage and the modified two-stage outfits.

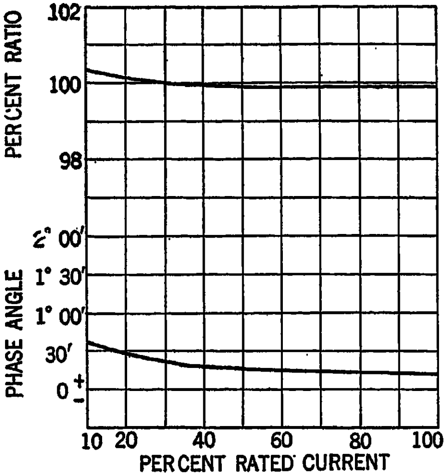

Fig. 2—Ratio and Phase-Angle Curves

150/5 A. ring-type current transformer

As to the question of core materials, undoubtedly the better the material, the better is the performance for a given circuit connection. Conversely, for a given quality of material, the two-stage principle will give far better results than the conventional single-stage operation.

Cathode Ray Oscillograph Study

Of Artificial Lightning Surges on the Turners Falls Transmission Line

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and

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Synopsis.—*A portable impulse generator delivering 400 kv. has been constructed. A cathode ray oscillograph of the cold cathode type has been so connected that impulses incoming over the transmission line to which it is connected will initiate the oscillograph.*

The results of studies on a 5.77-mi. section of a 66-kv. line of the Turners Falls Power Company are given. Impulses of definite forms are applied to the line. The theory of traveling waves is being checked experimentally, demonstrating the existence and magnitude of reflections, both for open ended lines and for lines closed through gaps or a combination of inductance and capacity. A simple description of the theory is given.

Some preliminary results are given for attenuation obtained by successive reflections from either end of the line.

The reduction of the incoming waves by an oxide film lightning arrester is given, the affected wave in one case having a front of six microseconds and in another case a front of 0.5 microseconds.

Further work to be done includes the effect of traveling waves on choke coils, transformers, and ground wires. Additional work is to be done to determine the laws governing attenuation.

As a result of the development of the klydonograph[1] and the surge-voltage recorder,[2] it has been possible to obtain records of lightning and switching surges on transmission lines, and a large number of data has been collected giving information concerning surges appearing on lines both as a result of lightning and of switching. These data[3] have given useful information concerning surge characteristics, such as magnitude, frequency of occurrence, polarity, approximate location of inception (in the case of lightning), and attenuation.

These studies using the Lichtenberg figures have not, however, given much information concerning the travel of waves through the transmission conductors or the effect of the various pieces of apparatus on these waves. Many of the data obtained are contradictory, which is to be expected since the effect of the apparatus on the traveling waves will in many cases depend on the character of the incoming wave, which may have in practise many different shapes. Furthermore, the recorder or klydonograph cannot distinguish between the original wave and a reflection—it is only capable of recording the crest values of transients.

Because of the limitations of this means of determining the characteristics of transients, two cathode ray oscillograph installations were made in the United States during the summer of 1928. The oscillogram[4] taken at Wallenpaupack showed a wave reaching its crest in about eight microseconds and having a total duration of about thirty-five microseconds.

Information of this kind is valuable, since it shows what form impulses due to lightning may take, but to study the reaction of apparatus or lines to natural lightning using cathode ray oscillographs, is both slow and costly. It is practically impossible to obtain a number of approximately identical transients caused by natural lightning, for the purpose of observing the effect of lines and apparatus on the shape of the wave.

It appears that a much more satisfactory way to study this phase of the protection problem, is to apply artificial lightning surges, which may be duplicated at will, and made to appear on the line whenever desired. The authors believe that this paper records the first application of impulses from a lightning generator to an actual high-voltage transmission line. This method of obtaining controlled impulses makes possible the continuation of the studies throughout practically the entire year, without depending on lightning storms which occur during the summer months only. The applied impulses may have various fronts and tails as desired, so that all factors may be studied.

APPARATUS

The equipment required consists essentially of cathode ray oscillograph equipment which can be set up and operated in the field, and an impulse generator which is easily moved along the transmission line as desired. It is of course necessary to have power available at both the oscillograph and impulse generator locations. Some form of communication between the men located at the impulse generator and at the oscillograph is desirable. At present it is easier to move the generator than to move the oscillograph; however, as oscillographic apparatus becomes further developed, it may become as truly portable as the lightning generator.

CATHODE RAY OSCILLOGRAPH

No detailed description of the cathode ray oscillograph will be given here as a rather complete explanation was given in a paper[5] before the A. I. E. E. in 1925. The oscillograph used in these studies, while

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*Both of the General Electric Co., Pittsfield, Mass.

[1] For all references see Bibliography.

Presented at the Regional Meeting of the Middle Eastern District of the A. I. E. E., Cincinnati, Ohio, March 20-22, 1929.

operating on the same principle as the Dufour oscillograph, is of the type described by Lee.[6]

This oscillograph is the cold cathode type which is not suited to continuous operation unless the cathode potential is kept low, which interferes with its ability to record high-speed transients photographically. Therefore, one of the first requirements in the use of this oscillograph for transient registration is, that means must be provided for exciting the cathode at the time of the occurrence of the transient. When using lightning generators in the laboratory this was accomplished readily because one could initiate the transient somewhat after potential was applied to the cathode of the oscillograph. When endeavoring to record lightning surges on a transmission circuit, it is necessary that the incoming transient set off a trip circuit which will excite the cathode and complete necessary connections so that the oscillograph is in operation within perhaps a millionth of a second after the voltage at the oscillograph begins to rise due to the incoming transient.

With a study such as that being described in this paper it is desired to obtain oscillograms of natural lightning transients as well as records of the man-made lightning transients and therefore the circuits are arranged to be operated either way.

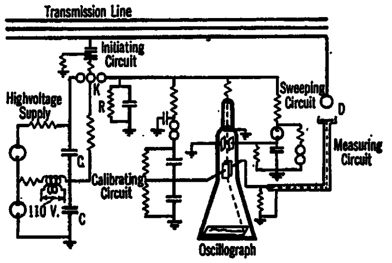

FIG. 1—CIRCUIT FOR AUTOMATIC INITIATION OF THE CATHODE RAY OSCILLOGRAPH BY INCOMING TRANSIENT

The diagram of circuits used for initiation of the oscillograph by the incoming transient is given in Fig. 1. When the oscillograph is about to be used a bank of capacitors *C* is kept charged to a potential sufficient for the excitation of the cathode of the oscillograph. This potential is applied between the two outside spheres of the three-electrode gap *K*, the middle sphere of which is held at a mid-potential by connection to the middle of the bank of capacitors. When the potential of the middle sphere begins to shift, due to the incoming transient, sufficient unbalance results so that the gaps arc-over and the potential of the charged capacitors appears across the resistance *R*. This same potential also appears between the cathode and the anode of the oscillograph, resulting in the establishment of the cathode stream, which is to be deflected by the incoming transient and leave a record of its wave shape on the photographic film.

The potential applied to the cathode is also applied to a capacitor in series with a kenotron and a resistor which controls the rate of voltage rise across the capacitor. The changing potential across the capacitor is applied to a pair of deflecting plates which results in the time scale or sweeping motion of the cathode spot on the photographic film. The rate of voltage change can be calculated but the time scale on the oscillogram is usually checked directly by means of superimposing at right angles the motion resulting from the application of a high frequency potential whose frequency is accurately known. The parallel resistor keeps the deflecting plates at zero potential except when the oscillograph is measuring a transient, while the parallel gap prevents the application of excessive potentials to the deflecting plates. The kenotron is used to prevent the plates from discharging and the cathode beam coming back across the film when the protective gap sparks.

The potential of the incoming transient is divided by the shielded gap *D* in series with the deflecting plates of the oscillograph which taken together constitute a capacity potentiometer. This circuit is shielded from stray influences by a grounded copper covering and arranged with inherent resistance such that local oscillations are minimized.

The circuit shown to the left of the oscillograph in Fig. 1 is arranged so as to apply to the deflecting plates a transient potential when the sweep has carried the spot nearly across the photographic film. Since the potential applied is known, a means of securing a voltage calibration is obtained which appears on the film as a superimposed rise in potential at the end of the time scale. This calibration circuit can be used or not at will. The use of this circuit does not appreciably affect the measuring circuit since the capacity relations are such that the potential across the calibrating capacity due to the impulse is about one per cent of that across the deflecting plates.

Cathode ray oscillograms taken with the circuit shown in Fig. 1 but without using the calibrating circuit are given in Fig. 7. The oscillograph was initiated by the transient, and the time delay in getting the cathode stream under way is indicated by the gap in the front of the wave. This method of operation of the oscillograph is the one which is being depended upon to secure oscillograms of lightning impulses on the transmission line.

Because this type of circuit loses a portion of the front, the circuit was modified as shown in Fig. 2. In this figure is also shown the circuit for the impulse generator set up to apply impulses to the transmission line.

With this method of operation the potential of the middle sphere of the three-electrode gap at the impulse generator is disturbed when the operator closes switch *S*. At the same time this applies to the lower conductor of the transmission line a small transient potential which disturbs the potential of the three-electrode gap at the impulse generator, which is held in a charged condition, with the result that the three-electrode gap sparks over and initiates a discharge of the generator, thus applying to the middle conductor of the transmission line a surge whose characteristics are determined by the constants of the generator and the line.

It is therefore possible for the operator at the oscillograph to initiate a transient at any time which he desires, applied at any point along the transmission line, and have the oscillograph already excited by the time the transient arrives, thus securing an oscillogram of the entire wave front. Such an oscillogram is shown in Fig. 10.

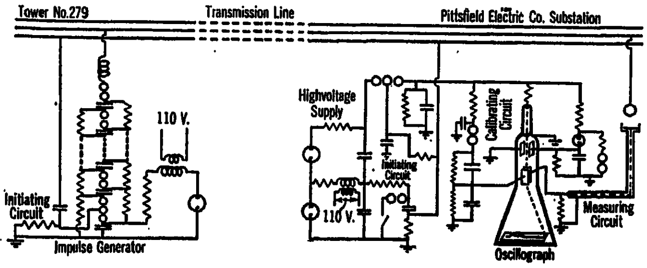

Fig. 2—Circuit for Applying Transients to a Transmission Line

Arranged to pre-excite the oscillograph so that the entire wave-front may be obtained

THE TRANSMISSION LINE

The transmission line on which these tests are being made consists of a double circuit, and extends from the Pittsfield Silver Lake Substation to the Cabot Station of the Turners Falls Power Company on the Connecticut River. The total length of circuit is 36.7 mi., passing over rather hilly country in western Massachusetts.

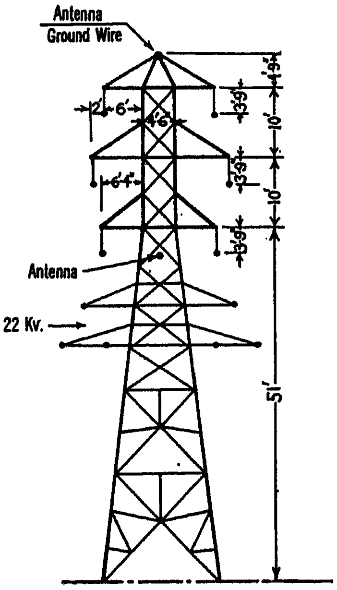

Fig. 3—Typical Tower

Showing location of line conductors, antennas, and ground wire

A typical tower is given in Fig. 3, showing the spacing of conductors and the single ground wire on the top of the tower which covers the entire length of line except for a distance 945 ft. on the Pittsfield end and 1400 ft. on the Cabot Station end, where carrier-current wires replace the ground wire. On the Pittsfield end a second carrier wire is installed as indicated in the figure. Two 22-kv. circuits are also carried on the towers from the Silver Lake Station for a distance of one mile. These are also shown in Fig. 3.

The line was built for 110 kv. operation, but is now operating at 66 kv. The 66-kv. line conductors are 2/0-7 strand hard drawn copper and the ground wire is 0.375-in. galvanized steel. The average height of line at the tower is 71 ft. for the top conductor. The line runs through rock, earth, and swamp country, the proportions being about equal on the section of the line used in tests thus far. At five points along the line disconnecting switches have been provided. Tests have been made only at Tower 279, which is located in a valley between two hills a distance of 5.77 mi. from the Silver Lake Substation.

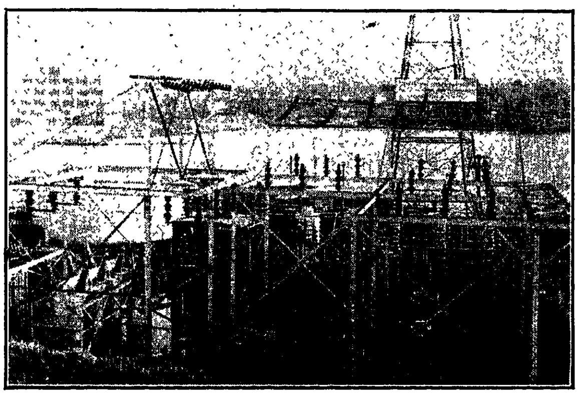

Fig. 4—Silver Lake Substation of Pittsfield Electric Company

Showing location of oscillograph and terminal equipment

CATHODE RAY OSCILLOGRAPH INSTALLATION

The cathode ray oscillograph is installed at the Silver Lake Substation in the smaller of the two buildings seen in the middle left in Fig. 4. Mounted on the roof of this building is the shielded gap which forms a part of the potentiometer for reducing the transients to values suitable for the oscillograph. On the roof of the building just beyond the one housing the oscillograph is mounted a two-way disconnecting switch so that connections to the oscillograph may be made to either side of the choke coil or to other points as desired. The photograph shows the location of choke coils and arresters and gives an idea of the station layout. The transformers at the left supply power to the General Electric Company while the circuit breakers and transformers of the Pittsfield Electric Company are further to the right and do not appear in the photograph. In all of the tests made, the circuit on the left side of the towers shown in Fig. 4 was used, the disconnects just over the circuit breakers on the right of the photograph were open, so that only the bus work shown in the photograph was in circuit during the tests. Although tests will probably be conducted at a later date with the line energized, all of the tests described in this paper were made with the line dead.

In some of the early work on this line the impulse generator, which was later moved out to Tower 279, was located just beyond the larger of the two small buildings shown in Fig. 4. In every case apparatus such as arresters or water tubes which were connected between line and ground in various tests were always located in the space just in front of the lightning arresters.

IMPULSE GENERATOR AT TOWER 279

The impulse generator was originally designed to deliver a crest potential to ground of 300 kv. This was later increased to 400 kv., using 16 oil capacitor units charged in parallel and discharged in series making use of the Marx[7] circuit as indicated in Fig. 2. The generator has a capacity of 0.0156 μf., the stored energy being 1250 watt-sec.

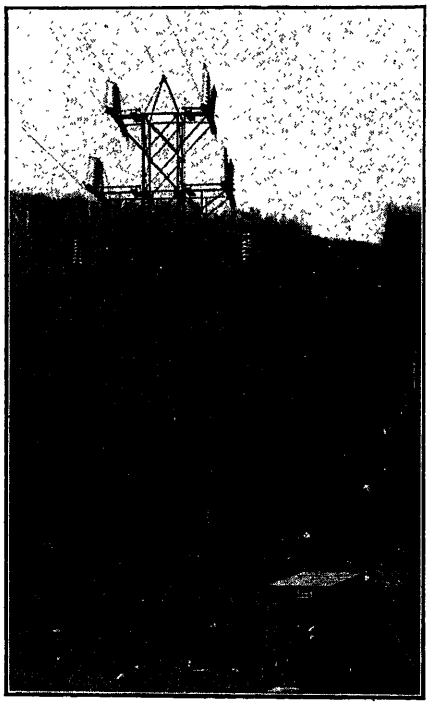

FIG. 5—TOWER 279—SHOWING PORTABLE IMPULSE GENERATOR

Line in foreground extends to Pittsfield

Tests have been made with two wave fronts, one a fast wave reaching its crest in about one-half microsecond, and the other a slower wave obtained by connecting an inductance of 0.0018 henrys between the generator and the line. This slower wave rises to its crest in about six microseconds. Practically all of the results to be given in this paper were taken with the slower of these two waves.

In all of the tests described in this paper, unless otherwise noted, the impulse generator has always been connected to the middle conductor of the transmission line, while the lower conductor has been used as a control wire to initiate the impulse immediately after starting the oscillograph so that the full wave could be obtained on the oscillogram.

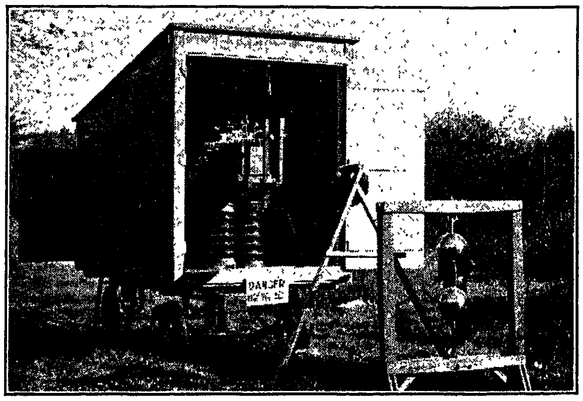

FIG. 6—PORTABLE IMPULSE GENERATOR AT TOWER 279

Open circuit voltage is 400 kv.

Fig. 5 shows the impulse generator at the base of Tower 279 with the connections made, and the disconnects open. The photograph shows the line extending over the hills to Pittsfield and gives a fair idea of the type of country traversed by this line. The middle and the lower conductors of the circuit on the right side of the tower are connected to the impulse generator.

Some idea of the construction of the impulse generator may be obtained from Fig. 6 which shows the inductance coil and a parallel sphere-gap (25 cm.) arcing over on the crest of the wave. The charging equipment is located at the rear of the building and cannot be seen in this photograph.

LINE CHARACTERISTICS

Considerable theoretical work[8] has been done with the travel of perpendicular waves on lines and the effect on these waves of different terminal conditions, such as open circuit, short circuit, and different values of resistance, inductance, or capacity, in combination or alone.

The problem for waves not having a perpendicular front is more complicated, but is probably the only case which occurs in practise, and is therefore of great importance. The reaction of devices such as lightning arresters cannot be calculated and must be determined by test under actual conditions as they exist in practise.

In order that a proper understanding of the principles involved may be obtained by the reader, two oscillograms are given in Fig. 7 which will be discussed in some detail. The circuit diagram is given under each oscillogram. It should be noticed in both cases that the impulse generator and the cathode ray oscillograph are both at the Pittsfield end of the line. Also in both cases the disconnects are open at Tower 279, 5.77 mi. away. In the upper oscillogram there is no apparatus connected at Tower 279 while in the lower oscillogram a sphere-gap connected between line and ground arced over.

The oscillograms are alike and may be superimposed up to the point where the gap sparked over. Beyond this point the reflection is negative, but otherwise is practically identical with the original reflection. These oscillograms show important characteristics, some of which should be mentioned before studying the travel of these waves in detail. They show how transients can be duplicated on successive discharges and how beautifully the line repeats its actions when subjected to the same influence. This means that in general if a line reacts differently under one lightning influence compared with another, it is probably due to a difference in the lightning impulse and not due to a line performing differently. These oscillograms also show the initiation of the oscillograph by the transient itself which is the reason for the missing portion of the front of the wave.

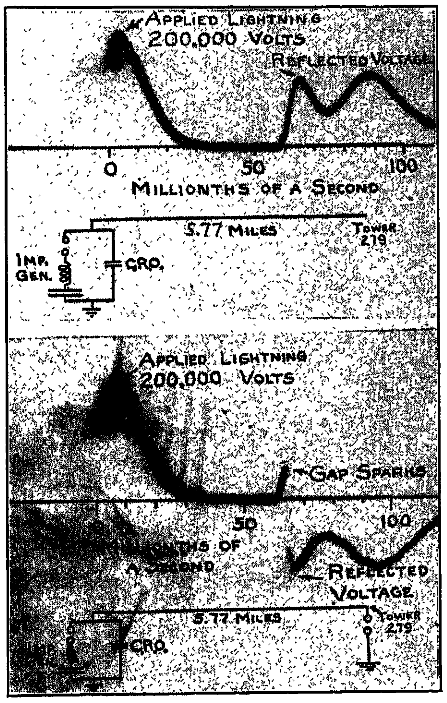

FIG. 7—OSCILLOGRAMS SHOWING INITIAL AND REFLECTED WAVES WITH IMPULSE GENERATOR AND OSCILLOGRAPH BOTH AT SILVER LAKE SUBSTATION

Upper oscillogram open ended line at Tower 279
Lower oscillogram line closed through gap at Tower 279

TRAVELING WAVES

A proper understanding of the oscillograms in Fig. 7 requires some knowledge of how such waves travel on transmission lines. An explanation made as simple as possible will be given without any attempt to assign any magnitudes to the various factors as the discussion is intended to be illustrative only.

A wave travels along a line by charging up the capacity of the line as the voltage is increasing on the front of the wave and discharging the line capacity as the voltage decreases on the tail of the wave. Such a process requires an increasing current flow through the line as long as the voltage is increasing reaching a maximum when the voltage has ceased to change. In other words, the wave-front of current will exactly correspond to the wave-front of voltage. Likewise, as the voltage decreases in the tail of the wave so also will the current decrease by the amount being discharged from the charged capacity of the line under the tail of the wave. Thus the voltage and current waves will have exactly the same shape both front and tail.

When such a wave is traveling along a line two equal energies need to be considered. One is electromagnetic and is stored in the magnetic field around the conductor due to the transient current, and the other energy is electrostatic and is stored in the electrostatic field between the conductor and ground. If the wave meets with an open circuit at the end of a line all of the energy at the open end becomes electrostatic and the potential at the end of the line is doubled. The returning reflected wave will be identical with the original wave, neglecting losses, at a distance away from the end greater than half the length of the original wave.

If the traveling wave meets a short circuit to ground the electrostatic energy at the short circuit becomes zero and the electromagnetic energy is doubled as is also the current in the short circuit. If the crest current of the traveling wave is I amperes, which value is obtained by dividing the crest potential of the traveling wave by the surge impedance of the line, then the current in the short circuit will be 2 I amperes.

In Fig. 8A is shown a simple method of determining the potential at the end of the line and visualizing, to some extent at least, the phenomenon of reflection. Wave W is traveling toward the open end of the line at X. Another hypothetical wave W' which is an exact reflection, is traveling toward the end of the line from the right. These waves are moved toward and through each other at the same rate and are combined by addition as seen in the case marked (Time II). It will be noticed that the potential at X at the end of the line will always be double, but the value at some other point as K in general will not be double. Thus with the type of waves usually found in practise the double potential occurs only at the point of reflection. If the wave has a flat top then the double potential would extend farther back depending on the duration of the constant potential. Finally the wave marked W' will be found traveling away from the point of reflection.

It should be noted that a cathode ray oscillograph out on the transmission line at a distance greater than the length of the wave would first give an oscillogram like

W followed by the reflection *W'*. These waves on the line exist both in time and in space. A cathode ray oscillograph at the end of the line would show a wave having the same length but with all of the ordinates doubled. This wave does not appear on the line in space, and is the record of successive voltage changes at the end of the line.

Fig. 8B shows the same wave as in Fig. 8A when the line is grounded. The reflected wave is identical with the original traveling wave except that it is reversed in potential. The wave in space is shown after two different times and finally the reflected wave will appear on the line identical with the original wave but reversed in polarity. The potential at the short circuit is always zero, while out on the line the oscillograph would show first the original wave and then its reversed reflection.

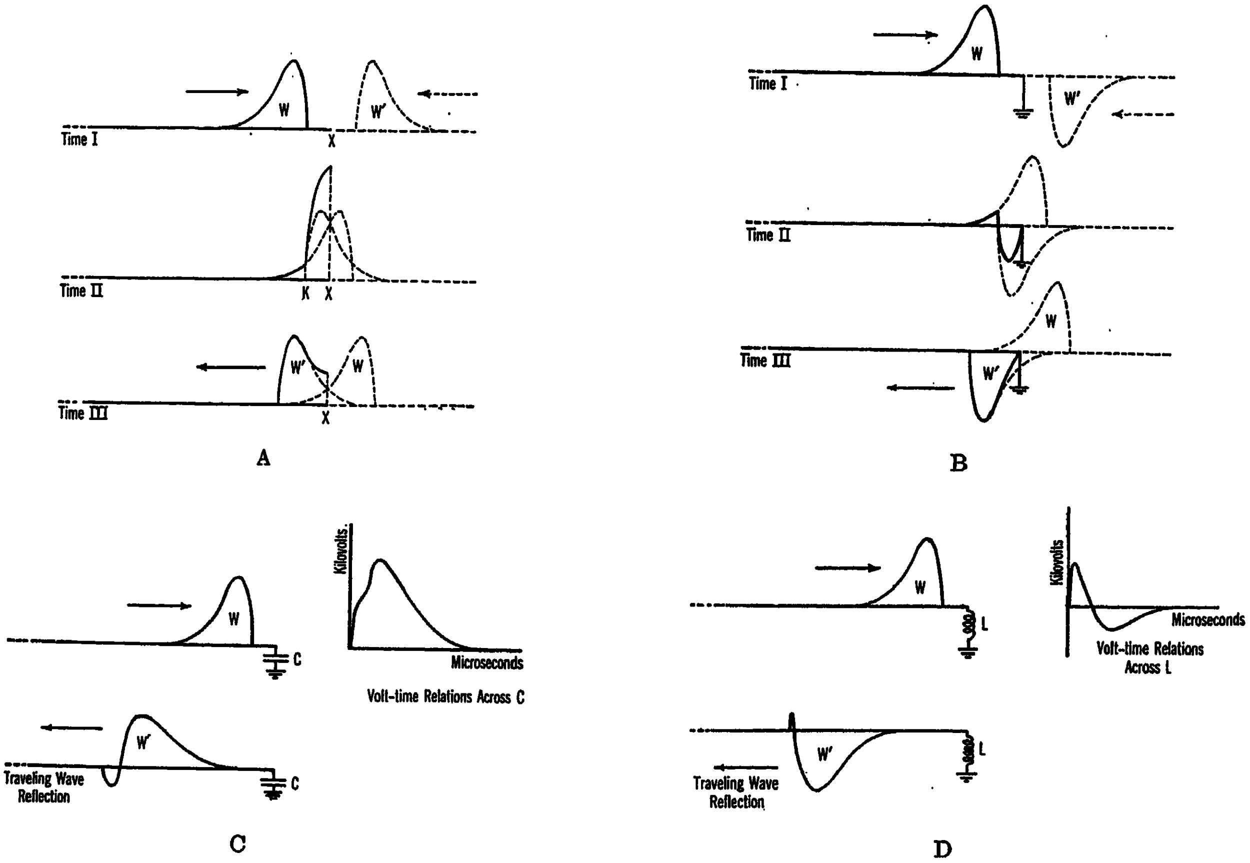

Fig. 8—Reflections for Different Terminal Conditions

Using a wave similar to test wave and *L* and *C* equal to the values used in the impulse generator

a. Open ended line
b. Line closed through zero resistance
c. Line closed through capacity equal to that of the impulse generator
d. Line closed through inductance equal to that of the impulse generator

It is important because of the use of apparatus having inductance and electrostatic capacity to consider the effect on traveling waves, first of capacity and inductance alone and then in combination. Therefore, Fig. 8C has been prepared showing the same wave used before, traveling along a line closed through an electrostatic capacity equal to the capacity of the impulse generator. The wave marked "traveling wave reflection" is the wave which would be measured by an oscillograph out on the line a wavelength away from the end. This wave appears in both space and time on the line. The summation of this reflected wave and the original wave gives the potential change across the capacity with respect to time and is what the oscillograph would show when connected across the capacity. Referring again to the reflected traveling wave, if the capacity is made larger, the portion below the zero line will become larger and the portion above the line will be correspondingly decreased. It may be noted that the capacity if small acts almost like an open ended line; if large it will act more like a short circuit, sending back into the line a large portion of the original voltage as a negative reflection, but the potential across itself will be small.

The effect of an inductance is shown in Fig. 8D, using a value of inductance equal to that used in series with the impulse generator. The traveling wave is shown and is somewhat similar to the reflected wave for the case with capacity at the end of the line, but is reversed. With inductance, the smaller the value of inductance the more it acts like a short circuit, and the greater the value of inductance the more it tends to act like an open ended line. The voltage variation across the inductance with respect to time is shown in Fig. 8D and is the summation of the original traveling wave and its reflection.

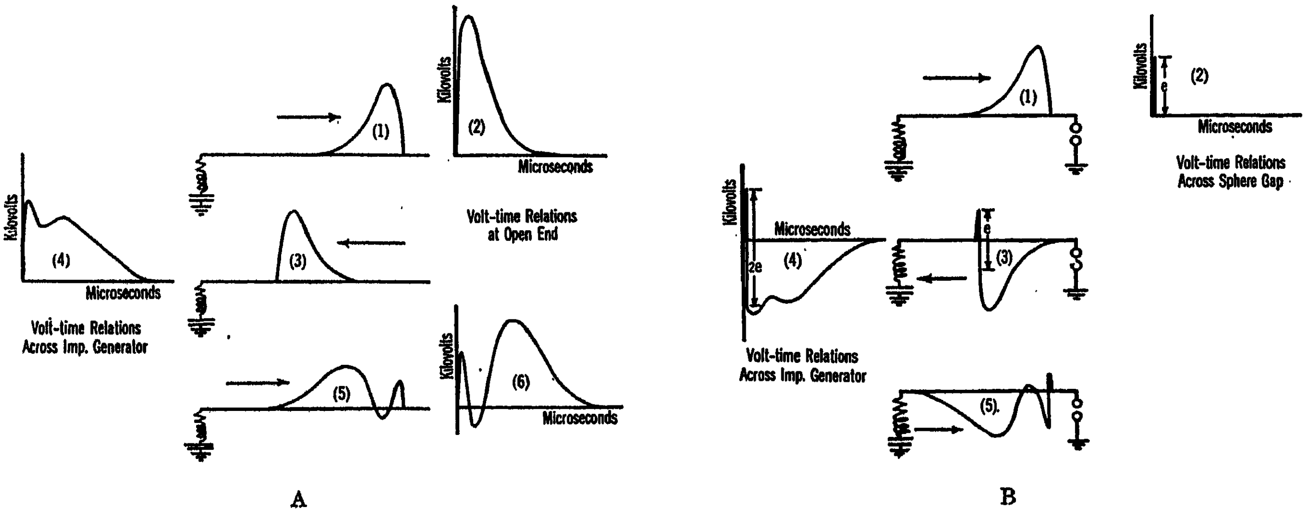

FIG. 9—REFLECTIONS WITH IMPULSE GENERATOR AT ONE END OF LINE

a. With open-ended line
b Line closed through a gap set for e volts

Using the methods similar to those used in Fig. 8, the two conditions seen in the oscillograms in Fig. 7 are set up and sketches of the waves are drawn for the waves on the line and the reflected wave from either end.

In the first tests the impulse generator consisting of series inductance and capacity with several small gaps in series was at the same end of the line as the cathode ray oscillograph, while later the impulse generator was moved to the other end of the 5.77-mi. circuit being studied. Thus, we have oscillograms taken at either end, although the capacity when at the opposite end of the line was reduced to 75 per cent of the capacity when the generator and oscillograph were together at the same end of the line. At the same time the capacity was reduced the potential was correspondingly increased. Fig. 10 is an oscillogram taken with the impulse generator and oscillograph at opposite ends of the line, and the form of the reflection is to be compared with that of the upper oscillogram of Fig. 7.

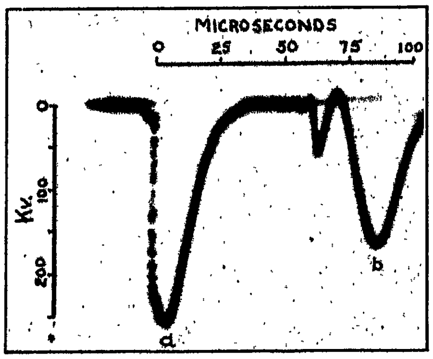

FIG. 10—FULL WAVE 6-8 MICROSECOND FRONT IMPULSE WITH GENERATOR AT TOWER 279

Referring to Fig. 9A, the same impulse discussed in Fig. 8 is seen traveling away from the impulse generator toward the open ended line (wave No. 1). An oscillograph at the end of the line would have shown a volt—time curve such as that shown at the right (2). After reflection, neglecting losses and attenuation, the wave will be the same as the original wave and will now be traveling back toward the impulse generator (3). With the line being considered this distance is 2 × 5.77 mi. or a time of about 62 microseconds is required for the original wave to return as a reflected wave. In this short space of time our tests have shown that the gaps at the impulse generator are still ionized with the result that the gaps continue to spark and (if the arc resistance is assumed constant) we may consider the impulse generator as inductance and capacity and resistance at the end of the line. The voltage to ground across the impulse generator with respect to time is shown at the left (4). This corresponds to the reflection shown in the upper oscillogram of Fig. 7. This curve is however the sum of the reflected wave (5) from the impulse generator and the reflected wave from the open end (3).

The wave (5) reflected from the impulse generator travels to the open end of the line where a reflection takes place as shown by (6). This may be compared with the reflections seen in Fig. 10.

Thus, except for the change in capacity and voltage of the impulse generator, losses and attenuation, one-half the sum of the two reflections *a* and *b* in Fig. 10 would give the reflection shown in Fig. 7. The reflection at the impulse generator results in a wave that is longer than the original wave. This lengthening of the wave is due to the storage of energy in the impulse generator and may also be seen in the case of either inductance or capacity considered in Fig. 8.

In Figs. 8 and 9 an attempt has been made to set up conditions which would check the oscillograms, approximately and also in such a manner as to be able to visualize how the reflections take place and show that inductance and capacity will give a form of wave similar to those observed. Mr. O. Brune has calculated these reflected waves by the methods of the operational calculus and his methods are outlined in a forthcoming article in the *General Electric Review*. He has been able to find quite satisfactory agreement with the oscillograms.

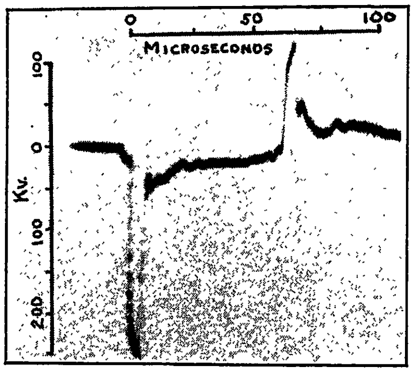

FIG. 11—CHOPPED WAVE-GAP SET AT 197-KV. CREST

It is possible to break up the original traveling wave in component perpendicular waves and treat each of these separately, adding the results together to find the effect of terminal apparatus on the original wave. This is treated at some length by Rüdenberg,[9] and Carson.[10]

It is interesting to consider the lower oscillogram of Fig. 7. Here the impulse generator and the oscillograph were at the same end of the line and a sphere-gap between line and ground was connected at the far end of the line. This condition is considered in Fig. 9B. Wave No. 1 is the original traveling wave, while No. 2 is the potential time curve across the sphere-gap. Up to the time of sparkover the line is open-ended; after that it is short circuited neglecting the resistance of the arc. Wave No. 3 is the reflected wave which reaches the impulse generator. The volt-time curve of potential rise across the impulse generator is shown in wave No. 4, which corresponds to the lower oscillogram of Fig. 7. Wave No. 5 is the reflected traveling wave traveling toward the gap which is still in an ionized condition. Therefore no potential will appear across the gap and wave No. 5 will be reflected back with reversed polarity.

In all the preceding discussion no account has been taken of losses or attenuation, which must be considered in the actual case in practise.

CHOPPED WAVES

Since in practise traveling waves are frequently cut-off by flashover of insulators, it is worth while to consider the effects of the line constants and apparatus on such waves.

Fig. 10 shows a so-called full wave which rises to its crest in six to eight microseconds as determined by the use of a high-frequency oscillator which makes it possible to measure the front accurately. The wave is apparently affected by corona at about 80 per cent of its crest value. This wave is of negative polarity but studies will be made later with opposite polarity. Since the impulse generator is at the far end of the line the oscillogram shows the voltage rise at the station end of the line and thus is probably close to double the voltage of the traveling wave.

In Fig. 11 all conditions were unchanged except that a sphere-gap was connected between line and ground at the impulse generator and set to spark at about the crest of the wave. It is interesting to note how steep the cut-off is and that the reflection comes back reversed to the station end where the oscillograph is located, which shows that the gap in parallel with the impulse generator is still ionized. The same sloping of the wave due to corona can be seen on the front of the reflected wave. When studying these oscillograms it should always be remembered that they represent voltage changes at the oscillograph with respect to time. From them with proper interpretation the traveling wave can be deduced. All of the tests to be discussed were made with one or the other of the waves shown in Figs. 10 and 11 unless otherwise indicated.

ATTENUATION

The attenuation on lines is a matter of considerable importance in the protection of lines and station apparatus. If it is known, for instance, that any impulse having a crest voltage up to the flashover value of the insulator for that particular wave will be so attenuated in a certain distance as not to be dangerous to station apparatus, then storms beyond a radius equal to that distance from the station need not be considered as dangerous to station apparatus and thus the problem of protection becomes more definite. It seems to have been quite definitely shown[11] that for dangerous voltages the attenuation depends largely at least on the effect of corona. If this is true then other factors in addition to potential will control the loss. All other factors being equal the corona loss will depend on the potential and for extremely steep waves perhaps on the wave-front.

Tests are being made on the Turners Falls line with the object of determining the factors which control the attenuation, particularly for potentials above the corona voltage. The effect of wave shape, polarity, maximum potential, ground wires, and other factors will be studied.

Some preliminary results are available on the short section of line worked with up to the present time. Later, the whole length of line (36.7 mi.) will be used which will give somewhat more accurate results than those presented here.

The method of making these tests was to apply at Tower 279 an impulse which reflected back and forth over the line which except for the small capacity of the oscillographic equipment and the station bus capacity was open at the oscillograph and closed to ground through the impulse generator at Tower 279. The first two reflections are shown in Fig. 10. The oscillogram in Fig. 12 shows eight of these reflections which corresponds to a length of line of approximately 85 mi. In Fig. 13 are given the successive reflections with chopped wave similar to Fig. 11 except that the parallel gap was set for 156 kv. instead of the 197 kv. setting used in Fig. 11.

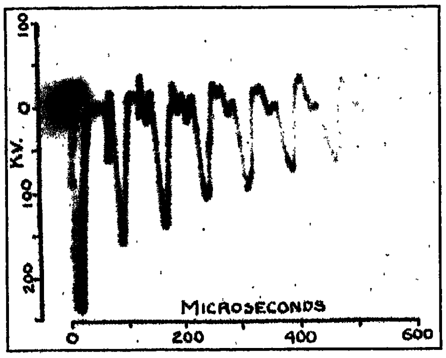

FIG. 12—SUCCESSIVE REFLECTIONS, FULL WAVE

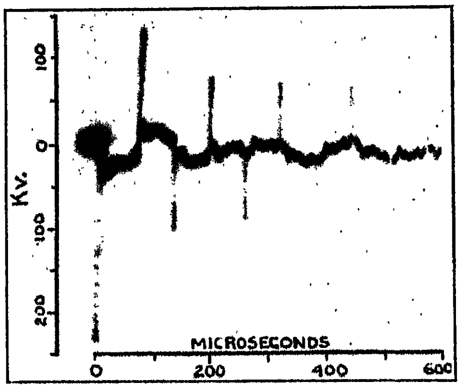

FIG. 13—SUCCESSIVE REFLECTIONS, CHOPPED WAVE

In Fig. 14 the crest values from Figs. 12 and 13 are plotted to show the change in potential with each successive reflection. When discussing this method of studying attenuation, compared with tests on a line of infinite length certain inaccuracies are present which must be considered. In the case of the full wave, Fig. 12, the wave form is very appreciably modified by the impulse generator and there is of course loss in the resistance of the arcs in the various gaps. It is expected that in the future means will be found of getting the generator off the line before the reflected wave returns to the generator.

In Fig. 13 the effect of the constants of the impulse generator do not appear in the reflection since the gap is kept in an ionized condition and acts as a line closed through a small but variable resistance.

There is good reason to believe that chopped waves will attenuate faster than full waves, which is supported by data given in Table I discussed in a following section. When the results are affected by the terminal conditions, it is probable that only the first reflection can be considered.

This method of study of attenuation is open to the objection not only of the effect of the terminal apparatus on wave form but also to the fact that the potential may be increased at one reflection point and made zero at another, as is the case in Fig. 13. This will result in different losses due to corona, for instance, which does not represent truly the travel of the wave along an infinite line.

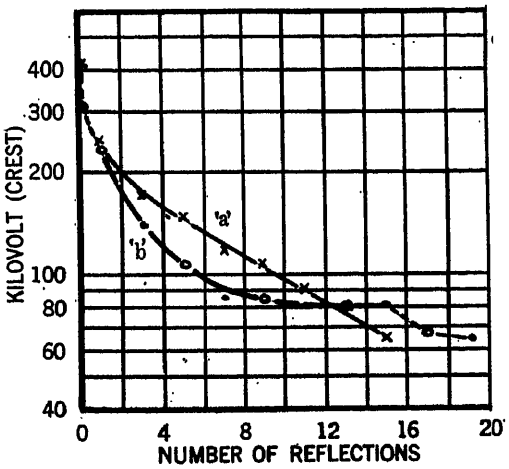

FIG. 14—CURVES SHOWING VARIATION IN CREST VOLTS WITH SUCCESSIVE REFLECTIONS

a. Full wave
b. Chopped wave

The calculated corona voltage on this line is 110-kv. crest, but the curves in Fig. 14 do not show any close correlation with this value. The ordinates given for the curves represent voltage at the oscillograph which is presumably double that on the line. With curve *b* after the first reflection the traveling wave was not much above the corona voltage, but at the oscillograph on reflection it was appreciably above the corona voltage which may explain the point of inflection in the curve at about 100 kv. For curve *a* increases in potential took place at both ends of the line and the wave was modified by the constants of the impulse generator so that no attempt will be made to draw any conclusion.

As the impulse generator is moved out along the line and successive oscillograms taken for increasing distances, data bearing more directly on the attenuation problem will be obtained without the complication of the effects of the ends. Increase in potential of the impulse generator above the corona voltage will aid in establishing the laws governing attenuation.

SURGE IMPEDANCE

Some preliminary work has been done in an effort to determine the surge impedance of the line. When the line is closed through resistance equal to the surge impedance no reflection will take place. By taking oscillograms of the voltage across the resistance its value can be adjusted until the reflection disappears. Two such oscillograms are given in Fig. 15. Fig. 15A shows a positive reflection indicating that the 530 ohms used was too high. On the other hand Fig. 15B with 320 ohms seems to give a negative reflection, indicating too low a resistance.

This method of determining the impedance does not give a very accurate result but does show that the surge impedance lies between 320 and 530 ohms. The calculated value of just the middle wire alone and one ground wire is 500 ohms. The effect of the other conductors is to lower the calculated value. The surge impedance may also be lowered by corona, and it is hoped to be able to secure data which will determine this point.

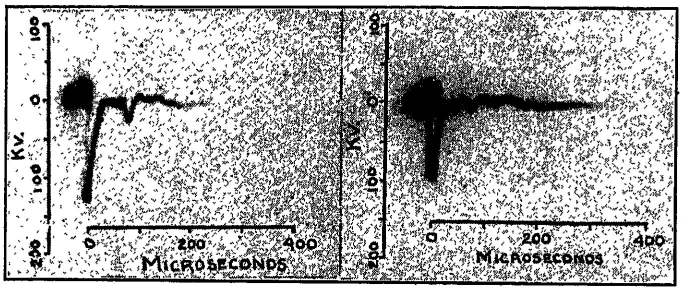

FIG. 15—VOLT-TIME CURVES ON WATER TUBE RESISTOR USED IN DETERMINING SURGE IMPEDANCE

a. 530 ohms
b. 320 ohms

Other methods which will give a better determination of the proper value of surge impedance are being developed and will probably be described in later papers.

With this line it will be possible to determine the effect of stopping the ground wire some distance from the station and also the effect of additional ground wires near the station. The magnitude of voltages and currents will be studied with their relation to ground resistance of the tower footing. Thus, it is expected to be able to give definite information regarding the benefit to be derived from additional ground wires from the standpoint of the effect on the surge impedance. It will also be possible to check the so-called theory of the short circuited secondary with regard to the effect of the ground wire on traveling waves.

INDUCTION WITH OTHER CONDUCTORS

The induction on the top and bottom wires was measured with sphere-gaps with the impulse being applied to the middle wire. In one case with 216-kv. crest applied at Pittsfield, a potential of 31 kv. was measured between the bottom wire and ground at tower 279, all of the disconnects being closed at tower 279.

In Table I are given the results obtained by applying full waves and chopped waves at tower 279 and measuring the voltages in the respective conductors at the Pittsfield end, using 25-cm. sphere-gaps. The disconnects at tower 279 were open.

TABLE I
INDUCTION ON TOP AND BOTTOM WIRES
IMPULSE APPLIED ON THE MIDDLE WIRE

Potential at Pittsfield	Full wave 205 kv. applied	Per cent	Chopped wave 186 kv. applied	Per cent
Middle wire...	240	100	198	100
Top wire......	56	23.4	40	20.2
Bottom wire...	45	18.7	29	14.6

No conclusions are being drawn from these results, as they should be considered as being preliminary, but they give approximate results which should be of value. Calculated values based on change in the electrostatic field assuming that the ground is at the surface of the earth give 23 per cent and 21 per cent for the top and bottom wires respectively. These values agree quite satisfactorily with the test results.

It is interesting to note that the carrier wire protective gaps arced-over each time an impulse was applied to the transmission line.

APPARATUS CHARACTERISTICS

The effect on the incoming impulse of choke coils, lightning arresters, bushings and transformers, outgoing circuits, and the effect of station bus work is to be studied. The work with regard to all other apparatus except the lightning arrester is in too preliminary a state to give any results in this paper. Some very interesting results on an oxide film arrester are available.

OXIDE FILM ARRESTER

Since voltages comparable with the maximum allowed by the line insulation of six disks were not available it was decided to reduce the size of the arrester to be tested to such a magnitude that it would have a potential applied comparable with what it might get in service.

The crest voltage shown in Fig. 10 for the full wave with the six microsecond front is 270 kv. This voltage represents about 25 per cent of the impulse arc-over of seven disks which is the normal insulation of a 110-kv. line. Since it is desired to have the voltage applied to the arrester in proportion to that which the normal line insulation in service would permit, an arrester should be used having a rating equal to 25 per cent of 110 kv., which is approximately equivalent to a 25-kv. nongrounded arrester which was chosen for the tests.

The oscillogram taken across the arrester is shown in Fig. 16, together with the wave of Fig. 10 without the arrester, as shown by the dotted line. The potential has been reduced by the arrester to 135-kv. crest and its duration has been greatly decreased. The voltage drops almost instantly to 70-kv. crest and subsequently decreases at a rapid rate. Therefore the arrester has changed a wave of dangerous potential and comparatively long duration into one of much shorter duration with a crest voltage safely below the strength of the parallel insulation.

Because of the fact that the arrester has absorbed some of the energy of the surge the reflected voltage is materially decreased. It is interesting to note that the first reflection peak due to the inductance of the impulse generator has not been changed much except in sign but the wave after leaving the arrester was sufficiently reduced in energy so that it was unable to charge the capacity of the impulse generator to a very high potential.

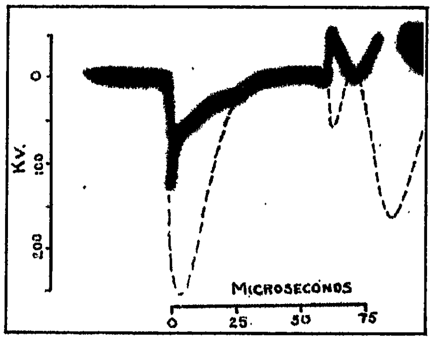

Fig. 16—Arrester Performance Characteristics on 6-Microsecond Front

Dotted curve—impulse at oscillograph without arrester. (See Fig. 10)
Oscillogram—taken across 85 *O F* cells

Oscillograms showing the arrester performance for a wave of steeper front are given in Fig. 17. This wave is obtained by shorting out the inductance coil of the impulse generator and it is found that the voltage at the oscillograph end of the line rises to a crest value of 285 kv. in an average time of 0.5 microseconds as measured on the time scale of a 100,000-cycle timing wave.

The potential across the arrester as shown by the oscillograph was 145-kv. crest. All voltages measured by the oscillograph were checked with the sphere-gap. It is felt that the two wave-fronts used in these tests are comparable to waves which would be found in practise.

The effect of the arrester on the chopped wave of Fig. 11 has been determined and the characteristics are similar to those of Fig. 16 except that the arrester discharges only for the length of time that the voltage is impressed which is a much smaller time than in the case of Fig. 16. Since the front of the wave is not affected by chopping the tail the potential allowed by the arrester is the same in either case.

SUMMARY

1. Theory of traveling waves is being checked experimentally, demonstrating the existence and magnitude of reflections, both for open ended lines and for lines closed through gaps or combinations of inductance and capacity. A simple discussion of the theory is given.
2. The effect on incoming waves of various pieces of apparatus, such as lightning arresters, choke coils, transformer bushings, and the effect of busses or other transmission lines connected to the bus are being studied.
3. The effect of ground wires on attenuation is being investigated which includes the effect of additional wires near the station as well as the effect of stopping the ground wire several towers out from a station.

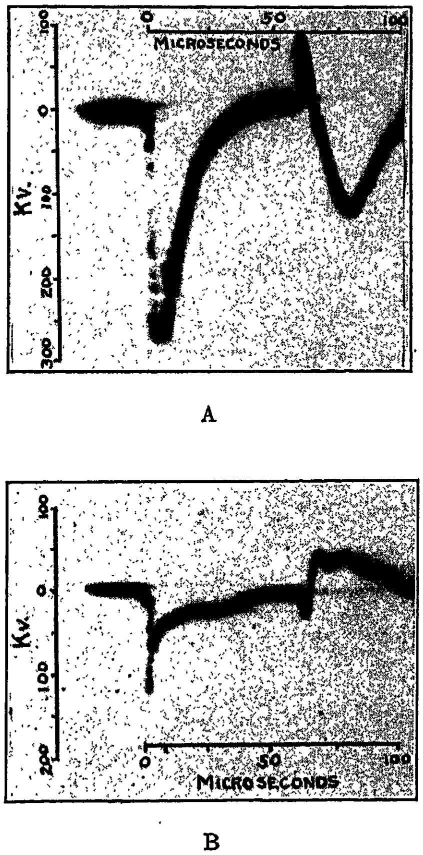

Fig. 17—Arrester Performance Characteristic on a 0.5-Microsecond Front

a. Without arrester
b. Across 85 *O F* cells

ACKNOWLEDGMENTS

The authors wish to express to Mr. F. L. Hunt and others of the Turners Falls Power Company and to Mr. Granville Whittlesey of the Pittsfield Electric Company their appreciation for the co-operation which

they have given in the investigation and without which the work could not have been done.

The work of the investigation has been carried on by Messrs. E. J. Wade, W. J. Rudge, Jr., O. Brune, and T. Brownlee. These men have all contributed largely in the design and construction on the impulse generator and the successful design of the oscillograph circuits and the operation of the equipment.

The authors express to these men their appreciation for the service they have rendered both in connection with the investigation and the writing of this paper.

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Discussion

C. L. Fortescue: I should like to call Mr. McEachron's attention to the initiating arrangement in Fig. 2. I don't know whether Mr. McEachron is aware that Mr. Harrington, who died recently, first used that circuit, and described it, I think, in the *Physical Review* in 1926, and later on in the *Electrical World*. We ourselves did not use it in the Rankin-Wilmerding tests, but it has been used in laboratory tests by the Westinghouse Company for some time.

I want to say that I think Mr. McEachron's work is of great value, but I don't agree with Mr. McEachron that we can find out all we want to find out about lightning surges from such tests. There are two things we want to do: We want to find out how the line behaves with traveling waves, but we also want to find out how steep a surge we can get from actual lightning, what its value may be if it is not chopped, and how long it lasts and the rate of diminution from its crest value.

Now Mr. McEachron's waves, or for that matter any wave we can place on a transmission line artificially at the present time, will not give the attenuation for a very high lightning wave, for the reason that the potential of a very high lightning wave goes up beyond the corona point of the line, and we have no figures for the attenuation in such a wave due to corona. We suspect that it may be very great.

I may say that we had to stop our Wilmerding-Rankin experiments before we got all the information we should have liked to have had. Last year we installed an experimental station in Tennessee using the new Norinder oscillograph. We have two locations there. This year we are extending those tests to get more authentic information on lightning. We are putting in three other cathode ray oscillograph stations. One is now in operation, or will be in operation in a few days. It is in Northern Illinois. We expect to have one in West Virginia, and another in New Jersey.

In the New Jersey tests we have had the good fortune to be able to obtain the use of a portion of a line which is now in construction, a 220-kv. line, which has the highest insulation of any practical line that has heretofore been made. It will be insulated with 18 units, and we are going to have a portable million-volt surge generator, and a portable cathode ray oscillograph down there to carry out these tests along the line that Mr. Eachron has been doing. We expect to get information on the actual value of the surge impedance of a transmission line, the mutual coefficients between the individual members of transmission lines, and we want to check up on surge impedance of tower footings, and so forth.

In the case of tower footings we measure the ground resistance by various schemes, but we have no assurance that the values we obtain by measurement are the actual surge impedances. We must find that out. This quantity is an important factor in line protection, and by means of the surge generator and the cathode ray oscillograph, as Mr. McEachron has used it, we expect to be able to obtain much important information.

I believe, and I have so expressed myself from time to time, that tests like these described by Mr. McEachron will be far more fruitful of real information than tests made in the laboratory with laboratory models. There are various factors that enter into the problem when using laboratory models which are generally ignored, but which are of vital importance. For example, the model may be reduced to one-hundredth the size of the cloud and actual transmission line, but the other dimensions are reduced in the same proportion, and therefore the time between reflections is reduced to one-hundredth; so that while before you were dealing with quantities like two microseconds between reflections at the tower points, now you will have to deal with quantities of the order of one- or two-hundredths of a microsecond. When dealing with quantities like that, cathode ray oscillograph will follow, but the photographic plate may be too slow; consequently all those values which are of so much importance are masked, and you get an answer which is incomplete and therefore false.

Now doing the work on an actual line in the field, doesn't entail a great deal more cost if you have a line which is in process of construction, and you obtain all these results unmasked. They are the actual facts that occur on transmission lines.

S. M. Hamill, Jr.: I should like to ask Mr. McEachron if the inductive reactance or the capacitance of the oscillograph is large enough to consider when you compare it with the constants of the remainder of the circuit, or is it so small that it is negligible?

Edward Beck: As the authors have pointed out, the way to investigate the behavior of transients in electrical circuits is to make the transients at will and then observe them at leisure with the proper tools. It is equally necessary to find out what kind of transients may occur. Probably many types of transients occur in service, caused by lightning. Possibly all or perhaps only a few of these types are hazardous to insulation. In order to determine the kinds of transients which may be expected on actual lines, tests are necessary such as were commenced last Summer and described before the Winter Convention of the Institute, particularly in the paper by Fortescue, Atherton, and Cox. This information augmented by data collected in such investigations as described by McEachron and Goodwin will construct a comprehensive picture of the lightning problem. However, this work will take time, particularly as the greater part of the burden must be borne by the manufacturers. The chief beneficiaries of investigations of transient phenomena are the operators, and the solution of their problem could be much accelerated if they would take an active interest in it.

These investigations again indicate the invaluable aid given by the cathode ray oscillograph. Several years ago tests similar to the ones described in this paper were carried on in Germany and described by Binder. The cathode ray oscillograph was not then available and the determination of wave shape was exceedingly laborious and left considerable chance for uncertainties. The oscillograph opened up the possibilities of thorough investigation of those transients.

The authors state in the beginning of their paper that they believe that it records the first application of impulses to an actual high-voltage transmission line. Because of the importance of this work and the comprehensive way in which it is being carried out, it will not detract in the least from its value to call attention to the earlier investigations which have been carried out along these lines. Besides the German work already mentioned, the Westinghouse Company undertook similar work on a 25-kv. line borrowed from the Duquesne Light Company in Pittsburgh in 1926. This work has been mentioned in the paper by Fortescue, Atherton, and Cox, already referred to, and is described in detail, showing oscillograms secured, by A. M. Opsahl in the March 26, 1929 issue of the *Electrical World*. In this investigation the behavior of artificial transients in a line 4.7 mi. long was studied, the effect of protective equipment including choke coils, determined, and the surge impedance of the overhead line accurately measured. Unfortunately this work had to be abandoned because the use of the line was required by the Duquesne Light, but the investigation begun in a preliminary way on the so-called Rankin-Wilmerding line in 1927 will be carried further on a larger scale during the coming Summer.

W. L. Everitt: This study of traveling waves is a very good one, and a method that is going to grow in the analysis of transmission lines. It seems to me that some of the methods which the communication engineer has developed for studying the action of the lines may well be applied to these transients upon transmission lines.

I wonder if any study has ever been made in predicting these wave forms by measuring the effects over a wide band of frequencies, and then trying to relate them to these different wave forms? A steep wave form, of course, means high frequencies are present with a definite phase relation, and with high frequency we have higher attenuation than we do at low frequency and a variation in their velocity, and therefore one would expect a steep wave front to be gradually reduced as it passes along the line.

K. B. McEachron: I think it is clear from the paper that we recognize two problems. One problem is the determination of what lightning is and what kinds of waves may appear as a result of lightning, and the other is the determination of what these waves may do after they get on the line, which is the part of the problem we are investigating. Both problems must be solved.

At the present time in the Pittsfield laboratory, apparatus is set up to catch lightning on the transmission line, and find out what it looks like, and these other laboratories which have been set up are arranged to do exactly the same thing, but we felt that while waiting for lightning storms, which might occur infrequently, we might just as well be busy studying the effect of surges on the apparatus and on the lines.

With regard to the application of voltages above corona limit, the corona limit on the particular transmission line which we are studying is 110 kv. We applied 400 kv. We have at the present moment a million-volt generator, which is entirely portable, which will go very much above the corona limit.

It is interesting to note that three or four years ago a million-volt generator would have required a building much larger than this room. To-day we have a million-volt generator on a truck 18 ft. long and 7 or 8 ft. wide, truly portable. I am glad to give credit to Mr. Harrington for the publication of a circuit similar to the one shown in Fig. 2, as pointed out by Mr. Fortescue. The circuit shown in the paper was developed prior to the publication of Mr. Harrington's work.

The disturbing effect of the cathode ray oscillograph depends on the potentiometer used. The capacitance used is generally small and the disturbing effect not great.

As a result of going from section to section telling this lightning story, I find that the average engineer must have this material explained to him very carefully, or it is not understood, not because it is so difficult, but because it is something with which he is unfamiliar. This paper was therefore written in an effort to make the subject as understandable as possible to all.

Concerning the use of frequency bands, it is fundamental, I should say as a rule in research work, to set up as nearly as possible the actual situation which you want to study. It is true that we might apply to this line or other similar lines, certain frequency bands and obtain certain results. That would give us information, but with regard to the major problem the use of high-voltage transients seems to be about the best method of attack.

I think the reason why the engineer puts transients in one class and 60-cycle phenomena in another, is because he has dealt with 60-cycle phenomena all his life and he is familiar with having the same voltage and the same current here as he has a mile away, but when dealing with transients, a new conception is necessary, since the potential may be zero and 100 kv. at points only 20 ft. apart in the conductor.

Developments in Network Systems and Equipment

BY T. J. BROSNAN[1] and RALPH KELLY[2]
Non-member Associate, A. I. E. E.

Synopsis.—This paper presents in brief review the development of the automatic network system in the seven years since its introduction. It covers the interesting features of design found in the different systems, the changes that have been made in the design of the apparatus as a result of experience, some of the problems that have been faced in the application of the system, and finally the lines along which thought is now being directed in the future development of the system.

THERE has been a great deal written in recent years in regard to the a-c. low-voltage network type of distribution system. The various requirements which caused such a system to be evolved, the fundamental principles of the design and operation of such a system, and the equipment used in it have been described in various technical papers. It is felt, however, that it would be interesting at this time to review the growth of the network system and to observe what developments have occurred in its design and to point out what changes have been made in the apparatus used as a result of the experience gained and in order to meet the requirements of various types of systems.

Prior to 1920 several attempts had been made to increase the reliability of the a-c. distribution system by connecting together the low-voltage sides of the distribution transformers. Fuses were generally used as the means of protection for clearing feeder and transformer faults. These fuses had several disadvantages and a more reliable method of protection was very desirable. The first improvement was made by the Puget Sound Power and Light Co. in Seattle,[3] which established a system having the distribution transformers connected to a common secondary through subway oil circuit breakers. These were equipped with relays which caused them to open only on a definite current flow in the reverse direction. The breakers were designed only for hand reclosing.

THE AUTOMATIC SECONDARY NETWORK SYSTEM

At the time when this system was being installed, the United Electric Light & Power Co. was working on the development of what is now commonly called the automatic secondary network system. In this system the energy is supplied through high voltage feeders and is stepped down to the service voltage through what are now called network transformers. A large number of transformers is connected solidly to each feeder without any protective device. The low voltage side of each of these transformers is tied to the secondary cable mesh through a special circuit breaker called a network protector.

The secondary cables are tied together at every convenient point to form a solid network and service mains for consumers are taken from the mesh at satisfactory locations. A single-line diagram of such a system supplied by three interlaced radial feeders is shown in Fig. 1.

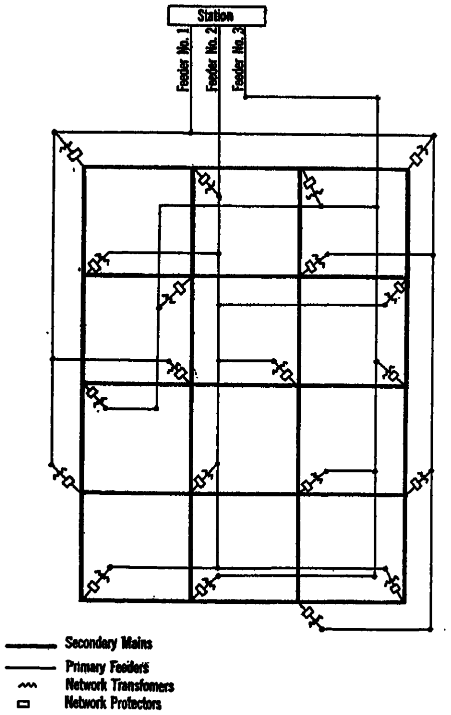

FIG. 1—DIAGRAM OF SMALL AUTOMATIC SECONDARY NETWORK SYSTEM SUPPLIED BY THREE INTERLACED RADIAL FEEDERS

In the United Company's system and in most others a common secondary network is used to supply both power and light. The transformers are connected in star on the low voltage side and have the neutral points solidly grounded and carried out as the fourth wire of a three-phase, four-wire system. Single-phase loads can then be connected between any phase wire and ground while three-phase loads are supplied from the outside or phase wires.

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[2] General Engineer, Westinghouse Electric & Manufacturing Co., St. Louis, Mo.

[3] "Merits of A-C. Underground Distribution," M. T. Crawford, *Elec. World*, Dec. 23, 1922.

Presented at the Regional Meeting of the South West District No. 7 of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

No protection of any kind is installed for a fault in the secondary cables and trouble of this nature is depended upon to burn clear. In case of a fault on any feeder or any transformer the feeder breaker at the substation opens. At the same time all transformers tied to the faulty feeder are automatically disconnected from the secondary network. This is accomplished by the opening of the corresponding network protectors which open when the current flows through them in the reverse direction. The faulty feeder with its transformers thereby is completely isolated from the remainder of the system. The system will continue to operate, receiving energy from the other feeders. It is so designed that no part will be overloaded when one feeder is out of service.

In order to simplify the operation of the system the network protectors are designed to reclose automatically as soon as the fault is removed and the feeder is energized from the station. To make the control from the station complete the relays are made sensitive enough to cause the protectors to open when only the magnetizing current of the transformers flows through them in the reverse direction. This means that all of the protectors on a feeder can be opened by simply opening the feeder breaker at the station and can be reclosed by closing the station breaker and adjusting the voltage on the feeder so that it is slightly higher than and approximately in phase with the voltage on the network.

After ten years of study and experimental work the United Electric Light & Power Co. installed the first system of this type in the up-town district of New York City in 1922.[4] The principle of operation was entirely new and the equipment which made it possible had to be developed. It is interesting, therefore, to note what changes seven years of operating experience have brought about.

In this country 22 companies now have a system of this type in operation, 6 are installing such systems, 5 have decided upon it, and it is being given serious consideration in at least 20 other localities. The network has been introduced in four foreign cities, one in Cuba, one in Mexico, and two in South America, and is being seriously considered by one other South American and one European city. In all of these systems the fundamental principles of the original design have been retained but the details of the layout have naturally been adapted to meet local conditions.

FEATURES OF VARIOUS SYSTEMS

All of the companies operating or installing an automatic network system, with one exception, have followed the original plan of using the same secondary mains to supply both power and lighting loads. In the Dallas system, polyphase loads are supplied from a three-phase, three-wire secondary mesh while the lighting and other loads are supplied from an entirely independent three-phase, four-wire secondary grid of cables.[5] These secondary networks are connected to different taps on the same step-down transformer banks which are star connected on the low voltage side. The ties from the transformer to each network are through separate network protectors.

Practically all of the companies in this country that have installed an automatic network system have adopted the star connection on the low-voltage side of the network transformers. The Puget Sound Power & Light Co. in changing over from the original system to the full automatic system have still retained the delta connection. In order to balance the loads on the different phases of the feeders the secondary system is divided into sections and the lighting load in each section is supplied from a different phase with its mid-point grounded. The only other exception to the star connected secondary is in the system of the New York Edison Company which retained the two-phase system which it had in operation and established a two-phase, five-wire secondary network. It is planned to change this over gradually to a three-phase four-wire system.

All but one of the systems now in operation use radial feeders. Two systems started operation with a loop feeder arrangement but have since adopted the radial type. The outstanding advantage of the radial feeder is that no high-voltage sectionalizing devices are required on it.

Over 80 per cent of the network systems now in operation in this country are supplied by feeders that come in either the 13.2-kv. or the 4-kv. class and there is just about an even division in the number of systems employing each of these two voltages. One company is operating a network supplied by 27-kv. feeders. Practically every standard size of cable between No. 2 and 500,000 cir. mils is used in the feeders of the different systems but the 4/0 and 300,000 cir. mil sizes stand out as the most popular. Most companies using large size feeders taper them down to smaller sizes in the laterals which connect to the distribution transformers. The 2/0 cable size is by far the most commonly used for this purpose.

The size of cable used to make up the secondary mesh of systems now operating varies from 4/0 up to 500,000 cir. mils. The 500,000-cir. mil size is being used in about half of the systems in this country.

A great deal of thought has been given to the question of secondary cable sizes, particularly with respect to burning faults clear. Operating experience and tests have shown that for sizes up to and including 500,000 cir. mils any type of fault that may develop on three single-conductor cables drawn into the same duct or on one three-conductor cable will burn clear if enough current is available at the point of fault. Very little is known for larger cable sizes. A few systems are using or are considering the use of two sets of cables of a smaller size in parallel instead of one set of 500,000 cir. mil size. The purpose is to reduce the time required to clear a very serious fault. However, most companies do not consider this arrangement necessary.

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[4] *Underground A-C. Network Distribution for Central Station Systems*, A. H. Kehoe, A. I. E. E. JOURNAL, June 1924, p. 844.

[5] "Standard Voltage A-C. Network," John Oram, *Elec. World*, September 1, 1928.

Three-conductor cable is used for the primary feeders on almost all of the systems. Two of those companies that are using three-conductor feeder cables change these to single-conductor cables before they reach the points at which the network transformers are connected. Single-conductor cable is used to form the secondary mesh of all systems with the exception of one in which three-conductor cable is used and one in which four-conductor cable is found. Single-conductor cable is popular because it minimizes the likelihood of copper-to-copper faults and it is easier to work on it while it is energized.

The most common practise up to the present time has been to use a solid secondary system with the connections from each network protector tied solidly to it. A few companies have, however, used sectionalizing boxes so arranged that it would be possible to divide the secondary mesh were it found necessary to do so. The use of fuses other than those for back-up protection in the network protectors has been avoided by most systems. One company has, however, installed fuses at the end of each street main to facilitate isolating a fault. Besides the cost of installing and maintaining the fuse boxes this scheme has the disadvantage that when a fault develops in a cable it is probable that the fuses on each end of this cable will blow and a block of load will be dropped. Without the fuses the trouble would have cleared at the point of fault and unless this happened to be at a point from which a service was taken off no load would have been lost.

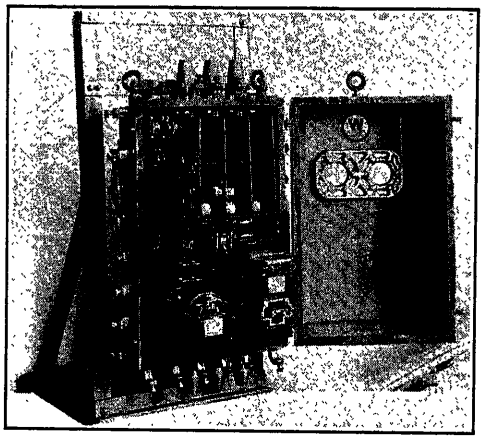

FIG. 2—NETWORK PROTECTOR WITH CONTROL PANEL CLOSED

Relatively little use has been made of service boxes in the systems now in operation, the common practise being to splice customer service cables directly to the secondary network cables in the service manholes.

In the construction of transformer manholes, the question presents itself as to whether a fire wall should be used between the transformer and network protector. The arguments in favor of the use of a fire wall are that in case of a transformer failure followed by an oil fire there is no danger of this damaging the network protector and the cables between the protector and the street mains and possibly causing a number of secondary faults. The increased cost of construction is alone responsible for the fact that the fire wall arrangement is not more generally used at present. One system has, however, gone to the exact opposite construction by mounting the protectors directly on the side of the transformers.

It is almost universal practise in subway construction to use only one cable per phase between the transformer and the protector. The use of more than one cable per phase would increase the number of wiped joints needed to bring these cables into the apparatus and would be virtually impossible of accomplishment with the present size and design of standard equipment. As a result the size of cable required between the popular 300-kv-a. transformer and 1200-ampere protector is 1,000,000 cir. mils. This cable is on the low-voltage side of the transformer and should it develop a fault the network protector will open but it is probable that the feeder switch at the station will not open. This section of cable is, therefore, a part of the secondary and since it consists of a size on which a serious fault will probably not burn clear every precaution should be taken to prevent such a fault from developing. One company is using cable with only weather-proof insulation for the connection between the transformer and protector even in manholes that are subject to flooding. The cable is carried along the side wall of the vault and is held in place by porcelain cleats. Since the cables for each phase are spaced about a foot apart there is little danger of a phase to phase fault and the elimination of the lead sheath prevents a fault between it and the copper. The transformers and protectors are equipped with porcelain bushings. In order to insure a water-tight seal at the points where the cables enter the equipment, a soldered joint between porcelain and metal is made by a recently developed process. This same arrangement is being used in some of the vaults of a network now being installed in the South and it is being given serious consideration by another company.

Those companies that are using three-phase transformers in manholes under the sidewalk, design the manhole so that two pavement slabs may be removed to form a hatchway through which the transformer may be lowered into place. The slabs are normally calked in place with pitch and the usual entrance to the manhole is through a round manhole opening.

NETWORK PROTECTORS

The piece of apparatus that has made the automatic network system possible is the network protector. Though there was no extensive experience on which to base the orignal design, it is worthy of note that with only minor modifications it has stood the test of service for a period of five years. During this period the protectors on one system were called upon to open or close a total of more than 200,000 times and the number of times that they failed to function properly amounted to less than four-tenths of one per cent. It should be remembered that in most of the opening operations included in the above the protector was functioning on the reverse magnetizing current of the transformers. The reports on the operation of the switches on other large network systems show even better results.

About a year and a half ago it was decided to make a complete redesign. A new network protector was brought out which retained all of the operating principles of the original unit but had incorporated in it the features that five years' experience had shown to be desirable. Two views of this type of protector mounted in a subway housing are shown in Fig. 2 and Fig. 3.

In the first place the arrangement of the equipment inside the protector was changed so that it became square in shape and then by replacing the sharp corners of the housing by flat portions the size became such as to require the minimum diameter manhole opening for installation. For sizes up to 1200 amperes the present design of network protector will pass through a 30-in. manhole opening. In keeping with the trend to structural construction, the new type of housing is made of 3/16-in. sheet steel with welded joints in place of the cast iron and aluminum housing that was formerly used. The housing is divided into two equal parts and the front half or cover is arranged so that it can be hinged to the base on either side, giving the maximum flexibility in the location of the protector in the manhole. By making the cover of the housing the same size as the base a large part of the protector is exposed when the cover is swung open which facilitates inspection and repair work.

Slate has been replaced by ebony asbestos as the material used in the main panel in order to reduce the possibility of breakage. The three main poles of the breaker are mounted on one side of the panel and the operating mechanism is mounted on the other side.

The circuit breaker type of construction was retained because of the good service that it had given in the first type of protector. A rather novel arrangement was adopted, however, by means of which the moving contact is made an integral part of the main copper bus and is simply laminated to give the needed flexibility. Both the stationary and moving main contacts are supplied with silver plates to reduce the contact drop. The arcing contacts are made of copper to avoid breakage of the carbon.

The advantages of a motor-operated closing mechanism over a solenoid type in the reduction of the jarring at the time of closing, the reduction of the current required to perform the operation and the consequent reduction in the size of the device required to break this current had been appreciated for some time. This type of device was not used on the original unit, however, because of the lack of a suitable low speed motor mechanism including a trip-free feature to permit the protector to open at any time without retardation due to the action of revolving flyballs. Such a mechanism was developed and the motor was adopted as the standard closing device in the new protector.

The action of the motor is transmitted to the breaker through a system of levers which has been so designed that the breaker is locked closed by the action of a toggle joint being drawn over center. As a result of this, no latching triggers are in motion while the breaker is closing and the danger of the breaker failing to remain closed is minimized. The only latch used in the entire design is in connection with the tripping mechanism and since this is not called upon to act at the end of the closing operation particular care may be taken to design it for its particular purpose of releasing the toggle joint.

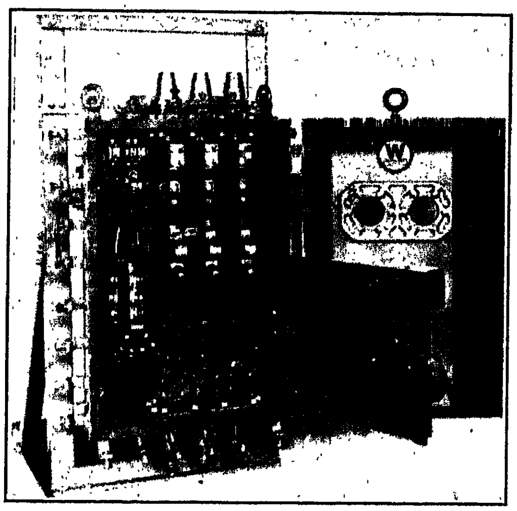

Fig. 3—Network Protector with Control Panel Swung Open

The original network protectors were designed with an undervoltage type of tripping device which had the advantage that the opening of the protector was a mechanical action and no energy was required at the time of tripping. The undervoltage trip gave splendid service on those systems where voltage conditions were steady but had the serious disadvantage that it caused excessive operation of the protectors on those systems which were subject to frequent disturbances of such a nature as to cause the voltage to drop to less than 30 per cent. The shunt trip device is more positive in that it does not cause the protector to open until energy is supplied to it by the action of the master relay. This same feature forms the disadvantage of the shunt trip for use in a network protector where the only energy available for tripping is from the system itself. If a fault were to develop in the transformer the only voltage that could be impressed across the shunt trip coil would be the voltage drop from the protector to the point of fault plus the drop due to the fault resistance. The shunt trip device was adopted as the standard in the new protector but only after the design of the entire mechanism was made such that as little as 15 volts impressed across the shunt trip coil will supply the energy required to cause positive operation. It is not likely that a condition will ever present itself where less than 15 volts will be available for tripping.

One of the biggest problems confronting those who designed the first network protector was the development of a type of relay in which could be centered the entire control of the protector. The satisfactory manner in which this design was worked out is shown by operating records which indicate that only about 25 per cent of the very small amount of trouble experienced with the original protectors can be charged against the relays. It is true, however, that the design was rather complicated and in redesigning the foremost purpose was to simplify and at the same time retain and improve the operating characteristics.

A three-phase relay was developed to take the place of the three single-phase relays that were formerly used. The three-phase relay has decidedly better closing characteristics than the single-phase type in that it will allow the unit to close in case of a heavy load on one or two phases even though the third phase may be very lightly loaded. The three-phase relay also gives somewhat better performance under certain secondary fault conditions.

The detailed design of the three-phase master relay represents a decided simplification. It has three electromagnets acting on two disks fastened to a common shaft and each electromagnet is equipped with a potential, phasing, and current circuit. A single-pole, double-throw contact arrangement has been adopted which makes it possible to control either a shunt or an under-voltage tripping arrangement. A new and considerably simplified means of adjusting the relay to close its tripping contact on the desired value of reverse current has been introduced in the new three-phase relay. It consists of a straight spring which is fastened to the moving contact support. As the moving contact approaches the stationary "trip" contact the spring comes in contact with an adjustable stop screw and the force acting to close the contacts must be sufficient to overcome the action of this spring. By adjusting the distance through which the contact must travel against the action of the spring, the value of reverse current required to close it may be adjusted.

The current coils of the master relay are supplied by saturating current transformers instead of the reactive shunts that were formerly used. This change, together with the others mentioned above, has made it possible to design a relay having wattmeter tripping and approximately wattmeter closing characteristics and still eliminates the holding coil, adjustable slide wire resistor, fixed resistor, and auxiliary contactor which were required in the first network protector relays. A view of the three-phase relay is shown in Fig. 4.

There are system conditions under which pumping, that is, periodic opening and closing of the network protector, might result with a three-phase relay as well as with the three single-phase relays. These conditions may be caused by the use of three-phase feeder regulators or single-phase induction regulators connected in delta. The pumping may also be caused by differences in the time of opening of the various protectors on a feeder, the charging current of which is considerably higher than the magnetizing current. To prevent pumping under any of these conditions a single-phase relay known as a phasing relay is supplied along with the three-phase master relay. In construction it is similar to one element of the master relay, differing from it in that it has only a phasing coil and potential coil and has a single throw contact arrangement. The contact of the phasing relay is in series with the closing contact of the master relay and both must be closed before automatic closing of the protector is initiated. When acting along with the master relay the phasing relay prevents the protector from closing under any conditions where the current that would flow after closing might cause the protector to reopen again and thereby absolutely prevents pumping action.

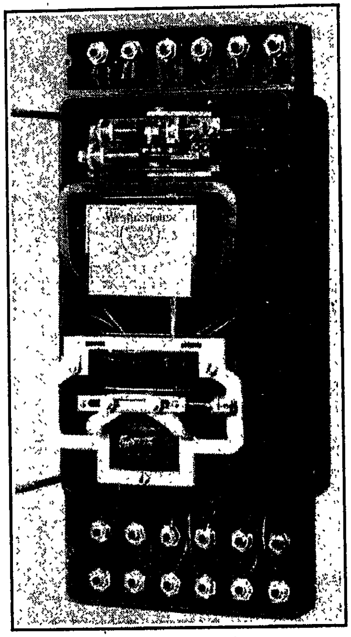

FIG. 4—THREE-PHASE NETWORK RELAY (GLASS COVER AND TERMINAL CHAMBER COVERS REMOVED)

The entire design of the new protector has been worked out on the basis of maximum accessibility for maintenance and repairs. In the first place everything is mounted on the front of the panel and the entire protector has been built up on the unit principle. Any pole of the breaker may be removed as a unit, the operating mechanism may be taken off as one piece without disturbing the rest of the apparatus and the tripping device may be removed as a unit. The relays and auxiliary control apparatus are mounted on a hinged steel panel in front of the main apparatus and all control leads are brought to a terminal block on this panel through special flexible wire. As a result this control panel can be readily removed and replaced. The method of connection is shown in Fig. 3. The terminal block of the relay is so arranged that the same set of terminal screws makes the electrical circuits to the various coils in the relay and connects the relay mechanically to the panel. If the terminal screw for any particular circuit in the relay is opened this circuit is broken and may be tested without removing the relay from the panel. This terminal arrangement is shown clearly in Fig. 4. The subway housing is equipped with two non-shatterable glass windows and directly inside one of these is mounted an operation counter and a position indicator while with the aid of a flashlight the fuses and breaker contacts may be observed through the other. Means are provided for electrically isolating the protector without removing the cables but should it be necessary to remove the cables it is possible to do so without breaking the wiped joint and special insulating sleeves have been provided to prevent grounding the energized cables against the housing during the operation of removing them. All of the control wiring has been so arranged that it can be easily traced.

NETWORK TRANSFORMERS

In the first network systems installed the transformers used were of the same standard type as those on the radial systems. Since that time, however, it has been found desirable to incorporate other features so that there has developed what is commonly called a network transformer particularly designed for service on this type of system.

Since the design of a network system is made up in such a way that a complete feeder may be taken out of service without overloading the remainder of the system under peak load conditions, one of the most important objections to three-phase transformers is removed because it is never necessary to operate a bank in open delta or to make a hurried replacement of a transformer that has failed in order to maintain service. The advantages of the three-phase transformer over three single-phase units are that it is somewhat cheaper in first cost, gives lower losses, takes up less space in the vault, gives a better vault layout in respect to the location of cables and such features, and has some advantages in respect to the mounting of auxiliary apparatus as will be brought out later. It is true, of course, that an opening must be provided in the vault of sufficient size to permit bringing in the transformer and the handling of the larger size unit is considerably more difficult than for the smaller single-phase transformers.

Although only one-third of the companies operating or installing network systems have thus far decided to use any three-phase network transformers, still this number is continually increasing and there is a definite trend toward the use of this type, particularly in those systems where new transformer manholes are being built and it is a relatively easy matter to provide the larger opening required.

The feeders supplying the average network system are interlaced and, therefore, supply transformers located at widely scattered parts of the area networked. It is possible for someone to close in manually a protector located at a point on the feeder remote from that on which a man is working and thereby to connect the feeder to the network. It is, therefore, considered desirable to locate at each transformer bank some device for grounding the feeder which supplies it. A man working on the feeder cable or any of the high-voltage apparatus connected to it can then make sure that the feeder is grounded in the immediate neighborhood of the section on which he is working. The first device brought out for this purpose consisted of a small switch mounted inside the transformer and equipped with an operating handle which extended through a stuffing box to the outside of the transformer tank. This switch was provided with an electrical interlock to prevent the possibility of its being thrown to the ground position with the feeder energized.

As operating experience grew it was found that it would be desirable to have some means of disconnecting each transformer from its feeder as an aid to testing and in order to keep any transformer out of service because of failure or for any other reason without keeping an entire feeder deenergized. This required some type of disconnecting device suitable for opening the circuit only when deenergized. In the first piece of equipment brought out for the purpose the disconnecting feature was incorporated in the grounding switch and made suitable for mounting inside the transformer. It was generally felt, however, that such a switch was too complicated for mounting inside the transformer, particularly for the higher voltages where considerable clearance is required, and for this reason a switch was designed for mounting on the outside of the transformer. The electrical interlock was retained as the means for preventing the feeder's being grounded while energized and in addition to this a mechanical device was arranged on the operating handle of the switch in order to insure still further against incorrect operation. Let it be assumed that the switch has been thrown over to the disconnect position and that while in this position the feeder breaker at the station has been energized. Since the electrical interlock derives its energy from the low voltage side of the transformer, it is ineffective when the switch is in the disconnect position and there is nothing to prevent the switch being thrown to ground even though the feeder is energized. To minimize the possibility of trouble from this source the mechanical interlock or sequence controller on the operating handle of the switch was so designed as to make it necessary to return to the transformer position from the open position before it is possible to throw the switch over to ground the feeder. Should the feeder be energized the electrical interlock becomes effective as soon as the switch is thrown to the transformer position and prevents the final operation to ground.

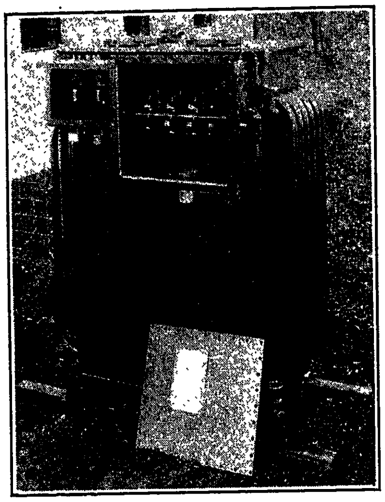

FIG. 5—THREE-PHASE NETWORK TRANSFORMER EQUIPPED WITH GROUNDING AND DISCONNECTING SWITCH

In a three-phase transformer all connections between the phases are made inside the tank and it is only necessary to bring out the three-phase leads. It is very desirable to bring these leads out through a terminal chamber so as to make it unnecessary to get inside the tank to connect or disconnect the transformer. By increasing the size of this terminal chamber it makes a convenient place in which to mount a three-pole switch having the features described. A three-pole, three-position switch mounted in the high-voltage terminal chamber is shown in Fig. 5.

In the case of single-phase transformers the advantages of mounting the switches in the terminal chambers still exist but a somewhat more difficult problem presents itself since it is necessary to mount a two-pole switch on each single-phase unit. Such a switch increases the dimensions of the transformer a considerable amount and the design is, therefore, made such that the entire terminal chamber may be removed from the transformer while it is being installed. However, the taking off and replacing of these chambers does involve a considerable amount of work.

The San Antonio Public Service Co. decided on the use of single-phase transformers in the network system that is now being installed and rather than use a separate switch on each transformer adopted the plan of using a three-pole switch having all of the features previously described but mounted apart from the transformer. The switch itself is oil filled and is arranged with compound filled terminal chambers at the top and bottom. The three-conductor feeder cable is brought into the top terminal chamber while the two cables from each single-phase transformer are brought into the other. The delta connection of the phases is closed in this chamber and three leads are brought from it up to the switch itself. A view of this switch with the front plates removed is shown in Fig. 6.

In all types used thus far the high-voltage grounding and disconnecting switch is oil filled. In the first type mounted on the outside of the transformer tank the common practise was to mount directly below it a compound filled terminal chamber to prevent oil siphoning through the insulation of the cable. The most recent development has been to replace this by a special compound filled pothead wiping bushing such as that shown in Fig. 5.

Tap changers which may be operated by handles brought through the cover of the tank are supplied in network transformers. It has been found desirable in service to have some control of the ratio of the step-down transformers and the tap changer has been adopted as the simplest means of accomplishing this with the transformer disconnected from the circuit.

One of the important questions that presents itself to any company designing a network system is the value of reactance that should be used in the network transformers. The problem is fundamentally one of the operation of transformers in parallel having in mind that in general the more fully the transformers share the loads on the system the more satisfactory will be the operation. The higher the impedance of the transformers as compared to the impedance of the secondary tie cables, the better will be the load division. Since a network is designed so that any feeder and all of

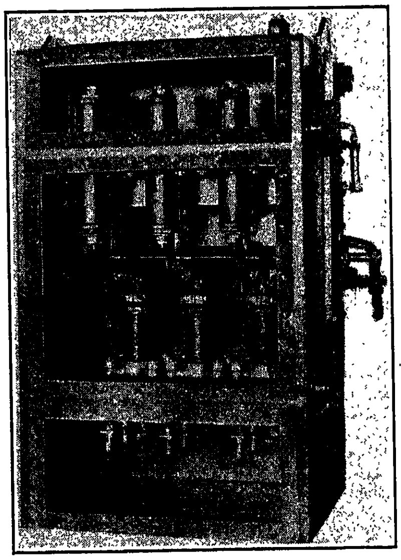

FIG. 6—SEPARATELY MOUNTED GROUNDING AND DISCONNECTING SWITCH

the transformers connected to it may be taken out of service and left out indefinitely, it is rather important that the remaining transformers share the extra load as much as possible. Besides improving the load division the high reactance also has the advantage of reducing circulating currents between feeders which in turn leads to more stable operation of the network protectors, particularly under light load conditions. These two considerations have caused the large majority of companies operating network systems to adopt the policy of using ten per cent impedance at the transformer banks.

The disadvantage of the higher reactance is that it increases the voltage drop. This is not as serious as it might appear to be on first thought, however, because the power factor of the network load is high at the time of full load when the drop is important and the drop caused by the current flowing through the reactance of the transformer is very nearly at right angles to the network voltage. In the average system the drop in voltage due to the addition of 5 per cent reactance will not be more than 1 per cent. However, the possibility of motor starting currents causing flickering of lights is increased.

If a company decides to use transformers having ten per cent impedance the next problem is to determine the most satisfactory manner in which to obtain the increased value. There are three methods that have been used up to the present time. The first method is to have the increased reactance built into the inherent design of the transformer by coil spacing and grouping. In the second method the increased value is obtained by the use of shunt iron paths in the transformer. As a third means transformers having standard reactance may be used and this can be raised to any desired value of the addition of small reactors in the cables coming out from the transformer.

Each of the three methods mentioned has certain advantages. Both the type of transformer having the ten per cent reactance as an inherent part of its design and that obtaining the increased reactance by means of a shunt iron path are self contained units while the third, or the method for which external reactors are required, has two distinct pieces of equipment. This means that the first two types give a unit that is somewhat simpler to install and this is particularly true in view of the fact that it is necessary to break the lead sheath of the cables over which the external reactors are slipped. Both of the first two have the important limitation, however, that the reactance is a fixed value and cannot be changed, while it is a relatively simple matter to change the reactance of the third type using external reactors. If the question were settled as to what is the most satisfactory transformer reactance to use in a network system and it was known that ten per cent at each bank represented the ideal condition, then this point would not be an important one, but this is far from being the case and it now appears that the conditions prevailing on each system and the design of the system will have a decided influence on the choice of transformer reactance. The feature of being able to change the reactance is also found of value in shifting load between banks if, for example, it is found that one transformer is carrying an overload with a feeder out of service while those around it are not carrying full load.

The type of transformer that has an inherent reactance of ten per cent maintains this reactance under short circuit conditions. In other words there is no saturation effect and the impedance drop of the unit when plotted against load forms approximately a straight line. In either of the other two methods, the shunt iron path or the external reactors saturate at about double normal load and have little effect at the time of short circuit. It must be remembered that the chief functions of the increased reactance are to improve load division and to reduce circulating currents. It is felt that for a system having the average load densities it is an advantage to have the added reactance saturate at the time of a short circuit in the secondary system so as to increase the current flowing into the fault and to facilitate burning it clear in the shortest amount of time. It may be that in systems having exceptionally high load densities the currents that could flow might reach such values as to prove destructive to cables, in which case the current limiting feature of the first design would prove advantageous.

Taking all factors into consideration it would appear that the use of transformers having not more than five per cent inherent reactance, with any additional amount desired obtained by means of external reactors, is the most satisfactory arrangement for the system having usual load densities.

Fig. 7 shows the low voltage side of a single-phase network transformer. A rather convenient method of mounting the external reactors is illustrated. These reactors also have applications elsewhere on the system. For example, it is rather common practise where two transformers are located in the same vault to install reactors on the cable bus joining these two transformers and to make these of such a size that they will introduce an impedance which is approximately equal to the impedance of the usual cable tie between banks located in different vaults. This improves the loading of the remaining transformer when one bank in the vault is taken out of service and also minimizes the possibility of trouble due to circulating currents. If the customer's load is split in such a way that the motor load is largely on one bank with the lighting load on the other, the reactor between the two will reduce the effect of motor starting on the lighting.

PROBLEMS OF DESIGN AND OPERATION

The highest tribute that can be paid to the network system is that it has given the performance expected of it by the companies that have installed it. There have been problems encountered and a review of some of these is of interest.

With the combined light and power system using the star connection it is impossible to supply the now commonly accepted standard voltages for both power and light. If the voltage to neutral is chosen as 115, then only 199 volts will be available between phases while with 120 volts from line to neutral 208 volts exist between phase wires. A large amount of discussion has been devoted to the relative merits of these two combinations and up to the present no definite decision has been reached. A few fundamental ideas seem to be crystallizing, however, which may point to the future trend. In the first place it is being rather generally accepted that the nominal voltage to neutral on the network system should not differ from the lamp and appliance voltage elsewhere on the system. The second point that operating experience seems to be establishing is that no very serious troubles are experienced from the operation of a network system, with the improved voltage regulation found in such a system, on either of the reduced values of voltage mentioned above as long as some effort is put forth toward educating customers on the new type of service that they are receiving. There are always some isolated cases of troubles, particularly when changing over apparatus which has been operating at higher voltage to the lower value but these can be readily corrected by minor changes in the apparatus itself or by the use of booster autotransformers.

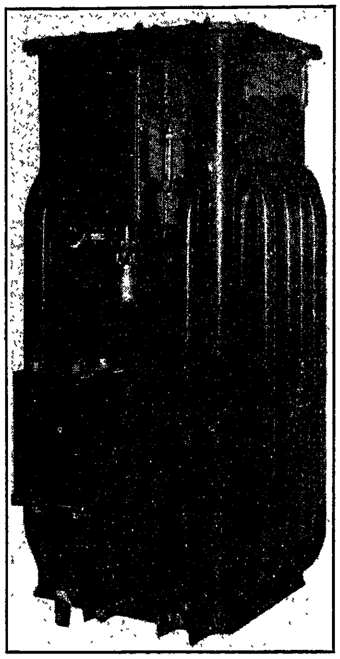

FIG. 7—SINGLE-PHASE NETWORK TRANSFORMER USING EXTERNAL REACTORS

Of those companies that are operating star connected combined light and power secondaries, it is found that just about the same number are maintaining 120/208 volts at the customer's service switch as those maintaining 115/199 volts. There are a few companies that are maintaining at the same point a voltage of approximately 117/203 volts. A check was made a short time ago of some of the companies that have been operating the largest networks for a long enough period of time to have obtained some valuable information on the question and it showed that three of the companies maintaining 120/208 volts find it unnecessary to use any boosters in supplying the motor load. The same is true of two of the companies holding 117/203 volts. On five of the large 115/199 volt systems an average of about 40 per cent of the motor load is supplied through booster transformers. In this latter class some companies have adopted a very liberal policy in regard to the application of booster transformers.

Another problem that presents itself is the determination of the size of motor that should be taken on the network system. In general, it might be said that any motor of a size normally found in the downtown area of a city may be supplied from a properly designed network. Special starters must be supplied on the larger motors to divide the starting current into increments that will not cause flickering of the lights. Various companies have adopted different limits in the starting current regulations but the trend seems to be to a limitation of 200 amperes per step at one-half second intervals. Test information might show that higher limitations would give satisfactory results on many systems.

One company started operation with the network feeders brought from the busses of two different stations. It was found that the variation in voltage conditions on the two busses was sufficient to cause excessive operation of the network protectors during periods of light load on the system. The condition was corrected by tying the two stations more firmly together on the high voltage side.

Some cases have been noted of excessive operation of network protectors in the immediate vicinity of elevator motors at the time of light load on the system. This may be caused by regenerative action of the elevator motors or by the relative voltage conditions of the feeder and secondary system being such as to cause the protector to remain open under the normal load being carried by the system but causing it to close as the starting current of the elevator motor lowers the voltage in the secondary grid. In either case the trouble can easily be corrected by raising the reverse current tripping adjustment of the relay. This will probably prevent this one protector from opening on the reverse magnetizing current of its own transformer but it will still trip when the feeder breaker at the station is opened because after all of the other protectors on the feeder have opened the magnetizing current of all the transformers connected to the feeder will be supplied through the one protector. When a sufficiently high adjustment cannot be obtained with the standard spring a special heavy spring may be obtained.

When the operation of network systems was first started it was felt that where feeder regulators with line drop compensators were used it would be necessary to supply some means of cross-compensation to insure stable operation of the regulators. It was believed that any action of the regulators that would set up a circulating current between the feeders would bring about a cumulative action which would cause some of the regulators to rotate to the full boost and others to the full buck position. To eliminate the possibility of this happening several simple means of cross connecting the line drop compensators were developed and some of these are now in service giving very satisfactory operation. It has since been found that if the line drop compensators are set properly it is possible in most systems to obtain stable operation of the regulators and proper voltage in the secondary system without tying the regulators together in any way. Several systems are now operating the regulators on their network feeders without any cross connections and have experienced no trouble.

PRESENT TREND OF DEVELOPMENT

Up to the present time only one company, the Knoxville Power & Light Co., is operating an overhead automatic network system of any appreciable size. The question of using networks in such areas has been given serious consideration by other companies and it is believed that decided advances will be made in the development of such systems within the next few years.

The first two of a group of large government buildings are now under construction at Washington. The automatic network system has been chosen as the most satisfactory from an engineering standpoint as well as the most economical type of system that could be used to supply these buildings. Each building will have a secondary cable grid tying together the transformer vaults located in that building and the secondary networks of all of the buildings will be connected solidly together by low-voltage tie cables.

This development is only the forerunner of a widespread application of network systems inside large buildings if the amount of thought being given the problem at the present can be taken as a criterion. Directly connected with this subject is the one of vertical distribution in tall buildings which is being actively considered by two companies at the present time. Preliminary plans call for the location of transformers on several different floors, the group of transformers on each floor being connected into a network which may be permitted to remain independent or may be tied in on the secondary side with the networks on other floors. Along with this development, consideration is being given to every detail of the vault layout and the design of the apparatus to minimize the possibility of a transformer explosion and to localize trouble of any kind.

The a-c. network is being given serious consideration as a system of distribution for large manufacturing plants. In this field such a system should find its greatest application in those factories where the process of manufacture is such that an outage of even a short duration has serious results. A special type of automatic network system is being considered for one manufacturing plant of this type at the present time.

The network system lends itself exceptionally well to the scheme of synchronizing generators at the load. One company that is operating a large network system is now making a thorough investigation of the feasibility of such operation.

The network stands today as a type of system that has stood the test of service and has given the results expected of it. In general, the continuity of service in practically every installation has been as dependable as the source of supply and most thought is now being given to possible means of increasing the dependability of the source.

Discussion

Frank E. Johnson, Jr.: Considerable difficulty has been experienced by the New Orleans Public Service in securing fuses of the proper characteristics for transformer protection. The function of these fuses, as I understand it, is to blow upon failure of the network protector to open for a primary cable fault involving back feed currents of a magnitude large enough to damage the transformers, and also for secondary cable faults (during which the protector is expected to remain closed) where such faults are of sufficient duration or severity to damage the transformers. Although it is necessary that as much current as possible be fed into a secondary fault to clear it, nevertheless this should not be entirely at the expense of the nearby bank, since a transformer failure is likely to involve a feeder outage and hence a reduction in network transformer capacity at a time when it is most needed. It would, therefore, appear advisable for this condition that the nearby bank or banks be properly cleared by fuses.

These two types of faults work against one another in so far as fuse size is concerned, because the ideal fuse to take care of the former fault should blow at current values just beyond the full-load rating of the transformer, while for the latter fault the larger the fuse the better are the results. The problem is further complicated by the different conditions brought out in the paper which exist for short circuits through the various types of inherent or external reactance transformers. In the fuse design it would seem then that preference should be given to primary feeder faults since in a carefully constructed secondary system, utilizing the proper type of cable, little or no trouble may be expected. There have been no secondary cable failures in the New Orleans system. Since various values of fault current will be encountered, a curve should be plotted between fuse current and the fuse blowing time. This curve, for the ideal fuse, would lie just below, but parallel to, the safe thermal-time curve of the transformer the fuse was designed to protect. Unfortunately, due to the great difference in thermal characteristics between the fuse and the transformer, this condition can only be approached. However, since the various makes of network transformers having the same inherent impedance appear to have approximately the same thermal-time curves, it does seem reasonable to believe that some standard fuse could be devised that would be interchangeable in all manufactured protectors to be used with a given size transformer. At present each new model of protector includes a different type or size of fuse.

A very helpful feature in our network protector operation has been the fine spirit of cooperation that has existed between the manufacturers and the New Orleans Public Service, Inc. Weak design features have been promptly rectified by immediately submitting detailed reports of any trouble experienced to the manufacturers. I think that a similar procedure on the part of all users of network protectors will bring the equipment to the highest state of reliability in the shortest space of time.

I should like to ask if any occasion has arisen where it was desirable that a tap changer on a network transformer be used so that the ratio on a particular bank or banks was different than elsewhere on the system, and if any trouble from circulating currents was then experienced.

In connection with allowable motor-starting currents, I will mention the fact that a 100-hp. slip-ring induction motor, with drum control switch, and used for hoisting purposes, was recently connected directly to the 500,000 cir. mil 1-conductor mains with entire absence of light flicker in the vicinity. The starting current, as read on a well damped ammeter, was 325 amperes. The installed transformer capacity (10 per cent impedance transformers), within a radius of 600 ft. from the building, is 2850 kv-a.

T. J. Brosnan: We have appreciated for some time the desirability of using fuses having the characteristics described by Mr. Johnson for back-up service in the network protectors. A considerable amount of work has been done on the development of such a fuse but as yet the results have not been very encouraging.

There are several companies that have had occasion to use the tap changers to change the ratios of all the transformers on the system but I do not know of any case where some of the transformers have been changed a greater or less amount than the others.

Standard Voltage A-C. Network

BY JOHN ORAM[1]

Non-member

Synopsis.—*The object of this paper is to present a description of the Dallas (Texas) underground network system. This system is of interest since it is directly opposed to the general trend of furnishing non-standard voltages in underground systems. The reasons for its selection and its advantages over the three-phase four-wire system are presented. A brief description of the installation of equipment in vaults and manholes is given. Another feature of this system is the use of standard reactance transformers together with an external reactor, shunted by a fuse, for purposes of overload protection to the transformers. In addition to the regular network this system has a single-wire manhole and vault lighting network which is also used as a control wire for turning on and off multiple street lamps. Another feature of this system is a complete underground telephone system with a telephone outlet in each manhole and transformer vault. This telephone system is also connected to float switches in transformer vaults which automatically notify the trouble office in the event of high water in a transformer vault. The routine tests made in connection with the operation of the network system are described and the method of locating primary faults is given in detail. Complete records are kept of the installation of conduit and cable. Photographic records of construction, operation, and cable failures, etc., are also kept. The record of this network system both as to efficiency and as to continuity of service is excellent.*

* * * * *

THE principal characteristic of the Dallas (Texas) underground secondary network system is that it provides standard 115-volt service for lighting consumers and 230-volt three-phase service for power consumers. This is done by adding an 18-volt secondary coil to the standard 115-volt transformer coil and bringing out three leads on each transformer. With three transformers connected in "Y", the arrangement gives 115 volts for lighting and 230 volts for power service with a common symmetrically grounded neutral.

On account of the high primary voltage used and the relatively short distance of approximately one mile from generating station to load center no feeder regulators are required. The range of voltage regulation is about five per cent from no load to full load and the control of the generating station busbars by Tirrill regulators is adequate for this purpose. Since the major portion of the load follows quite closely that of the underground system and all other lighting service is controlled by individual feeder voltage regulators in substations, no interference with the remainder of the system is caused.

The Dallas network system covers an area of approximately 0.75 sq. mi. and consists of five 13-kv. underground feeders serving eighty-two 300-kv-a. transformer banks which supply two sets of secondary network cables, one being the lighting network and the other the power network. Automatic network units are used between transformers and the secondary network system. The principal load in the underground district consists of office buildings, theaters, department stores, and hotels. The average load density is 20,000 kv-a. per sq. mi. The greatest concentrated load is a hotel which consumes an average of 10,000-kv-a. hours every 24 hr.

The network system at present is designed for a normal maximum loading of 80 per cent on each transformer bank and an emergency loading of 100 per cent. This arrangement will permit the removal from service of any one of the five feeders without overloading the remainder of the system. The normal voltage in the secondary mains is 115 volts for lighting and 230 volts for power. The emergency voltage will average 110 and 220 volts for lighting and power, respectively.

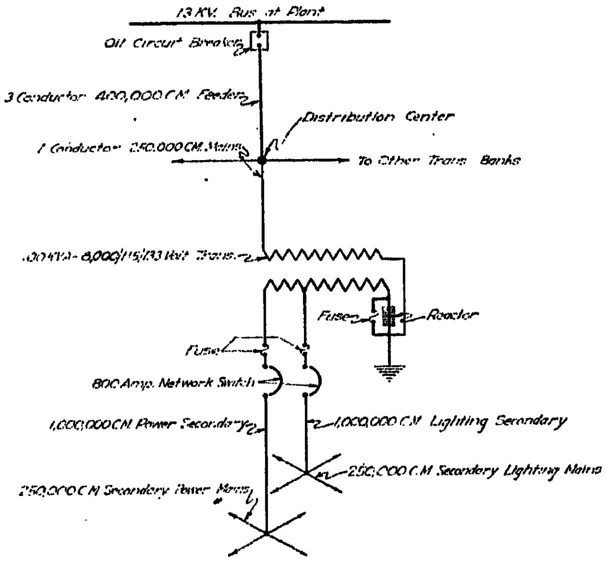
13 KV. Bus at Plant
Oil Circuit Breaker
3 Conductor 400,000 C.M. Feeder
Distribution Center
1 Conductor 250,000 C.M. Mains
To Other Trans. Banks
100 KVA- 8,000/115/133 Volt Trans.
Fuse
Reactor
Fuse
800 Amp. Network Switch
1,000,000 C.M. Power Secondary
1,000,000 C.M. Lighting Secondary
250,000 C.M. Secondary Lighting Mains
250,000 C.M. Secondary Power Mains

FIG. 1—SCHEME OF DESIGN OF DALLAS UNDERGROUND NETWORK

The system is composed of three-conductor, 400,000-cir. mil 13-kv. primary feeders, 250,000-cir. mil 13-kv. primary mains, 100-kv-a. 8000-115/133-volt transformers, automatic network units, 250,000-cir. mil secondary power cables, 250,000-cir. mil. secondary lighting cables, and a 500,000-cir. mil bare neutral.

The decision to use two sets of mains was made after a careful consideration of investment costs, operating costs, and quality of service to be rendered. It was primarily because of the superior quality of service provided that the seven-wire system was adopted

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[1] Chief Engineer, Dallas Power & Light Company, Dallas, Texas.

Presented at the Regional Meeting of the South West District of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

Previous to the installation of the underground network system the customers of the Dallas Power and Light Company had been furnished with standard 110-volt lighting and 220-volt power service. With the thought of good service and public relations in mind, some secondary network system was sought which would continue to furnish normally standard voltages on both light and power mains. Investigation showed that the additional costs of the secondary portion of the system were offset by the better service provided and the saving in customers' equipment. Fig. 2 shows the comparative investment costs between the several major portions of the underground network system.

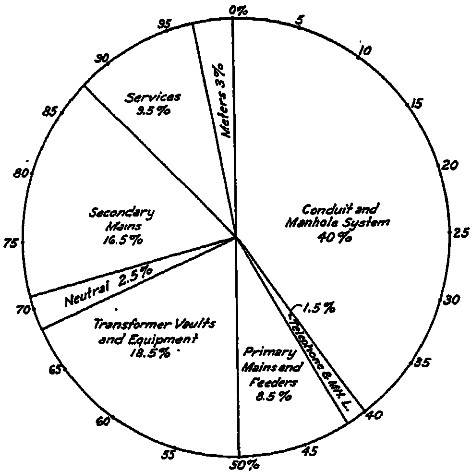

FIG. 2—CHART SHOWING DISTRIBUTION OF COSTS OF DALLAS UNDERGROUND NETWORK SYSTEM

In addition to furnishing standard voltage to both lighting and power services, the seven-wire system has the advantage that, since two separate sets of mains are used, fluctuations due to the power loads do not cause flickering of the lights. The absence of flicker on the lighting system may be observed from the voltmeter charts shown in Fig. 3, taken simultaneously on lighting and power services in the same distribution box. There is no limit to the size of motor that may be operated from the network system, and motors up to and including 30 hp. may be started directly across the line without the use of starting compensators. In this way the economy resulting from the simplicity, high performance, and balanced design of the common squirrel-cage motor is retained, together with greater flexibility in application where remote control is desired.

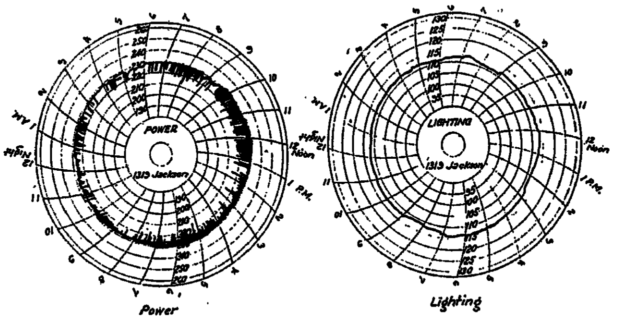

FIG. 3—TWENTY-FOUR HOUR VOLTAGE CHARTS ON POWER AND LIGHTING MAINS TAKEN AT SAME LOCATION

Another feature of this system is that standard reactance transformers are used with an external reactance installed in the neutral leg of each transformer, so arranged that the reactor is normally shunted by means of a fuse, thereby providing good regulation and lower secondary losses during normal operating conditions. When, however, an overload occurs on a transformer bank, due to feeder outage or other abnormal conditions, the fuses blow and thereby insert the external reactance in the circuit and relieve the transformer of part of its load.

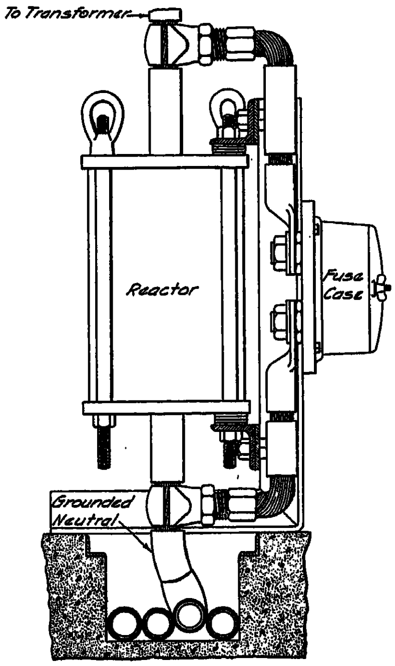

FIG. 4—FUSED REACTOR FOR TRANSFORMER PROTECTION

The transformers are installed in vaults either built by the Dallas Power and Light Company, or if in building basements they are in accordance with Dallas Power and Light Company specifications. In either case the only opening to the vault is through a manhole cover, 20 in. in diameter, built into a removable concrete panel 4 ft. wide, by 5 ft. long, placed in the sidewalk next to the curb. The castings in the panels allow men and small equipment to enter the vault with ease. If necessary to replace transformers or network switches the panel is removed. Each vault is ventilated by two 28-in. square ducts with openings in opposite ends of the vault, one near the ceiling, the other near the floor. Both ducts terminate on the sidewalk next to the curb and are covered with gratings.

The equipment in a transformer vault consists of three transformers, two network switches, primary cables, secondary cables, three reactors, a condulet each for telephone and lighting service, and miscellaneous equipment such as a ladder, manhole guard, and covering for cable trenches. The primary cables enter the vault in one of the lower ducts and are racked on wall insulators to the transformers. The transformers are placed on T-rails imbedded in the concrete floor. The secondary leads are trained vertically from the transformer terminals to the secondary cable trench in the floor. The secondary power and lighting cables extend in separate trenches to the wall on which the network switches are installed. The 1,000,000-cir. mil secondary cables are extended from the upper terminals on the network switch to the secondary network in the street. The secondary cables are racked separately from the primary cables.

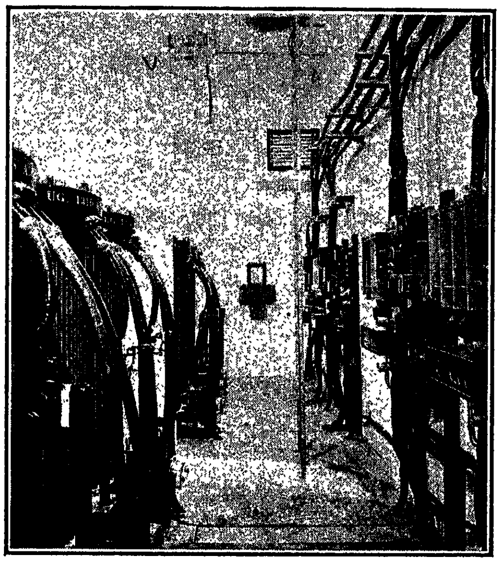

FIG. 5—TYPICAL DOUBLE TRANSFORMER VAULT

There are at present 84 submersible and 80 non-submersible network switches in operation on the Dallas system. Of the submersible units 72 are solenoid operated and 12 motor operated; all use single-phase relays in air bells which are mounted on the wall of the vault. This arrangement facilitates maintenance and provides better ventilation of the relays. No phasing relays are used on the foregoing units. Of the non-submersible units 78 are solenoid operated equipped with single-phase relays and phasing relays, and two are motor operated equipped with three-phase relays and phasing relays. Few difficulties in the operation of the network switches have been en-encountered from unbalanced loads and elevator regeneration. These have been overcome by using three-phase relays or otherwise.

Although secondary network system in general consists of 250,000-cir. mil lead covered cables on cross streets or between important loads, additional 500,000-cir. mil cables are sometimes installed. The secondary neutral is grounded in each transformer vault and serves as a primary neutral.

The secondary services are extended to the customers building line in 4-in. fiber conduits, one each for power and lighting services where both are required. The conduits are terminated in a distribution box placed in the outside building wall, or iron conduit of reduced size is carried up the building wall about 12 ft. and terminated with a condulet. A panel is installed in the distribution box and both power and lighting services are connected to this panel. If it is not possible to install the distribution box, the customers' outlets are grouped on the outside wall at the first floor ceiling line. The cables used for services are single-conductor, 600-volt, paper insulated, and vary in size from No. 6 A. W. G. to 500,000 cir. mil.

In addition to the secondary and primary mains in the duct lines there are telephone and manhole lighting cables with outlets in each manhole and vault. The manhole lighting cables are No. 6 A. W. G. and form a secondary network with 115 volts to ground which is used by the workmen for lights, blowers, soldering pots, and other general conveniences. This network is connected to the main secondary lighting network through a fuse and switch, and is so arranged that in addition to serving as a manhole lighting system it will also serve as a pilot wire for controlling a proposed multiple street lighting system in the underground area.

The telephone system is in the same conduit as the manhole lighting cable. It consists of two 26-pair cables which extend from the switchboard in the Trouble Division's office to central locations, and branch out into two-conductor No. 14 lead covered cables. These two-conductor cables are installed on the main streets so that each manhole and vault has a telephone outlet.

The telephone system is also used as a signal system to give warning of water in transformer vaults. This is accomplished by connecting a coil of No. 4 bare copper wire in a porcelain housing to the ungrounded side of the telephone circuit. The device is installed in transformer vaults about six inches above the floor. When water reaches a level of six inches or more the telephone cable is short-circuited and a light signals the operator in the Trouble Office.

In the operation and maintenance of the network system certain routine tests and inspections are made. The vaults are inspected regularly and all dirt, paper, and other accumulations removed from the ventilator pits and floor. Oil samples from all transformers are taken once each month and tested for dielectric strength. The oil level in the transformers is observed and if necessary oil is added. The oil in each transformer is filtered at least once a year regardless of its dielectric strength. Each network switch is inspected weekly and the number of operations recorded. An inspection of network switches is made each month, They are cleaned and adjusted, and repairs are made if necessary.

To test the operation of network switches, feeders are removed from service once each week during the light load period, by opening the circuit breakers at the Generating Station. If the feeder is not automatically deenergized when the breaker is opened, a trouble man is sent out to see which network switch has failed to open. A signal system installed in each vault saves the trouble man much time in locating the faulty switch. This signal system consists of a 125-volt lamp installed in the ventilator so that it can be seen by the trouble man without getting out of his car. One terminal of the lamp is connected to ground and the other to a lighting phase through an auxiliary switch that closes only when both network switches are open. A signal lamp not burning indicates that a network switch is closed. If faulty the switch is locked open until repaired.

In the location of primary faults special equipment is used. The apparatus is installed at the Generating Station and consists of a 50-kv-a. 440/8000-volt step-up transformer and 8000-volt, 15-ampere reactor. The procedure followed in locating a fault is to manually lock open each network unit on the feeder in trouble, disconnect the feeder at the Generating Station and connect the reactor to the faulty conductor until it is broken down to ground. With a split-core instrument transformer and low reading ammeter the current sent out over the cable is traced to the fault location.

Detailed records of underground installations are kept under five general classifications, *viz.*: (1) Duct Line Records, which are in the form of drawings showing a plan and profile of the duct lines together with the location of all substructures adjacent thereto; (2) Manhole Records, which are also drawings giving the location and details of construction of each manhole; (3) Cable Records, which give the location, size, date installed, and other data pertaining to each cable in each manhole; (4) Transformer Records, which gives the purchase specifications together with a complete history of each transformer; and (5) Service Records, which is a section map of the city showing the location and size of each lighting and each power service.

Records are kept of all equipment failures which includes a detailed description of the failure together with a photograph of the faulty apparatus where possible. Included in these records are cable failures, transformer failures, network unit failures, etc.

During the year 1928 the average efficiency of the underground network system, as calculated from watt-hour meter readings at the Generating Station and those on customers' premises, was 95.5 per cent. The average annual power factor was 0.90 and the load factor 0.50. Approximately 36 per cent of the energy distributed was power and 64 per cent lighting.

The operation of the system since its installation has been successful. The voltage regulation has been good and there have been practically no complaints. The record with reference to continuity of service, like that for voltage regulation, has been satisfactory. The completed network system has been in operation nearly two years and during that time no customer has been out of service. A number of faults on primary cables has occurred, but in each instance the trouble has been automatically and successfully cleared without interruption to service. Secondary cable faults burn clear without noticeable effect on the service.

Automatic Reclosing of High-Voltage Circuits

BY E. W. ROBINSON[1] and S. J. SPURGEON[1]

Non-member Non-member

Synopsis.—The experience of the Alabama Power Company with the reclosing of high-voltage circuits is described, this covering numerous installations on 22-, 44-, and 110-kv. circuits. No definite reclosing cycles are recommended but instead this paper deals with various typical applications of high-voltage reclosing oil circuit breakers on the Alabama Power Company system.

INTRODUCTION

ALABAMA Power Company operates over a wide expanse of territory where the load density is generally low. About 1922 a large expansion program started which immediately developed the desirability of more primary substations, where the 110-kv. system might be tapped and voltage reduced to 44 kv., and of more switching stations where the existing 44-kv. lines might be interconnected. The cost of attendance at these primary substations and switching stations intervened, however, as a serious obstacle and brought about an active interest in automatic reclosing switching equipment which might allow the substitution of a daily or weekly inspection of such stations for continuous attendance.

GENERAL DISCUSSION

No suitable reclosing relays or devices were commercially available at the time so the power company started a series of experiments and developments which resulted in 1922 in the application of a reclosing device to a standard 50-kv. oil circuit breaker controlling a 44-kv. feeder. This device is of interest as being a self-contained mechanism including a motor driven breaker operating device, reclosing relay, and lock-out relay. It required a minimum amount of power, could be operated from any available a-c. or d-c. source, and was applicable to any standard breaker by minor alterations. As originally built the interval between breaker operations was approximately one minute and the device could be set to lock-out after one, two, or three reclosures. The original mechanism was far from perfect but continued in operation for several years.

About this same time various manufacturers put on the market reclosing relays adapted for use with standard breaker operating mechanisms. Among these were the induction type, motor-driven drum type, and motor relay type. These were all more or less unreliable in the beginning, but they, and also the so-called "reclosing device" developed by Alabama Power Company, were rapidly developed and have reached a high degree of reliability. Considerable numbers of each type of reclosing installations were put in service and it is interesting to note than in 1928 all types showed better than 85 per cent of correct operations, and the two preferred types in excess of 95 per cent.

On the Alabama system originally there was a uniform period of one minute between breaker operations. Experience showed that this was satisfactory and subsequently the initial interval was reduced to 45 sec. and in some cases to 30 sec. Later intervals were also reduced in some instances to 45 sec. Originally two reclosures before lock-out were provided for and this schedule has been followed consistently. Two reclosures gave ample time for the controlled line to be restored to service if this were possible and there seemed to be no valid argument for a greater number as it was extremely unlikely that a line would show up bad on two trials and good on the third, while on the other hand, a larger number of reclosures prolonged system disturbances, vastly increased the duty on the breakers, and resulted in increased cost of maintenance and inspection.

The engineers of Alabama Power Company have not been impressed with the necessity nor desirability of a standard reclosing cycle for high-voltage breakers. Instead they have preferred to treat each case separately taking into account the character of the controlled line, peculiarities of the load supplied, possible short circuit duty on breaker, rupturing capacity of breaker for various duty cycles, and class of service to be maintained. In most instances the reclosing cycle is adjustable and in fact is changed from time to time depending on operating conditions. With proper consideration given these factors satisfactory applications will result. This is borne out by the experience of Alabama Power Company where, in a six year period, there have not been in excess of two failures of reclosing high-voltage breakers which might possibly have been avoided had the substations where these failures occurred been attended and breakers non-reclosing.

A better conception of the application of reclosing high-voltage breakers on Alabama Power Company system may be obtained from brief descriptions of various typical installations. Therefore, descriptions and illustrations are included for one or more examples of each of the following typical applications of automatic reclosing high-voltage oil circuit breakers.

a. To plain radial feeders—44 and 110 kv.

b. To 44-kv. networks fed from one source.

c. To 44-kv. networks fed from several sources.

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[1] Both of Alabama Power Co.

Presented at the Regional Meeting of the Southwest District of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

d. To very long 110-kv. transmission lines.

e. In conjunction with specially designed accessory switching equipment.

These examples follow:

RADIAL FEEDERS—44-KV. AND 110-KV.

The application of automatic reclosing switches to radial feeders is the simplest and most common of all the applications. This type of installation was made on the 44-kv. installation at the North Auburn Primary Substation. All the 44-kv. oil circuit breakers at this station are operated by self-contained, motor driven, automatic reclosing devices. There is also an 110-kv. installation at the Flomaton Primary Substation. Here the breakers are operated by solenoid mechanisms which are in turn controlled by reclosing relays of the induction type. All the circuit breakers installed at the North Auburn and Flomaton Primary Substations are arranged for one minute reclosing intervals and lock out after two reclosures. No unusual maintenance problems have been encountered on these breakers.

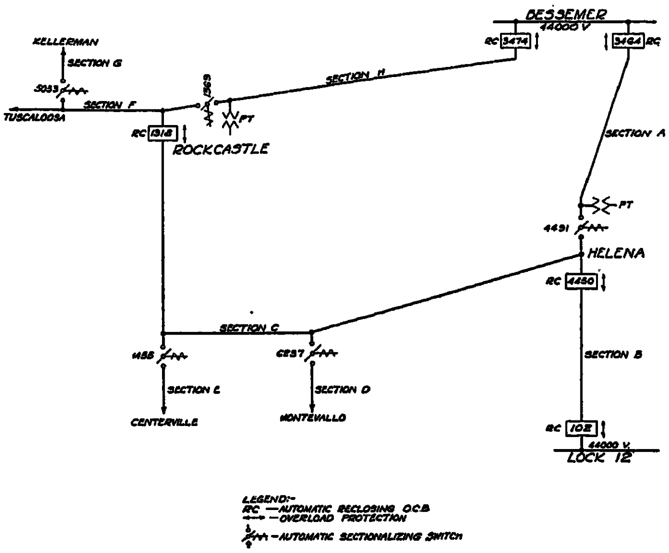

FIG. 1—BLOCTON LOOP AUTOMATIC SWITCHING SCHEME

44-Kv. Network Fed from One Source. The application of reclosing breakers to networks fed from one source presents a more difficult problem, especially when various switching stations are remotely located with respect to the source of power and attended stations. The application becomes even more difficult when it is desired to automatically isolate trouble in relatively small sections of the entire network. A good illustration of this type of network is the Blocton Loop, shown in Fig. 1. This network feeds a great many important customers distributed over a large area. The network is tied together with reclosing oil circuit breakers and numerous automatic sectionalizing switches specially designed for the purpose. These circuit breakers and automatic sectionalizing switches are so arranged and timed that any section of line in trouble is isolated automatically from all the other sections of line and service restored to all the network except the section in trouble. The particular type of automatic sectionalizing switch used in the Blocton Loop network is designed to reduce the number of automatic reclosing oil circuit breakers necessary to secure complete sectionalization, thereby decreasing the initial installation cost. These switches are essentially spring actuated automatic gang operated air-break switches. Each phase includes a high-voltage solenoid connected through an insulating member to an inverse time-limit circuit closing relay. These switches operate in conjunction with special timing relays. Their function is to open a line automatically after it has carried excess current momentarily a given number of times within a given period of time. They are so designed that they cannot open ordinarily when the line is energized, though in some cases it is necessary to add low-voltage relays to make this positive.

The following examples describe the functioning of the Blocton Loop in case of permanent trouble. Transient disturbances are not considered, as they result in no permanent derangement of the Loop.

Assuming trouble in Section *A*, Fig. 1, automatic reclosing oil circuit breakers 3464, 1318 and 4450 open. This "notches up" the timing relays and starts a timing cycle on the automatic sectionalizing switches 1369 and 4491. In one minute all three of the above mentioned oil circuit breakers reclose and open again immediately and the timing relays on switches 1369 and 4491 "notch up" again and start a new timing cycle. After another minute has elapsed, oil circuit breakers 3464, 1318, and 4450 again reclose and again open immediately The timing relays on switches 1369 and 4491 then "notch up" the third time, close their trip contacts and start a new timing cycle. When the trip contacts on the timing relay on switch 4491 are closed the trip circuit is completed through an undervoltage relay connected to a 44,000/110-volt potential transformer and the switch opens. Breakers 3464, 1318 and 4450, which are set for three reclosures (four openings) before lock-out, then reclose. Breaker 3464 opens immediately after reclosing the third time, and locks out, isolating Section *A*. Service is then restored to all other sections through oil circuit breakers 1318 and 4450. Switch 1369 does not open since oil circuit breaker 3474 does not open during the sectionalization and consequently there is always potential on the line between breaker 3474 and switch 1369, thus keeping the trip circuit of 1369 open at the contacts of an undervoltage relay.

Since all the automatic sectionalizing switches used in the Blocton Loop operate the same as those described above, in case of trouble, a detailed explanation of the different cycles of operation of the other automatic sectionalizing switches in the loop is not given in the following discussion but, instead, it is confined to an outline of the breakers and switches which operate to sectionalize trouble in any given section.

When trouble develops in Section *B*, the automatic reclosing oil circuit breakers 4450 and 102 lock-out after the third reclosure (fourth opening) and the trouble is thus isolated. Trouble developing in Section *C* is isolated by oil circuit breakers 4450 and 1318 and automatic sectionalizing switch 4491. Switch 4491 opens after the second reclosure (third opening) of oil circuit breakers 3464, 4450, and 1318. Oil circuit breakers 4450 and 1318 and 3464 reclose again and 4450 and 1318 open and lock-out after the third reclosure (fourth opening), thus isolating the fault in Section *C*. Breaker 3464 restores service to Section *A*. Trouble in Section *D* is isolated by the automatic sectionalizing switch 6237 after the first reclosure (second opening) of breakers 3464, 4450, and 1318. Automatic sectionalizing switch 1455 isolates trouble on Section *E* with exactly the same operation of oil circuit breakers. Trouble on Section *F* is isolated by automatic sectionalizing switch 1369 and oil circuit breaker 1318. Automatic sectionalizing switch 1369 opens after the second reclosure (third opening) of the reclosing circuit breakers 3474 and1318.

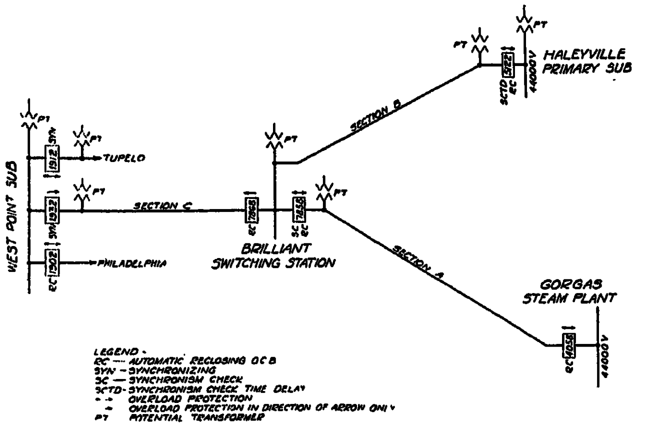

FIG. 2—ALABAMA-MISSISSIPPI AUTOMATIC SWITCHING SCHEME

Breaker 1318 opens and locks out after the third reclosuse (fourth opening) and 3474 recloses and restores service to Section *H*. When trouble develops in Section *G* automatic sectionalizing switch 5033 isolates it after the first reclosure (second opening) of reclosing oil circuit breakers 3474 and 1318. Breakers 3474 and 1318 reclose to restore power to all sections affected except Section *G*. Trouble in Section *H* is isolated by automatic sectionalizing switch 1369 and breaker 3474. Automatic sectionalizing switch 1369 opens after the second reclosure (third opening) of oil circuit breakers 3474 and 1318. Breaker 3474 recloses, opens, and locks out on the fourth opening. Breaker 1318 recloses and restores service to Section *F*.

While this loop is fed from Bessemer and Lock 12 it is considered to have one source of power supply as there is little chance of the two stations being out of synchronism and no provision is made for automatically checking synchronism either at Bessemer or Lock 12.

44-Kv.-Network Fed from Several Sources. The application of reclosing breakers to networks fed from several sources is complicated by the necessity for automatic synchronizing or synchronism checking. Two good illustrations of this type of network are shown in Fig. 2 and Fig. 4.

The Alabama-Mississippi network, shown in Fig. 2, has three sources of power,—Gorgas, Haleyville, and West Point. The power that can be furnished the network from West Point is not sufficient to carry much load in Alabama and the normal power flow is from Alabama to Mississippi. The chief sources of power are from Haleyville and Gorgas.

This network has three special features, all of which are necessary for a complete automatic switching scheme of this kind. These special features consist of an automatic synchronizing device for the control of breaker 1932 at West Point, synchronism check devices at Brilliant on breaker 7858 and at Haleyville on breaker 5122, and a special timing relay at Haleyville on switch 5122. The automatic synchronizing device at the West Point Substation will close breaker 1932 only when there is potential on both the bus and line terminals and then only when both systems are in synchronism.

The synchronism check schemes at Brilliant and Haleyville on breakers 7858 and 5122 consist of a number of relays which allow the breakers controlled to close only when the two sources of power are in synchronism for a period of several seconds. There are other relays used in connection with the synchronsim check schemes which automatically select the breaker across which synchronism is to be checked. For example, if for any reason breakers 4058 at Gorgas and 7858 at Brilliant open, breaker 7858 does not reclose until after breaker 4058 at Gorgas recloses. This scheme then allows synchronism to be checked across breaker 7858 at Brilliant. If, however, Section *B* is deenergized by the opening of breakers 7858, 5122, and 1932, breaker 7858 becomes automatic reclosing and synchronism is then checked across breaker 5122 at Haleyville. The synchronism check scheme on breaker 5122 is further controlled by a timing relay which is the third special feature mentioned above. The function of the timing relay is to prevent breaker 5122 from reclosing until breaker 7858 has restored power to the Brilliant side of breaker 5122, or until breaker 7858 has locked out because of trouble. This timing relay is set to operate for four minutes before closing its contacts. The closing of the timing relay contacts makes breaker 5122 full automatic reclosing. If, however, permanent potential is established on the Brilliant side of breaker 5122 before the four-minute timing cycle of the timing relay is completed, the timing cycle is terminated immediately and the synchronism check scheme functions to close breaker 5122 if the two sources of power are in synchronism. Momentary restorations of voltage across breaker 5122, due to operations of breaker 7858 during trouble, do not affect the timing cycle of the timing relay nor the functioning of the synchronism check scheme on breaker 5122. All breakers in this network are normally closed.

Assuming a permanent case of trouble occurs in Section A, automatic reclosing oil circuit breakers 4058 and 7858 open and clear the trouble. After one minute, breaker 4058 recloses, but opens automatically again immediately. It then recloses again after one minute, opens, and locks out. This isolates Section *A* as breaker 7858 does not reclose, being prevented from doing so by the synchronism check scheme, on account of lack of permanent voltage on the Gorgas side. Had voltage been restored, breaker 7858 would have reclosed.

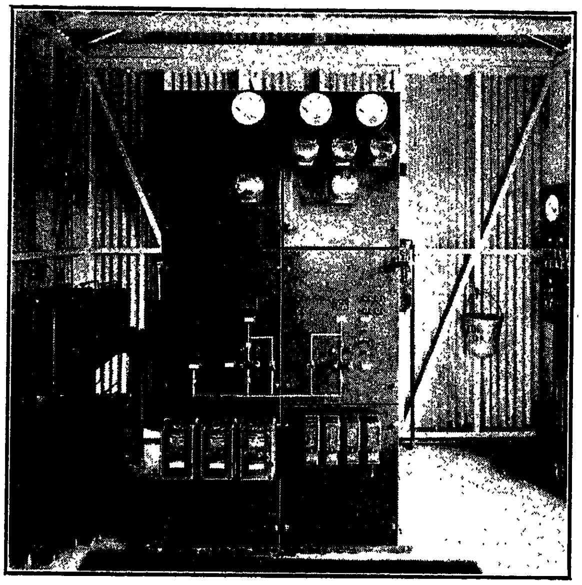

Fig. 3—Brilliant Automatic Switching Station Switchboard

Assuming temporary trouble occurs in Section *B*, breakers 1932, 7858, and 5122 open. Breaker 7858 closes in one minute and restores service to Sections *C* and *B*. Breaker 1932 then synchronizes and closes immediately. The timing cycle on breaker 5122 is terminated as soon as permanent voltage is established from Brilliant. The synchronism check scheme then checks synchronism and closes breaker 5122, thus restoring the network to normal. In case of permanent trouble in Section *B*, breakers 1932, 7858, and 5122 open. Breaker 7858 recloses and opens twice and locks out on the third opening. The timing relay on breaker 5122 completes its four minute timing cycle and at the end of that time breaker 5122 becomes straight reclosing and locks out after the second reclosure. Since breaker 1932 does not synchronize on a deenergized line, the trouble is isolated.

Assuming trouble in Section *C*, breakers 7868 and 1932 open. If the fault is temporary, breaker 7868 recloses and breaker 1932 synchronizes and closes. If a permanent fault, breaker 7868 locks out after the second reclosure (third opening) and breaker 1932 does not close until potential is again reestablished from Brilliant.

Fig. 3 shows the switchboard and relays at the Brilliant Switching Station. It is interesting to note that all the reclosing mechanisms used in this network, except at Gorgas, are of the type originally developed by the engineers of the Alabama Power Company.

The Montgomery automatic switching scheme shown in Fig. 4 employs the synchronism check scheme and a timing relay similar to that installed at Haleyville on breaker 5122, but is much more complex. Almost the entire Montgomery load is carried through the Gas Plant Substation.

Montgomery is normally served from Upper Tallassee and the Montgomery Primary Substation. All breakers in the network are normally closed except breaker 2808 at Montgomery which is not closed except in emergencies. The automatic reclosing oil circuit breakers 2828, 2868, 2822, 3216, and 3210 do not play any part in the operation of the network and are omitted from the discussion.

Breakers 8656 and 8646 at Upper Tallassee open for trouble on lines designated as Sections *A* and *B*. Breakers 2836 and 2816 are automatic reclosing and lock out after the second reclosure (third opening). If for any reason potential fails on both the lines from Upper Tallassee, breaker 3204, which is non-reclosing, opens on reverse power. The primary substation will then carry the 11-kv. load in Montgomery. When breaker 3204 opens, the 44-kv. bus is deenergized and breaker 2808 closes after a few cycles and breakers 2836 and 2816 both open if not already in the open position. These provisions are made for isolating the power supply from the primary substation since the entire load of Montgomery cannot be carried from that

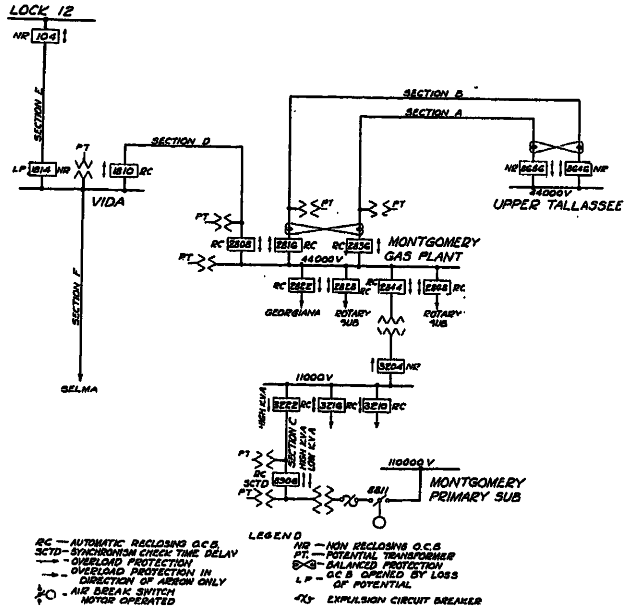

Fig. 4—Montgomery Automatic Switching Scheme

source and the source of power from Lock 12 through Vida is not adequate to carry the Montgomery load and the load on all the 44-kv. feeders. When potential is again restored to the Upper Tallassee side of breakers 2836 and 2816, they again reclose. All switching at Montgomery is brought to the attention of the local organization through a signal system, and after the automatic swiching as described above is performed, an operator goes to the station and closes breaker 3204 and opens breaker 2808, thus restoring the station to normal.

Any trouble in the transformer bank at the Gas Plant Substation is cleared by breaker 2844, which locks out after the second reclosure (third opening) and breaker 3204, which is non-reclosing.

A fault in Section *C* is isolated by automatic reclosing breakers 3222 and 8906. Breaker 8906 is equipped with a synchronism check scheme and a timing relay as explained in the above discussion. Ordinarily, if not interfered with by the synchronism check scheme or timing relay, breaker 3222 closes in one minute. If the fault is temporary, the four minute timing cycle of the timing relay on breaker 8906 is terminated as soon as permanent voltage is established from the Gas Plant Substation. The synchronism check scheme then checks synchronism and breaker 8906 closes, thus restoring the network to normal. If the fault is permanent, breaker 3222 locks out after the second reclosure (third opening). The timing relay on breaker 8906 then completes its four minute timing cycle, and at the end of that time the breaker becomes straight automatic reclosing and locks out after the second reclosure (third opening). It is noted that breaker 3222 opens on overload toward the primary substation. This breaker is set to open at a higher kv-a. and longer time than breaker 8906 for a power flow toward the 110-kv. bus. This allows breaker 8906 to open in advance of breaker 3222 in case the 110-kv. line is deenergized, or when trouble occurs in the primary substation. Under these conditions, breaker 8906 does not reclose until power is again applied to the 110-kv. line and then only after synchronism is checked. Breaker 8906 is also opened by overload relays set for relatively high kv-a. and operates only on trouble in Section *C*.

When trouble occurs in Section *D*, breaker 1810 opens on overload. Breaker 2808 immediately closes on failure of potential on Section *D*. If trouble is only temporary, power is restored by breaker 2808, and in one minute, breaker 1810 recloses. An operator then goes to the Montgomery Gas Plant Substation and opens breaker 2808 to restore the system to normal. If, however, trouble persists after the first and second reclosure, then both breakers lock out after their second reclosure (third opening) and the trouble is isolated in Section *D*.

When trouble occurs in Section *E*, breaker 104 opens and clears the trouble. This opens breaker 1814 on failure of voltage. Breaker 2808 then closes and restores service to Sections *D* and *F*.

When trouble occurs in Section *F*, the same operations take place as when trouble occurs in Section *E*, but the operator at Lock 12 is able to restore service to Section *E* when breaker 1814 opens upon failure of bus voltage. When breaker 2808 closes, it provides a new source, so that breaker 1810 may open three times and lock out, leaving Section *D* supplied through breaker 2808.

None of these extensive applications of reclosing breakers has developed any particular or fundamental difficulties in reclosing high-voltage circuits.

VERY LONG 110-KV. TRANSMISSION LINE

A very good example of the application of automatic reclosing oil breakers to very long 110-kv. transmission lines is the Jordan Dam-Hattiesburg line shown in Fig. 5. This line is 231 mi. in length, and has four 110-kv. oil circuit breakers in series, two of which are automatic reclosing. It serves a total of seven primary substations, but inasmuch as only four of these are of interest from the standpoint of the application of reclosing oil breakers to 110-kv. lines, only four of them are discussed.

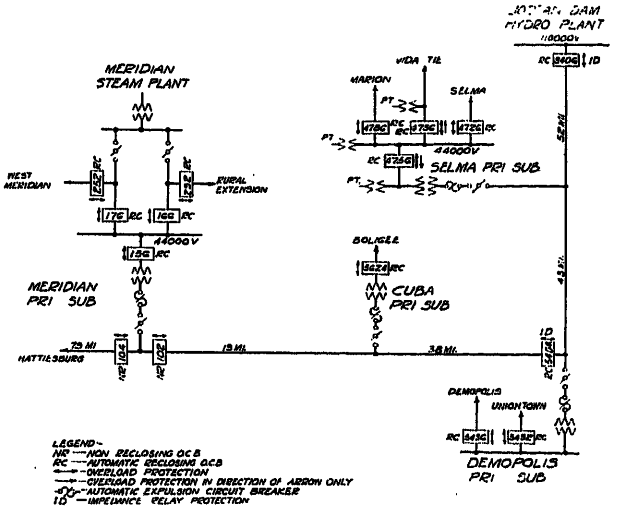

FIG. 5—JORDAN DAM-HATTIESBURG-MISSISSIPPI 110-KV. LINE

The Jordan Dam hydroelectric plant is the normal source of supply. Breaker 8406 at Jordam Dam is automatic reclosing and is equipped with impedance relays which depend on both the line current and voltage drop in the preceding section of line for their operation. This breaker is also equipped with overload relays for back-up protection.

The Selma Primary Substation includes four 44-kv. automatic reclosing breakers which lock out after their second reclosure (third opening). This station has a source of power from Vida, hence the transformer bank breaker 4756 is equipped with reverse power protection in addition to overload protection. The normal source of power is from the 110-kv. line and the automatic reclosing breaker 4736 is normally open. In addition to overload relays, this breaker is equipped with reverse power relays for opening the breaker when power flows toward Vida. When the power supply fails on the 44-kv. bus, breaker 4756 opens and breaker 4736 closes and restores power to the 44-kv. bus from Vida, unless there is a fault on the 44-kv. bus, in which case it locks out after the second reclosure (third opening). When voltage is again restored to the source side of breaker 4756 it closes and breaker 4736 is opened, thus establishing normal service to the load. The reclosing oil circuit breakers 4786 and 4726 control straight radial feeders.

Demopolis is 95 miles from Jordan Dam. Breaker 5404, which is in the main line, is automatic reclosing and locks out after the second reclosure (third opening). This switch isolates trouble between Demopolis and Meridian and is equipped with impedance relays and with overload relays for back-up purposes. The low-voltage side of the bank is protected with automatic reclosing oil circuit breakers 5436 and 5432, both of which control radial feeders.

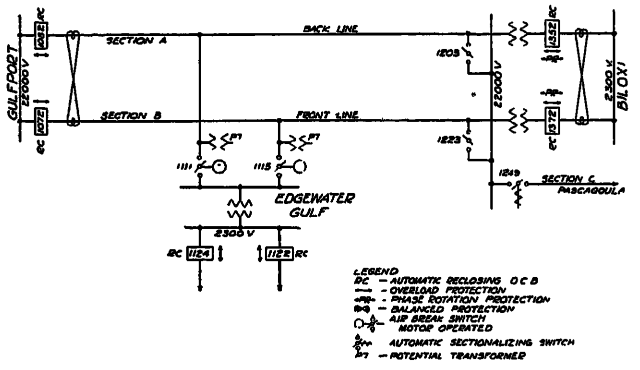

Fig. 6—Gulfport-Biloxi Automatic Switching Scheme

The Cuba Primary Substation transformer bank is protected by one 44-kv. automatic reclosing oil circuit breaker which is set to lock out after the second reclosure (third opening).

Breakers 102 and 104 at Meridian are equipped with overload protection, but no reclosing features. There are several 11-kv. automatic reclosing breakers installed in the Meridian Primary Substation on the radial feeders.

In Conjunction with Specially Designed Accessory Switching Equipment

The use of automatic reclosing high-voltage oil circuit breakers in conjunction with specially designed accessory equipment is common on the Alabama Power Company and affiliated systems. Fig. 6 covers an interesting application at Gulfport, Mississippi.

The automatic switching scheme described includes three distribution substations,—Gulfport, Edgewater, Gulf Hotel, and Biloxi,—with two lines connecting Gulfport and Biloxi as shown in Fig. 6. The source of power is Gulfport and the network is designed to eliminate interruptions, in so far as possible, to Edgewater Gulf Hotel and Biloxi. The loop is normally closed at Biloxi, through the two transformer banks and their low-voltage automatic reclosing breakers 1372 and 1352. A transfer bus at Biloxi allows Pascagoula to be served from either the "back" or "front" line through the gang operated air-break switches 1203 or 1223. Normally, switch 1223 is open and switch 1203 is closed. The line to Pascagoula is equipped with automatic sectionalizing switch 1249, which isolates trouble on that line after the "back" line terminal breakers 1352 and 1082 have opened twice within a one and one-half minute period.

The Edgewater Gulf Hotel normally is fed from the "back" line through motor operated air break switch 1111. In case of trouble on this line, switch 1111 opens and motor operated air break switch 1115, which is normally open, closes at the end of 30 sec. and service is restored to the Edgewater Gulf Hotel. When permanent potential is again restored to the "back" line, switch 1111 automatically closes and switch 1115 opens, thus restoring the network to normal again. The Edgewater Gulf Hotel does not suffer an interruption in the transfer from the "front" to the "back" line. The automatic switching scheme at the Edgewater Gulf Hotel does not transfer from a deenergized line to a deenergized line. Neither does it transfer from an energized line to a deenergized line. If, for any reason, both lines become deenergized simultaneously, then the switches correctly transfer the load to the first line to become energized.

The proper sequence of the automatic switching operations of the loop is better explained by assuming individual cases of trouble on the three different lines. If trouble develops on the "back" line, breakers 1082 and 1352 open. In 30 sec. motor operated air-break switch 1111 at Edgewater Gulf Hotel opens and 1115 closes, providing there is power on the "front" line, thus restoring power to Edgewater Gulf Hotel. One minute after their initial opening breakers 1082 and 1352 reclose. If the trouble is temporary, power is thus restored to Sections *A* and *C*. One and one-half minutes after breakers 1082 and 1352 reclose, switch 1111 at Edgewater Gulf closes and switch 1115 opens, thus restoring the loop to normal. If, however, the trouble is of a permanent nature, breakers 1082 and 1352 lock-out after their second reclosure (third opening) and the Edgewater Gulf Hotel load under these circumstances is not transferred from the "front" line to the "back" line until the fault is removed and power is restored to the "back" line.

Assuming a permanent case of trouble on the "front" line, both the automatic reclosing oil circuit breakers 1072 and 1372 act as straight automatic reclosing breakers which lock out after the second reclosure.

When a permanent fault occurs on the line to Pascagoula, the same automatic switching takes place as when trouble occurs on the "back" line except the automatic sectionalizing switch 1249 isolates the trouble after the second opening of breakers 1082 and 1352 and service is restored to the "back" line by the reclosing breakers 1352 and 1082, after which the Edgewater Gulf Hotel is transferred to the "back" line as previously explained.

CONCLUSIONS

There is nothing experimental about automatic reclosing of high-voltage circuits. Selection of breakers and choice of reclosing duty cycles should be made separately for each application, based on the various factors outlined. Highly reliable reclosing relays or reclosing devices are now available from various manufacturers. While attendants at stations and substations are made unnecessary by automatic reclosing of circuits, extremely careful and regular inspection and maintenance of breakers and reclosing equipment are essential. This is vastly more important than in the case of attended stations or substations. Even at points where attendants are stationed it is sometimes economical and desirable to install reclosing equipment, either to improve service or to reduce attendance.

Discussion

R. J. Wensley: This paper gives a brief historical résumé of automatic reclosing and to avoid the possibility of a false impression from that history I should like to give just a few facts I happen to know from my own personal experience.

The first use of periodic reclosing for service restoration, so far as I know, was quite a while prior to Mr. Spurgeon's experience. We installed a periodic reclosing equipment at Lancaster in 1916, built under the patents of F. E. Ricketts of Baltimore, Maryland, and a little later another type.

Then in 1919 was developed the Type *G R* periodic reclosing relay which, with very minor modifications, is still the standard of the Westinghouse Company. The first specification of that was made in 1919 for the Guggenheim Copper interests, and in the same year for the Pennsylvania Power and Light Company at Allentown.

The Alabama Power Company and their corps of engineers have done a most excellent job of applying automatic reclosing equipment to complicated networks, and in fact they lead the entire country in such application. It would, however, be unfair to the history of the art to say that the principal manufacturers just started the development of these devices in 1922.

At the present time this motor-driven relay is in very extensive use in this country and abroad. One of the latest developments might be of interest. In this development ordinary d-c. solenoids are employed for opening a-c. breakers. The direct current is supplied by means of Rectox copper-oxide disk rectifiers connected to the a-c. system. This eliminates the more complex a-c. equipment and allows use of the simple d-c. solenoid in connection with the *G R* relay.

That emphasizes something that the principal manufacturers are always striving for,—the use of standard apparatus. The development of special devices for specific purposes is frequently very expensive and when we can take a standard circuit breaker without alteration, and apply a few special pieces to enable automatic reclosing, or any other special function, we feel we have gained something for the industry.

Bare Wire Overhead Distribution Practise

BY M. C. MILLER[1]

Non-member

Synopsis.—An appreciable saving may be made in construction of overhead distribution systems by the use of bare conductors in the place of weatherproof conductors. The reliability of the system will be considerably increased with lower maintenance and operation costs. Bare conductors have practically unlimited life and when removed can be reinstalled as new wire resulting in a reduction of depreciation.

This paper outlines the advantages of using bare wires and gives also advantages of medium-hard-drawn as compared with annealed conductors.

FOR the purpose of furnishing the electrical industry with data concerning the use of bare wire and to assist in choosing between bare and insulated wires for overhead distribution systems, this paper was prepared. It shows relative costs of construction, maintenance, and depreciation with condensed data relative to unit weights and costs, and cost of lines and systems with bare and weatherproof wire.

To give a comparison of cost for conductors of the sizes most frequently used in distribution circuits the following costs per 1000-ft. of bare and weatherproof wires are calculated assuming bare at 20 cents and weatherproof at 22 cents per pound.

No. 6 Solid W. P.	$24.64	No. 6 Solid bare	$16.00	Saving $8.64	35%
No. 4	36.08	No. 4	25.20	10.88	30%
No. 2 Str. W. P.	59.04	No. 2 Str. bare	41.00	18.40	31%
No 1/0	93.28	No. 1/0	72.40	20.88	22%
No. 4/0	176.00	No. 4/0	130.60	45.40	26%

An average saving on a number of distribution systems would be about 25 per cent in cost of copper conductors if bare wire were used instead of triple braid weatherproof wire.

The installation costs of bare wire as compared to triple braid weatherproof do not vary directly with the weight. On short extensions of one or two or three spans the labor costs would be about equal, with some savings up to 15 per cent or 20 per cent on longer lines where the lesser weight of bare wire saves on transportation, splicing and setting up reels, etc.

Construction men who have been installing and maintaining bare copper conductors for lighting and power secondaries, series street lighting circuits and primaries, 2300-volt to 12,000-volt, express a very definite preference for it over the weatherproof wire.

To operate satisfactorily it must be put up right, sagged correctly, and guyed sufficiently, but once put up right it does not stretch and cause excessive sag which periodically must be pulled up or re-sagged, resulting in a large saving in maintenance costs.

No increase in horizontal spacing is necessary as the smaller sag required for bare conductors reduces the possibility of their swinging together. It has been found desirable, however, to provide additional vertical spacing where bare conductors are installed on vertical secondary racks and 12-inch separation eliminates any possibility of conductors getting together. Eight-inch spacing has been used in many places with no report of trouble from this cause. This increased spacing uses up a foot more of pole height but very seldom requires a five foot higher pole.

The life of weatherproof wire is comparatively short and it is often necessary to replace it with new wire after 10 to 20 years in service. The life of bare copper wire is unlimited. When weatherproof wire is removed from service after 10 to 20 years it usually has to be scrapped as the weatherproofing is deteriorated to such an extent that it is not practicable to reinstall it. Bare copper wire removed is worth practically the same as new wire and may be reinstalled. These two features alone will justify the use of bare copper wire.

In many communities there is an ordinance requiring the use of insulated wires and weatherproof wire is often accepted by inspectors in such communities as meeting these requirements. When such ordinances exist steps should be taken to have them changed. The word insulation might be assumed by the courts to mean insulation sufficient to protect against voltage carried by the conductors. This was practical when the industry was in its infancy and many of the existing electrical ordinances were made at that time. Bare wire has been used in many instances even where insulated wire is required. This has been permitted as it is realized that it is impractical to comply economically with such an ordinance and install wires insulated for voltages from 2300 to 13,000 volts.

Comparative estimates for constructing an 11-kv. distribution system in a small town to serve 60 customers show a saving of 9.5 per cent in total construction cost by using bare in place of weatherproof wire. The saving in cost of conductors is 30 per cent, poles 9.4 per cent, and labor 5 per cent. An estimated cost of a mile of No. 4 weatherproof medium hard drawn copper three-phase, 2300-volt line using 35-ft. poles spaced 200 ft. is $1625.00; No. 4 medium hard drawn bare using 35-ft. poles spaced 300 ft. is $1133.00, or a saving of $492.00 or 30 per cent in total cost of bare as compared to No. 4 weatherproof conductors. To summarize the above advantages in using medium hard drawn bare conductors for general distribution construction:

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[1] Distribution Design Engineer, Texas Power & Light Co., Dallas, Texas.

Presented at the Regional Meeting of the South West District of the A.I.E.E., Dallas, Texas, May 7-9, 1929.

Saving in initial investment cost of conductors may be from 15 per cent to as high as 30 per cent; poles and attachments 5 per cent to 10 per cent; labor 5 per cent to 20 per cent; with a total saving of from 15 per cent to 30 per cent on rural lines and 5 per cent to 10 per cent on complete distribution systems.

Yearly maintenance costs are 15 per cent to 25 per cent less for the reason that conductors do not have to be resagged frequently and there are less outages caused by breaks and conductors swinging together and burning down.

Life of conductors is conservatively three or four times that of weatherproof.

Practically no depreciation occurs in conductors; when salvaged they have practically the same value as new.

Hazard to linemen is not increased as they are necessarily more careful. Hazard to general public is greatly reduced as lines do not break as frequently.

General appearance is greatly improved.

Medium hard drawn has decided advantages over annealed copper wire, having greater strength allowing greater spacing of poles, and considerable savings in construction cost.

The use of bare medium hard drawn copper for overhead distribution systems conductor is worthy of consideration. It has been used on a number of systems in small towns and cities for a sufficient length of time to prove its worth.

ADVANTAGES OF MEDIUM-HARD-DRAWN OVER ANNEALED CONDUCTORS

In designing or replacing distribution lines another point arises which is worthy of consideration, namely the economy of using medium-hard-drawn instead of annealed wire. Medium hard drawn copper conductors have many advantages over annealed wire, principally on account of the greater strength, being about 25 per cent in No. 1/0, No. 2/0, and No. 4/0 sizes and 35 per cent to 45 per cent in No. 8 to No. 2 sizes. This increase in strength in the smaller sizes allows an increased spacing of poles in urban and rural districts, and in densely built up districts provides an added factor of safety which is very desirable as it affects hazard to the general public as well as reliability of service and maintenance costs.

There is a very slight difference in electrical characteristics, the resistance of medium hard drawn being 1 to 1.5 per cent greater than that of annealed copper wire, which is negligible in distribution system calculation.

Joints in or taps to medium hard drawn copper wire used for overhead lines should not be soldered because annealing of the copper and decreasing its strength would result. There is a number of solderless connectors available. The cost of these installed is usually less than that of a good soldered connection. Various types have been in use for a number of years and no trouble has been experienced from these connectors working loose and causing loose connections. They are more flexible then soldered connections and are of great assistance in replacing conductors, disconnecting taps, services, etc.

The increased working tensions of medium hard drawn copper conductors require more adequate guying to eliminate possibilities of anchors and guys creeping and conductors becoming slack.

Discussion

G. A. Mills: The use of bare conductor for distribution has received a great deal of attention. Mr. Miller and his associated engineers are probably the leaders in this section of the country in the use of bare wire.

I think we have to be careful in the application and rather divide the distribution system into several sections. For instance, rural sections and small towns should have one standard of construction, and the larger communities have another problem. We have found in our larger cities especially, we cannot take advantage of the longer pole space. Narrow lots and the density of houses require a certain pole spacing to take care of the service. Therefore, I fail to see in some of the cases that we can make or save any money on the pole-spacing feature.

I personally am in favor of insulated wire for secondary. I base this on experience in the loss of transformers due to contact between the conductors and on account of the legal requirements, feeling that ordinary weatherproof will offer enough mechanical separation to give insulating value. On primary lines I agree with Mr. Miller, on account of the fact that only under unfavorable circumstances will the insulation give protection in case of a fallen line, which is our greatest hazard.

The companies I am associated with have been experimenting with this problem. They are using a hard-drawn weatherproof wire to try to get half way between the soft-drawn and bare-wire situation. With hard-drawn wire we feel we can maintain our clearances substantially as discussed by Mr. Miller.

We are also experimenting with a lead-oxide covering. It is too early to prophesy what we will find out but according to the theory presented lead oxide will reflect the violet ray which is responsible for the deterioration of the compound which holds the fiber together to make up the insulation and mechanical separation.

J. E. Rose: In the country along the Gulf Coast we have salt foam, shell dust, and dirt of various kinds blowing over the wires and there is no insulation that will hold.

We have been using bare wires for quite a while. We have found the main trouble with the bare secondaries is that small boys occasionally throw pieces of wire into them. We have never had any other case of trouble with bare wire. On our field work of a very temporary nature we are using bare weatherproof triple-braided wire because that will stand up for about six months to a year, and the men are offered some protection by it.

We have used bare wire for all our primary lines. Our men are told that any wire that has no insulation is unsafe and we make them all safe by keeping them bare.

J. B. Thomas: With regard to the advantages and disadvantages of using bare wire, this is primarily an economic problem. We must first, however, be sure that we do not, by the use of bare wire, increase the hazard to either the public or our employes.

Some four years' experience has shown that with proper spacing, proper guying and construction methods in general, you can make bare-wire installation just as safe, in my judgment, as the weatherproof-wire installation can be made. You are working on the very sound principle that a man may take hold of a weatherproof wire thinking that the insulation has value and be seriously injured but the bare wire speaks for itself as being a thing that must be handled carefully and with discretion. We find that our accidents to employes, as nearly as we can tell, are lessened rather than increased by the use of bare wire and so far, it appears that accidents to the public in general have been materially lessened because the construction is fundamentally stronger and the job generally better done.

Having taken care of safety requirements, then, we come back to the basic economics of the problem and Mr. Miller's paper dwelt largely on the first-cost features. Those are important and we are all working on lowering the first cost of all our construction.

In addition to the first cost there is the cost of upkeep. The serving of the average residence customer results in costs which are principally maintenance, operation, accounting, and service matters so that the cost of energy supplied is a small portion of the total cost. Experience shows that often in the life of an ordinary distribution system built with weatherproof wire you have to go back and pull slack and take up the stretch. You can get around that to a large extent by using weatherproof hard-drawn wire.

Also about every ten years in this section of the country it is necessary to take down a large part of the insulated distribution conductor because the insulation has become loose and hangs down in shreds and carries rain water from one conductor to the other which makes trouble. It is quite general practise to figure on a substantial rebuilding of weatherproof wire distribution systems in, say, ten years. But on a properly constructed transmission line where bare wire of adequate strength is used, there is very little cost arising out of conductor maintenance.

This is a question which has to be worked out carefully and there are many problems. In my organization we have had some four years experience in bare-wire distribution, some in cities of 50,000 and 60,000 people, without substantial troubles. We are installing our distribution systems at lower costs and the very considerable mileage already installed is being reflected in a decreasing curve of the cost per mile of maintaining distribution lines and services.

S. M. De Camp: I should like to ask what effect crystallization would have on hard-drawn wire. Will the wire be crystallized by swinging under the force of the wind?

M. C. Miller: We have had no trouble from that cause so far. We are using the medium-hard-drawn wire rather than the real hard-drawn.

A. C. Scott: It would seem to me that the use of bare copper wire for the primaries might be very advisable but in considering the use of secondaries where there are numerous trees, for example, and opportunities for short circuiting, I should think bare wire would be likely to cause trouble.

In connection with damage suits I have always found it difficult to convince the legal profession that the weatherproof insulation was placed on the wire for mechanical reasons rather than for protection. They always take the position that it should be a complete insulator. I think that if the primaries were universally of bare wire the question that always arises in that respect would be well answered to begin with, as it now is for high-voltage circuits.

M. C. Miller: In some of the towns in which we have distribution systems there is a large number of trees. The secondaries are not far below the primaries and it is necessary to keep the primaries above the trees. So we trim the trees, if necessary, or put in higher poles so we can get the secondaries also above the trees. With secondaries run through trees, especially if they are dense, weatherproofing doesn't last. Where we have gone through small trees, with increased spacing of 12 in. between conductors, the conductors don't get together as readily as weatherproof would. There is a little less slack in them and it is a little harder to pull them together. We haven't had much trouble, but where possible we keep them out of the trees.

Interconnection in the Southwest

BY GEO. A. MILLS[1]
Member, A. I. E. E.

Synopsis.—Interconnected electric service in the Southwest has kept pace with the country as a whole. Eight states are now connected by a continuous transmission system, operated at a voltage of 60,000 volts or more. This system extends 720 mi. from East to West and 810 mi. from North to South. A description of the system is given in this paper.

A few years ago the supply of electrical energy in the Southwest centered around the larger communities, obtaining such supply from steam plants located within the boundaries of the city. The nearby communities were served by moderate voltage transmission lines radiating from these plants.

In the arid sections of the district where condensing water is scarce, oil engines were used extensively as a basis of power generation and for load development.

In the past few years development has progressed with the expansion of existing power plants and the extension of transmission networks around the larger urban areas. In the northern section of the district this development has practically covered a majority of the territory; however, in the southern portion there still exist many scattered small plants and low voltage transmission systems.

The great industrial development which started a few years ago and is now under way, has brought about a change. The small plants and low voltage networks were not only unable to furnish the capacity needed but were unable to serve economically the large industries demanding a power supply. This brought about a consolidation of the smaller properties into larger operating groups, the constructing of higher voltage transmission lines, and the development of larger base load plants.

Such development, in providing an ample supply of economical energy, has stimulated general expansion in all lines of activity such as agriculture, by providing an energy supply for irrigation pumping, gins, oil mills, creameries, condenseries, cheese factories, and general service to the farm; refrigeration; mining, smelting, and quarrying; saw mills, paper mills, and other wood product industries; the oil industries, in the electrification of drilling and well pumping equipment, refineries, pipe line pumping stations and compressor stations; and last but not least, has improved the standards of city life in industrial, commercial, and domestic activities.

The general territorial growth has been so rapid that at the present time the larger operating groups have covered the intervening territory so the border transmission lines have been interconnected, thus bringing about the present interconnected network. These interconnections have been a natural procedure since the operating executives and transmission engineers have in most cases selected similar distribution voltages from sixty to seventy thousand volts with the idea of interconnection in the future. This selection was made necessary on account of the great distances between load centers and the scarcity of favorable plant locations near the load centers, requiring a high distribution voltage to take care of the situation properly.

The skeleton transmission map, Fig. 1, shows the present interconnected lines operated at 60,000 volts and higher voltage. The map also shows the location of points of interconnection between various operating groups and the major power stations.

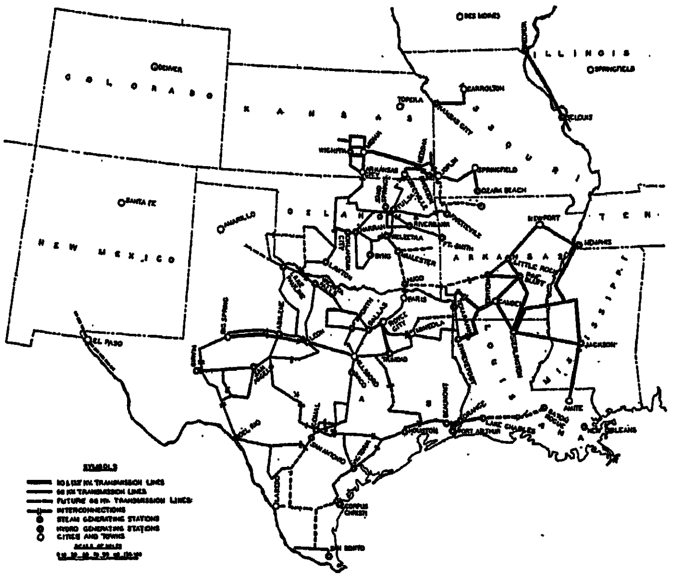

FIG. 1—INTERCONNECTED TRANSMISSION SYSTEM OF THE SOUTHWEST

The great distances between power supply points is evident by an inspection of the map. It is to be noted that the distance of continuous interconnected line stretching from Memphis, Tennessee, to the Ozark hydro plant in Missouri, is 1500 miles. The interconnected territory from east to west is 720 miles wide and from north to south is 810 miles in extent.

INTERCONNECTION CONTRACTS

The first interconnections made in the Southwest were the result of smaller operating companies purchasing energy from the larger companies having excess prime capacity. The contracts covering these interconnections and the rates used therein were based entirely on the purchase of prime energy. During the past two years several new interconnections have been made under contracts wherein it has been desirable to utilize the excess capacity of a company which has installed a large power unit ahead of its own customers' requirements. These contracts provide for the mutual benefits of interconnection in meeting operating emergencies and provide for emergency operation of standby plants to hold total plant investment to a minimum cost.

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Executives and engineers are fast recognizing the desirability of such mutual contracts since they provide a method of keeping the reserve capacity to a minimum and allow the installation of larger, more economical prime movers, resulting in greater economies and better service to the customer than if the developments were made independently.

POWER SUPPLY

Table I lists the principal operating plants in the Southwest connected to one interconnected network. It is to be noted that of 1,415,475 kilovolt-ampere capacity, only 22,500 kilovolt-ampere is hydro.

TABLE I
PRINCIPAL OPERATING POWER PLANTS

Name or location	Capacity in kv-a.	Prime movers
Ozark Beach, Mo.	11,250	Hydro
Riverton, Kan.	72,000	Steam turbine
Neosha, Kan.	50,000	Steam turbine
Wichita, Kan.	37,500	Steam turbine
Harrah, Okla.	81,250	Steam turbine
River Bank, Okla.	28,125	Steam turbine
Byng, Okla.	21,900	Steam turbine
Oklahoma City, Okla.	12,500	Steam turbine
Bell Island, Okla.	17,250	Steam turbine
Sand Springs, Okla.	14,700	Steam turbine
West Tulsa, Okla.	37,500	Steam turbine
Weleetka, Okla.	18,750	Steam turbine
McAlester, Okla.	10,600	Steam turbine
Lawton, Okla.	11,250	Steam turbine
Lake Pauline, Tex.	18,750	Steam turbine
Abilene, Tex.	6,250	Steam turbine
San Angelo, Tex.	12,500	Steam turbine
Girvin, Tex.	22,500	Steam turbine
Wichita Falls, Tex.	13,500	Steam turbine
Leon, Tex.	38,000	Steam turbine
Ft. Worth, Tex.	55,000	Steam turbine
Dallas, Tex.	103,000	Steam turbine
Trinidad, Tex.	50,000	Steam turbine
Waco, Tex.	15,000	Steam turbine
Shreveport, La.	37,500	Steam turbine
Texarkana, Ark.	9,375	Steam turbine
Sterlington, La.	101,850	Steam turbine
Little Rock, Ark.	16,500	Steam turbine
Pine Bluff, Ark.	13,375	Steam turbine
Remmel, Ark.	11,250	Hydro
Memphis, Tenn.	67,500	Steam turbine
Jackson, Miss.	6,875	Steam turbine
Comal, Tex.	75,000	Steam turbine
San Antonio, Tex.	36,125	Steam turbine
Victoria, Tex.	11,250	Steam turbine
San Benito, Tex.	26,250	Steam turbine
Deep Water, Houston, Tex.	125,000	Steam turbine
Gable Street, Houston, Tex.	35,000	Steam turbine
Neches Plant, Beaumont, Tex.	64,750	Steam turbine
Port Arthur, Tex.	9,800	Steam turbine
Sahira River, Orange, Tex.	9,000	Steam turbine
Total	1,415,475	

The largest and most recently constructed plants in this territory have been laid out with the idea of utilizing the interconnected lines in disposing of the excess energy. In several cases single unit plants were operated, using the interconnected lines to other plants as reserve capacity until the load reached a point justifying a second unit. At the present time four such plants shown in the lists are operating on this basis.

The future power supply will be from the expansion of present base load steam plants located where condensing water is ample and fuel economical, and from future hydro plants in west Texas, northwest Arkansas, southern Missouri, eastern Oklahoma, and south Texas. All of these developments are made possible on account of interconnections since the projects offer capacities in excess of local requirements.

The proposed hydro developments are of unusual interest since the lakes created will be utilized for flood control and in south Texas the water impounded will be of untold value in providing water for irrigation downstream from the dams. The largest sites are in the Texas Big Bend country of the Rio Grande, miles away from any industrial development.

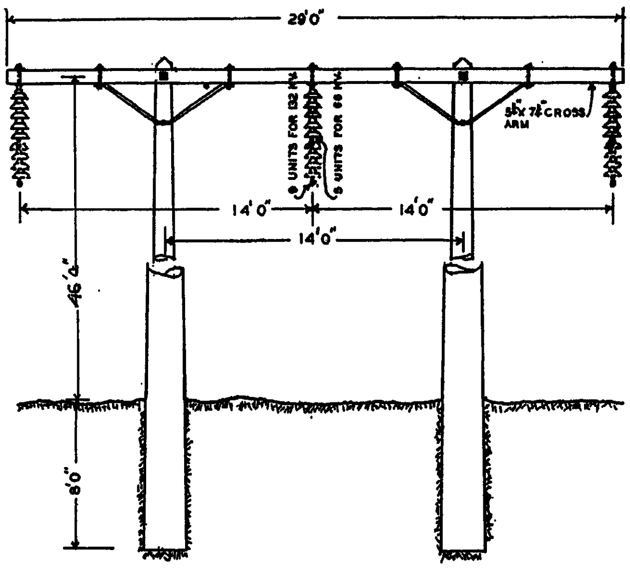

FIG. 2—TYPICAL STRUCTURE USED IN THE SOUTHWEST FOR 66-KV. AND 132-KV. TRANSMISSION LINES

LINE CONSTRUCTION

The present tendency in interconnecting line construction is the use of not less than No. 4/0 conductor supported on H-frame creosoted Southern pine pole structures, spaced for 132,000-volt operation, located on private right-of-way following the shortest path between power supply and load centers or interconnecting points. A typical pole structure is shown on Fig. 2.

Steel towers in the Southwest are only used for special crossings, multiple circuits away from plants and substations, through congested urban areas, and in one or two cases, for double circuit lines.

OPERATION

Successful interconnection operation between several operating companies under different ownership has been accomplished in the Southwest. The total interconnected network is normally operated in four groups, Arkansas, Louisiana, Mississippi, and eastern Texas in one group; the balance of Texas and southwestern Oklahoma in a second group; the balance of Oklahoma and northwestern Arkansas in a third group; and Kansas and Missouri in the fourth group. The largest group is that in Texas where seven large operating companies under five different ownerships normally operate connected.

The rapid industrial growth of Texas has been the result of this interconnected system, especially in the arid regions of west Texas where transmission service is necessary to transmit the energy from distant plants located where circulating water is ample and an economic supply of fuel is obtainable.

FUTURE PROBLEMS

On account of the great distances covered by single circuit or loop transmission lines and the remote location of base load plants, greater dependence on transmission line service is a necessity. The future engineering development must take into account the delivery of energy to the customer at the same general costs experienced elsewhere by increasing the reliability of the present and future transmission systems as well as the development of more economic plants to overcome the longer transmission distance between power supply and consumption.

The engineers in the Southwest are actively at work on these problems and through studies now in progress are keeping pace with similar work throughout the country to determine the proper construction to render the maximum service.

FUTURE INTERCONNECTION

The future interconnections in the Southwest will require greater capacity lines and voltages in the 132,000-volt class to shift the increasing blocks of energy; lines designed with higher flashover voltage characteristics; use of regulating transformers with tap changers which may be adjusted under load and the use of synchronous condensers to deliver the voltage regulation required on such long lines as well as the design of more economic plants, and most important of all, interconnection contracts which will make the maximum economic use of the combined facilities.

The future holds forth more reliable transmission service, increasing number of interconnection points, higher voltage and higher capacity per single transmission circuit, better voltage regulation methods, more steam plants with larger economic units located where circulating water and cheap fuel is ample, large hydro plant developments, and better contractual cooperation in the operation of the combined plants to give the customer the maximum service at lowest possible cost.

Discussion

J. B. Thomas: Perhaps it would be of interest to point out that the density of load on the transmission network in Texas is approximately 1½ kw. for each square mile of the whole territory involved. The cardinal plants listed in Texas aggregate 350,000 kv-a., that is, plants over 5000 kv-a. capacity.

The area of our state is about 254,000 sq. mi. Of course, there are many places of greater load density but with the widely spread load, with limited cooling water, and with fuel supplies not closely spaced, we are primarily up against a transmission problem. Our experience has shown that we can operate in this area to the advantage of customers by cutting down the investment in plants through means of the interconnections which afford reserve sources of power supply and which obviate the necessity of duplicate transmission lines where large areas are dependent almost entirely on transmission-line service.

The determining factor in operating an interconnecting system as large as that of Texas is the good sense and neighborliness of those who own the various properties that interconnect, and it is only by broad constructive thinking and action that this interconnection is made possible, and without the interconnection it would be impractical for this area covered by Mr. Mills' paper to have anything like the power supply available that it now has. We are very proud that in this area a substantial industry can get electric power at whatever point in the area its market and raw material and labor supply indicate that the plant ought to be located.

It occurs to me to mention also that with these long transmission distances, (there are only ten of these larger plants in these 250,000 sq. mi.), our studies show that the availability of service in the area compares very favorably with the availability of service anywhere in the country and is exceeded only by the availability in the metropolitan centers. Out on these long transmission lines where important loads are served, hundreds of miles from generating stations, 99.66 per cent of all the hours that elapse carry with them full electric service on this network.

R. J. Wensley: In Mr. Mills' discussion of interconnection he speaks of the physical and financial advantages, which are unquestioned. It happens that when interconnections become very large and the connection is very intricate, many engineering problems arise. Mr. Mills stressed the need for reliability of service and apparently Texas is getting a very high degree of reliability, but looking for still better.

Many people think that interconnection simply means building transmission lines and connecting them together. There is much more to interconnection than just that. For instance, there is the control of voltage at the various operating centers without causing disagreeable circulating currents or very poor power factor to some of the plants. The manufacturers have been cooperating on the problem of interconnection by building large transformer units with tap changers which can change the voltage ratio under load so that the ideal voltage and power-factor conditions can be secured at every important distribution center. So, in planning for such interconnections do not forget the advisability of voltage control in the transformer units.

Then we have the question of system stability which grows more important with every tie, every increase in the size of the system. There, again, the manufacturers have been cooperating with the development of quick-response excitation systems which will maintain the synchronous operation of generating and condensing equipment so that the systems will not fall out of step under severe surge conditions.

A great deal of very expensive development work is being carried on at the present time. Exciters are being built with totally laminated field structures and voltage ranges up to six or eight times the normal excitation voltage so that tremendous excitation can be thrown on generator fields to secure this kind of stability. So the excitation must be watched to see that adequate means are provided to maintain this desired stability.

Then, tremendous amounts of money are being spent on the development of relay systems to secure effective sectionalization of interconnected systems. It hasn't been very many years since selective timing was considered adequate for all relay practise and the induction disk relay became very prominent because of its ability to secure that selective timing. Today selective timing is a small part of it and the impedance-type relay and many other forms of new relay methods have been developed to permit such successful sectionalizing of the transmission network. So, wherever you plan to tie together two big systems with one or more ties, do not forget to have a careful relay analysis made because if you don't you are liable to find poorer service after you get the tie put in.

One of the most important things in the operation of a big interconnection is the ability of the dispatching center of each of the systems to be able to communicate freely and dependably at all times with the dispatching centers of the other connected systems. No better means of communication exists than the transmission lines themselves, and there again the manufacturers are cooperating by carrying on extensive developments in the guided radio types of communication systems permitting the use of the transmission-line networks for the carrying of the voice of the dispatchers to distant points. When we first started on this work a few years ago it was thought that all that was necessary was to couple a radio transmitter to a line and then hook on a receiver anywhere and we ought to be able to hear. The phenomena of standing waves, of short spurs with their reflection losses, and differences in lengths of connecting tie lines with the resultant phase angles of the received energy were not thought of, and on many a line when a receiver was hooked on and a transmitter was put on the other end, nothing was received and it wasn't possible to find out why.

We have now reduced the analysis of transmission networks to an exact science, and given the precise measurements of transmission systems we can predict in advance by calculation the exact results to be obtained. We had a case not long ago where we applied a certain type of filter choke to a line supposed to be 13.5 mi. long. The thing wouldn't work. We said that the survey was probably incorrect. We ran a frequency run on the line and said it was 12.3 mi. long. The line was surveyed again and proved to be 12.35 mi. long. The electric survey using the variable-frequency method was more precise than the map the utility company was using. So you can see we have reached a degree of precision of such means of communication as to permit of real service. That is another thing to remember when planning your interconnection. Don't forget the necessity for very dependable communication.

G. A. Mills: There are many things which are going on that cannot be presented in a short paper. Mr. Wensley has brought these to my mind. We don't know yet how we shall solve all of them but we hope to solve them soon.

Electrification of Oil Pipe Lines
in the Southwest

BY D. H. LEVY[1]
Associate, A. I. E. E.

Synopsis.—This paper briefly reviews the history of electricity as motive power for oil pipe line operation and notes some of the reasons for, and the results of, its rapid development in the Southwest. Some attention is paid to the method of serving these loads from transmission lines and the tendency of modern pipe line practise is shown. The last five years have seen rapid advances in the methods of underground transportation of petroleum and the paper reviews some of the present tendencies in design of the modern oil pipe line station.

* * * * *

INTRODUCTION

DURING the last two decades, most of the crude oil produced in the United States has been moved to the refineries, ports, and markets through underground steel pipe lines. The oil was forced through these lines by means of reciprocating pumps driven by steam engines or oil engines.

About three years ago, a number of factors developed which allowed, and in fact required, the extensive employment of electric power for pipe line pumping and this system has now proved highly successful.

The chief factor in this adoption of electric drive was the recent discovery of vast quantities of oil in new fields in Texas and Oklahoma. This required immediate and quick construction and operation of more and larger pipe lines.

Electrical pumping was the solution for several reasons. The proposed pipe lines were to pass through rough and dry territory having few highways and fewer railroad facilities. This condition was favorable to the electrical equipment as it is lighter and more easily transported than Diesel or steam equipment. Furthermore, the available water was generally unsuitable for Diesel engine cooling. Steam for heating the oil was not necessary in the moderate climate in this region. Moreover, electrical equipment had given very satisfactory service in other applications in producing oil.

Considering all these conditions, one pipe line company electrified 150 mi. of 10-in. line and the results were very satisfactory. Later, when the immense Permian Basin fields in southwest Texas were discovered, electrical pumping was installed by several companies operating in that area.

One of the companies electrified 400 mi. of 10-in. line, extending from McCamey, Texas to Healdton, Oklahoma. This line is served by the transmission systems of three power companies and at this writing has been in successful operation for six months, meeting all expectations as to continuity of service and economy of operation. A second company built and electrified 125 mi. of line from McCamey to Del Rio, Texas with the same satisfactory results. A third company extended their lines from Crane through DeLeon, Texas to the Gulf, a distance of 650 mi. This line has 19 stations, 14 of which are completely electric-driven, and the remainder partially so. A fourth company has now under construction 450 mi. of 10-in. line with 11 motor-driven centrifugal pump stations and this line has been in operation since January 1, 1929.

The first pipe line company mentioned is now engaged in building a line from McCamey in West Texas to tide-water at Houston, a distance of approximately 425 mi. According to present information, this line is to be motor-driven.

In most of the cases a specially developed centrifugal pump is used. The centrifugal pump has the advantages of non-pulsating delivery, ability to handle oils of various viscosities, ideal characteristics for booster service, and low first cost.

Several years ago, before this type had been developed for oil-line pumping, the only available centrifugal pumps for the deliveries and heads required were boiler-feed pumps. These were tried and it became immediately evident that they were unsatisfactory. Pump efficiencies were 55-60 per cent and this alone precluded their use in oil line pumping. The centrifugal pump manufacturers attacked the problem from all angles and the so-called "high efficiency" type of multi-stage centrifugal pump came into being. New theories of centrifugal pump design were developed; casings and impellers were redesigned; internal losses were analyzed and reduced as far as possible and the mechanical details received much attention. Stuffing boxes and bearings were redesigned, and from it all, came the present oil line centrifugal pumps with higher efficiencies than had even been hoped for just a few years previously.

As to the electrical features of the typical motor-driven pipe line station, there is no departure from modern substation practises. A wiring diagram of a typical station is shown in the accompanying illustration (Fig. 1). Greater attention perhaps has been paid to the construction of flame-proof installations. Some of the companies favor separating the motor from the pump by means of metal fire walls; others install

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[1] Electrical Engineer, Magnolia Petroleum Co., Dallas, Texas.

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them all in one room. It seems that personal opinion of the engineers of each company determines the policy to be pursued.

Squirrel-cage induction motors are used in all cases. Motors have been of all standard voltages but the present trend is toward the use of 2200 volts on all motors above 50 hp. Oil circuit breakers are usually installed on incoming lines and these are provided with the usual overload, undervoltage, and over-pressure protection.

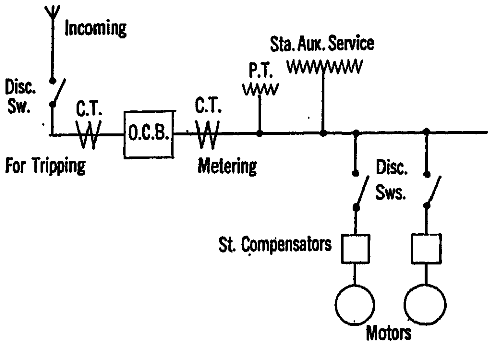

Fig. 1.—Typical Oil Pipe Line Pump Station

Oil line wiring diagram

As to the actual cost of moving oil through pipe lines by means of electric power, it is impossible to give any accurate figures unless all the conditions of service are known. The various factors, such as efficiency of motors, efficiency of pumps, friction of station piping, rate schedules, discharge pressures, etc., all enter into the final analysis.

Below will be found a tabulation of electric pipe line station performance, based on crude oil of 80 Sec. Saybolt Universal viscosity at 60 deg. fahr. and at a discharge pressure of 700 lb. per sq. in.

PIPE LINE "A"

Reciprocating Pumps

Station	Months	Barrels pumped	Total kw-hr.	Barrels per kw-hr.
1	9	7,625,286	1,959,700	3.88
2	9	7,658,579	1,959,500	3.91
3	9	7,633,049	1,829,900	4.17
4	9	7,653,154	1,819,500	4.22

PIPE LINE "B"

Reciprocating Pumps

Station	Months	Barrels pumped	Total kw-hr.	Barrels per kw-hr.
5	6	4,756,768	1,202,900	3.96
6	10	6,538,992	1,634,000	3.98
7	6	4,480,040	1,122,600	4.05
8	10	6,519,459	1,616,100	4.03

PIPE LINE "C"

Centrifugal Pumps

Station	Months	Barrels pumped	Total kw-hr.	Barrels per kw-hr.
1	12	6,761,903	2,618,800	2.58
2	9	7,138,178	1,652,700	4.32
3	12	8,132,583	2,444,600	3.31

To meet the demands on their various generating stations and transmission systems, power companies serving the Southwest have had to make extensive additions, improvements, and extensions. Interconnections have been worked out and the service rendered has been, as a whole, very satisfactory to the pipe line companies.

In laying out transmission lines to serve pipe line loads, the importance of the absolute necessity to maintain continuous service must be considered. Pipe line systems having small ballast or surge tanks at each station do not provide storage capacity sufficient for an appreciable shut-down and a continuous supply of energy must be maintained at all times. To supply this demand, if at all possible, pipe line services should be supplied from a system having a two-way feed, with necessary switching apparatus to quickly sectionalize all transmission line faults.

As a source of constant revenue, pipe line loads are very desirable to the power companies. Revenues are the maximum on account of the high load factor. The average monthly load factor of one of the southwestern pipe lines for the last twelve months has been 86.6 per cent and from present indications it will continue as high for the next four years. These loads are desirable also on account of the ease of service. No extensive distribution facilities are necessary to secure the revenue. In many cases in Texas and Oklahoma, station locations are within sight of the transmission lines. As to the power factor of these loads, if the station is correctly designed, the motors driving the pump should be fully loaded and the power factor should be between 76 per cent and 87 per cent. Certain high-speed centrifugal pump motors, tested under field conditions, showed a power factor of 87 per cent. If the rate schedules contain a power factor clause with a penalty and premium rider, the use of condenser equipment by the pipe line company would be justified in most cases.

CONCLUSION

It is felt that the electrification of oil pipe lines is completely justified by the success of present installations. The time of installation is only half that required for Diesel engine equipment. The operators are easily trained to handle the electrical installations. Their operation on units of a transmission system having several sources of power is just as satisfactory as the present oil engine-driven stations. In general, shut-downs due to transmission line faults are less frequent than shut-downs due to failure of mechanical equipment. Cost of operation compares favorably with oil-engine driven stations. The near future will see many advances in pump construction and in electric pump station design and a consequent increase in over-all efficiencies.

Discussion

W. C. Dreyer: Mr. Levy mentioned that synchronous motors and squirrel-cage motors are used for this application. It seems to me there is a very definite field for the use of the wound-rotor motor. There are operating conditions on pipe lines where they want to run the station at reduced capacity. Much higher efficiency may be obtained at reduced capacity with wound-rotor motors than if the discharge valves of the centrifugal pumps are choked down to reduce the pressure.

Regarding the synchronous motors on centrifugal pumps, I don't believe they will be used as extensively in the future as they have been in the past. When these original major pipe line installations were made there was a very great difference in efficiency between the squirrel-cage motor and the synchronous motor. About the best efficiency on squirrel-cage motors of that size was 91.5 to 92 per cent. During the last year or two, by using silicon iron laminations and special end rings on the rotor, it has been possible to raise the efficiency to about 94.5 per cent. This cuts down the margin of efficiency of the synchronous motor, which was 95.5 or 96 per cent on motors of that size, so that the lower cost and greater sturdiness of the squirrel-cage motor is not compensated for.

On the subject of choice of motors there is a feature that hasn't been mentioned, automatic control of pipe lines. The squirrel-cage motor will be much more adaptable to automatic control when it becomes general, as only a starting operation is necessary. It is a simple matter to operate a station entirely automatically by the pressure in the line or by remote control. Though this is done it will be a long time before the labor at the station will be entirely eliminated. The investment and the combustibility of the fluid will make it very desirable to have one man at a pipe-line station subject to call by an alarm system or some such device.

D. H. Levy: I should like to answer Mr. Dreyer's question about synchronous motors. The nature of the fluid, as he mentioned, is inflammable. The slightest leak at a flange or valve, with 700 lb. of pressure, produces an oil vapor ready to fire. With a standard synchronous motor you have some arcing at the collector rings with attendant liability of an explosion. You also have to have an exciter, increasing the maintenance of another piece of rotary apparatus. You do, however, gain two or three points in efficiency.

To use synchronous motors successfully as applied to direct connected oil pipe line pumps, you should have fire walls between the drive and the driven unit, necessitating two different rooms with packing glands on the shaft between them. You have one operator operating these two stations and he is going to be on one side or the other of the fire wall. He doesn't get a whole view of the situation when he stands at the switchboard or gage board and operating troubles may result, due to the human factor. Our experience has been that we can install induction motors as closely as we want to the centrifugal pump with practically no danger of fire hazard.

Some companies are starting to install control apparatus in separate rooms. That defeats the purposes of inspection or observation. The normal crew of a pipe-line station, electric driven, is four men, chief engineer and three engineers, with no oilers or operators. These stations have no telegraph operators, using the telephone only for dispatching. That leaves one man always on duty.

Our company has gone so far as to try to perfect an automatic control for pipe-line stations controlling centrifugal pumps cut in to the line, by-passed by check valves. One unit is started by a rise in pressure on the incoming line. If the station ahead happens to put another unit on the line to bring up the pressure to 700 lb. (or other final pressure) in turn the pressure will be reflected at the station in question and No. 1 unit will start. It will have a tendency to pull down the suction and increase the pipe-line discharge pressure. This is held for 15 sec., using a timing relay. If conditions are still stable or become so, then No. 2 unit starts and pulls down the suction again to normal. We have four of these in Texas that were not entirely successful on account of the station ahead being a reciprocating station driven by oil-engine units. If one of the oil engines happens to miss fire two or three times, the speed decreases and down goes the pressure and the unit is stopped, then it picks up and comes back again. We try to minimize the surge conditions (at least the starting condition, due to surges), by installing more timing devices. So far we have found these fairly practicable.

The ideal condition would be to have one originating station and a series of booster stations cut in to the lines with only ballast tanks. We do install over-pressure protecting devices on all our electric stations. Prior to the electrification work we had lost many lines due to excessive pressure. If a man happens to switch from one tank to another and forgets to open the valve before he has closed the valve on the line, up goes the pressure to approximately 1000 lb. I have seen as much as 22 ft. of lineal rip in an 8-in. line with consequent loss of time and oil. So we put over-pressure protecting devices set 25 to 50 lb. above normal operating pressure. Pipe lines used for 700 lb. of pressure have an ultimate strength for 1000 lb. The thickness of the walls of the pipe is about ¼ in. Consider the erosion of two or three years and you see how close to the safe limit we operate. The tendency now is to use slightly heavier pipe and higher operating pressures.

As to the use of wound-rotor motors, we would still have the same trouble as to fire hazards. The use of totally enclosed motors prevents the desired inspection. Our company prefers to use the induction motor with automatic and hand starters. We try to limit as far as possible the number of sizes, speeds, and voltages. All except the motors used in oil production work, are now of the standard 2200-volt type.

Lightning Studies of Transformers
by the Cathode Ray Oscillograph*

BY F. F. BRAND[1] and K. K. PALUEFF[1]
Member, A. I. E. E. Associate, A. I. E. E.

***Synopsis.**—Study of transformers has been under way for some time to coordinate the strength of transformers and transmission line insulation under lightning conditions. This study has taken two forms: first, the transient dielectric strength of the line end of the winding and second, the distribution of transient voltages and therefore the stress caused by them throughout the winding.*

Theoretical studies and spark-gap tests of transient voltage distribution in transformer windings have been previously published by the Institute. Since then an extensive study of transient voltage phenomena has been made by the cathode ray oscillograph on power transformers connected to a short transmission line and subjected to artificial lightning waves sent along the line.

The effects of the transmission line, concentrated inductance, and transformer entrance bushings on traveling waves were studied, as well as the effect of traveling waves of various service conditions in producing internal oscillations in ordinary transformers.

The non-resonating transformer was studied under similar conditions.

A striking agreement between the oscillographic records and theoretical conclusions previously published was found, sufficient to establish beyond any doubt the following conclusions:

1. Very high voltage oscillations occur throughout the entire winding of even a grounded neutral transformer.

2. Points of the winding near the grounded neutral may rise to 95 per cent of the crest voltage of a very short traveling wave (three microseconds long).

3. Entrance bushings have a negligible effect on the shape of the incoming traveling wave.

4. In case of sudden voltage changes, concentrated inductance in series with the transformer, unless by-passed by a suitable device, causes rise of voltage across the transformer terminals as well as internally in the windings.

5. Arc-over of line insulators by a traveling wave produces severe oscillations in a transformer the amplitudes of which are roughly proportional to the arc-over voltage of the line insulators.

6. Grading the insulation between high voltage and low voltage and ground in ordinary transformers with grounded neutral is a dangerous practise when the transformers are subject to lightning.

7. All the above conclusions apply to concentric winding core type as well as interleaved and shell type transformers. From theoretical studies the non-resonating type of transformer has been developed and its action checked by tests. This type of transformer eliminates voltage oscillation within the winding and therefore local concentration of transient voltage.

* * * * *

INTRODUCTION

THIS paper, which deals particularly with the effect of lightning and other transient voltages on transformers, is one part of an extensive coordinated investigation covering all of the various phases of the lightning problem which has been under way for a number of years in the high voltage engineering laboratory at Pittsfield. Other papers of this series on other phases of the problem have already been published.[2]

Part I

Lightning, and to some extent other transient voltages such as are caused by switching surges and arcing grounds, have always been a serious source of failures of high-tension lines and apparatus, including transformers.

Transformers are in general reliable pieces of apparatus; in fact, their reliability is probably higher than that of any other electrical apparatus, but a small percentage of failures does take place and the majority of these is probably due to lightning.

This is not only because surges are frequently of very high voltage, but in regard to their effect on transformers and similar apparatus, it is because lightning voltage waves are of steep front, while switching surges and arcing grounds are often of relative high frequency.

The effect on transformers of high-frequency oscillations and steep front waves, are to some extent similar and a discussion of the effects of lightning in a measure covers the effects of other high-voltage transients.

Previous studies of these phenomena have been either theoretical or tests by spark-gap measurement. Since the cathode ray oscillograph is a new tool which enables one to record values almost instantaneously and obtain complete wave forms of phenomena lasting a few microseconds, it seemed desirable to use this device for such a study.

Part II

LIGHTNING FAILURES OF TRANSFORMERS

Transformer failures sometimes occur directly through the major insulation from high-voltage winding to low voltage or ground, either near the terminal or in the main body of the winding.

Failures are more liable to occur between turns or coils of the same winding or in auxiliary parts connected to the winding, such as taps, tap terminal boards, or ratio adjusters.

Failures which occur through the major insulation near the line terminal are due to excessive voltage applied to the transformer.

Failures occurring internally in the winding or auxiliary parts are due not only to the voltage to which

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*With an Appendix, *Present Status of the Cathode Ray Oscillograph*, by H. L. Rorden and J. C. Dowell.

[1] Both of the General Electric Company, Pittsfield, Mass.

[2] F. W. Peek, *Lightning*, A. I. E. E., TRANS. Vol. 43, p. 1205, Vol. 47, p. 1131; Vol. 34, p. 1857; A. I. E. E. Quarterly TRANS., Vol. 48, April 1929, *Frank. Inst. Jl.*, Feb. 1925.

Presented at the Regional Meeting of the South West District No. 7 of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

the transformer is subjected but to the nature of the surge and its effect in the transformer.

It is obvious that irrespective of what might be called the internal effect of the surge on the transformer, some limit must be set to the voltage which may be applied to the transformer. If this is not done failure will occur in any transformer.

Part III

LIMITS OF VOLTAGE ON TRANSFORMER INSULATION

To cause breakdown of any insulation requires energy. It is therefore a question of time as well as of voltage. The shorter the time, the higher the voltage required. Since lightning voltage waves last a relatively short time (from a few to possibly 100 microseconds), the voltage required to break down insulation by lightning is always higher than that required for long time duration of the voltage.

The voltage required to cause breakdown by lightning is higher the greater the energy required for a particular kind of insulation. It is fortunate that the oil and most solid insulations require greater energy to cause breakdown than air, and therefore higher lightning voltage for the same 60-cycle strength.

The shape of the insulation, or rather of the dielectric field, has of course a marked effect on the voltage required to cause breakdown.

If the shape of the dielectric field, or in other words, the potential gradient, changes with varying frequency or wave form of applied voltage, it will cause a variation of the ratio of applied voltages to cause breakdown.

To determine the lightning breakdown voltage of even the simplest insulation structure, requires a knowledge of the wave form of the applied voltage. For this reason, data on lightning breakdown by various authorities are not always comparable since the wave form assumed or used for test purposes may vary considerably.

When lightning sparkover of insulators is referred to it is usually based on data published by F. W. Peek, on the insulator sparkover under artificial lightning produced in the Pittsfield Laboratory, for a wave rising to approximately maximum voltage in one-quarter of a microsecond and tapering down to one-half its crest value in five microseconds. This wave form was used long before any actual measurements of lightning wave form were obtained, and it was assumed to correspond to quite severe lightning. The tests were made in such a manner as to utilize the full wave above the half crest value. Below half voltage this wave is not effective because it then corresponds to the 60-cycle value. This particular wave may be said to have an effective length of 5 microseconds. The effects of waves having effective lengths of 5 to 80 microseconds have also been published.[3]

Field tests have since shown that this assumed wave is remarkably close to that obtained under severe conditions.

Based on this wave form, a string of suspension insulators requires a voltage of approximately 1.8 to 2.0 times the 60-cycle dry sparkover voltage, to cause lightning sparkover. Sparkovers caused by natural lightning have given the same range.

Corrugated or petticoated bushings such as are used for outdoor service, require a lightning voltage from 2.0 to 2.5 times the 60-cycle dry sparkover voltage depending upon design.

Oil, and other solid fibrous insulation such as that used in oil-immersed transformers, require from 2.8 to 3.5 times the lightning voltage compared to 60-cycle one-minute breakdown voltage. These ratios are crest to crest value. Effective or r. m. s. values are not used in dealing with lightning waves.

Giving the ratio of r. m. s. 60-cycle one-minute breakdown of oil and solid insulation against crest value, lightning breakdown by the Pittsfield wave results in a value of 4 to 5.

This ratio does not hold good for the ratio of standard factory test voltage at 60 cycles to the breakdown value under lightning of completed transformers due to two causes:

1. The completed transformer has a margin of safety over the factory test. This margin varies with voltage rating, usually being higher in the lower rated voltage apparatus.
2. The ratio holds for a simple insulation structure and does not take into account change of voltage gradient in the transformer with varying frequency.

It is obvious from the different ratio of lightning and 60-cycle strength of different apparatus, that transformers, for instance, comprising oil and solid insulation, may have a lightning strength equal to the strength of bushings, or line insulators having a very much higher 60-cycle sparkover value. For instance, a transformer for a 115,000-volt circuit given a factory test of 231,000 volts, may be stronger under lightning conditions than line insulators having a 60-cycle dry sparkover for 400,000 volts. This conclusion might be correct as regards the insulation to ground of the line terminal end of the winding. The conclusion would not necessarily be correct for the whole transformer due to the effect of the lightning wave form on the distribution of voltage within the transformer winding.

Part IV

EFFECTS OF TRANSIENT VOLTAGE

If we consider the case of a transformer with one terminal grounded such as Y connected banks with grounded neutral, the normal frequency voltage distribution is a uniform fall of voltage from line to neutral.

In a transformer each turn and coil has capacitance to ground and to adjacent turns and coils.

Under the impact of a traveling voltage of steep front on account of the inductance of the winding the inductance current cannot be established instantly and therefore only the charging current of the winding capacitance to ground will flow at the first moment.

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[3] F. W. Peek, Jr., *Lightning*, A. I. E. E. Quarterly TRANS., Vol. 48, April 1929.

In a simple way the circuit of capacity elements of the winding (as shown on Fig. 1) can be likened to that of a string of suspension insulators and therefore the flow of charging current establishes voltage distribution along the winding, similar to that of the insulator string as shown by curve 1 of Fig. 2.

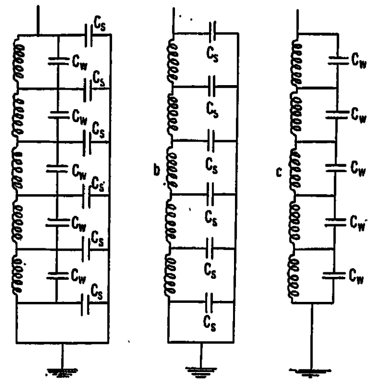

FIG. 1—EQUIVALENT CIRCUIT OF AN ORDINARY TRANSFORMER

a. Complete circuit
b. Circuit effective at low transformer harmonics
c. Circuit effective at high transformer harmonics

This results in a great concentration of voltage on the end coils and turns at the time of impact of the traveling wave. Seventy-five per cent of the voltage may be consumed across 5 to 10 per cent of the winding adjacent to the line.

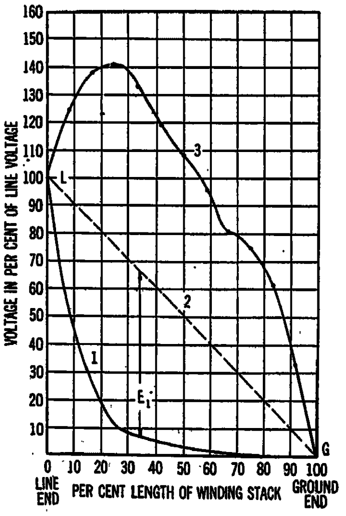

FIG. 2—VOLTAGE PRODUCED BY SINGLE TRAVELING WAVE (CALCULATED)

1. Initial voltage distribution
2. Final voltage distribution (or line of equilibrium)
3. Maximum voltage to ground

With voltage maintained across the transformer constant for a sufficient time (say 50–100 microseconds), the inductance current will be established throughout the winding and with it uniform voltage distribution, as the inductance current, in contrast to the charging current flowing at the first instant of impact of the wave, is one and the same throughout all the winding. The voltage distribution is shown on curve 2, Fig. 2, and is called final.

Since there is a difference between the initial voltage distribution (see curve 1, Fig. 2) and final distribution (curve 2, Fig. 2) a transient will take place during which the voltage distribution will readjust itself from initial to final value. On account of the low damping factor of transformer windings the transient will be oscillatory.

The voltage of each point of the winding between line and ground terminals will oscillate about the point of its final voltage, reaching maximum voltage to ground as shown by curve 3, Fig. 2.

Part V

MECHANISM OF OSCILLATION

The curve of difference between the initial and the final voltage distribution along the winding, gives the initial amplitude of oscillation of different points of the winding and is called the initial space wave or initial voltage distortion curve.

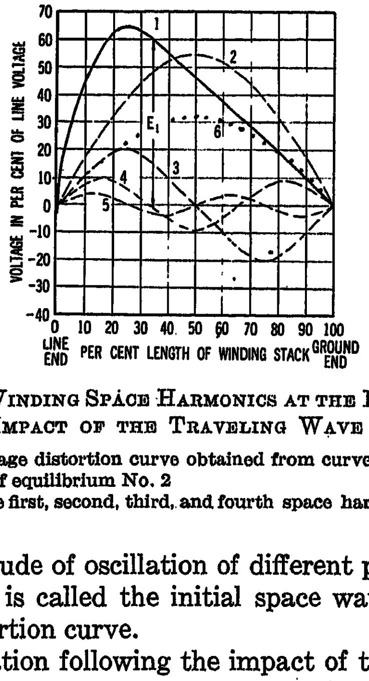

FIG. 3—WINDING SPACE HARMONICS AT THE INSTANT OF IMPACT OF THE TRAVELING WAVE

1. Initial voltage distortion curve obtained from curve 1, Fig. 2 by referring it to line of equilibrium No. 2
2, 3, 4, and 5 are first, second, third, and fourth space harmonics of curve No. 1

The oscillation following the impact of the traveling wave may be looked on as a collapse of this space wave on line *L. G.* with its inertia carrying it on the other side of this line. The line *L G* therefore is the line of equilibrium.

The initial distribution curve is generally of the character of that shown in curve 1, Fig. 2. With such an initial distribution the initial space wave will consist of the fundamental frequency space wave and various harmonic space waves.

When these are all superimposed on one another the result will be the curve of initial voltage distortion or the initial space wave.

Curve 1, Fig. 3, is the space wave obtained on a 6600-kv-a. transformer and is drawn from the initial voltage distribution curve 1 of Fig. 2. It consists of fundamental, second, third, and fourth space harmonics of magnitude shown and of seven higher harmonics of smaller amplitude which are not shown.

The frequency of oscillation of the first few harmonics is approximately proportional to the second power of their order.[4] That is, the second harmonic oscillates four times as fast as the fundamental and the third nine times as fast. Taking the fundamental frequency space wave shown by curve 1, Fig. 4, and any two points of the winding symmetrically located in respect to the middle point of the winding, the difference in potential of the fundamental frequency of each point and the nearest end of the winding is the same in magnitude but opposite in sign. This potential difference will force current from each point to the respective ends of the winding. Thus the current in the halves of the winding will be in opposite direction.

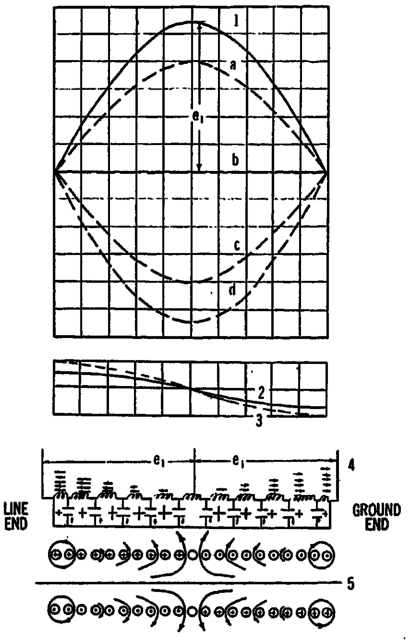

Fig. 4—Current and Voltage of the First or the Fundamental Harmonic

1. Simultaneous first harmonic voltage along winding to the line of equilibrium *L G* of Fig. 2, at moment of impact of traveling wave

a, b, c, d simultaneous voltage to line of equilibrium $^1/_8$, $^1/_4$, $^3/_8$, and $^1/_2$ of a cycle of fundamental frequency later

2. Voltage between adjacent elements of winding stack
3. Current in various parts of the winding during oscillation
4. Voltage and current distribution in equivalent circuit *b* Fig. 1
5. Magnetic flux distribution produced by current No. 3

Curve 2, Fig. 4, shows that the fundamental space wave produces maximum voltage between adjacent elements of the winding, such as turns or coils, at the ends of the winding; therefore the inductance current of oscillation of this wave will be the maximum at the ends as shown by curve 3, Fig. 4. The magnetic fluxes set up by these currents will be as shown on Fig. 4.

Following the same train of thought the curves of voltage distribution produced by other harmonics can be easily delineated. Fig. 5 illustrates the second harmonic.

The examination of these figures reveals that the ratio of the frequencies of the first few space harmonics to that of the fundamental must be nearly equal to the second power of their order.

Where in a transformer series capacitance C_w is not negligible the ratio of frequencies of first three harmonics instead of being 1:4:9 may be 1:3:8 or even 1:2.5:6.5.

The considerations outlined previously and above lead to the following conclusions.

With uniform windings:

1. The impact of a single traveling wave striking the winding causes it to oscillate. The oscillation starts simultaneously throughout the entire winding. It is composed of a number of space harmonics.

2. The higher the harmonic the smaller its amplitude.

3. On account of the symmetry of distribution of harmonics the oscillations cause high voltage not only near the line end but also at many other points including that near the grounded end.

4. The amplitude of oscillation depends upon voltage of the applied wave, the steepness of the initial (or electrostatic) voltage distribution, the steepness of the front of the traveling wave, and the length of the tail of the wave. The damping factor of the winding is not sufficient to prevent the oscillation.

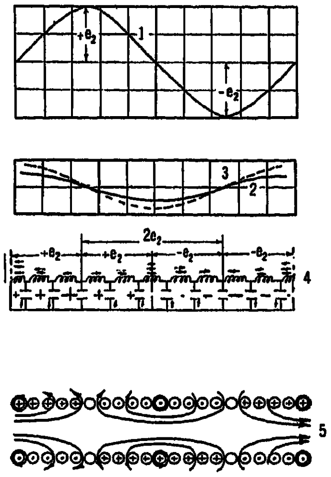

Fig. 5—Current and Voltage Distribution of Second Harmonic

1. Simultaneous second harmonic voltage to line of equilibrium *L G* of Fig. 2 along winding stack at the moment of impact
2. Voltage between adjacent parts of winding stack at the moment of the impact
3. Current in various parts of the winding during oscillation
4. Voltage and current distribution in equivalent circuit *b* of Fig. 1
5. Magnetic flux distribution produced by current No. 3

It is obvious that in non-uniform windings, such as interleaved types, in addition to the oscillation involving the entire winding, local oscillations are present at the points of discontinuity.

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[4] See *Effect of Transient Voltage on Power Transformer Design*, by K. K. Palueff, A. I. E. E. Quarterly Trans., Vol. 48, April 1929.

Part VI

TESTS BY CATHODE RAY OSCILLOGRAPH

In order to check previous theoretical studies of the behavior of transformers under lightning and tests made by spark-gap measurements, a series of tests by an artificial lightning generator in connection with a short transmission line has been made by the cathode ray oscillograph.

Where convenient, several oscillograms were recorded on the same film. In such cases some waves were taken above the zero line, then the polarity of the oscillograph was reversed and the remainder of the waves recorded downward below the zero line. The waves obtained with reversed polarity are marked *R*. The crest value of the wave at the transformer end of the line (see Fig. 20) was considered as unity and crest values of all other waves expressed in per cent of this value.

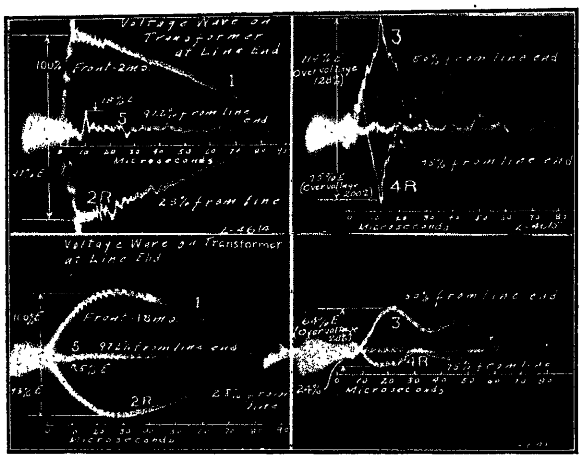

FIG. 6—EFFECT OF FRONT OF TRAVELING WAVE. 6600-KV-A. TRANSFORMER

Top

"Exceedingly steep front" and long tail

1. Voltage across transformer

2, 3, 4, and 5 are voltages to ground at points 97.2, 50, 25, 2.8 per cent away from ground terminal with crest values of 97, 114, 75, and 18 per cent of applied voltage, respectively

Bottom

"Steep front" and long tail

1. Voltage across transformer

2, 3, 4, and 5 are voltages to ground at 97.2, 50, 25, and 2.8 per cent away from ground end with crest values of 93, 64, 24, and 5 per cent of applied voltage

The voltage of any given point of the winding to ground is considered "normal" if it is the same percentage of the crest of the applied wave as it is under normal operating frequency excitation. Excess over this value is called "overvoltage" and is expressed in per cent of the "normal" voltage.

The tests were made on an overhead transmission line over two miles long having a surge impedance of 350 ohms. The following apparatus was used at various times in the tests:

6600 kv-a-110-kv. Circular coil type transformer
1000 kv-a.- 44-kv. Shell type transformer
Air core inductances of 1.0 and 0.24 millihenry
110-kv. solid insulation transformer bushing having a capacitance of 0.00024 microfarads.
Various protective spark-gaps with and without resistance in series.

To date more than 600 records have been obtained representing a very wide range of service conditions affecting the voltage transients.

The agreement between theoretical predictions[4] and experimental results is striking.

THE EFFECT OF WAVE FRONT

If a given transformer has electrostatic voltage distribution such that the voltage distortion curve contains *n* space harmonics, a single traveling wave of rectangular front will produce oscillations containing all these harmonics. See top of Fig. 6. A wave of slanting front will produce only those harmonics whose duration of one cycle is in excess of the length (or duration) of the front of the wave. See bottom of Fig. 6.

This means that the front of the wave must be at least as long as one cycle of the fundamental frequency of oscillation of the winding to cause no voltage distortion in the winding.

For this reason waves of even 100 microseconds front may cause dangerous voltages in large transformers. Fig. 7 illustrates the relation of wave front to maximum voltage to ground at two points in the winding.

It is suggested that the wave front be called "steep" when it produces oscillations of fundamental frequency only, "very steep" when oscillations of fundamental and second harmonic are produced, and "exceedingly steep" when harmonics of a higher order are produced.

With the wide variation of high-frequency characteristics of transformer windings it is obvious that the exact range of the length of the wave front cannot be stated, but appears to be as follows:

"Exceedingly steep."	Of the order of 3 microseconds–0.57 miles.
"Very steep"	Of the order of 10 microseconds–1.9 miles.
"Steep"	Of the order of 50 microseconds–9.5 miles.

EFFECT OF THE LENGTH OF THE WAVE

The internal oscillation will reach maximum values to ground at all points of the winding, if the transformer is subject to a traveling wave of exceedingly steep front and of appreciable amplitude for a length not less than the duration of ½ cycle of the fundamental frequency of the winding.

On account of the relation of the harmonics, such a wave will produce severe overvoltage at both the line and the ground quarters of the winding.

Fig. 8 shows the relation between wavelength and voltage to ground of the middle part of the winding.

In discussing wavelengths, it is important to designate them in their relation to the natural frequency of the transformer. Thus, waves may be considered as "very long" if the length is at least equal to a full cycle of the fundamental frequency of the winding, as "long" if at least equal in length to ½ cycle of the fundamental, as "short" if the length is between ¼ and ½ cycle, and "very short" if less than ¼ cycle.

If expressed in time or length, the order of the various waves would be:

Very long wave	25 to	100 microseconds	5	to	19	miles
Long wave	12 to	50 microseconds	2.3	to	9.5	miles
Short wave	6 to	25 microseconds	1.15	to	5	miles
Very short wave	1 to	4 microseconds	0.29	to	0.76	miles

EFFECT OF THE TAIL OF THE WAVE

The effect of the steepness of the tail of the traveling wave is similar to that of the steepness of the front, because the transient effect is essentially the same, whether the terminal voltage suddenly rises from zero to a given value and thereby creates a "positive" transient or suddenly falls from that value to zero and creates a "negative" transient.

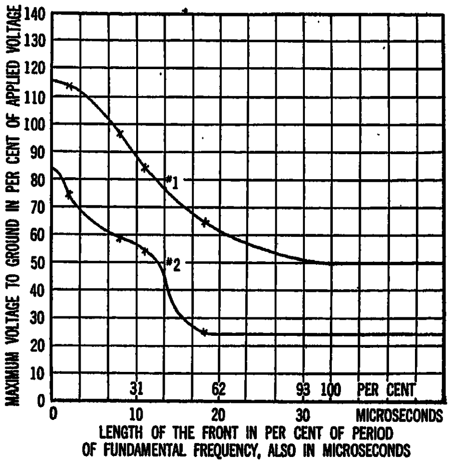

FIG. 7—EFFECT OF STEEPNESS OF FRONT OF THE TRAVELING WAVE ON VOLTAGE TO GROUND (TEST)

1. Point of winding 50 per cent away from ground terminal
2. Point of winding 25 per cent away from the ground terminal

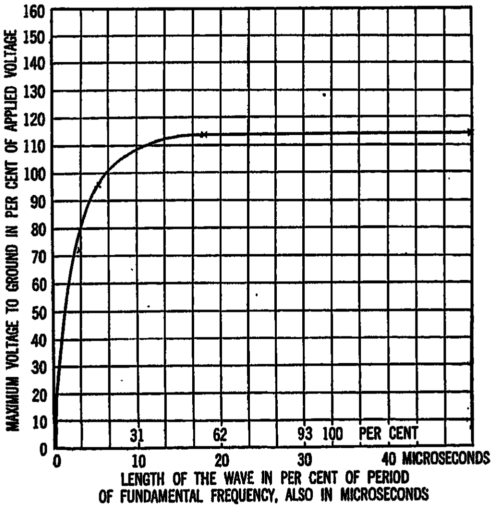

FIG. 8—EFFECT OF THE LENGTH OF TRAVELING WAVE ON VOLTAGE TO GROUND OF POINT OF WINDING 50 PER CENT AWAY FROM GROUND TERMINAL (TEST)

However, on account of the fact that at the time the end of the traveling wave reaches the transformer terminals, the winding is in a state of positive oscillation, sudden fall of the terminal voltage superimposes a negative oscillation, and the combination may create local stresses in excess of those caused by a wave of steep front and of slanting tail.

It follows that to prevent internal oscillations from reaching dangerous values, protective devices must be set so as to reduce the crest value of the incoming wave well below the ultimate strength of the transformer.

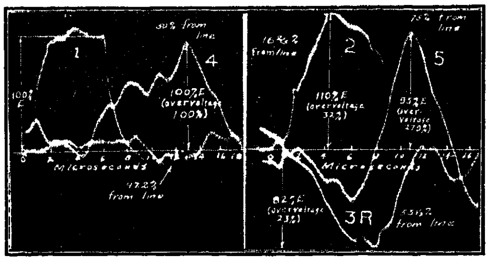

FIG. 9—VOLTAGES AT SEVERAL POINTS IN TRANSFORMER WINDING

1. Voltage at transformer terminal produced by traveling wave of approximately five microseconds

2, 3, 4, and 5 are voltages at points 83, 66, 50, and 25 per cent from ground terminal with crest values of 110, 82, 100, and 95 per cent of applied voltage respectively

As shown by Fig. 6, curve 4 *R*, a long wave with slanting tail produces at a point 25 per cent away from ground a voltage 75 per cent of the terminal value, whereas a short wave, Fig. 9, curve 5, produces at the same point a voltage 95 per cent of the terminal value.

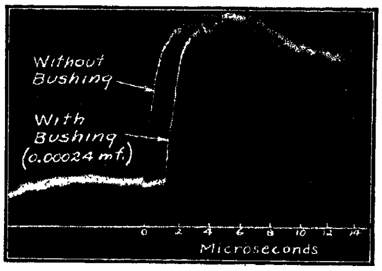

FIG. 10—EFFECT OF BUSHING ON SHAPE OF TRAVELING WAVE

(To avoid confusion two waves were displaced along time coordinate as shown)

EFFECT OF THE BUSHING

It has sometimes been stated that the capacitance of the bushing has a marked effect in slanting the wave-front and therefore decreasing the severity of voltage distortions. It has been shown previously[4] that very long wave fronts are necessary to avoid voltage distortion.

The bushing capacitance (of even a "high capacitance" bushing) has a negligible effect on the wave front as shown by Fig. 10 and therefore cannot affect the transient phenomena in the transformer.

Effect of the Transformer on the Incoming Wave

The effective capacitance of a power transformer is too small, and its surge impedance is too great to have any appreciable effect on the shape or crest value of the incoming traveling wave.

To illustrate this, a traveling wave was sent along the transmission line and records were made at the far end with the transformer first connected to and then disconnected from that end. The wave shape was practically identical in spite of the fact that the waves were only two microseconds long, see Fig. 11. The 2 per cent difference in crest value is well within the limits of error of measurement.

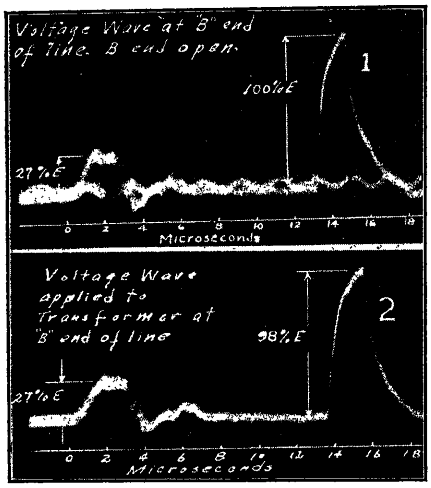

FIG. 11—EFFECT OF TRANSFORMER ON INCOMING TRAVELING WAVE

1. Wave at the end of transmission line with transformer disconnected
2. Wave at the end of transmission line with transformer connected (voltage across transformer)

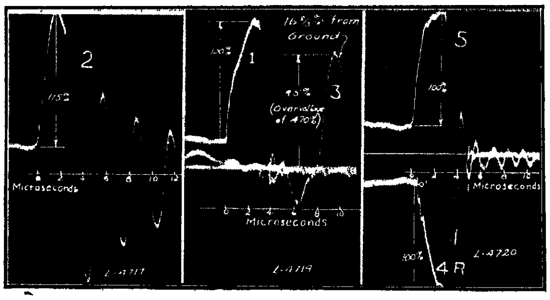

FIG. 12—EFFECT OF SERIES INDUCTANCE IN CASE OF TRAVELING WAVE OF VERY STEEP FRONT AND TAIL

A Inductance of 1.0 μ h.

1. Voltage applied across transformer and inductance in series

2. Voltage across transformer. Crest value 115 per cent of the applied wave (Curve No. 1). Frequency of oscillation 333 kilocycles

3. Voltage to ground at point of winding 17 per cent away from the grounded end. Crest value 95 per cent of applied voltage

Note, front of the wave No. 2 is steeper than No. 1. Approximate effective capacitance of the transformer winding 0.00206 μ f.

B. Inductance of 0.24 μ h.

4*R.* Voltage applied across inductance and transformer

5. Voltage across transformer. Crest value 100 per cent of the applied voltage. Frequency 710 kilocycles. Note, front of the wave No. 5 the same as No. 4*R*. Approximate effective capacitance of the transformer winding 0.0019 μ f.

Effect of Inductance in Series with Transformer

Inductances such as current limiting reactor or current transformer if not bridged by a proper by-pass device, enter into oscillation with the effective capacity of a transformer to which they are connected.

Such oscillations may create very severe overvoltage inside of the transformer and cause voltage across the transformer in excess of the crest value of the incoming wave. These oscillations also take place in case of arcing grounds or switching.

Fig. 12 illustrates what happens when a line insulator arcs over at the crest of a 3-microsecond wave. The initial voltage across the transformer without inductance in series and with an inductance of 0.24 millihenry in series are identical.

The voltage from line end of the inductance to ground shown in Fig. 12, curve 4*R*, is recorded downward.

The voltage across the transformer with this inductance in series is recorded on the same film but upward and is identical to that of the incoming wave up to the moment of insulator arc-over. Thereafter the voltage across the transformer oscillates at about 710 kilocycles.

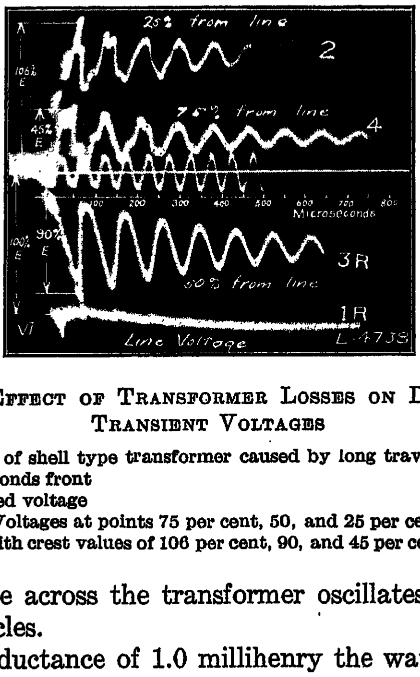

FIG. 13—EFFECT OF TRANSFORMER LOSSES ON DAMPING OF TRANSIENT VOLTAGES

Oscillations of shell type transformer caused by long traveling wave of eight microseconds front

1 *R.* Applied voltage

[illegible] *R*, 4. Voltages at points 75 per cent, 50, and 25 per cent away from [illegible] end, with crest values of 106 per cent, 90, and 45 per cent

With inductance of 1.0 millihenry the wave applied across inductance and transformer is shown in Fig. 12, curve 1. The voltage across the transformer rises to 115 per cent of the applied value and after insulator arc-over, oscillation takes place at 330 kilocycles.

The severity of the oscillation inside the transformer produced by such a transient can be judged by Fig. 12, curve 3. It shows that the voltage of that part of the winding 16.6 per cent away from its grounded end, instead of being at 16.6 per cent above ground, rises to 95 per cent of the applied voltage. Stated in terms of the normal voltage of this point, this is an overvoltage of 470 per cent. This is good evidence of the danger of reducing the insulation between high voltage and low voltage and ground near the grounded neutral.

Inductances of much smaller values have negligible effect on incoming wave but also enter into oscillation, overstressing the line end of the transformer winding.

EFFECT OF DAMPING AND TYPE OF TRANSFORMER

That the damping effect of the transformer is negligible in preventing the high voltages produced by the first few cycles of the oscillation is evident from examination of Fig. 13. It shows the oscillation in a shell type transformer at various points in the winding obtained with a wave of 8 microseconds front. The rest of the oscillograms given in the paper were made on the core type transformers and also illustrate the negligible effect of damping.

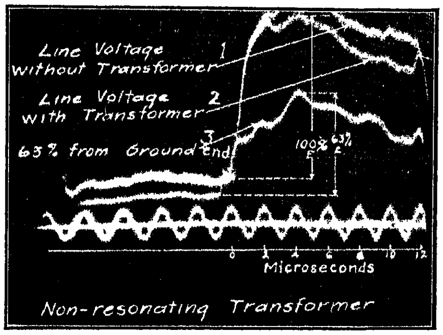

FIG. 14—NON-RESONATING TRANSFORMER

1. Voltage wave at the end of transmission line with transformer disconnected
2. Voltage wave at the end of the transmission line with transformer connected (voltage across transformer)
3. Voltage at 63 per cent point. Crest value 63 per cent of applied voltage

Note No. 3 a practical duplicate of the shape of No. 2 in spite of the fact that the transformer was out of oil and electrostatic unbalance was created thereby

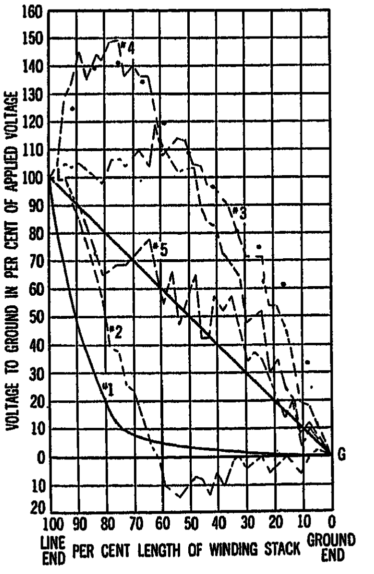

FIG. 15—SIMULTANEOUS VOLTAGE TO GROUND PRODUCED BY A SINGLE TRAVELING WAVE (TEST)

1. At the instant of impact ("initial" voltage distribution)
2, 3, 4, and 5.—3, 14, 18 and 24 microseconds later respectively

Dots show results of calculation (same as curve 3, Fig. 2) of maximum voltage to ground

CONCLUSIONS

By theory and test, it has been shown that:

1. The initial voltage distribution due to a wave of steep front results in the most of the voltage being concentrated on that part of the winding near the line end.

2. The effect of the concentration of voltage in the end turns under the initial voltage distribution is to increase the potential gradient to ground at the end of the winding due to change in the shape of the dielectric field.

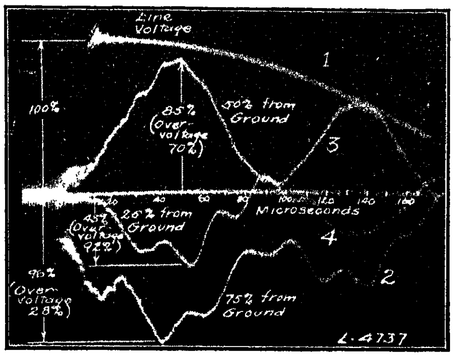

FIG. 16—OSCILLATIONS IN SHELL TYPE TRANSFORMERS DUE TO TRAVELING WAVE OF 14 MICROSECONDS FRONT

3. At normal operating frequency, steepness of the potential gradient is chiefly due to stress to low voltage winding and ground, the stress between coils being low. Under lightning waves, the stress between coils is increased very much and sharpens the potential gradient. It is as if the ground potential surface had been moved up close within the end of the winding.

The oscillations resulting in adjusting initial to final voltage distribution usually result in higher than normal stress to ground at points intermediate to line and ground.

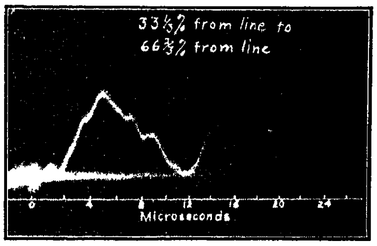

FIG. 17—HARMONIC ANALYSIS

Voltage between 33.3 per cent and 66.6 per cent points of winding produced by wave No. 1 of Fig. 6. The principal oscillation is that of second space harmonic because the voltage of fundamental harmonic produces no potential difference between these points and the amplitude of third harmonic is zero for both points.

4. These increases of potential gradient therefore result in lowering the ratio of impulse to 60-cycle voltage breakdown of such a winding to a value lower than the ratio for a simple piece of insulation unless the insulation is properly proportioned throughout the transformer. Thus the line terminal insulation impulse

ratio may vary for different transformer designs depending on the amount of local concentration of voltage near the line end of the winding.

5. If the transformer is subject to transients consisting of a series of damped oscillations such as are caused by arcing grounds, or arcs during the opening or closing of switches, etc., the voltage distribution in the ordinary winding becomes a very complex matter depending on the constants of the transformer and the frequency of the applied surge.

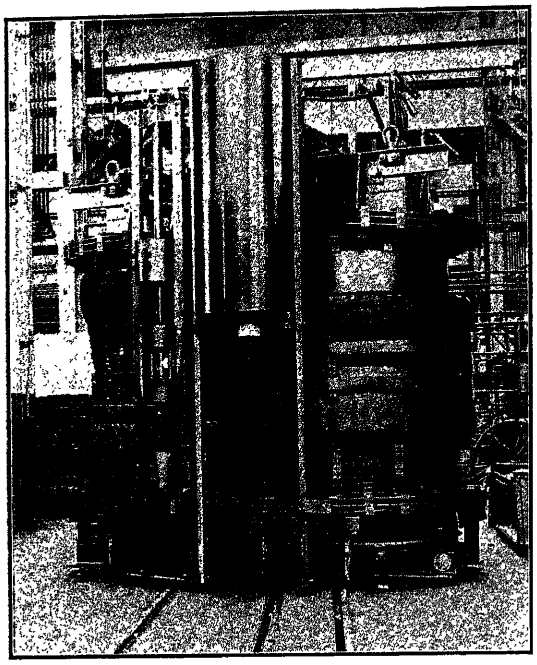

FIG. 18—220-KV. AND 37,000-KV-A. (EQUIVALENT CAPACITY) THREE WINDING NON-RESONATING TRANSFORMER

500-kv. insulation test

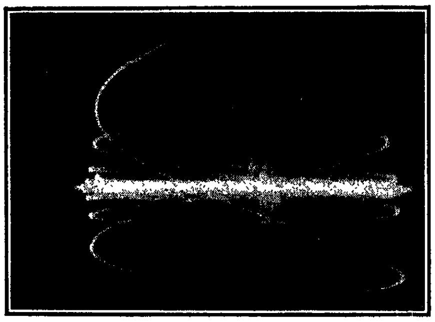

FIG. 19—AN EXAMPLE OF A WAVE RECORDED WITH OSCILLATORY SWEEP

Several cycles of single-frequency damped oscillation

6. The oscillations resulting from such surges may cause high local voltages in the transformer winding at many different points, and the position and value of these voltages change with the frequency as well as the amplitude of the impressed voltage.

7. If the frequency of the surge is in the neighborhood of the resonating frequency of the transformer windings as a whole or in parts, voltage to ground as high as twice the impressed voltage of the surge may occur at some point within the winding.

8. If, for grounded neutral operation, the insulation to low voltage winding and ground is graded somewhat in the order of the normal frequency voltage stress to ground, overvoltages due to oscillations may overstress the graded insulation to low voltage or ground at various points in the winding. Graded insulation in ordinary transformers subject to single waves may therefore be unsafe.

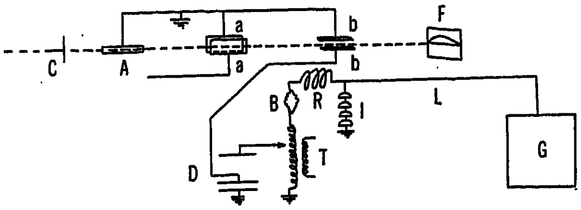

FIG. 20—DIAGRAM OF TESTING CIRCUIT

A and *C* anode and cathode of the oscillograph *aa* "time" and *bb* "voltage" plates of the oscillograph, *F*—Film or screen, *D*—Potentiometer, *T*—Transformer under test, *B*—Transformer entrance bushing, *R* inductance coil, *I*—Line insulator, *L*—Transmission line, *G*—Lightning generator

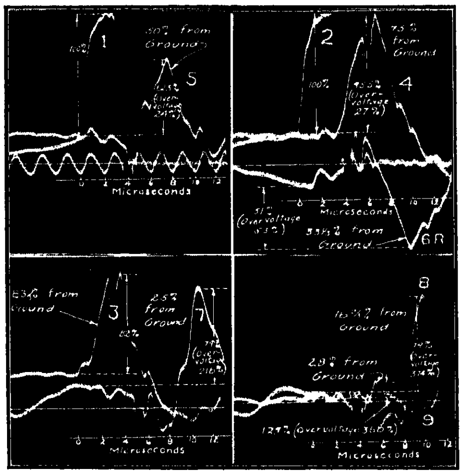

FIG. 21—OSCILLATIONS ALONG WINDING DUE TO SHORT TRAVELING WAVE OF "EXCEEDINGLY STEEP" TAIL, PRODUCED BY SPARKOVER OF LINE INSULATOR

1. The traveling wave applied to transformer
2. Point 97.2% from ground end—Crest 100% of applied voltage
3. Point 83.3% from ground end—Crest 82% of applied voltage
4. Point 75% from ground end—Crest 96% of applied voltage
5. Point 50% from ground end—Crest 62% of applied voltage
6. Point 33% from ground end—Crest 51% of applied voltage
7. Point 25% from ground end—Crest 79% of applied voltage
8. Point 16% from ground end—Crest 79% of applied voltage
9. Point 2.8% from ground end—Crest 13% of applied voltage

9. It is general practise to reenforce the insulation between turns and coils near the line end of the winding, but the oscillation of the winding as a whole or in parts, may produce very high voltages between turns and

coils at points remote from the line end and cause local failure between turns or coils or across taps, etc., unless the permissible lightning voltage to which the transformer may be subjected is reduced below that set by the terminal insulation alone.

It is apparent that the strength of the transformer winding to resist lightning voltages can be much increased, if the difference between the final voltage distribution and the initial distribution can be eliminated, because:

A. No change of shape of the dielectric field or change of gradient with a given voltage impressed takes place on the end of the winding if the voltage concentration on the end turn and coil is eliminated. It is the same as at normal operating frequency.

B. Overvoltage to low voltage and ground due to oscillations in the winding building up a higher voltage than "normal" with uniform distribution are eliminated.

C. Voltage stresses between turns and coils would be reduced to the ratio of the lightning and normal frequency terminal voltage only.

D. The insulation to low voltage winding and ground could be graded in the order of the normal voltage stress.

E. The terminal voltage stress would therefore be the only lightning voltage limit and this would bear the same ratio to 60-cycle voltage as the impulse ratio of the insulation used alone.

The factory induced voltage test could therefore be made a real measure of the ability of such a transformer to withstand lightning, which is not true with ordinary transformers.

Part VIII

THE NON-RESONATING TRANSFORMER

The reason that the voltage distribution in a transformer under surges is not uniform and that oscillations take place may be briefly stated to be:

Because the electrostatic charging current of the ordinary transformer has to flow through the winding.

If the electrostatic charging current is directly supplied to the various parts of the winding in the proper proportion, then unequal voltage distribution and oscillations will be eliminated.

This principle is the basis of design of the non-resonating type of transformer. At the present time in commercial designs, it is limited to transformers operating with neutral grounded directly or through a moderate impedance.

The electrostatic charging current of the various elements of the winding is supplied by means of carefully rounded and thoroughly insulated metallic surfaces connected to the line terminal, and adjacent to the winding, so that the discharging current of the coils to ground can be supplied directly, without having to flow through the impedance of the winding.

Therefore the proportions of the applied voltage of each part of the winding, given by its position in the dielectric field at any frequency, is the same as the proportion of voltage generated in that part at normal frequency operation. The initial and final voltage distribution is therefore the same and no transient or oscillation can take place.

Thus by properly proportioning the electrostatic characteristic of the transformer, the voltage distribution throughout the winding is made a straight line falling from line terminal to ground irrespective of the frequency or wave shape of the applied voltage.

In practise, the non-resonating winding is uniform and concentric with the low voltage winding. The coils are relatively narrow. Standard circular coils are used, connected together successively at the inside and outside turns, so that cross connections from inside to outside of coils are avoided.

The mechanical design of the transformer follows long established practise, lending itself readily to assembly and handling, and embodies the usual desirable characteristic such as free oil circulation and ability to withstand short circuit stresses.

In the new transformer, voltage stresses between turns and coils under lightning conditions are reduced in the order of from 10 to 1 to 100 to 1. The overvoltage to ground caused by oscillations being eliminated, results in decreasing the stress to ground sometimes as much as 6 to 1. The elimination of local excess voltage between turns and coils reduces the probability of arc-over of tap terminal boards, ratio adjusters, etc. It is well known that the spacing or dielectric strength of taps, ratio adjuster contacts, etc., has to be made much greater than that required for the normal or factory test voltages in the ordinary transformer, due to the possibility of building up very high local voltages during lightning and other transient voltages.

Graded insulation in the order of the normal frequency stress to ground is possible and safe with the new type of transformer, since overvoltage to ground at points in the winding between line and ground ends no longer occurs due to transients.

The proportion of voltage stress in all parts of the winding under all conditions of voltage and frequency remains the same as that during the factory insulation test, which therefore becomes a real measure of the ability of the transformer to resist lightning. This never could be tested correctly before.

In such a transformer, since all voltage stresses are under complete control and are made uniform throughout the winding for all frequencies, the reliability, naturally, will be much greater than in ordinary transformers.

Appendix

PRESENT STATUS OF THE CATHODE RAY OSCILLOGRAPH

on the Measurement of Transients

BY H. L. RORDEN[5] Non-member AND J. C. DOWELL[5] Associate, A. I. E. E.

Since there are several methods of operation of the cathode ray oscillograph, it is felt that the method applied in this case should be briefly reviewed.

In Fig. 20 the cathode beam is made to pass between two pair of deflector plates; "time" plates, *aa*, and "voltage" plates, *bb*, and to impinge upon the fluorescent screen or photographic film enclosed in the vacuum bell of the oscillograph. Since this beam is susceptible to deflection by transverse electrostatic fields, it may be made to sweep across the film if voltage is applied between one of the pairs of plates.

If an oscillating voltage is applied to the "voltage" plates, a vertical straight line will be traced on the film. Should a uniformly rising potential be applied to the time plates which are perpendicular to the first pair, it will sweep the beam in a horizontal direction at a uniform rate, and thus record a voltage-time relation of the voltage wave applied to the first pair of plates. By this method, therefore, the shape of the voltage wave is recorded in rectangular coordinates which of course is the best form of record.

Instead of applying to the time plates a voltage which rises at a uniform rate, some of the investigators are using a sinusoidal sustained oscillation of high frequency which gives an extremely complex record as shown in Fig. 19. Comparison of this record with other records given in the body of the paper, illustrates the superiority of the uniform sweeping method.

Formerly the principal difficulty involved in this method was the comparatively slow speed of sweep obtainable, which allowed a maximum speed of sweeping of 15 to 20 microseconds per centimeter on the film. However, the speed of the sweep has now been increased 200 times so that a wave of a fraction of a microsecond in duration can be spread over several centimeters of the film, permitting detailed analysis of the shape of the wave.

There are two problems peculiar to the study of transformer voltage transients: First, there is the necessity of measuring voltage between two points, both of which are above ground potential. Second, there is the necessity of synchronizing the voltage waves obtained at different points of the winding, produced by identical waves applied several times in succession. Both problems are easily solved. Fig. 17 serves as an illustration of the solution of the first problem, Fig. 15 was made possible by solution of the second. The family of curves was obtained from synchronizing oscillograms of 36 points of the winding and then scaling them at various time intervals as indicated on the figure.

Another recent development is the automatic initiation of the oscillograph, whereby it was made possible to record the wave of natural lightning[3] with the loss of but a small part of the front of even very steep waves.

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[5] Both of the General Electric Company, Pittsfield, Mass.

Discussion

PROGRESS IN LIGHTNING RESEARCH[1]

(PEEK)

LIGHTNING STUDIES OF TRANSFORMERS BY THE CATHODE-RAY OSCILLOGRAPH

(BRAND AND PALUEFF)

DALLAS, TEXAS, MAY 9, 1929

J. F. Peters: For many years it has been recognized that lightning voltages do not distribute uniformly in transformers, that the potential gradients near the terminals may considerably exceed normal operating voltage. To provide for that, the insulation at these places is reinforced. Until recent years we were obliged to depend upon experience, spark-gap measurements, and calculations to guide us in the design of transformers. It is gratifying to find that results obtained by the cathode ray oscillograph check the earlier values obtained by spark gaps so well.

The cathode ray oscillograph has added some very interesting details to our knowledge of voltage within transformers and it, along with other devices, is yielding valuable information on the nature and magnitude of actual lightning voltages encountered, which information is highly important in considering the necessary insulation within a transformer. We find that if the voltage is applied abruptly to a transformer the initial voltage distribution within the transformer is a function of the physical portions of the unit. The initial distribution can be controlled very considerably by properly selecting the proportions of the winding. When the impulse is maintained for a considerable time, 10 to 20 microseconds, the voltage will penetrate into the winding and may overshoot the values beyond a uniform distribution. The amount the voltage overshoots depends to a considerable extent upon the initial distribution which in turn is a function of the physical proportions of the design.

The maximum voltage that can reach a transformer is determined by the line insulation adjacent to the transformer. The flashover voltage value of line insulation varies with the length of voltage application. The line impulse flashover for impulses lasting 2 or 3 microseconds is approximately two times the 60-cycle crest flashover. For impulses lasting 15 microseconds or more the impulse flashover is approximately 1.2 times the 60-cycle crest values. A transformer should be insulated to meet these extreme conditions. The high but short impulses do not produce large oscillations on account of the time being too short. The longer impulses produce larger oscillations but the impulse is lower to start with. This state of affairs should be taken into consideration when determining the proper insulation to use.

S. W. Marshall, Jr.: The developments set forth in the Brand and Palueff paper may well be considered to be classical. The check, by the cathode ray oscillograph, of the calculated initial voltage distribution along the transformer winding caused by a single traveling wave, gives a definite insight as to the "whys" of some transformer failures.

Why is it that the second harmonic oscillates four times as fast as the fundamental?

The paper states: "The impact of a single traveling wave striking the winding causes it to oscillate. The oscillation starts simultaneously throughout the entire winding . . . ." Does this oscillation exist in both primary and secondary windings, or only in the winding conductively connected to the incoming wave?

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[1] This paper with other discussion of it was published in the A.I.E.E. Quarterly TRANS., Vol. 48, April 1929, pp. 436 and 468.

So much capital is invested in the conventional core-type transformer that I shall confine my concluding questions to this type. Consider a two-winding transformer connected to a transmission line. Is it likely that an incoming wave from the transmission line would flash over the low-tension transformer bushing, or low-tension substation apparatus of normal insulation value? What is the general wave form which is best communicated to the low-tension winding from such an incoming wave?

D. W. Roper: The paper by Messrs. Brand and Palueff is a notable contribution to our knowledge of the effects of lightning transients on transformer windings. It is interesting to note that, as a result of this knowledge, it is possible to alter the design of the transformer so as to make the insulation less vulnerable to lightning disturbances. The paper does not state definitely the sizes or types of transformers investigated, but the general description of the windings indicates that the investigation and perhaps also the modified designs apply to the larger sizes of transformers used on transmission lines. It is hoped that they will later be able to apply the same revised principles of design to transformers of the distribution sizes.

The elimination of lightning as a disturbing factor of electrical supply systems is such a vast and complex problem that it must be solved by cooperation among a great many investigators, including representatives of the manufacturing companies and the operating companies. The problem can best be solved by each investigator applying himself to that particular portion of the problem to which he is best fitted by reason of his circumstances and facilities, and then, on occasions like the present, presenting the results of his investigations to his fellow workers in this field.

The paper is an excellent example of such cooperation. It is also, perhaps, an example of the fact that the manufacturers can do a somewhat better job in solving their portion of this problem than can the operating companies as, with the modern equipment available for laboratory tests, they are able to reproduce in their factories the phenomena occurring in the field and to observe the effects of the phenomena on the apparatus which they manufacture. On the other hand, the operating companies must secure sufficient data so that they can eliminate the lightning as a variable.

Other investigators in the field have reported that there is not only a wide variation in the maximum voltage of such transients, but also a difference in the shape of its voltage wave, and further, that the lack of information on the latter point is probably the reason why it is impossible to reconcile the results reported by the several investigators on the effects of lightning on transmission lines.

The final test of the effectiveness of changes in the design of apparatus connected to electrical supply lines as well as the insulation and the protective devices connected to such lines is to be found in the operating records of the electrical supply companies. Within the past year several companies have reported to this Institute the results of the investigations that have been made on their transmission lines, and it appears probable that a similar paper regarding lightning protection on distribution lines will soon be available. It is the writer's experience that the manufacturers have always been ready and willing to cooperate in the solution of the problems in this connection, and there is no indication whatever that this cooperation is likely to be reduced. In fact, several new investigations have been started within the past year requiring cooperation between the manufacturing companies and the operating companies in order to add to our information on the subject. The increasing number of papers on the subject to this Institute in recent years indicates that the importance of the subject is fully appreciated by all interested parties and that the subject is receiving proper attention, in spite of some very drastic comments to the contrary in the technical press recently.

Edward Beck: During the past few years there have been interesting and valuable advances in the art of cathode ray oscillography. In the last year the Norinder type of automatic oscillograph was brought to this country and used in the field for the recording of natural lightning. The advantages of this type of automatic oscillograph in permitting the complete recording of a lightning transient are well known. The instrument was described in the Fortescue, Atherton, and Cox paper at the last Winter Convention. A new oscillograph of this type has been designed and several have been constructed. This device may be operated as a Dufour, the original laboratory type developed in France during the War, or, by the addition of a chamber containing the Norinder relay scheme, as an automatic instrument for use in the field on transmission lines. This makes the instrument extremely flexible and provides practically the same instrument for those who wish to use it in the laboratory only and those who at times may wish to make automatic records. The addition of the deflecting-plate system comprising the Norinder relay converts the instrument of the Dufour type into one that is entirely automatic within itself, not depending on the operation of external relays to set it in operation on the arrival of a transient.

This latest development in the field of cathode ray oscillographs presents several other novel features, among which is the embodiment in the device of an electron-filtering scheme which removes from the recording cathode beam those electrons which are the chief cause of fogging of the film. This permits the obtaining of a very sharp, well-defined record. The proper use of magnetic fields for focusing the electron beam further assists in the securing of sharp records. The oscillograph case consists of welded steel shells, thereby combining extreme strength with light weight. The deflecting-plate system may be adjusted from the outside through vacuum-tight cone joints. The entire oscillograph may be taken apart and reassembled very easily.

In our work with these oscillographs we are planning to use the high-frequency oscillator scheme for timing the recorded phenomena. In the first work done in this country on an actual transmission line with surge generator and cathode ray oscillograph three years ago, we came to the conclusion that this method of recording is the most satisfactory because it records considerable detail of each part of the transient besides giving a very long time scale because of the doubling back and forth of the timing axis. For instance, using this method of recording, it was possible to obtain in great detail pictures of four and five successive reflections on a 4½-mi. transmission line. While records taken by this means may appear complicated, they are easily intelligible with very little experience. Other methods of timing will produce simpler oscillograms, but it is thereby necessary to sacrifice either detail or length of the record.

J. H. Cox: Mr. Peek has reported the progress and status of the lightning problem from his point of view. He has reported the work being done by his group. I should like to add to the picture and make it more complete by reporting the work being done by the group with which I am associated.

Last year we manufactured three Norinder cathode ray oscillographs and applied two of them in the field on a 154-kv. transmission line of the Aluminum Company of America in Tennessee. As a result of this application one oscillogram was obtained. In the beginning of the work it was decided to use a rotating film drum which provided a maximum of 1000 microseconds per 2½ in. of film. With this time scale the surge which we obtained was somewhat cramped in space since it had a duration of 50 microseconds. This left the time of front somewhat indefinite but a definite value of duration was obtained. In all future work an oscillating time scale will be used with which any desired value of speed may be obtained.

The wave recorded at Tennessee having a duration of 50 microseconds, compares quite closely with that reported by Mr. Peek which had a duration of 40 microseconds. They are of the same order of magnitude. However, it would be dangerous to draw conclusions from only two authentic records of actual lightning. Although, in general, lightning surges may be of this order of duration and shape, they probably vary considerably and a great many more data are needed. In the effort to obtain such data our activities for the present season have been expanded and we will have five Norinder oscillographs in operation, ready to record lightning on actual lines. Two of these are those installed last year in Tennessee; a third is now installed on the 132-kv. line of the Public Service Company of Northern Illinois near Chicago; a fourth is on the 132-kv. line of the American Gas & Electric Company near Charlestown, West Virginia; and the fifth is on the 220-kv. line now under construction of the Public Service Gas & Electric Company of New Jersey. With the activities now going on, undoubtedly more actual records of lightning will be obtained before the season is over.

There are two sides to the lightning problem; one is the determination of the actual lightning surge as generated, and the other is the determination of how it performs on an actual line. The performance of traveling waves has been studied theoretically and has been calculated, but there are certain quantities which must be included in such calculations which are not definite. Such quantities are the value of conductor and earth resistances under traveling wave conditions, the effective size and spacing of the conductors and their images which determine the self and mutual surge impedance, the value of terminal ground resistance, etc. The mutual surge impedance between conductors determines how ground wires act on line conditions, and data on mutual surge impedances under varying conditions are needed. Also, information as to the effect of apparatus on surges must be determined. Mr. Peek has reported that waves as they travel along a line are attenuated rapidly by the effect of corona. This attenuation has been roughly determined by the klydonograph but more accurate information is needed. In view of the small number of actual lightning surges which may be expected at any one point, all of the above factors may be studied much more efficiently by the use of artificial surges.

Mr. Peek has mentioned the work combining a surge generator and cathode ray oscillograph on actual lines as being done by Mr. McEachron of his company. This type of work was first started in this country three years ago in a test on a 4½-mi. length of 33-kv. line between Rankin and Wilmerding, Pennsylvania. Due to the limitations of time, apparatus, and insulation, the amount of actual data secured was limited but many of the above mentioned points were determined under the conditions used and a great amount of experience was obtained which will prove extremely valuable in the more elaborate tests now under way.

As mentioned by Mr. Peek, Mr. McEachron has gone forward extending this type of test to a 66-kv. line of considerably greater length. Naturally he has made his apparatus more flexible for this type of work. We are now planning to extend the work to a 220-kv. installation. The 220-kv. line in New Jersey on which a Norinder cathode ray oscillograph is being installed for the purpose of studying lightning, as mentioned above, being in the construction period and therefore free from the complications imposed by having dynamic voltage and load on the line, presented a rare opportunity for the study of the performance of surges on a line by means of a surge generator and cathode ray oscillograph. Therefore, in this test the cathode ray oscillograph installation has been made portable and a portable 1,000,000-volt surge generator is being added. This makes the entire set-up very flexible as either unit can be moved and connected to any point on the line desired. As a result of this work and that of Mr. McEachron, before the year is out most of the questions outlined above should be settled with considerable accuracy. This will be but a small fraction of the time that would have been required had actual lightning surges been waited for.

Mr. Peek mentioned that direct strokes generally arc over the insulation and that the usual surge with which we have to contend on transmission lines is the result of an induced stroke. We see no logical basis for the belief that direct strokes are necessarily so severe as to be hopeless on our higher voltage lines and we have a great amount of evidence which indicates that this is not necessarily so. In fact, from the data we are led to believe that the majority of the surges with which we have to deal on transmission lines and which have been recorded in klydonograph tests are the result of direct rather than induced strokes.

The fact that with three groups working only two surges of actual lightning on transmission lines were obtained last year, stresses the point that lightning surges do not occur frequently at one point on a line and that no large number of records can be expected. This points out the need for greatly expanding the activity in this direction in order to obtain the necessary data in a reasonable time. The only way to obtain data is by measurements on actual transmission lines. It is relatively easy to obtain lightning surges on short antennas since on such conductors high potentials are induced by strokes even though at some distance from the location. However, such records mean very little. The fact that many occur on antennas and extremely few occur on continuous lines determines conclusively that there is a wide fundamental difference between the conditions and that by the time an antenna picture is interpreted in terms of a surge on a continuous line not much reliance can be placed on the interpretation.

It has been suggested that the present activities by the different groups, each working on their own problem, is the best way of solving the lightning problem. I must take exception to this point of view since I believe it is an established fact that nothing is done by scattered individual effort as well as by an organized engineering study and I believe that a real coordinated study of lightning is needed. Much of the study should be done on a greatly expanded scale and a great many more installations are needed. An organized effort should provide more funds and avoid duplication of work. When we consider that $150,000,000 per year is being spent on transmission lines alone, certainly the $500,000 to $1,000,000 per year necessary to do this work properly is amply justified for research of so fundamental a problem. This amount of money is not being spent at present because the whole industry is not cooperating. Only those who have immediate needs, together with the manufacturers, are putting forth any effort and more is needed. It is believed that the necessary cooperation and support would be forthcoming if the work were headed up by some authoritative organization.

F. W. Peek, Jr.: I do not feel that I need go into an elaborate closing discussion since the results of my own work are well covered by my paper. There are a few points, however, that I must clarify.

I am quite in sympathy with the views expressed by Mr. Roper. We have had a coordinated plan of study for the solution of the lightning problem under way for a number of years. Operating engineers have collaborated with us in this plan and have been of great assistance. Scientific and engineering problems are generally solved by those who are extremely interested and enthusiastic rather than by a committee. Our general plan has been to observe lightning in the field, reproduce it in the laboratory and coordinate the results with a mathematical and physical study. The progress of such an investigation is slow at first, but becomes very rapid as data are accumulated and better instruments are developed. Within the last five years "lightning" has been put on an engineering basis. As an instance of recent progress, during the past year 5,000,000 volts have been directly generated in the laboratory; natural lightning waves recorded by oscillograms have been reproduced and used in research on insulators and apparatus; waves have been sent over lines and their effects in their travels on the insulators, inductance coils, open lines, etc., recorded by the oscillograph; the waves have now been followed into the transformer and Messrs. Brand and Palueff have shown how high internal voltage concentrations occur in the ordinary transformer and how they can be prevented in the new non-resonating transformer.

One of the discussors has implied that I have expressed the opinion that most lightning failures are caused by induced voltages. It was not my intention to give this impression. I did say that most lightning voltages that happen on transmission lines are caused by induction. Direct strokes to transmission lines undoubtedly occur and may be the main cause of trouble in some of the very highly insulated lines. However, there is also evidence that induced voltages may be very high. This applies particularly to those induced voltages caused by direct strokes in the vicinity of the line. I might also point out here, as I have before, that high induced voltage can only result from a very rapid collapse of the electrostatic field, or what is the same thing, a very rapid discharge of the cloud. The laboratory curves for the lightning flashover voltage of insulators were obtained with all types of waves, since all types of waves must occur on transmission lines. However, the wave used as a standard is the one that gives the same arcover voltage for insulators as natural lightning does under the worst conditions on actual high-voltage lines. These measurements on insulator flashovers afford a check on the effective duration of the waves occurring on transmission lines. The lightning waves already measured are what would be expected from the hundreds of flashover voltages recorded. As brought out in my paper, a total of five oscillograms of waves was obtained last year. A more elaborate program is under way this year and a number of cathode ray oscillographs is connected to lines.

The cathode ray oscillograms shown in the paper by Messrs. Brand and Palueff, are extremely interesting and important. In fact, it is my opinion that the non-resonating transformer is one of the most important developments that have been made in high-voltage transformers in many years.

K. K. Palueff: It is gratifying to learn that oscillographic study of transformer oscillation conducted by Mr. Peters during the last few months, confirmed the theory and the oscillographic records presented to the Institute by the present writer at the last Winter Convention.

Thus, the fact that a single traveling wave is capable of producing complex and severe oscillations within the winding, causing it to rise above ground potential far in excess of the uniform voltage distribution, is no longer disputed.

Mr. Peters' present discussion shows that there is still some difference of opinion in regard to the relation between the amplitude of internal oscillations produced by long and short waves.

Mr. Peters states:

(a) Short waves of the order of 2 to 5 microseconds do not produce oscillations which overshoot the uniform voltage distribution.

(b) Long waves, of say 15 microseconds, do produce such oscillations.

(c) If the arcover value of the line insulation adjacent to the transformer is *E* for the short wave, then for a long wave (15 microseconds) it is 0.60 *E*.

(d) Thus, generally speaking, the internal voltages, if expressed in volts instead of fractions of applied voltage, produced by a long and a short wave, are about the same and do not overshoot the uniform voltage distribution drawn for the applied voltage of *E*.

The present writer's views on the above paragraphs are as follows:

(a) Harmonic analysis as well as the oscillographic data, of the relation between length of the wave and the amplitudes of the internal oscillations, indicate that the voltage to ground of approximately one-third of the winding stack near the grounded end, caused by a short wave, like those produced by flashover of line insulators by a lightning traveling wave, overshoot very substantially the uniform voltage distribution. (See oscillogram No. 5 of Fig. 9, and oscillograms 6, 7, 8, 9 of Fig. 21, and also Fig. 8 of the present paper.) These oscillograms show, for example, that a wave cut to a length of approximately 3 microseconds by an insulator flashover, causes the voltage of a point of the winding 16 per cent away from the grounded end to rise to 79 per cent of the crest value of this short wave, instead of being not over 16 per cent as Mr. Peters states.

(b) In this statement, Mr. Peters confirmed the writer's view as expressed in the present and previous papers.

(c) Examination of the oscillographic data presented to the Institute on the variation of impulse ratio of line insulators with variation of the length of the traveling wave, shows that there is only one paper that gives the numerical relation of these factors. This paper was the one presented by Mr. Peek at the Winter Convention. Fig. 11 of this paper shows that if the line insulation arcover voltage for a short wave (5 microseconds) is *E*, then for a long wave (20 microseconds) of the same front as the short one, it is approximately 0.93 *E* and not 0.60 as Mr. Peters states.

(d) From the above, it follows that both short and long waves are capable of producing internal voltages far in excess of the uniform voltage distribution. If the amplitudes of oscillation produced by short and long waves are expressed in volts instead of in fractions of amplitude of the applied voltage, then generally speaking, two-thirds of the winding is brought to higher potential above ground by the long waves than by the short. For the other third of the winding, at the grounded end, however, the voltage to ground is high even with short waves.

Mr. Peters further suggests that amplitude of the internal oscillations "can be controlled to considerable extent by properly selecting the physical proportions of the windings." Nothing can be said against such a statement if the physical proportions actually used in power transformers of all types are not taken into consideration.

Very unfortunately, practical designs allow such a narrow variation of the proportion that no radical benefit can be derived from change of the width and the length of the winding stack of any given type of transformer.

It appears that in practical transformer design, the high-frequency characteristics can be substantially modified only by use of electrostatic shields.

I think it would be of considerable interest to the Institute if Mr. Peters shows us the proportions of the winding of a practical power transformer that will give substantial improvement in transient voltage stresses. I think that the statement of quantitative relation between these proportions and the amplitudes of transient voltages would be of particular value as a general statement in such a problem may be misleading.

Answering Mr. Roper's discussion, I should like to say that laboratory transient voltage tests have been made on the entire range of transformers, including distribution, instrument, and power transformers, of a variety of sizes and types. The test data published were obtained, as stated on the fifth page, on a 6600-kv-a., 110-kv. concentric-type transformer, and a 1000-kv-a., 44-kv. shell-type transformer, in conjunction with a transmission line.

Mr. Marshall touches a very interesting phase of the phenomenon. In the original form, this paper contained a rather detailed explanation of the reasons that make the frequency of the successive harmonics of the transformer windings increase faster than their order. In the attempt to shorten the paper, these explanatory remarks were omitted, and only Figs. 4 and 5 were retained in the hope that closer examination of these two figures by the inquisitive reader would be sufficient to explain this rather unique relation.

The reason for such an unusual relation can be explained as follows:—The oscillation of the first few harmonics takes place essentially between the inductance of the winding and its capacitance to *ground.* And it is these harmonics, therefore, that are principally responsible for the rise of the potential of the winding above ground, after the impact of the traveling wave. For this reason, the equivalent circuit of the transformer winding for the first few harmonics can be simplified by omission of the series capacitance C_w, thereby converting it into circuit *b* of Fig. 1.

Fundamental Harmonic. In spite of the fact that all condensers of this circuit are the same, the simultaneous voltage to ground of a given harmonic is different for different condensers C_s and follows the space waves shown by sine wave No. 1 of Fig. 4.

For instance, the potential difference forcing into oscillation the condensers of the middle of the winding is 54 per cent of the line voltage while it is 38 per cent for condensers at 25 and 75 per cent points and zero for those at the ends (see Fig. 4). For this reason, the effective capacitance of the winding to ground, entering into an oscillation of fundamental frequency, is not equal to C, (the sum of all condensers of the equivalent circuit, *b* of Fig. 1), but only to a certain fraction thereof, for instance $K_1 C$.

Examination of the fundamental-frequency space wave, shown by curve No. 1, Fig. 4, reveals that voltages between adjacent elements of the winding are maximum at the ends of the winding, and zero in the middle. These voltages are of such polarity that the currents flow in opposite directions in the two halves of the winding, with maximum current at the end of the winding and zero current in the middle. (See 2, 3, 4 of Fig. 4.)

The magnetic flux distribution of fundamental frequency shown on 5, Fig. 4 indicates that the effective inductance for the fundamental harmonic oscillation is a certain fraction, say K_2, of inductance L obtained by connecting two halves of the winding in series and bucking. The frequency of oscillation of the fundamental harmonic, therefore, will be $f_1 = \dfrac{A}{K_1 K_2 \sqrt{L C}}$

where A is a constant independent of the transformer characteristics.

Second Harmonic. As the space wave of the second harmonic consists of two halves of the sine wave shown by wave No. 1 of Fig. 5, each half covers only one-half of the length of the winding and therefore the effective capacitance of each half equals half of the effective capacitance for the fundamental harmonic of the entire winding. In addition, we find that the second harmonic voltage of the line half of the winding is positive and that of the ground half is negative. For this reason, the effective capacitances of the two halves of the winding may be considered as connected in series, thus making the effective *capacitance* of the entire winding for the second harmonic equal to *one-fourth* of the effective capacitance for the fundamental harmonic.

As these elementary condensers discharge into the inductance of the winding, currents will be created as shown by curve 3 of Fig. 5 and by the arrows on the equivalent circuit diagram 4, Fig. 5. The magnetic flux distribution will be as shown on 5 of Fig. 5. Comparison of the current and flux distribution of the second harmonic with that of the fundamental shows that the effective *inductance* for the second harmonic is very nearly equal to *one-fourth* of the fundamental.

It follows that frequency of the second harmonic f_2

$$= \frac{4\,A}{K_1 K_2 \sqrt{L C}} = 4 f_1.$$

In a similar manner, it can be shown that the frequency of the third harmonic is approximately $9\,F_1$.

With increasing order of harmonics, the voltage of each quarter of a cycle appears across smaller and smaller parts of the winding. For instance, the full value of the tenth harmonic appears across every 2.5 per cent of the winding. Therefore, the series capacitance between adjacent coils of the winding becomes more important than their capacitance to ground. Hence, for higher harmonics, the equivalent circuit of the transformer winding can be simplified by the omission of condensers to ground C_g of circuit *a* of Fig. 1, converting it into circuit *c* of the same figure, and the ratio of frequency of oscillation will be directly proportional to the first power of the order of the harmonics.

On account of the variation of the ratio of the capacitance between coils, and capacitance to ground for different transformers, in many cases the series capacitance cannot be neglected when calculating the frequency of even the first three harmonics and the ratio of their frequencies therefore, instead of being 1:4:9, may actually be 1:3:7, or even 1:2, 5:6.

As far as the transformation of high-frequency oscillations from the high-voltage winding to the terminals of the low-voltage winding and the bus structure, is concerned, I feel that space limitations will not permit me to answer them completely at the present time. I should like to say, however, that high-frequency oscillations originating on the high-voltage side, after going through considerable modification, appear on the low-voltage side. The wave shape of the low-voltage winding oscillation varies with transformer characteristics as well as with that of the apparatus and bus structure connected to the low voltage. Their magnitudes may, and do, reach, under certain circumstances, dangerously high values, sometimes sufficiently high to arc over the bus structure, or damage the low-voltage network.

The transients that are best communicated from the high side to the low, are those of relatively low frequency. The traveling wave that is best communicated to the low-voltage side is one of considerable length, but of course it appears on the low-voltage side, not as a traveling wave, but as a complex oscillation, the shape of which depends not only on the transformer characteristics and those of the low-voltage network, but also on the shape of the traveling wave.

It generally can be stated, however, that with low-voltage windings located concentric with high-voltage winding, the high-frequency voltages transformed to low-voltage terminals are not as severe as in case of two windings being interleaved.

Flying Field and Airway Lighting

BY H. R. OGDEN*
Non-member

Synopsis.—This paper discusses methods of lighting airports and airways and describes various types of lamps and equipment used for this purpose. In the information on airport illumination are included beacons, obstruction lights, boundary lights, illuminated wind direction indicators, field and building flood lights, signal lights, and ceiling illumination. The portion on airway lighting covers principal and intermediate beacons and emergency field lighting. An extensive bibliography is included.

During the last few years many fundamental problems of illumination for night flying have been solved by both civil and military organizations. Although the future may see considerable divergence between military and civil lighting practise, at the present time both are similar and in fact commercial aviation has largely followed the trend of military design.

The components of a complete lighting system for an airway and its terminals have gradually increased in number as the need for their existence has become evident through experience. Particularly is this true of the equipment at the terminal airports. Here is a list of those components which should be considered in planning a system for lighting a terminal flying field, given in the order of importance: (1) beacon, (2) obstruction lights, (3) boundary lights, (4) illuminated wind direction indicator, (5) flood lights, (6) building flood lights, (7) signal lights, (8) ceiling projector. The components of a lighting system for an airway will be given later. There is some controversy as to the the relative order of importance of the items listed above, [1, p. 11] but the question is unimportant, for experience indicates that the lack of even the least important item may cause a fatal accident. A night mail pilot with insufficient fog flying experience took off into a low fog, stalled his airplane, and dove into the ground to his death. A positive knowledge of the ceiling through the use of a ceiling projector might have forestalled the flight.

The difficulty of night flying lies not so much in taking off from an airport to fly as in locating an airport and landing safely upon it when once in the air. Some experienced pilots prefer to take off without flood lights so long as the horizon is visible, for the reason that the pupils of their eyes will not have been contracted by any artificial light. A forced landing outside the field immediately after the take-off, in which insufficient altitude has been gained to make a return glide to the landing field advisable, might be more serious than would otherwise have been the case with eyes accustomed to the darkness.

If the problems confronting the night pilot desiring to land are based upon a succession of observations and decisions, we may well attack those problems in the order in which they occur to the pilot.

Airport Beacon. No mariner ever experienced greater satisfaction in identifying a lighthouse than a night pilot feels when he is uncertain of his exact location and finally espies the aerial beacon for which he has been searching the horizon. The majority of night flights must be completed before dawn due to lack of sufficient fuel to remain aloft. A landing field must be located before the fuel is exhausted.

What sort of light can pilots most easily pick up, a white light or colored, a fixed, flashing, or occulting light? The question of a colored beacon was brought up with the development of the red neon light and its comparative value as a beacon is still unsettled, although an official government test [2, p. 2] indicates that in clear weather the range of a neon beacon is no better than street lights. Extravagant claims have been made for the fog piercing quality of the neon light, especially in Europe and England,[4, pp. 2-5] but the marked advantage of this light source has not been proved.[3, p. 4] In view of the fact that even the position of the sun cannot be determined through more than a few hundred feet of fog, human efforts to overcome the fog hazard by artificial lighting are probably wasted so far as beacons are concerned.

Now in regard to the problem of using fixed, flashing, or occulting beacons, this problem was solved for the aviator hundreds of years ago by the navigators of the sea. Flashing or occulting beacons are absolutely essential, not only because they can be located more easily, but because of the prevalence of fixed lights throughout the country which would be confusing to a pilot searching for a fixed beacon light.

A short flash followed by a longer period of eclipse will be referred to as "flashing," while a long period of illumination followed by a short period of eclipse will be referred to as "occulting." The department of commerce recommends a flashing beacon having a flash duration not less than 1/10 second, luminous period not less than 10 per cent, and eclipse period not greater than 10 seconds.[1, p. 11]

The writer once experienced a search for a beacon on a night flight unguided by intermediate route lights. The beacon was a rotating projected beam light of approximately 10 deg. uniform divergence flashing for a duration of about ¼ sec. every 10 sec. It was noted that that period of eclipse was much too long, and the flash much too short, requiring an unreasonable period of concentration upon the light to establish its identity. The subject of the most suitable periods of flashing and eclipse commensurate with a reasonable cost for the beacon might well be made a matter of experiment.

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1. For references see Bibliography.

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An authority on night lighting equipment states that ". . . . the periods of illumination of a flashing light should have a duration of at least one second and for similar reasons the period of eclipse in an occulting light should have at least the same duration. On the other hand, it is undesirable that the period of eclipse in a flashing light or the period of illumination in an occulting light should be too long, especially where many other ground lights or other aviation beacons are in the vicinity, as in cloudy weather or bumpy air it is difficult to establish the identity of a beacon having such long periodicity."[5, p. 10]

There are various types of airport beacon in use at the present time. The most common in the United States is the rotating searchlight type with a 24-in. dioptric lens, 1500-watt incandescent lamp, and parabolic reflector. An electric motor in the base rotates the light. A recent development of the light includes a lamp changer mechanism which automatically brings a new lamp into position and incandescence, at the same time giving a warning of the burned out lamp at the control office.[5, p. 2] The beam has a vertical divergence of 3 to 4 deg., and a horizontal divergence of 10 to 40 deg. The greater horizontal divergence increases the length of flash, but decreases its intensity. This difficulty of securing great intensity and length of flash is the only criticism that can be made against this beacon. Even with a 40-deg. horizontal divergence, the length of flash is only 1 1/9 sec. at 6 rev. per min. But the more serious fault of this periodicity is the eclipse of 8 8/9 sec. Equal periods of flash and eclipse would be more suitable, but a beacon furnishing such a light appears to be more costly. A 360-deg. lighthouse lens with revolving reflector would provide such a light.

To secure proper distribution of the lower edge of such a beacon beam along the horizon, some method of adjustment ought to be incorporated in the light, either by an adjustable pivot or an adjustable base. The latter appears to be more practical. The French are using tripod type of mount for this purpose on some of their lights.

The problem of securing visibility of the rotating searchlight beacon over the entire sky is solved in some cases through the use of a vertical slit on the face and transversely along the top of the drum, thus projecting a slender beam to the zenith. The upper edge of a 4 deg. diverging beam, whose lower edge is on the horizon, passes the 1000-ft. level at 3 mi., 3000-ft. at 8 mi., and 5000-ft. at 13 mi. from the source. A pilot flying over the beacon within the inverted cone formed by the upper edge of the beam would not see the main beam during 26 mi. of flight if he were flying at 5000 ft. altitude.

If a water tower is located in the vicinity of the airport, it may be fitted up as a satisfactory short range beacon by painting the wall of the tank white and flood lighting it by means of signboard flashing lights surrounding the periphery. Since such a tower would otherwise have been wired for obstruction lights to safeguard pilots from colliding with it, the cost involved comprises only the lamps, fixtures, signboard flasher, and white paint. A sufficient number of light sources would provide against interruption of the beacon caused by a burned out lamp.

If for any airport lighting system the range of the water tower beacon is not considered sufficient, the tower may still be used to good advantage to mount on its peak any other type of flashing beacon. There are two advantages gained in mounting the flashing beacon on a tower high above the landing field. First, it allows the lower edge of the light beam to be adjusted below the horizontal, in line with the new horizon. This gives greater range of the beacon to a low flying airplane forced to fly near the ground by low clouds. Setting a beacon 100 ft. above the ground increases its range at ground level 13 mi. Second, a flashing beacon near the ground is disconcerting to a pilot about to land. When the intense rays of the beacon strike his eyes, the pupils converge and handicap his vision. To mount an airport beacon high is therefore a distinct advantage.

At the Croydon airport, London, a unique beacon has been tested and been found to possess unusually good visibility in thick atmosphere. It consists of a moving chain passing through a solution of strontium and then through an oxy-acetylene flame. The resultant light is very intense, but its superiority to an electric beacon does not appear to be sufficient to warrant its general adoption. In direct comparison with the strontium beacon, the Croyden airport is equipped with a truncated cone neon beacon, which is reported to pierce a small amount of fog better than any other beacon at the field.[3, p. 4]

The last beacon, and probably the favorite of night pilots, is the lighthouse type of beacon wtih 360-deg. Fresnal lens units. For the same power input, these beacons do not have the range of the searchlight type due to the fact that the light is diffused over 360 deg. as compared to 10 to 40 deg. for the searchlight type. But on the other hand, the lighthouse beacon can be designed to give any type of flash desired. The largest beacons of this type are located in France, one near Paris, and one near Dijon. Each has 1,000,000,000 cp., sufficient to give a range of visibility of 100 mi. in clear weather.

To sum up the present practise in the use of airport beacons, the projected beam type is most common in the United States, the Fresnal lens lighthouse beacon is favored in France,[4, p. 1] while the flashing neon beacon is extensively used in Germany.[4, p. 3] England, also, is partial to the neon beacon due to the claim for its fog piercing quality. Since the Fresnal lenses are expensive to manufacture, while the neon tubes are expensive to operate, it is evident that European practise in night lighting does not consider cost so important as superiority of equipment.

Obstruction Lights. After a pilot locates his landing field through the aid of the airport beacon, he approaches the field and observes obstructions which he must avoid while examining the landing area, the wind direction indicator, and ships about to land or take off. Crashes which occur through colliding with other airplanes near the ground or with obstacles while still in flight are usually fatal to the occupants. The obstruction lights are therefore very important in any plan for lighting an airport. A careful analysis must be made of the probability of an airplane colliding with any obstacle within a half mile of the field, and other obstacles farther away which are on the direct line of an airway. It is reasonable to assume that no pilot will fly lower than 100 ft. altitude until actually making his approach for a landing. Ordinarily, no obstacles less than 50 ft. high, which do not immediately surround the field, need be lighted. However, each field presents special problems, and no set rule can be made. It should be borne in mind that an airplane glides at an angle never steeper than 1 to 5. This will indicate roughly what structures or natural objects in the vicinity of an airdrome should be considered obstructions. A compromise between ultra safety and cost must always be made. Poor pilots will sometimes hit obstacles in spite of everything that can be done to safeguard them.

The universal practise in Europe and America follows the recommendations laid down at the International Air Convention for marking obstacles with red lights. The Material Division of the Air Corps recommends the use of 50-watt lamps in ruby globes of high light transmission qualities.[6, p. 4] The lights burn steadily, and may be connected with the field lighting circuit as will be discussed later. The lamp fixtures should be so placed that the obstacle light can be seen from the horizontal to the zenith in all directions, and from the ground anywhere on the airdrome. Flag poles and chimneys may be lighted conveniently by the use of unattached fixtures raised to the top by lanyards carried over a pulley at the top. The light can be exchanged at sunrise and sunset with a flag in the case of a flag pole, and can remain constantly in place on a chimney save when lamp replacement or cleaning is necessary.[6, p. 4] Lights too far from the airdrome to make practical a connection with the field circuit can be installed with a separate meter, attached to the circuit of some other party and rent be paid therefor, or, in the event that no power lines are conveniently near, it may be more economical to install an automatic gas light to burn constantly until necessary to replace the gas cylinder at the end of several months.

Boundary Lights. When the pilot of an airplane has observed all the obstructions about a field upon which he intends to land, his next observation is usually to determine the extent and nature of the field. Boundary lights can answer both of these questions. The Department of Commerce has made a careful study of the boundary light problem and has published the results in Aeronautics Bulletin No. 2.[1, p. 12] That bulletin should be consulted in drawing up a plan for an airport lighting system.

The general practise for placing boundary lights is to outline the field completely except along the hangars, outline just the corners of the field, or outline the runways. The last method is the most expensive, and, in the case of raised lights, invites collision from a rolling airplane. However, with lights installed flush with the ground or raised only a few inches, permitting an airplane wheel to roll over without damage, this system has a distinct advantage. It enables the field manager to indicate clearly the best approach and best place to land under various wind conditions by merely lighting the one runway which should be used.

A succession of white neon tubes end to end, spaced intermittently along each side of a runway, enclosed and protected by a concrete trough covered with sidewalk glass, has been suggested for experiment. Some such arrangement has evidently been tried at Croyden, but no report on the results obtained is available. Or a succession of incandescent lamps spaced 100 to 200 ft. apart, and placed in a suitable concrete base, would answer the same purpose, but would not be so visible in heavy atmosphere. Sometime radio is likely to solve the danger of fog flying, and when that time comes some supplementary lighting of the landing runway will be necessary. Two solid lines of red neon tubes may be the solution.

A special plea is made for the design of boundary lights which are fairly indestructible. The present practise of installing a lamp 1 to 3 ft. above the ground on the top of a pipe set in concrete[1, p. 12] has one insurmountable drawback. The installation attracts the attention of curious people, and invites malicious people or undisciplined children to break or pilfer the globes and lamps. No measure short of a constant guard near such boundary lights will protect them from depredation. Landing fields are too large for a manager on one side of the field to be responsible for the protection of the boundary lights on the opposite side. The design of lights flush with the ground will not attract so much attention, and it is believed they can be made fairly indestructible, at the same time fulfilling the purpose for which they are installed.

Another advantage can be gained by using boundary lights flush with the ground. Instead of establishing a row of lights on the very edge of the landing area, such a row could be placed 300 ft. from any edge along which a telephone or power line was located. Thus, a normal glide to the edge of the boundary lights would insure the pilot's clearing the poles and wires by a safe margin. In addition, lights placed away from the edge of the field would be less conspicuous and less likely to be molested.

In Europe and in England the use of automatic gas operated lights for both obstruction and boundary lights has been found to be the most economical installation,[4, p. 9] but no data on the relative cost of gas and electric operated boundary and obstruction lights in America are available at this writing. Undoubtedly, an electric system which is integral and controlled from the field administration building is convenient from the standpoint of control. From the standpoint of reliability, the individual gas lights are preferable. It is simply the story of not having all the eggs in one basket.

At Croydon airport the gas boundary lights are of the flashing type, 1/10 sec. flash and 9/10 sec. eclipse. This may be an advantage in distinguishing the boundary lights from the obstruction lights, but the practise hardly seems necessary. A more economical distinguishing feature would be the use of white lights for lighting the boundary. In the case of lights flush with the ground, white lights would be necessary to secure sufficient intensity.

Illuminated Wind Direction Indicator. Normal landings are always made into the wind in order to reduce the ground speed of the airplane as much as possible and retain directional control after the airplane is rolling on the ground. To indicate the direction of the wind to the pilot about to land, several devices are in use. For daytime operation, a wind cone of fabric attached at the larger end to an iron ring free to rotate about a vertical axis, and tapering from about 2 ft. diameter at the ring to 1 ft. at the smaller end, has been in use since the beginning of heavier-than-air flying. The first illuminated wind direction indicators naturally were developments of the wind cone. One authority maintains that illuminated wind cones are not satisfactory,[6, p. 5] but a number of internally illuminated wind cones have been in service at a large Air Corps field for nearly a year and have given uninterrupted satisfactory service.[7] Such indicators are unquestionably the cheapest to install and maintain.

Another type which is popular in England and Germany is a system of lights sunk flush with the ground level, so controlled by either a wind vane switch or manual operation in the administration building that an illuminated *T* is visible to the pilot.

The best military practise in America calls for an externally illuminated *T* situated on the ground in front of the operation office of the field. It consists of a skeleton framework built in the shape of a *T*, carrying a vertical fin at the base of the *T* to hold the head of the *T* into the wind, all covered with airplane cloth, and suitably finished.[6, Fig. 6] It is supported horizontally above the ground on a pivot located at the center of gravity. The illumination is provided by two rows of 10-watt lamps located on top of the *T*. The current is transmitted through a brush arrangement on the supporting bearing. For night visibility, the best color for the *T* is white, whereas for daytime visibility the most satisfactory arrangement developed to date is a chrome yellow *T* riding a cement or wood background painted black. A good compromise would be a white *T* on a black background.

The best practise seems to indicate a single wind direction indicator suitable for both day and night, either an illuminated wind cone or illuminated *T*. The latter is visible a greater distance, and therefore is more satisfactory to the flight leader of a formation of airplanes. The former is more economical, and ought to fulfill the needs of most civil airports when properly designed.

Flood Lights. No component of airport lighting systems has received so much attention and been the subject of so much experiment in this country as the airport flood lights. The problems connected with flood lighting have been many. At the same time, an airplane properly equipped for night flying is less dependent upon flood lights than upon several of the airport lighting components generally considered less important. Landing lights now available for installation on airplanes, developed by the Material Division of the Air Corps, make landing at night reasonably safe when the landing field is properly boundary lighted. However, flood lights must be available in an emergency, and flood lights also improve the ability of a pilot to judge his height above the ground.

Since the manner in which a pilot should pass flood lights in landing is an important factor in determining where the lights should be placed, considerable experimenting has been done along this line. Some pilots recommend that the pilot glide down to a landing over the light and parallel to the beam, others that he glide diagonally across the beam from the rear, and still others recommend that a landing be made perpendicular to the beam. However, the last recommendation is based upon the use of a single light source of the 180-deg. Fresnel lens lighthouse beacon type, or a battery of projectors arranged along the side of the field. The disadvantage of the first method is that when the airplane enters the beam near the ground, the pilot is suddenly confronted with the shadow of his airplane extending far out in front of him. This disadvantage becomes less annoying as the path of a landing airplane is changed to a perpendicular to the light beam. No one recommends landing into the glare of a light.

For several years the students at the Air Corps Advanced Flying School have been landing straight down, or diagonally across from the rear of the beam from a 500,000,000 high intensity arc, whose beam had a 40-deg. lateral divergence and a 4-deg. vertical divergence. No difficulties have been experienced, and the embryo pilots found the landings quite easy. However, recent experiments at Wright Field, the home of the Air Corps Material Division, indicate that even better results can be obtained by greater lateral divergence of the beam and landings made perpendicular to the principal axis of the beam. If the foregoing premise is accepted, then the flood lighting of any field to take care of various wind directions resolves itself into the use of two light sources or two batteries of light sources on adjacent sides of the field. Only the light from one side of the field will be operating at any one time and landings can be made either parallel to that side or diagonally away from it. The arrangement of flood lights is discussed and illustrated thoroughly in the Department of Commerce Aeronautics Bulletin No. 2.[1, p. 14]

The practise followed in the design of flood lights has been in the direction of two entirely different types of light units. Experiments in America for a number of years were restricted, for lack of funds, to the searchlight type. Many of these units were manufactured during the World War, and were available for experiment. A 36-in. drum with a parabolic reflector and a high intensity arc light source was fitted with a dioptric lens which diverged the light beam laterally to a 40-deg. spread. This unit required the constant attention of an operator, the light emitted was not always steady, and when the arc had to be replaced a serious interruption of the light ensued. The first improvement of the unit consisted in substituting a 10-kw. incandescent lamp for the arc. The new light source did not have the same cp. as the arc, nor was the color of the light so good, but the rays were steady, a prompt lamp change could be made, and the quality of the light was satisfactory. The last major improvement was the incorporation of a lamp changer mechanism which automatically brought a new lamp into position and incandescence, and gave a warning signal in the control room when a lamp burned out. As a single narrow beam, single source flood light, this unit is very satisfactory.

However, such a unit lights only a limited portion of the landing area. The advent of formation flying at night will require a wider distribution of light. To secure a greater light spread would require a battery of such lights, the combined overlapping light of which would be much more intense than necessary. To secure a sufficient spread of light at a reasonable cost would require either a new type of single source light with a larger lateral divergence, or a battery of smaller projectors.

European practise invited the attention of American engineers to the 180-deg. Fresnel lighthouse lens flood light, which has been tested comparatively with various types of flood lights[12] and found to be superior for the operation of airplanes in formation. A low mounting of the light, with a sharp cut-off on the upper edge of the beam secured by louvres, produced the most favorable illumination. One objectionable feature of this installation is that irregularities in the surface of the landing field produce shadows.

To overcome this serious objection on irregular fields, a battery of six smaller 24-in. drum projectors with parabolic reflectors, 1500-watt lamps, and 40-deg. dioptric spread lens can be mounted on 10- to 15-ft. standards and spaced 200 to 300 ft. apart along the side of the field. This equipment is much less expensive than the single source lighthouse lens flood light, is preferable for irregular surfaced fields, and is therefore to be recommended for moderate sized airdromes from which airplanes operate one at a time.[12, p. 6] The cost of lighthouse lens units will probably limit their use to military airdromes and major terminals, while the coming host of smaller municipal airdromes will be equipped with batteries of 24-in. projectors. The present equipment which is being manufactured for flood lighting needs only to be installed properly to give entire satisfaction. Many refinements will be made in the future, but at least the fundamental problems have been solved.

Signal Lights. Although the night traffic at municipal airports in this country has not yet reached the volume at which a system of signal lights is necessary, yet the day when such lights will be required is not far off. The Air Corps Advanced Flying School has been using signal lights controlled from a command tower for several years. The installation consists of rows of blue incandescent lamps of small wattage mounted along the entire edge of the two ends of two adjacent hangars. Switches on the command tower permit any number of the four rows to be illuminated at one time. The signals that can be given are limited. Some system will no doubt be developed to use the Morse code. Weather reports will constitute the majority of messages to be transmitted, and the signals should therefore be visible at a high altitude. Perhaps neon tubes will offer clear cut light signals.

It will be seen that the foregoing electric lighting equipment will entail a careful study to work out a satisfactory wiring and control system. The Material Division of the Air Corps has worked out a series circuit to include all of the lights at an airport.[6, p. 18] To insure continuous operation an auxiliary power generator must be included in the equipment. In addition, a safety precaution worth taking is to keep on hand at the administration building several magnesium flares. In case of snow or fog or failure of the electric circuit, these flares would be invaluable in case an airplane should arrive unexpectedly.

Airway Lighting. The components of an airway lighting system are: (1) principal flashing beacons, (2) intermediate beacons, (3) emergency field lighting. Practise in airway lighting in Europe and America has been widely different, but as time goes on, the respective systems are growing more similar. Europe began lighting its airways with widely separated beacons of tremendous candlepower. Two beacons are 1,000,000,000 cp. each, located at Mount Valerien and Mount Afrique in France. Moderate sized intermediate beacons are located at the terminal airports,[4,p. 20] and still smaller beacons are being installed along the routes. In Germany, the majority opinion favors comparatively low power lights at frequent intervals from 1 to 2 mi. apart.[4,p. 4] On the Berlin-Konigsberg and Berlin-Amsterdam routes are used double series of neon lights which burn continuously night and day. Operation cost of each light is claimed to be about one dollar per month.

American practise began with the installation of beacons spaced close together, though not so close as in the present German practise. And now larger beacons are gradually being used at major terminals to supplement the small beacons.

England has favored the automatic unattended gas lights[4,p. 11] for aerial route beacons, and the Bureau of Aeronautics in our Department of Commerce has installed these beacons on some of the air mail routes where electric power is not available and the lights must operate unattended for long periods of time.

Emergency landing fields are being established at approximately 25-mi. intervals along the air mail routes. Most of the flashing beacons are located at these emergency fields. The lighting equipment in addition to the beacon consists of boundary and obstruction lights, and an illuminated landing *T*.

Future Developments. The impetus which Congress gave to aviation through the establishment of a Bureau of Aeronautics in the Department of Commerce has been very great. The increase in night flying, encouraged by the night air mail and the installation of airway lighting, has been remarkable. Although the fundamental problems of aviation lighting have been solved, much important work lies ahead. As already mentioned there will probably be a continual divergence in practise between the requirements for civil aviation and military aviation. Two factors dictate this divergence: utility and cost. Night flying in the Army and Navy will develop into the operation of groups of airplanes as opposed to airplanes operating singly, so that flood lighting systems will probably be designed along two different lines. Also, military aviation will require mobile lighting units as well as permanent peace time installations, and therefore the development of dual equipment will require larger expenditures for military lighting than for civil. However, as will be noted in this paper, those who are charged with the design and installation of lighting equipment for private, municipal, and federal airports and airways will continue to draw upon the knowledge acquired and the experience gained by the pioneering work of the military services, and particularly in this country by the Material Division of the Air Corps. It is vitally necessary for the military services to pioneer in the development of material which will improve the system of national defense. It is also reasonable that where such development is useful to civil progress, the knowledge gained through the expenditure of public moneys should be extended freely to the public when not inimical to the security of the nation. Such are the conditions surrounding the development of almost all aviation lighting equipment by the U. S. Air Corps. When the business of furnishing lighting equipment to civil enterprises grows to larger proportions we can look for great experimental work to be done outside of the Army and Navy. That such a time will come is certain. The Army has for several years been drawing upon the ideas of commercial airplane designers for improvements in airplanes, and the day is not far distant when very superior aviation lighting equipment will be designed for civil use.

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Electrical Features of the Kansas City New Water Works

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Synopsis.—This paper describes the main electrical features of the new works recently constructed for additional water supply for Kansas City, Missouri. It enumerates the chief features of the power contract under which the city is purchasing its power. It states the type and capacities of the major electrical equipment and touches briefly on the wiring and lighting systems. It does not treat the efficiencies or the costs of the equipment.

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UNTIL comparatively recently standard waterworks practise was confined almost exclusively to the plunger type of pumps driven by reciprocating engines. They afforded economic operation over a wide range of conditions, but the floor space requirements and the capital investment were considerable. When it became expedient, in many cases, to install additional pumping capacity in a limited space and with limited funds, the turbo centrifugal pump received recognition. Its successful performance paved the way for electric motor centrifugal pumps which offered still further economies in capital investment. At the time when Kansas City was formulating its decision on the important question of steam-turbine or electric motor centrifugal pumps, standard practise, especially in the case of high-head pumpage, was strongly in favor of the former. In fact, of the thirty-five largest cities of the United States, only two or three used the electric motor as a prime mover, and one of these operated on "off-peak" power using storage reservoirs. Inasmuch as Kansas City is on a direct pumpage basis and cannot avail itself of the benefits of "off-peak" power rates, conditions are not exactly comparable.

Another factor which militated against electric drive in Kansas City was the fact that the city would be totally dependent, for its water, on purchased electric power with all the hazards, real and imaginary, that are involved in seven or eight miles of feeder lines, in addition to other likely causes of service interruptions. In consequence, the electrical layout had to be such as to give maximum assurance of continuity of service but still be kept within limited capital investment since the latter was one of the chief assets of electric drive. Inasmuch as a comparison of over-all costs, that is, fixed and operating charges, between steam and electric drive was made, it was necessary to have low power rates to offset increases in fixed charges.

POWER CONTRACT

At this stage it might be interesting to discuss the power contract between the city and the local public utility which contains some unusual features and which had some bearing on the selection of the type of equipment chosen. Some of the outstanding features of the contract are these:

1. That the city shall have first call on the power plants of the company.
2. That no substations shall intervene between the pumping stations and the power plant.
3. That duplicate feeders of very liberal capacity shall connect the stations with the power plant by two separate and independent routes.
4. That no other consumers shall be served by these feeders.
5. That the "off-peak" demands be calculated on a monthly average of daily half-hour peaks rather than on an individual half-hour.
6. That peak demands due to large fires, or emergency due to breakage of large mains, etc., not be considered in establishing the maximum demand.
7. That the peak months of water pumpage—viz.—June, July, and August, not be considered in establishing the maximum demand but be treated as individual months.
8. That a credit or debit be allowed or charged depending on the cost of fuel relative to a certain base price.
9. That a reduction in the demand charge be made based on 80 per cent power factor. Maintenance of 100 per cent power factor reduces the demand charge by 20 per cent.

The following are the rates:

Demand Charge:

$1.56¼	per month per kw., plus $3000.00 per month to cover charges on feeder lines.

Energy Charge:

$0.007	per kw-hr. until the demand at the high-lift station exceeds 2800 kw.
$0.00675	per kw-hr. until the demand at the high-lift station exceeds 4250 kw.
$0.0065	per kw-hr. until the demand at the high-lift station exceeds 5600 kw.
$0.00625	thereafter.

The life of the contract is twenty years but it may be terminated upon the giving of due notice by the city should the company fail to give first-class service.

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CHOICE OF MOTOR EQUIPMENT

At the low-lift stations the range of head was very wide and since better pump efficiencies could be obtained with a change in speed, the two winding motors received considerable thought. A more careful examination of the head fluctuations disclosed the fact that the extremes in head variation occurred during only 20 per cent of the time. Based on the narrower limits of head variation, it was decided to use synchronous rather than two-winding induction motors. The former had the advantage of better efficiencies during 80 per cent of the time, better power factor at all times, less operating complications, and slightly less cost; while the latter afforded better pump efficiencies for 20 per cent of the time. Besides, synchronous motors would raise the power factor made low by the many small induction motors in the purification works. The power factor clause in the power contract was a determining factor in the decision.

Accordingly, unity-power-factor, synchronous motors were selected for the low head stations and 0.8 power factor (leading) motors for the high head station.

OUTDOOR SUBSTATION EQUIPMENT

Power for the low-head stations is bought from the Kansas City Power & Light Company at 13,200 volts and is transformed to 2300 volts in an outdoor substation which is the property of the city. Two feeders, one overhead and the other underground, serve the station by two separate routes. The substation equipment comprises two 2500-kv-a. banks of transformers connected delta-delta. Each transformer is rated 833 kv-a., 13,200/2300 volts, single-phase, 60-cycle. These capacities were chosen on the basis that when overloaded 25 per cent one bank would carry 90 per cent of the ultimate load. Transformers are equipped with oil conservators and thermometers.

The two incoming feeders form a loop and are protected by directional relays. The overhead line is provided with a hairpin loop and oxide film arrester. Each bank of transformers is controlled by an oil circuit breaker on the high-tension as well as on the low-tension side and is differentially protected. A tie-line breaker on the high-tension side provides flexibility while the low-tension side is connected to a bus which is sectionalized by disconnecting switches. From this bus four feeders, two to each station, supply the low and secondary station. Each station is fed from both sections of the bus.

The oil circuit breaker equipment comprises three 400-ampere, 25,000-volt and two 1200-ampere, 15,000-volt breakers, the latter of which are enclosed in switch houses. All breakers are hand operated with d-c. automatic trips. The direct current is furnished by a portable battery located in a meter house and connected to a battery charger.

At this station the power is metered on the high-tension side. Duplicate sets of metering transformers, one for the company and the other for the city, are installed on the structure. Duplicate sets of meters are located on the switchboard in the secondary pumping station. The metering equipment consists of recording demand and power-factor meters and integrating watthour meters.

LOW-LIFT STATION

Water is pumped from the Missouri River to the purification works, about 2000 ft. away. The pumping station in which this initial pumping is done is known as the Low Lift Station. Each of the four main pumps in this station is driven by a 400-hp., unity power factor, synchronous motor operating at 2300 volts, three-phase, 60-cycle, 360-rev. per min. The excitation for these motors is supplied by two 30-kw. motor-generator sets. Each set is capable of furnishing excitation for five 400-hp. motors. These sets may be used for lighting in case of failure of the lighting transformer.

The control for this equipment comprises a fourteen-panel switchboard on a gallery floor 28 ft. above the pump room floor. A mechanical remote-control system through bell cranks and pull rods operates the oil circuit breakers which are supported on a pipe framework and are located on another gallery 12 ft. below the switchboard floor.

The main bus is sectionalized by two oil circuit breakers. In the section between these breakers there is connected a station service feeder which serves power, lighting, and heating transformers. Each outside section of the bus is fed by an incoming feeder, each feeder having the capacity of the entire station. One section serves two motors, while the other serves the remaining two motors and the future unit.

Motor starting is accomplished by means of compensators and a starting bus. This bus is sectionalized by disconnecting switches, each section being served by a compensator. Each compensator is of sufficient capacity to permit of four starting operations within 10 min. without excessive heating of the compensator. In this station there are eleven 600-ampere and four 400-ampere, 7500-volt oil circuit breakers.

Auxiliary power for operating vacuum pumps, sump pumps, motor-generator sets, crane and gate valves in this building as well as the revolving screens in the intake building is obtained from three 25-kv-a., 2300/230-volt transformers connected delta-delta. The lighting transformer is rated at 15 kv-a., 2300/230/115 volts.

Both the intake building and the low-lift station are heated electrically and are served by a 150-kv-a., three-phase, 2300/230-volt transformer. Industrial oven type, 220-volt ribbon element heaters suitably mounted and backed with reflecting and insulating material and provided with grill work in the front are placed along the gallery in a manner similar to steam radiator practise. In most locations two elements are used to form one radiator but they are individually controlled by local switches so that flexibility in controlling the heat may be obtained. All the transformers in this station are controlled by one oil circuit breaker but are protected by individual S & C fused disconnecting switches.

SECONDARY-LIFT PUMPING STATION

After the water has been purified it is pumped to reservoirs at the high-head stations. This being the second stage of pumping gives to the station the name of secondary lift pumping station. The pumping capacity of this station is the same as that of the low-lift station and since the heads are about equal, the electrical equipment is a duplication of that at the low-lift station. In addition to the main pumping units and the vacuum pump motors there are three 75-hp., 2300-volt, 1800-rev. per min. motors driving wash water pumps, two 60-hp., 2300-volt, 1800-rev. per min., and two 25-hp., 220-volt, 1800-rev. per min. motors driving house service pumps. The 75-hp. motors are controlled by automatic "across-the-line" contactors and push buttons. Each motor may be controlled from any one of three points. The 60-hp. motors are started across the line by closing the oil circuit breaker. The 25-hp. motors are controlled by starting compensators.

In addition to these pump motors all the power used in the chemical house, chlorine house, filter house, Dorr clarifiers, and the lighting for all these buildings and the grounds are controlled from the switchboard in the secondary-lift station. In the chemical house there are about 150 hp. in motors ranging in size from ¼- to 50-hp. Four 7½-hp. motors operate the Dorr clarifier mechanisms.

The switchboard comprises 21 panels and the system is that of mechanical remote-control, with the main oil circuit breakers and bus located on the lower floor in a switch-room beneath the switchboard. The starting bus, its compensators, and oil circuit breakers are located at the back of the switchboard. The busses are sectionalized in a manner similar to those at the low-lift station, but between the sectionalizing breakers there are two feeders, one for power and one for light. This building is not electrically heated. In the chemist's laboratory, however, there are ovens, sterilizers, refrigerators, distillers, etc. Another small difference between this station and the low-lift station is that the field rheostats for the main motors are electrically controlled in this station. The 36-in. discharge valves in both stations are electrically operated.

There are seventeen 600-ampere and four 400-ampere, 7500-volt oil circuit breakers in this station.

The auxiliary power transformers comprise three 75-kv-a., 2300/230-volt units connected delta-delta. Multiple lighting is served by a 75-kv-a., 2300/230/115-volt transformer, and series lighting by an 8-kv-a. constant-current transformer.

LIGHTING AND WIRING

Lighting in almost all the buildings is laid out to give at the working plane an intensity of six foot-candles exclusive of a liberal allowance for dust and aging of the lighting units. In the filter galleries, this intensity is maintained but over the filters and in the pipe galleries a lower intensity is used. A generous number of wall receptacles is installed. Glass and fancy fixtures are omitted, porcelain enameled steel reflectors being used instead. The installation is substantial and put up to stay for many years. Galvanized iron conduit and condulets are used throughout. Wire is 30 per cent para, double braided, except in damp places where triple braided weatherproof wire is used. Waterproof fixtures are used in such places.

Around the uncovered basins flood lighting projectors, standing two feet above the ground, are used. Thisi s done to reduce the number of bugs which swarm around lights, die, and fall in the water. At each flood light location there are three waterproof condulets, one for connecting the projector, another housing a switch for controlling the projector, and the third, a receptacle. The receptacle permits the concentration of more than one projector at a location during periods of cleaning of the basin. The projectors are fed by armored cable laid from 12 to 18 in. in the ground. At various manholes, lights, switches and receptacles are located and connected to the same cable system. About other parts of the purification works, there are distributed lighting standards for general illumination. Along the roads leading to the pumping station there are similar standards. They are all connected to a series system which is served by a constant current transformer in the secondary-lift station and operated at 6.6 amperes.

POWER CABLES

All power cables are insulated with varnished cambric and are lead covered where they are installed in pump-room floors or other places that are likely to be damp or flooded with water. Standard 5000-volt insulation is used on 2300-volt circuits and 17,000-volt insulation on 13,200-volt circuits. On 2300-volt circuits 7500-volt potheads are used. The control wiring for the East Bottoms Station is lead covered.

UNDERGROUND CABLE SYSTEM

Between the outdoor substation and the low- and secondary-lift pumping stations as well as between the secondary station and the chlorine and chemical houses the feeders are three conductor, varnished cambric insulated, lead covered cables installed in fiber ducts surrounded by a concrete envelope. Cables vary in size from No. 6 A. W. G. to 800,000 circular mils. Conduits vary from four to ten ducts. Concrete manholes, properly drained, are installed at frequent intervals.

EAST BOTTOMS STATIONS

The new electrically operated high-head station is known as the East Bottoms Station and is located about five miles from the purification works. The present main motor equipment consists of two 2000-hp., 0.80 power-factor (leading) 13,200-volt, three-phase, 720-rev. per min. motors equipped with direct-connected exciters. They are provided with temperature detectors and enclosing bell ends. The station is planned for six such motors.

The switchboard comprises 14 panels at present but will ultimately be expanded to 21. The system is that of electrical remote control, some operations being performed by manually operating the control switches while others are automatically controlled by relays and interlocks. The front of the switchboard faces the pump room, while access to the back is had from the switch house. All busses and switching equipment, oil circuit breakers, instrument transformers, compensators, etc., are enclosed in a monolithic concrete cell structure.

The station is fed by two 13,200-volt lines direct from the public utility's power-house bus. Ultimately there will be four such feeders. The main bus in the station is in duplicate but may be made a continuous bus by use of a tie breaker at one end of the bus structure. Indicator lights are located over the disconnecting switch compartments of the cell structure to show the position of the controlling oil circuit breaker.

The oil circuit breaker equipment comprises one 600-ampere, and seven 400-ampere, 15,000-volt units each rated at 14,000 r. m. s. amperes at 15,000 volts, and four 400-ampere, 15,000-volt units each rated at 1500 r. m. s. amperes at 1500 volts. All but two breakers are of the phase per cell type. The breakers are all solenoid operated and the control current is supplied by a storage battery. A small motor-generator set furnishes the trickle charge for the battery.

Motor starting is accomplished by compensator and starting bus as in the other stations but in addition neutral breakers provide a smoother transition from starting to running voltage. Motors have 85 per cent pull-in torque and full voltage is applied before the field circuit is closed.

At this point, the sequence of operations in starting a motor might be interesting. First, the neutral breaker is closed by closing its control switch, which operation automatically closes the compensator breaker. Next, the starting breaker control switch is closed. An accelerating relay controls the next steps. As it closes, it trips the neutral breaker which in turn closes the running breaker. The field breaker control switch is then closed and the compensator breaker control switch, which is the same as the neutral breaker control switch, is opened, which automatically trips the starting breaker. If the field breaker be not closed within ten seconds after closure of the running breaker, the field failure relay trips the running breaker. This time limit has been set on account of hydraulic conditions.

With the closure of the running breaker, a solenoid valve on the automatic check valve is closed. This will be described more fully later. Due to the fact that this solenoid valve closes only after the running breaker is closed, the motor "pulls in" against a closed discharge because the check valve cannot open while the solenoid valve is open. This reduces the required pull-in torque of the motor.

In this station there is a 30-kw. motor generator set which is capable of performing four important functions. It can supply excitation for any one or two of the main motors. It provides a source of direct current for lighting; it can give a full charge to the battery; it provides current for the control circuit while the battery is out of service or being charged.

Auxiliaries are fed from a bank of three 25-kv-a., 13,200/230-volt transformers. Lighting is supplied by a 25-kv-a., 13,200/230/115-volt transformer. Emergency lights are fed from the battery and go on automatically whenever the lighting transformer is out of service.

The description of this station would be incomplete without a word about the automatic stop and check valve. It opens and closes automatically with unbalance of hydraulic conditions in the valve. The necessary unbalance of hydraulic conditions may be controlled by a solenoid-operated valve in the blow-off line of the main valve. The solenoid circuit is interlocked with the running breaker and a control switch on the switchboard is inserted in the circuit. With the closing of the running breaker the solenoid valve closes and permits the main valve to open as soon as the pressure on the pump side of the valve becomes greater than that on the line side. As soon as the running breaker trips out, the solenoid valve opens and the main valve closes. The control switch on the board permits the operator to open the solenoid valve circuit, thereby opening the solenoid valve, which in turn closes the main valve even with the pump operating at normal speed. The operation of the main valve may be noted by the pressure-gage reading. This permits of the checking of one of the most important automatic features of the station without disturbing normal operation.

The use of 13,200-volt motors and electric radiators for building heating are radical steps in waterworks practise. It must be realized that the city is entirely dependent upon electrical power for its water. Moreover, this electric power is being purchased from the local public utility. It is a great tribute to the reliability of electric public utility service and to the electrical industry as a whole.

Meeting Long Distance Telephone Problems

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Synopsis.—*There have been presented before the Institute numerous papers describing various technical and apparatus developments of value in providing long distance telephone service. Several papers have also appeared covering specific transmission or operating problems or dealing with the advance planning of the telephone plant. Feeling that it might be of interest, particularly to the young engineering graduates, the writers have prepared this over-all sketch of the general problem of actually providing year by year the extensions and additions to a comprehensive network of communication channels necessary to keep pace with a growing public demand for long distance service. Since the writers are most familiar with the area served by the Southwestern Bell Telephone Company the discussion will be restricted to that territory.*

* * * * *

THE Southwestern Bell Telephone Company operates in the states of Missouri, Arkansas, Kansas, Oklahoma, Texas, and a small section of Illinois which is socially and economically associated with greater St. Louis. This is primarily an agricultural region but there are several large manufacturing centers and extensive areas devoted to oil and mineral production. The great difference in climatic and soil conditions which exist over such a wide territory naturally brings about a diversity of agricultural interests. Local, seasonal, peak demands for long distance service occur at the times when crops must be marketed and moved.

Oil, on account of its fortuitous occurrence and the sudden and imperative demand for long distance service attending its discovery, creates the most difficult problems. Through some perverse trick of fate the deposits have been found uniformly in areas otherwise of no great economic importance and having either no or very little telephone development.

Contributing more than any other item to the problem of keeping abreast with the demand for long distance service is the fact that the Southwest is still immature in the social and economic sense and is growing lustily and at a rate which rarely fails to exceed the most optimistic forecasts.

Beside these purely local peculiarities which contribute to the problem there is the generally increasing demand for long distance service on the part of the public. Based fundamentally on the increase of wealth and prosperity, traffic growth has been accelerated by better service and reduced rates. New and improved apparatus and methods become available from time to time—and not the least of the tasks of the field engineer is the more or less continuous modernization of an existing plant.

With this picture of the factors affecting the demand for service, it will be interesting to note the extent and complexity of the long distance network required. The map of Fig. 1 shows the principal towns and cities of the Southwestern area, and the single lines connecting them represent circuit routes. There may be a large number of communication channels on each route, of course. Not all of the cities and towns, and not all of the circuit routes shown are Bell owned and operated. Long distance service must be universal, and considered independently of who owns and operates the plant.

In addition to the network shown on Fig. 1 there is the long distance telephone plant of the Long Lines Department of the American Telephone and Telegraph Company. This department is organized to supply the extreme long distance facilities necessary between the several associate company areas in order that universal service may be realized. This paper does not include consideration of the Long Lines Department's plant or its problem.

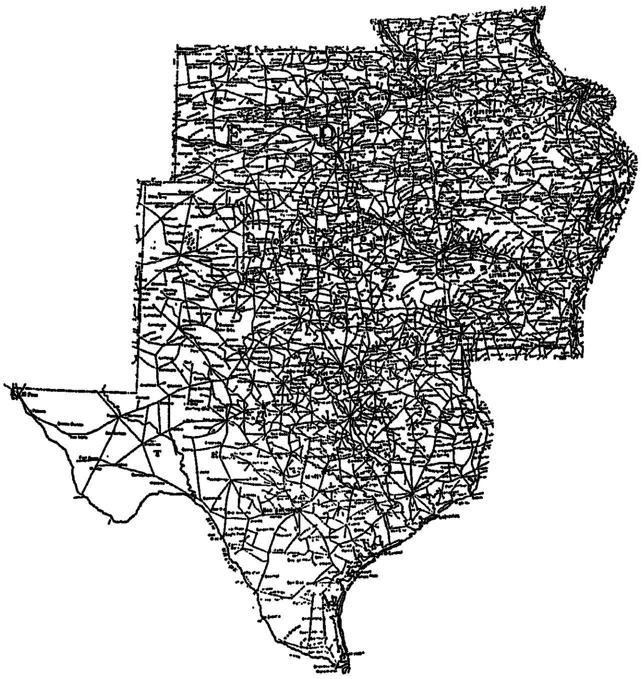

FIG. 1—LONG DISTANCE LINES

Confronted with an extensive existing plant which is meeting the present day requirements, it is the task of the field engineer to plan for continuous extensions, reinforcements, and betterments. The various technical developments that have been made in the art of

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1. Southwestern Bell Telephone Co., St. Louis, Missouri.
2. Southwestern Bell Telephone Co., Dallas, Texas.

Presented at the Regional Meeting of the South West District No. 7 of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

communication, the engineering practises that have emerged, and the advance planning that has been done must be practically applied and coordinated with a working system.

The fundamental requirements of any link of a system of long distance facilities is that it must talk satisfactorily. It must be capable of transmitting speech easily and without undue effort on the part of the users. To accomplish this there must be:

1. A sufficient volume so that speech may be received with adequate loudness.
2. Freedom from distortion so that speech will be satisfactorily intelligible.
3. Freedom from extraneous interference.

Each circuit together with all of its associated equipment must be designed to comply with these basic requirements. Not only must it furnish adequate transmission as an individual unit but it must also perform satisfactorily as a link of the whole system. In the planning of long distance facilities it is necessary always to envisage each unit of the plant as a part of a nation wide system and to insure its proper performance for any function it may be called upon to play.

In addition to regular long distance telephone communication it is necessary that certain special services be furnished. The radio public demands the chain broadcasting of programs and the frequent picking-up of programs at points remote from the radio transmitting stations. This requires the setting up of very reliable and extremely high quality telephone channels. Extraordinary freedom from distortion and noise is necessary since even with the perfection of modern amplifying and loudspeaking devices the quality of a radio program must exceed that of a simple transmittal of intelligible speech in order that it may prove enjoyable. Also, the radio listener is not in position to interrupt and request the repetition of a misunderstood word or phrase. Fortunately telephone circuits used as an adjunct to radio broadcasting may be operated as one-way circuits and the limiting effects of echoes are thus avoided. The simultaneous operation of telegraph over wire facilities primarily placed for telephone service has long been practised. Telegraph facilities, then, are a by-product of the telephone business, and economy dictates that these be made use of as fully as practicable. The demand for telegraph service is always sufficiently great to call for weighty consideration in planning for telephone facilities. Picture transmission is still before the public for trial. Should a widespread demand develop it will be necessary to set up a network of communication channels surpassing all previous standards as regards constancy, for a very minute deviation during transmission will spoil a picture.

Types of Facilities Used

There are three types of facilities available for providing long distance circuits. These are open wire, carrier current systems (superposed on open wire), and cables. The first two are closely interrelated and are of service in the early and intermediate stages of development, while the last is the final goal of progress at the present stage of the art.

Open wire was the first and is still the main reliance for providing circuits in this country. In a large area of low population density such as that of the Southwest, open wire must constitute the bulk of the plant for a number of years to come. Three sizes of copper wire now find general application. These are designated 104, 128, and 165, the figures indicating the respective diameters in thousandths of an inch. Improved transmission through the use of copper wire larger than 165 is unduly expensive, while sizes smaller than 104 do not possess the requisite mechanical strength for resisting the stresses of storms. Copper open wire circuits possess electrical characteristics so ideal as to endear them to the hearts of transmission engineers. With the aid of suitably spaced telephone repeaters they may be extended to practically any distance. Iron wire has lost its place in a plant designed for universal service. Having when new a transmission efficiency only one-fourth that of the same size of copper, iron wire becomes progressively worse from the day of its installation due to corrosion. This instability renders it wholly unsuited to the application of telephone repeater improvement and thus so seriously restricts its flexibility that it has been determined to eliminate iron wire completely from the long distance plant. The application of the principle of loading to open wire facilities has been rendered totally obsolete on account of the better performance and greater flexbility of non-loaded open wire with repeaters.

Along with its many and manifest advantages open wire construction possesses a number of serious drawbacks. Poles, crossarms, pins, and wire cannot withstand the assaults of exceptional wind and sleet storms, and occasional service interruptions from this source must be expected. By their very nature open wire circuits are of maximum susceptibility to both mechanical and inductive interference. It is thus generally impossible to continue in open wire up to the central office in a city. In such instances it is the practise to terminate the open wire out beyond the city congestion and to bring the circuits into the office over specially designed entrance cables. These entrance cables must often be of considerable length and add materially to the transmission losses. Through the application of the proper systems of loading the cable circuits may be made to match the impedances of the open wire circuits, and thus preserve the full possibilities of telephone repeater operation. With the increasing lengths of telephone circuits which modern service requirements demand, the problem of limiting the cross-talk among open wire circuits has called for greater and greater care in the planning of transposition schemes for them.

For several years now carrier current systems have been available by means of which additional telephone channels may be derived from existing and suitable open wire facilities. For distances beyond certain minimum figures these carrier systems yield telephone channels more economically than possible through the placing of additional open wire. The outstanding usefulness of carrier systems, however, lies in the increased capacity they confer upon an existing open wire structure. It is thus possible to defer heavy expenditures for major relief. The flexibility of carrier systems is apparent. They may be installed speedily to meet an emergency condition, and may be readily recovered and removed to another territory when the emergency has disappeared.

There are two types of carrier telephone systems in general use. One provides three additional telephone circuits above the one furnished by the pair of wires over which it must operate. Under favorable conditions it is economical as compared with stringing of wire for distances as short as 150 miles, and by the use of carrier repeaters spaced at regular intervals it may be extended to any distance. The other provides one additional talking circuit and finds its application with distances between 50 and 200 miles. The three-channel system employs frequencies from 4 to 28, and the single channel system frequencies from 4 to 10 kilocycles. Both of these systems utilize grouped frequency arrangements whereby different frequencies are used to transmit in the two directions.

At the higher frequencies of the carrier systems the attenuation offered by the open wire circuits is greatly increased over that experienced at voice frequencies. When it is remembered that cross-talk effects increase almost directly with frequency and are a function of the energy level differences between the disturbing and the disturbed circuits it will be realized that the application of carrier systems to an open wire lead may present serious difficulties. Transposition arrangements quite satisfactory for voice frequencies are totally inadequate for the carrier range. Special systems of transpositions must therefore be employed which, in addition to making use of extremely elaborate patterns, must be spaced with great precision. The retransposal of existing open wire leads is a large item of expense which must be carefully weighed when the use of carrier systems is being considered.

The necessary entrance cables which complicate voice frequency transmission problems constitute a proportionately more serious problem at carrier frequencies. To reduce attenuation and to limit reflection effects the cable conductors must be suitably loaded for carrier operation. For the frequency range employed by the three channel system loading coils must be placed at 930 foot spacing. In the case of underground construction it is rarely possible to realize this ideal spacing on account of manhole locations. It then becomes necessary to place the loading coils at something less than the theoretical spacing and to build out the short sections to the required capacity by means of special stub cable connected in shunt with the loaded conductors. In order to avoid carrier cross-talk in entrance cable it is necessary to maintain a degree of segregation among the conductors assigned to carrier operation. Where long entrance cables are involved it may well occur that the largest single item of expense in connection with a carrier installation will be the preparation of these for carrier operation.

Despite elaborate preparations in the form of special transposition of the open wire and loading and segregation of entrance cable conductors, it is still not possible to operate carrier systems indiscriminately on any open wire lead. Serious energy level differences and opposite directions of transmission with the frequency groups must be avoided. All the systems on a lead cannot always be coterminous, for a lead may be hundreds of miles in length, while carrier systems over it will serve various intermediate and overlapping lengths. At any intermediate point it may be desirable to introduce a system terminal with its high output level to operate in parallel with other systems whose energy has been greatly attenuated in transmission from distant terminals. Such differences in level have to be minimized either by separation on the pole head or by the introduction of repeaters to raise the lower energy levels. Junctions with other open wire lines and the generally complex nature of the open wire network give rise to situations such that it is practically impossible to realize completely the theoretical carrier possibilities.

With these various restrictions controlling the situation, the average carrier possibilities that can be realized provide for a total of 24 carrier channels on any long four-arm lead. Further exploitation of the remaining carrier potentialities on open wire leads is blocked by the use of phantom circuits which definitely limit the transposition refinements needed to restrict the cross-talk. For important open wire leads consideration is therefore being given to abandoning phantom circuits on all wires of the first four arms, except the pole pairs. A further simplification of the cross-talk problem can be made by reducing the spacing between the wires constituting a pair from 12 to 8 inches and correspondingly increasing the spacing between pairs from 12 to 20 inches. By these arrangements the carriers' possibilities can be increased from 24 to 48 channels, but as 8 phantom circuits are sacrificed the net gain is only 16 talking circuits. It is expected that this refinement will be applicable only to pole lines carrying principally very long haul circuits. Where there are numerous short circuits on a lead, the necessity of terminating wires at frequent intervals destroys the flexibility of the arrangement.

The nature of the long distance telephone development and the growing requirements of the Southwest have furnished a particularly favorable field for the application of carrier telephone. The distance between many of the more important cities is from two to three hundred miles, and the three-channel system has been made large use of in spanning these. The single-channel system has found equally widespread application. While carrier telephone has been used in the Bell System for about a decade, the installations previous to 1925 were of a transitional nature in the evolution of a commercially practical system. It is of local interest to note that with the rendering available of the standardized three-channel system the first commercial installation in this country was placed in service between Dallas and Houston in the summer of 1925. It is also of interest to remark that the first commercial installation following the field trial of the single-channel system which is now standard was placed in service between Amarillo and Borger in the early days of 1927. Borger was a new oil town that had sprung up from nothing overnight. From this auspicious beginning the use of carrier telephone has grown until the present day finds some 35,000 miles of telephone channel obtained by this means in the Southwest. The map of Fig. 2 shows the extent of this development, which represents about one-sixth of that of the entire United States.

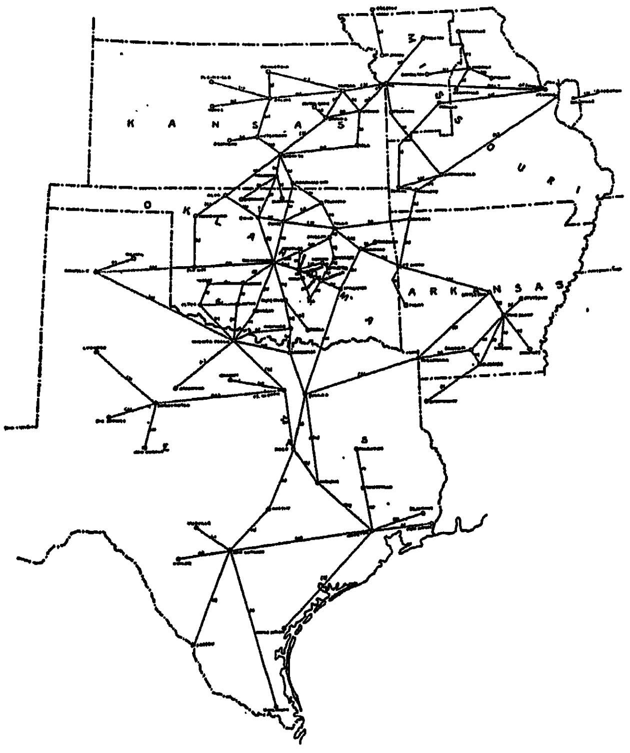

Fig. 2—Carrier Circuits in Southwest

From the discussion so far it is clear that with any open wire lead the condition will be reached ultimately when the maximum wire load is in place and the fullest possible use of carrier has been made. It then becomes necessary either to build additional open wire lines or else have recourse to some other means. Where the circuit growth is sufficiently rapid economic studies rule in favor of cables in place of building one or more supplementary open wire lines. In addition to proving in from a strict economy standpoint, cables offer many advantages as a medium for long distance circuits. Increased freedom from interruptions due to physical failure of the plant, lower susceptibility to external interference, and more stable operating conditions are some of the benefits accruing. In the territory of the Southwest the condition is rapidly approaching when cables will be needed between nearly all of the important centers. To meet this situation a cable program has been adopted for the period 1929-1933 which will connect most of these points. This is shown on the map of Fig. 3. Of the cables in the program the sections between Oklahoma City and Tulsa, Oklahoma City and Holdenville, and Dallas and Cisco are under way and will be completed during 1929. As it now stands, the program provides for a total of approximately 2600 miles of cable for the five years.

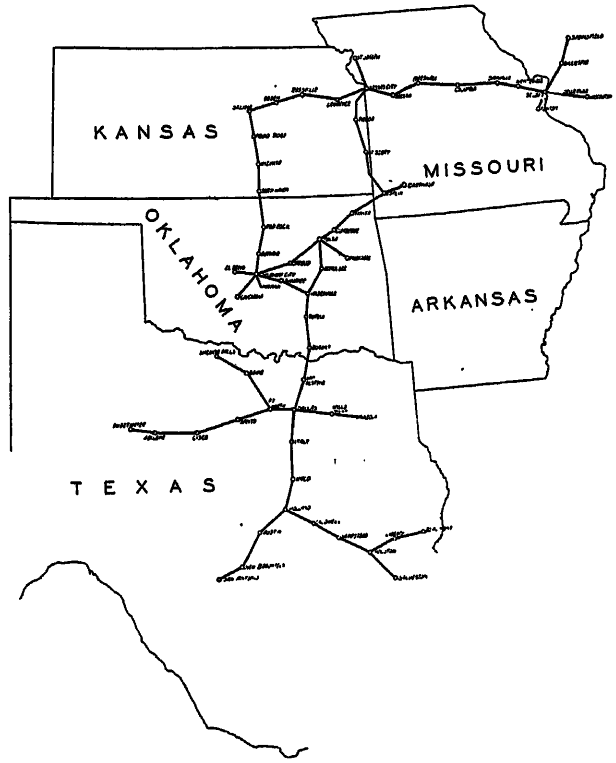

Fig. 3—Toll Cable Program for Period of 1929-33

The Tulsa-Oklahoma City and the Fort Worth-Cisco sections of the cables mentioned above will represent a unique departure from the previously standard practises of this country. The usual lead covered cable is to be manufactured and then given a protective coating including several layers of jute and tar and two servings of steel tape armor. A trench is to be dug and the cable simply placed therein and covered up. When it is realized that a standard 750-foot length of this armored cable together with its reel will weigh about five tons and that the route is to be strictly cross-country, it will be appreciated that some interesting construction difficulties must be met and solved. It is planned to present the story of these installations to the Institute at a later date.

Whether the channels for long distance circuits are derived by means of open wire, carrier systems, or cables, elaborate installations of equipment are required at the circuit terminals and at the intermediate repeater stations. The placing of this equipment and the providing of the necessary building space for housing it constitute major problems. Building additions must be made or even complete new buildings erected, new equipment must be installed and placed in service, and existing working equipment must be relocated to best advantage,—all without interrupting the service. Also, irrespective of the means used to derive the communication channels, arrangements must be made at the terminal offices whereby any two circuits may be connected together and a built-up connection thereby established between two terminals not having direct circuit connection. The problem comes in arranging for such interconnections with the result that built-up connections shall satisfy the same fundamental transmission requirements as are demanded of the individual links. To meet the volume requirement it is essential that a definite gain be inserted whenever two circuits are connected together. The practise of the past has been to provide with repeaters certain of the cord circuits appearing at the switchboard positions which handled switched traffic. The fundamental weakness of this method lay in the fact that the insertion of the proper gain was left in the hands of a very human operator, and there was always the possibility of its being omitted. The latest practise eliminates this element of human fallibility. The switchboard positions are equipped with but one type of cord circuits. When a connection is established between two long distance circuits the proper gain is automatically provided; when a connection is established between a long distance circuit and a local telephone no gain is inserted since none is necessary. The distortion and interference requirements are met for built-up connections simply by causing the individual links to meet sufficiently high standards in these respects so that a chain of four or five in tandem will still yield a satisfactory grade of over-all transmission.

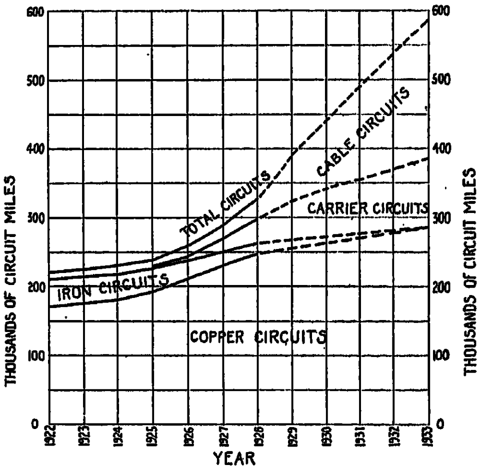

Fig. 4—Growth of Long Distance Circuits

An interesting review of the growth of the various types of facilities used to derive long distance telephone circuits in the Southwest is shown on the diagram of Fig. 4. The rise and fall of iron wire, the abrupt rise to prominence of carrier systems and cables, and the rapidly mounting total are significant of the present trends.

DETERMINING THE PROCEDURE

Periodically the field engineer must review his entire long distance telephone plant. On the basis of the years in service and the stage of deterioration, plans must be formulated for the systematic replacement of poles, crossarms, insulators, and wire. In connection with such plans the estimated growths for several succeeding years and the probable future destinies of the pole lines must be taken into consideration, since these will determine the extent and character of the replacement work. Entrance cable conditions, building situations, and central office equipment requirements must be reviewed in the same light. Where it is indicated that even with the relief which carrier systems can afford the ultimate capacity of a pole line will be reached within the next few years, comparative cost studies must be made to determine the economic advisability of going to cable as against placing additional open wire lines. Such technical developments and improvements in the art as have become available recently must be injected as far as practicable into all plans.

From such considerations the field engineer each year evolves a list of projects for the next succeeding five years. This list is submitted to the company executives in order that the necessary financing may be arranged for. The material requirements of this list are submitted to the manufacturers in order that they may be prepared to fill the orders. Such projects as are given approval constitute the definite program of active work for the immediate future. For each approved project detailed plans and an estimate must then be prepared and submitted to the executives for final approval. Upon securing this, the project becomes a construction or installation job, and the engineer's final task is to see that it is executed in accordance with his plans.

In addition to this somewhat orderly program of activity, the years are always spotted with unforeseen situations of an emergency character. A new oil field is discovered at Seminole, a Democratic Convention is announced for Houston, an extraordinarily large cotton crop is realized in Texas, or floods in Arkansas carry away miles of pole miles,—all these call for prompt action with an insistence which cannot be denied.

CONCLUSION

The writers have prepared this non-technical discussion with the hope that it might prove of some interest to engineers as an indication of the range of problems encountered in this particular field of endeavor. Its purpose will have been accomplished if it has shown something of the zest and interest, something of the wide range of technical acquaintance, and something of the harassment arising from continuously changing conditions which attach to the task of meeting the demand for long distance telephone service in a large area. For it is true that the battle of progress is waged no less actively, and broad technical understanding, initiative, and good judgment are no less necessary in the field outposts than in the more sheltered seclusion of the research laboratories.

Discussion

G. H. Quermann: This paper gives a very complete picture of the problems presented in the design of long distance telephone plant, and although it does not specifically touch on the long lines department's problems, they are in general very similar to those discussed in the paper.

The facilities of the long lines department must of course be so designed to operate satisfactorily with shorter haul toll circuits and local exchange facilities and since they constitute a trunk-line system, which extends practically throughout the United States, they must of necessity be of the highest grade in order to provide satisfactory service on all connections.

With long lines facilities extending over the greater portion of the United States with connections into Canada, Mexico, and Cuba, the furnishing of messsage telephone service together with telegraph service over these facilities for press, government, and commercial private-wire use presents a service problem of no small proportion.

The furnishing of suitable facilities for program transmission for national broadcasting is also one in which the long lines department is vitally concerned.

Picture transmission is now offered between eight cities in the United States, namely, Boston, New York, Cleveland, Atlanta, Chicago, St. Louis, San Francisco, and Los Angeles. These stations are equipped with apparatus for the transmitting and receiving of pictures. St. Louis is the only point in the territory of the Southwestern Bell Telephone Company. The circuit used for this service to the Pacific Coast is at times routed through this territory by way of Little Rock and Dallas.

Open-wire plant at present constitutes the major portion of long lines circuits in this territory. This is not true in the East where the requirements of long distance telephone service has exceeded the capacity that can be economically provided by open-wire plant. There, cable circuits form the major portion of long-distance communicating circuits. However, as has been outlined the situation in this territory is such that the provision of toll cables between the more important centers now appears to be the next practicable means of providing adequate long-distance facilities.

In the territory west of the Mississippi, extensive use is being made of 3-channel carrier telephone systems. At the present time one of the longest carrier systems in service operates through this territory with terminals at St. Louis, Mo. and San Bernardino, Cal. It is 2100 mi. in length and has intermediate repeaters at nine points, including Little Rock, Dallas, and El Paso.

The problem of coordinating the carrier systems operating over an open-wire line is a very important one. As the long-lines carrier systems are frequently operated on lines used jointly with the associated companies, this becomes a mutual problem.

The paper pointed out that in some cases consideration is now being given to abandoning phantom circuits on some of the wires on a lead and reducing the spacing between wires constituting a pair from 12 to 8 in., correspondingly increasing the spacing between pairs from 12 to 20 in., in order to increase the carrier possibilities and obtain a net increase in circuits. The long lines department is now doing on a trial basis, some work of this type in certain sections of its plant extending from Wichita, Kans. to Denver, Colo., Joplin, Mo. to Tulso, Okla., and Oklahoma City, Okla. to Texola, Okla.

C. W. Mier: The paper mentions the difficulties sometimes encountered in meeting the long-distance problems in connection with the discovery of oil. Having been very closely associated with this problem in Oklahoma during the past few years, I should like to tell briefly of the growth in traffic in the vicinity of the famous Seminole Oil Field.

Oil was first discovered in this field about July 1926, and reached a peak production in July 1927. Previous to July 1926, the long-distance telephone traffic out of Seminole amounted to about 800 calls per month and was handled over two circuits by one operator. By January 1927, this traffic had grown to 3688 calls per month, an increase of 360 per cent, and the operating force had increased to 30. This traffic increased by leaps and bounds until in March 1929, when the total out calls for Seminole were 34,574 per month and the operating force had increased to 60. This is an increase of about 4000 per cent in two years and nine months, as compared to an average annual increase under normal conditions of about 10 per cent.

There are at the present time 43 long-distance circuits radiating out of Seminole and, after the completion of the Oklahoma City-Shawnee-Seminole-Holdenville cable, there will be 57 circuits.

The growth in toll traffic at Shawnee, 20 mi. distant from Seminole, has been almost as great. In December 1927, there were 33,205 calls handled out of Shawnee per month. In December 1928, this traffic had increased to 50,247 calls or an increase of about 50 per cent as compared to an increase under normal conditions of about 6 per cent or 7 per cent.

As an indication that the experience of Seminole will be repeated, oil was recently discovered in the vicinity of the town of Maud, located about 17 mi. from Seminole. Until June 1928, this office handled about 868 out long-distance calls per month with an operating force of 5. In July 1928, immediately after the discovery of oil, this traffic had increased to 2467 calls, an increase of 180 per cent, and the operating force consisted of 17 people. In December 1928, the traffic increased to 10,766 calls, an increase of 1200 per cent over June 1928, and in March 1929, this office with an operating force of 47 people was handling 12,851 out calls per month, an increase of 1800 per cent in less than a year.

This, it will be appreciated, throws an unusual task and strain upon the operating forces. That the traffic is being properly handled is indicated by the following figures. The percentage of offered toll calls completed for 1928 was 91.2 for the 39 largest toll offices and 92.4 for all offices in Oklahoma, the highest results yet obtained.

The faster handling of toll calls has resulted largely from the extension of the combined line and recording operating method to all offices in this territory and to its application to an increasingly greater percentage of the traffic handled. Combined line and recording operating provides that the long-distance operator who records the subscriber's call handle it to completion instead of simply recording it and then passing it to another operator to complete. For 1928, the average interval from the time a call was placed until conversation began or a definite report was received was 2.9 minutes. For the last four years the average speed of service was as follows:

1925......6.4 minutes
1926......5.7 minutes
1927......4.2 minutes
1928......2.9 minutes

It is of interest that the percentage of the total toll traffic on which the speed of handling required more than ten minutes has been decreased from 13.1 per cent in 1925 to 2.2 per cent in 1928.

Through the traffic engineering work much improvement in the handling of this traffic has been realized and still further improvements can be made. Wherever possible, direct circuits are installed, thereby reducing the number of offices through which calls must be switched. Routes quoted for the operators' use are regularly checked to be certain that they are the best available, and through analyses of difficulties encountered on this class of traffic the combined efforts of the traffic and engineering forces are directed toward the routes or situations that are producing the most trouble.

E. T. Mahood: The existing long-distance plant represents a highly complicated, well balanced structure, gradually developed to its present form throughout the years of its building. No radical changes can be made in any one part without disturbing the whole. The fitting in of new instrumentalities has become increasingly difficult as the complexity of the long-distance network has increased.

But what does the future hold? Twenty-five years from now, will we regard the long distance service of today as we now regard that of twenty-five years ago? Of course only the future holds the answer to this question, but if past experience means anything, it certainly indicates that the limits have not yet been reached. Everything seems to point to a continued development of the country, with greater and greater business and social intercommunication. Long-distance communication facilities must keep pace with any such development. What the long-distance telephone plant will then consist of, how it will be used, the character and quality of the service, and the engineering problems involved in providing the necessary facilities challenge the imagination.

L. E. Adler: As I understand the design of the two-way telephone repeater, the gain is chiefly limited by two factors—one, the absorption of a certain amount of gain in the artificial line of network; and the other that after a certain point is reached self-oscillation occurs. What progress, if any, has been made toward the design of two-way repeaters of the full-gain type or the so-called "22-type" in which the limitations are largely removed?

H. R. Fritz: The limitation on the gain of the particular repeater mentioned by Mr. Adler, the so-called 22-type, is controlled by the maximum gain that can be obtained without the system singing. There is a return path through the repeater circuit and the total gain is limited by the best balance that can be obtained between the artificial network and the line.

If this were not a controlling factor, we might add one or more stages of amplification to the present repeater. For four-wire cable circuits in which a separate pair of wires is used for transmitting the voice in one direction and another for transmitting it in the other direction, a one-way amplifier not subject to these limitations is used. It has a maximum gain about twice that of the 22-type, which for other reasons cannot always be profitably used.

Telephone Transmission Networks

Types and Problems of Design

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Synopsis.—*A brief résumé of the nature of telephonic signals is given, showing how the qualities of wave composition which distinguish signals from other electrical waves set the requirements on networks and provide a basis for their design.*

The principal functions of wave filters, equalizers, telephone transformers, line balancing networks, and artificial lines are outlined. In order that these networks may be used in conjunction with other apparatus in the telephone system they must provide efficient transmission, low distortion, good impedance balance, stoppage of longitudinal currents, stable characteristics with current variations, low external coupling, and low reflection coefficient. In addition to these requirements the network must not cross-talk into associated circuits and must have desirable impedance characteristics in the attenuation range of frequencies as well as throughout the transmission range.

An illustration of the use of transmission networks in a typical three-channel carrier telephone system is given describing the functions of the line filter sets, the directional filter sets, band filters, and equalizers.

Some of the engineering limitations on the design and construction of networks are discussed.

* * * * *

THE past thirty years have witnessed many important developments in electrical communication. Looking back, it appears almost as though the various dynamic sciences, which had rapidly grown in scope under influence of Maxwell, Helmholtz, and others, having been for a time pent up, were fairly bursting in their eagerness to spill over into the communication field innumerable facts capable of being put to great practical use. The writings of Heaviside are clear evidence of the transition, pointing to many useful relationships while making important advances in electrodynamics itself.

Subsequently, striking applications of the new knowledge, as typified by inventions and discoveries, went constantly hand in hand with a codification or systematization of the knowledge found useful. Thus, paralleling the origination of the loading coil by Pupin, the thermionic vacuum tube by De Forest, and the wave filter by Campbell, has been the systematizing work of Campbell, Kennelly, Blackwell, and others. Out of the latter process has grown what is often called "telephone transmission theory." One of the most important and striking branches of "telephone transmission theory" is that which relates to the properties of electrical networks used for transmission purposes, called *transmission networks*.

There is not space available here to enter into a complete discussion of the uses to which transmission networks are put*, and for purposes of illustrating such uses we may confine ourselves to certain aspects of long distance telephony. This choice of illustration will serve perhaps better than any other to bring out the physical and economic attractions which the networks can have for engineers.

Long distance circuits are of various kinds. There are open wire,[2] cable, and radio facilities. These impose very different requirements on apparatus used in transmission. From an electrical standpoint, they may provide telegraph, normal telephone (both side circuit and phantom circuit),[3] superimposed carrier current (telephone and telegraph), telephotograph, and radiotelephone transmission.

In Fig. 1 there are shown the routes of some of the more important repeatered[4] open wire circuits of this country. Each route on the accompanying map (Fig. 1) represents not one circuit only but generally a large number of circuits, depending on traffic conditions. In Fig. 2 is a similar diagram of the routes of the long distance carrier telephone and telegraph facilities which are superimposed on open wire lines and cables. Circuits such as these together with long distance cables form the trunk lines, veritably the "backbone circuits," which tie together from a transmission standpoint the lines of the entire Bell System. At the present time such long distance circuits are about half open wire and half cable, with the percentage in cable rapidly growing. Of the open wire circuits about one-fifth are carrier circuits. As examples, the route through Omaha, Denver, and Salt Lake City is

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[1] Both of the Bell Telephone Laboratories, New York, N. Y.

*A short bibliography is appended.

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[2] *Open wire facilities* are those whose circuits employ wires separately strung overhead on pole lines, in contrast to *cable facilities* which use wires from among those provided by a cable. The electrical characteristics of the two types of circuits are generally quite different.

[3] *Normal telephone transmission* is that which uses directly the ordinary frequencies of speech and music. In carrier current transmission, a modulation is employed in order to permit using a different range of frequencies for transmission purposes. See section 4.

A phantom circuit is one which straddles two pairs of wires, one pair acting as a conductor in either direction. In this case, each pair is also used for transmission in which one wire of the pair works in one direction and one in the other. Each of the pairs is said to comprise a side circuit and to afford side circuit transmission.

[4] *A repeatered circuit* is one which contains repeaters providing amplification for the purpose of overcoming line losses.

on the "Central Transcontinental Route" and that through Dallas, El Paso, and Phoenix is on the "Southern Transcontinental Route." The economic importance of such circuits is evident.

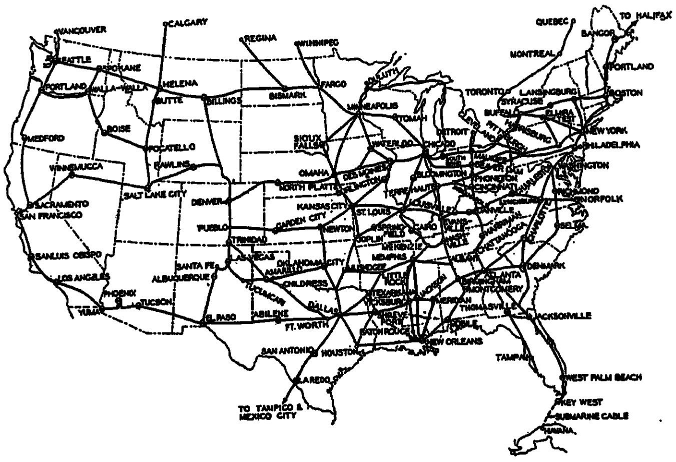

FIG. 1—SOME OF THE MORE IMPORTANT REPEATERED OPEN WIRE CIRCUITS IN THIS COUNTRY

The length of trunk line circuits, together with the high standards of transmission they require, and the desirability of employing them intensively by multiplexing messages, means careful consideration of the fundamental factors influencing transmission and of the numerous functional objectives which component apparatus must attain. This is particularly so if the service provided is to have universal standards, regularity of operation, and low costs of maintenance and operation.

Taking, at a repeater station on one of the routes plotted in Figs. 1 and 2, one side circuit of a pair of intensively occupied phantomed circuits, and schematically outlining its possible transmission channels as in Fig. 3, something may be seen of the orderly functional arrangements of apparatus required. It is not to be supposed that all long circuits provide this number of kinds of message facilities. But on the other hand, they must be considered as potentially capable of being so occupied.

With this situation in mind, we may consider in some detail just what the manifold functions of typical networks are and may illustrate them concretely in a typical system. The illustration will be that of a three-channel (*i. e.*, three-conversation) carrier telephone system. The functions necessarily divide themselves according as they have to do (1) with the primary purposes of employing the networks, and (2) with the conditions to be met by networks as integral parts of systems, working harmoniously with other apparatus. As a prerequisite to such a discussion we shall start with a survey of the ideals of signal transmission, and as a logical closure we shall indicate the directions in which physical limitations are to be found in the construction of networks to have prescribed characteristics.

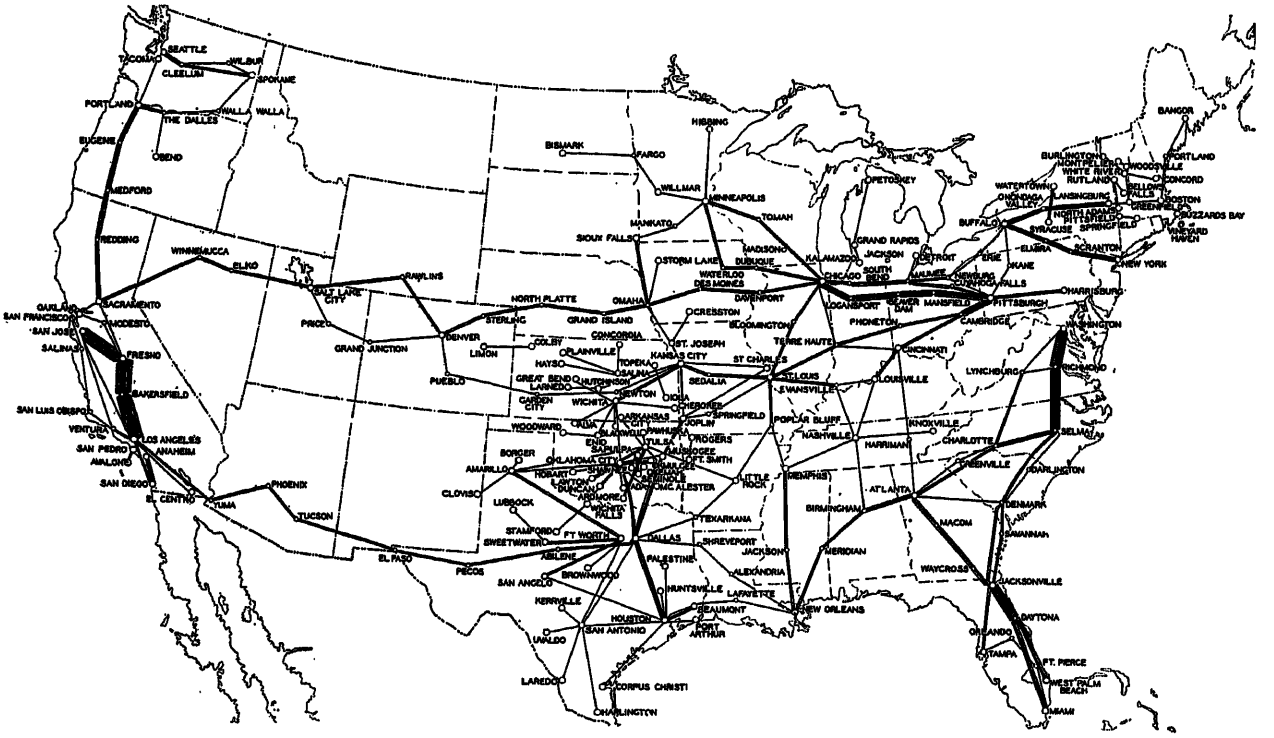

FIG. 2—ROUTE OF THE LONG DISTANCE TELEPHONE AND TELEGRAPH FACILITIES WHICH ARE SUPERIMPOSED ON LOCAL WIRE LINE AND CABLE

The remainder of this paper will therefore be devoted to: (1) signals and their transmission, (2) primary functions of transmission networks, (3) secondary functions imposed on networks by the necessity of coordinating parts of systems, (4) a specific illustration of their use in a three-channel carrier telephone system, and (5) engineering limitations on network design and construction.

Only the more important types of transmission networks will be mentioned under (2). These are: *wave filters, equalizers, telephone transformers, line balancing* (*i. e., simulating*) *networks*, and *artificial lines.* Under (4) the use of but the first two of these types will be shown.

I. Signals and Their Transmission

Signals, whether of systems of telephony (carrying speech and music), telegraphy, or telephotography, have qualities of wave composition which differentiate them from other electrical waves. These qualities determined largely the performance required of networks which aid in their transmission, and thus provide much of the physical basis of transmission network design methods. It is obvious from the purpose and circumstances of signals that they are essentially of a *temporary* character. They are used at a particular time to convey information and they cease as soon as their message has been transmitted. We may consider as pertinent examples of this: articulate speech, music, and telegraph signals.

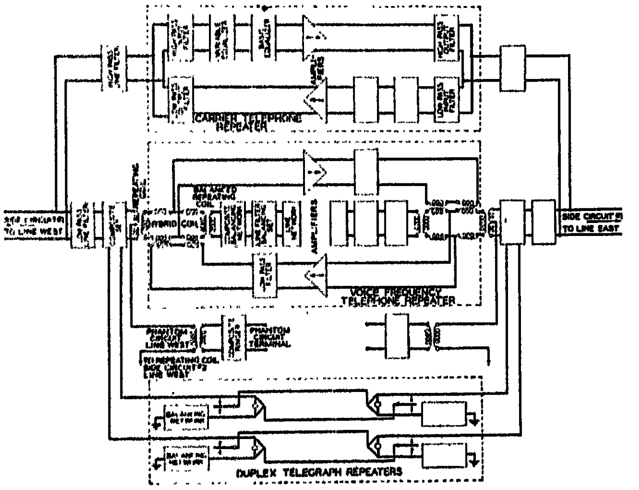
CARRIER TELEPHONE REPEATER
AMPLIFIERS
SIDE CIRCUIT TO LINE WEST
HYBRID COIL
BALANCED REPEATING COIL
AMPLIFIERS
VOICE FREQUENCY TELEPHONE REPEATER
SIDE CIRCUIT TO LINE EAST
PHANTOM CIRCUIT LINE WEST
PHANTOM CIRCUIT TERMINAL
DUPLEX TELEGRAPH REPEATERS

Fig. 3—The Possible Transmission Channels at a Repeater Station

Consequently, even the most elementary of signals, such as a "single-frequency tone," maintains an approximate steady state merely for a limited period. Such elementary signals can be used to convey only a very small number of different ideas. For the free interchange of ideas through signals we must employ either a complex code of artificial groups of signal elements to represent the characters of a written language, or the much more complex signal elements (vowels, consonants, and notes) required in spoken language and music.[5] In the case of telegraphic signals, because of economic considerations, the duration of and time between pulses is made so short that the transient state predominates, so that the distinguishing characteristics are largely transient in character. In the case of speech and music, owing to changes in expression much of the distinguishing characteristics are also transient in character. To put this in another way, the envelopes of the waves which *modulate with time* the characteristic frequencies which the sounds, if sustained, would tend to possess, have an important bearing on the meaning conveyed by the signal. Preservation of these envelopes is a matter of transient state transmission. But transient states really involve the transmission of steady-state frequencies grouped as *continuous bands* along the frequency scale. *Preservation of the entire content of a signal therefore involves consideration of the transient properties of networks, i. e., involves their steady-state properties over a somewhat wide frequency range.*

The initial and final decay periods of a sound would clearly arise from the exigencies of time,—from the temporary character of sounds and the sequential arrangement of syllables in speech and notes in music. They are, however, attended by psychological peculiarities which have a bearing not only on the superficial meaning of the signals but on the recognition of identity of the speaker (*naturalness* includes this) and on shades or variations of meaning to express ideas pointedly. These peculiarities lead to considerable differences in wave form among examples of the same nominal sound.

In spite of this and of the variations which exist between voices of different persons, it is found possible to evaluate experimentally the relative importance of different frequency ranges in the transmission of sounds and to determine what imperfections of signal waves are permissible, in the aggregate, in transmission. This is usually done by removing, with electrical wave filters, various frequency ranges from a transmission system capable of transmitting the entire audible frequency range. Not only is the importance for the interpretation of speech sounds determined but also the importance for conveying the more subtle and psychological aspects of both speech and music.

The imperfections which may creep into the transmission of signals may be classified under three headings: improper signal *volume*, *distortion* of signal content, and *interference* by waves foreign to the signals. That is, if waves are received of appropriate intensity, if the important frequency ranges are uniformly transmitted, and if nothing but the desired waves are received, the ideals of transmission will have been realized. Waves may be so strong as to cause physiological difficulties in reception, or so weak as not to be recognized. This means that efficiency of transmission is an important factor although, when amplifiers are used, partly an economic one. Signal distortion is of two kinds: *amplitude distortion* and *phase distortion.* Amplitude distortion arises from the transmission of different frequencies with unequal efficiencies and is highly important in all types of signals, but its aggregate importance depends on the importance of the frequency components that are efficiently transmitted. Phase distortion results from different frequencies traveling with different velocities such that their relative arrival times differ from their relative starting times. It is ordinarily less important in the transmission of speech than in other kinds, but the accumulation of phase distortion which may occur on long telephone circuits gives rise to undesirable transient effects. Interference may be of two kinds: It may arise from the introduction of energy into the communication circuit from outside, through electrostatic or electromagnetic coupling, or it may result from the imperfect separation of several simultaneously transmitted signals or from the generation of extraneous frequency components through modulation.

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[5] The signals corresponding to pictures resolved for transmission purposes are somewhat similar in form to telephone and telegraph signals. On the other hand, depending on the speed at which they are sent, they may require the use of a frequency range greater even than that of speech or music.

In connection with this section see "Transmission of Information" by R. V. L. Hartley in the *Bell System Tech. Journal*, July, 1928.

These three types of imperfections should be kept in mind in considering the functions of different types of transmission networks.

II. PRINCIPAL FUNCTIONS OF TRANSMISSION NETWORKS

The principal functions of the more important types of networks may be described as follows:

Wave Filters

It is characteristic of telephonic signals that their transmission requires the use of a continuous band of frequencies whose width may be perhaps 2000 cycles as a minimum and 6000 cycles as a maximum (both approximate). Where, as an example, due to economic factors it is desirable to transmit simultaneously two or more telephonic frequency bands, whether this be done entirely conductively along wires, or partly by radiation through space, discriminating means are required to make the transmission of a particular communication channel efficient to that band, or those bands, of frequencies which are desired, while rendering the circuit highly inefficient at other frequencies. To put this in another way, means are required which will *pass freely* desired bands of frequencies, while *highly attenuating* or extinguishing neighboring undesired bands of frequencies. This is the essential function which the wave filter serves. The frequencies attenuated may, however, be either true signals or unintelligible noises and as a rule comprise both. On the other hand, should disturbing frequencies lie within the frequency band of the desired signals in the same circuit, the two groups cannot be separated by filtering action.

Equalizers

In any telephonic signal which is to be transmitted efficiently, proper audition requires that all component frequencies be treated alike, in so far as efficiency of transmission is concerned, in order that some frequencies may not be unduly emphasized to the detriment of others. This uniformity of efficiency of a band of frequencies is required, of course, primarily as a characteristic of the overall system traversed by the signal. Some parts of a transmission system are inherently not capable of closely uniform transmission. Wave filters, for example, tend to discriminate against frequencies near the edges of their transmission bands due to dissipation of energy in the effective resistances of their elements and to reflection losses at their terminal junctions. Loaded telephone lines behave similarly and unloaded lines have an attenuation which rises gradually with frequency because of the nature of the capacity effects present. The equalizer cannot restore the loss of effectiveness which occurs to some frequency components due to such distortion. Amplification of power requires such generative devices as vacuum tubes. What the equalizer can do is to attenuate efficiently transmitted frequencies in such a manner and to such an extent that all desired frequencies suffer or prosper in transmission alike.

Telephone Transformers

The telephone transformer is a rather different device, in its electrical structure, from the power transformer. Two chief differences in operating requirements exist. The impedance of the terminal apparatus of power transmission circuits is generally low in comparison with the characteristic impedance[6] of the lines and the lines are electrically short, this explaining why "stepping up" voltages on power transmission systems usually leads to increased efficiency. In telephone circuits, on the other hand, the impedance of the terminal apparatus is comparable in magnitude to the characteristic impedance of the lines, and the lines are electrically long. Efficiency of transmission is under the latter conditions governed largely by "matching of impedances" at junction points in a circuit. Consequently the effectiveness of telephone transformers in properly transforming the impedances of a circuit is of primary importance. Secondly, because of the relatively high frequencies employed in telephone work and because a rather wide band of frequencies needs to be transmitted, inherent internal impedances, such as those of winding capacities, become very important and require that the telephone transformer be considered primarily as a network. In the case of transformers working into the very high input impedances of vacuum tubes or providing band transmission at high frequencies, we may say that the frequency range of transmission and the transformation ratio of telephone transformers is as completely dependent on the magnitude of inductance and capacity elements as are the characteristics of wave filters.

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[6] The *characteristic impedance* of a line is one to which the impedance of connecting apparatus should be approximately matched for maximum power output conditions. It is principally determined by the distributed inductance and capacity of the line.

Line Balancing Networks

The two-way repeater circuit has essentially the form of an a-c. bridge, in which a balanced condition depends both on the similarity of certain transformer (hybrid coil) windings and upon the equality of impedance of the two halves of the repeater output circuit. In the case of the 21-type (two way, one amplifier) repeater, the two halves are the telephone lines joined by the repeater. In the case of the more stable 22-type (two way, two amplifier) repeater, an output circuit exists at each end of the repeater and each telephone line is matched in impedance against a "dummy" line, or balancing network. The function of this network is solely to present the same impedance to the repeater over a range of frequencies as is presented by the line, *i. e.*, to simulate the impedance of the line. No *through* transmission is required of it. The design of such networks involves a choice of configurations of resistances, capacities, and inductances which bear a close physical relationship to such impedance elements of the line as dominate its total impedance, and a determination of impedance values of the network elements in accordance with network theory.

Artificial Lines

The function of artificial lines is to exhibit some transmission property or properties of a real line, at one frequency or over a range of frequencies, so that there may be compactly constructed a network which will serve for certain purposes involving wave propagation, in place of a real continuous line. The artificial line is generally a network of lumped, discrete impedance elements whose constants are computed from network theory. One of the most common forms of artificial line is the variable *attenuator*, which is used as a basis of comparison in attenuation and transmission loss measurements. It is usually constructed of resistances so that over a wide range of frequencies it displays uniform attenuation. Other forms of artificial line, however, display over a range of frequencies the varying attenuation (and perhaps phase) properties of real lines and circuits and involve properly disposed inductance, capacity, and resistance units. In contrast to the equalizer, they simulate, rather than compensate for, frequency-transmission characteristics.

III. SECONDARY FUNCTIONS OF TRANSMISSION NETWORKS

The functions of transmission networks outlined in section 2 show the ordinary reasons for their employment in communication systems. When so employed, however, since they must fit into the transmission scheme of systems and work harmoniously with other apparatus, they may be faced with numerous additional requirements. Occasionally, the latter duties may even provide the more difficult design requirements. The more important of these possible secondary requirements follow:

Efficient Transmission

This requirement applies particularly to transformers, wave filters, and equalizers. Each of these types of apparatus could fulfill its principal duties while consuming an *undue portion* of the strength of signals entrusted to it. Under some conditions, amplification of signals by the use of vacuum tubes could offset this consumption from a transmission standpoint at an economic sacrifice and the problem would then be one of economic balance; but in other cases the amplification would be objectionable because accompanied by such factors as amplification of noise, impoverishment of circuit balance, and loss of circuit flexibility. Efficient transmission is secured by proportioning materials so as to limit dissipation of energy.

Low Distortion

It is not possible, of course, to secure entirely uniform transmission of all signal frequencies, and indeed refinements of transmission beyond a certain point have in this direction economic limits. To the extent, however, that distortion exists, the quality of signals is impaired. When distortion is excessive, equalizers often provide a desirable way of reducing it, but do so at a sacrifice in over-all circuit efficiency.

Impedance Balance

There is a number of kinds of impedance balance commonly found in communication circuits. All of them are akin to balances in a-c. bridge circuits. They need to be maintained over bands of frequencies. Balancing arrangements offer a kind of selectivity which differs from filter selectivity in that, in general, it keeps apart throughout systems two trains of waves arising from different sources and drawn off to different destinations, instead of separating on a basis of frequency difference waves which have mingled in a common circuit. An example is the balance necessary in a side-circuit, and therefore imposed on transformers or filters located in the latter, in order that currents traveling a superimposed phantom circuit may not interfere with side-circuit transmission. A second kind of balance is the repeater balance, mentioned in section 2, which may lead to requirements on, say, filters located in the output circuits. Here the separation is between incoming and outgoing speech signals using the same frequency range. In other cases, circuit balance is relied upon to prevent interference currents set up between circuit wires and ground from entering the circuit of the former. To obtain a high degree of balance, apparatus located in a balanced circuit must be quite symmetrical in its circuit impedances with respect to the associated circuit from which it is protected by balancing.

Stoppage of Longitudinal Currents

Longitudinal currents are those which travel both sides of a circuit in the same direction as parallel paths and return by some other path, generally a ground circuit. They are objectionable because (1) they tend to enter the transmission circuit whose sides they travel, in this case called a transverse circuit, through irregularities of balance in the latter and to unite with transverse currents; and (2) in many cases they correspond to objectionable voltages of considerable strength. They may be eliminated in two ways: (1) by providing for them a short-circuit path to ground in the longitudinal circuit and (2) by causing virtually an open-circuit gap to occur in the longitudinal circuit, each of the methods being so employed as not to affect transmission in the transverse circuit. Transformers are required by either method.

Transmission in Closely Associated Circuits

When the same wires are used to provide both side circuit and phantom circuit transmission, it is a requirement of apparatus (such as transformers and filters) inserted in side circuits that they shall not impair transmission in the phantom circuit, and vice-versa. This means that apparatus in one circuit must not insert impedance in another circuit and requires close coupling between those impedance elements of networks which are located in series with the line wires.

Stability of Characteristics with Current Variations

If the inductances, resistances, or capacities of a network are subject to variations in value with varying current strength, as is the case, for example, with the inductance and resistance of coils using unstable core materials, two detrimental results are possible. First, the frequency characteristics of the network will be distorted as the current varies from its mean value. Second, a kind of modulation will occur, resulting in the generation of new frequencies whose presence will interfere with interpretation of signal waves traversing the network. Stability of characteristics is, obviously, to be secured by the use of materials whose permeability, resistance, and dielectric constant do not vary with current strength.

Low External Coupling

It is desirable that electrostatic and electromagnetic coupling between a network and parts of its own or other circuits be kept low, that is, that the network be self-contained electrically and receive and supply energy only through its normal input and output terminals. When coupling exists to some other circuit, (1) energy introduced from the latter circuit may interfere with signals or (2) energy derived from the network may cause interference in the circuit to which it is coupled. When coupling exists to other parts of the circuit in which the network is located, (1) impedance balances may be upset, (2) large attenuations, as of a filter, may be nullified by bypass circuits, and (3) circulating currents of considerable strength may be set up when vacuum tubes are involved in the circuit. Electrostatic and electromagnetic shielding permits control of external coupling.

Low Reflection Coefficient

Where networks connect directly or through other transmitting networks to a line, it is desirable that the impedance offered by the network to line currents be such that wave reflections are not set up in the line by impedance mismatches at its ends. Reflections result not only in undesirable transient waves in the circuit in which they originate but cause interference into other circuits and complicate line transposition problems. There is thus usually prescribed for a network an impedance characteristic.

Modulator Circuit Impedances

When networks connect to vacuum tube modulator (or demodulator) circuits, they are required to have certain types of impedance characteristics even in the non-transmitting range of the networks, in order that modulation may be as complete as possible and yet undistorted. In certain modulator circuits the networks must present a high, and in other cases a low, impedance to the modulator if certain frequencies which the tubes tend to supply are to be eliminated.

Parallel or Series Operation

It is a requirement particularly of groups of wave filters selecting signals for a number of communication channels that the impedance of each in the transmitting ranges of the others be such as not to interfere with the operation of the latter. In the case of filters operated in parallel, this means a high shunting impedance; in the case of filters operated in series, a low series impedance. It is possible to meet this requirement because in general numerous configurations of filter elements can be arranged which will meet given transmission requirements but allow wide latitude of choice of impedance characteristics.

A number of considerations should be noted with respect to the above requirements. First, in any given circuit conditions, the requirements can be expressed in numerical measure so that networks can be designed to meet them quantitatively and measured accordingly when constructed. Secondly, the various requirements which might be imposed on a network by circuit conditions, tend for the most part to be conflicting, since they restrict choices of design, and must be evaluated and balanced from an economic standpoint. Finally, the ability of a network to meet the above requirements is subject to physical limitations which will be discussed in section 5.

IV. ILLUSTRATION OF THE USE OF TRANSMISSION NETWORKS IN A COMMUNICATION SYSTEM

Filters and Equalizers Required in a Typical Three-Channel Carrier Telephone System

As an example of the way in which a number of functional requirements may be simultaneously imposed in practise on networks used in communication systems, let us consider the more important of the filters and equalizers which enter into the scheme of operation of a typical three-channel carrier telephone system. The example chosen is that of a "Type C" carrier system, designed for use over relatively long distances and employing the so-called[8] "suppressed carrier, single sideband" method of transmission. It represents, of course, but one set of possible circuit conditions and the numerical constants or characteristic curves which give the required performance of the networks would be greatly influenced by such factors as: grade of transmission desired, gage of wire available, economical distance between repeater stations, and interference conditions. Hence the example should be considered to illustrate principles rather than to define practise closely.

In Fig. 4 are outlined schematically the terminals and one of the repeaters of a Type C system, so drawn as to emphasize the filters and equalizers used therein. The upper half of the diagram relates to transmission from left to right; the lower half to transmission in the opposite direction (and to non-carrier facilities). Beginning at the left-hand end of the diagram, currents derived from the subscriber's line S and traveling to the right, pass through balancing apparatus H which separates on a directional basis incoming and outgoing currents of the carrier system, and thence into a modulator M which raises them in the frequency scale. For each channel the desired sideband is selected by a modulator band filter (for the channel whose modulator is shown, band filter No. 5) and is sent into an amplifier A common to all three channels. Here the strength of the signals is increased so that even the attenuation of a considerable stretch of line may leave them strong enough to override disturbances. The "high pass" directional filter, arranged so as to pass currents in the upper half of the frequency range used by the system, transmits the signals to a high pass "line filter" and this in turn delivers them to the line.

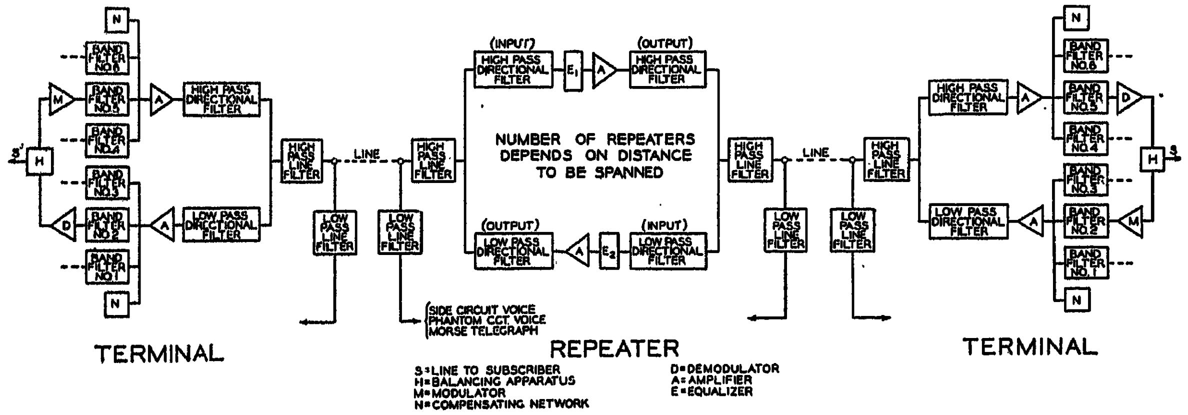

FIG. 4—SCHEMATIC OUTLINE OF THE TYPE *C* CARRIER SYSTEM

Upon arrival (in an attenuated condition) at a repeater station, the carrier currents are conducted through a high pass line filter to the repeater proper. A separation of high and low frequency carrier currents, corresponding respectively to the two directions of transmission, is effected by directional filters; consequently, currents arriving from the left go through the "input" high pass directional filter, reaching equalizer E_1. The purpose of the latter is to correct for the signal distortion introduced by the line and by directional filters. An amplifier A, next in order, raises the level of strength of the signals of all three channels. Finally, an "output" high pass directional filter leads the amplified signal currents to a high pass line filter and thence to the next stretch of line over which they are to travel, This process may be repeated a number of times, depending on the total distance spanned by the system.

When the signal currents reach the receiving terminal on the right, they are lead successively through a high pass line filter and a high pass directional filter to a terminal amplifier A where the channel currents are strengthened prior to demodulation. Demodulator band filters (No. 5 for the channel shown) select for each channel the desired group of frequencies and reject the others so that the demodulator D may not be hindered in its operation. The latter restores the channel currents to their original position in the frequency scale and delivers them through the balancing apparatus H to the receiving subscriber's line S.

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[8] The reader is doubtless familiar with the fact that in carrier modulation, as in radio systems, a "carrier frequency" and "two sidebands" are produced. For demodulation and recognition of signals the carrier frequency and one sideband may be suppressed, the other sideband being transmitted and a local carrier frequency supplied at the receiving end of the system. By this method a smaller frequency range is required for communication. For a discussion of methods of operation of carrier systems, reference may be made to *Carrier Current Telephony and Telegraphy* by E. H. Colpitts and O. B. Blackwell, A. I. E. E. TRANS., 1921, which discusses generally the progress which had been made in carrier systems up to that time; also to various later papers which may be found in the *Bell System Tech. Journal*, such as that by H. A. Affel, C. S. Demarest, and C. W. Green, July, 1928, entitled "Carrier Systems on Long Distance Telephone Lines."

Transmission in the reverse direction is accomplished in a similar fashion, except that low pass directional filters, transmitting currents in the lower half of the frequency range of the carrier system, replace the high pass directional filters of the upper half of the diagram.

We may discuss in order the duties of four groups of apparatus: (1) the "line filter sets," each comprising a high pass and a low pass line filter connected in parallel at the ends next to the line, (2) the directional filter sets, each comprising a high pass and a low pass directional filter connected in parallel at the ends next to the high pass line filter, (3) the band filters, arranged in groups of three at the terminal stations and provided with "compensating networks," and (4) the equalizers located at repeater stations, one being used for each frequency range of transmission.

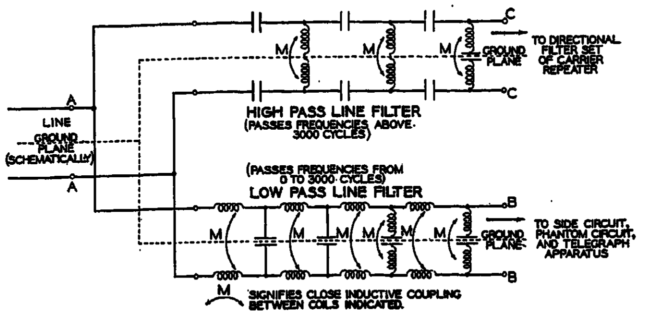
C
M
M
M
GROUND PLANE
TO DIRECTIONAL FILTER SET OF CARRIER REPEATER
C
A
LINE
GROUND PLANE (SCHEMATICALLY)
A
HIGH PASS LINE FILTER (PASSES FREQUENCIES ABOVE 3000 CYCLES)
(PASSES FREQUENCIES FROM 0 TO 3000 CYCLES)
LOW PASS LINE FILTER
B
M
M
M
M
M
M
GROUND PLANE
TO SIDE CIRCUIT, PHANTOM CIRCUIT, AND TELEGRAPH APPARATUS
B
M SIGNIFIES CLOSE INDUCTIVE COUPLING BETWEEN COILS INDICATED.

Fig. 5—Schematic Arrangement of a Line Filter Set

Functions of Line Filter Set

Referring to Fig. 4, it is apparent that a line filter set is employed wherever carrier currents are put on, or taken off, a stretch of line in the system. The reason for this is that the line is normally employed for the transmission of side circuit voice currents and perhaps phantom circuit voice and low frequency telegraph currents as well, and the line filter set provides the apparatus by means of which the carrier currents may be superimposed on the same line without interference.

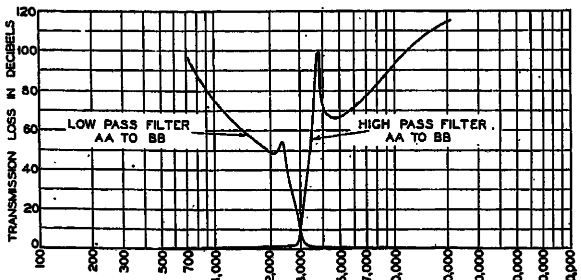

Fig. 6—Transmission Loss Curve Corresponding to the Line Filter Set of Figs. 5 and 7

The superimposition is accomplished in two steps: first, a low pass filter inserted in the circuit of the normal facilities permits passage of the normal currents but is designed to reject from the normal circuit high-frequency carrier currents; second, a high pass filter transmitting carrier currents is inserted in the carrier circuit so designed as to reject low-frequency currents from the normal facilities. A schematic arrangement of a line filter set is shown in Fig. 5 and the transmission loss curves corresponding to it in Fig. 6. A photograph of a typical line filter set is given in Fig. 7. Fig. 6 shows the reduction in strength suffered by currents because of the presence of the filters between *A A* and *B B*, and between *A A* and *C C*, respectively. This reduction (called transmission loss) is plotted in *decibels*[9] (*d b*) which are logarithmic in character but the corresponding current ratio values are shown by the ordinates at the right of the figure.

We may now observe:

(1) That if telegraph facilities are maintained, the low-pass filter must yield desired telephone current characteristics while carrying substantial telegraph currents.[10]

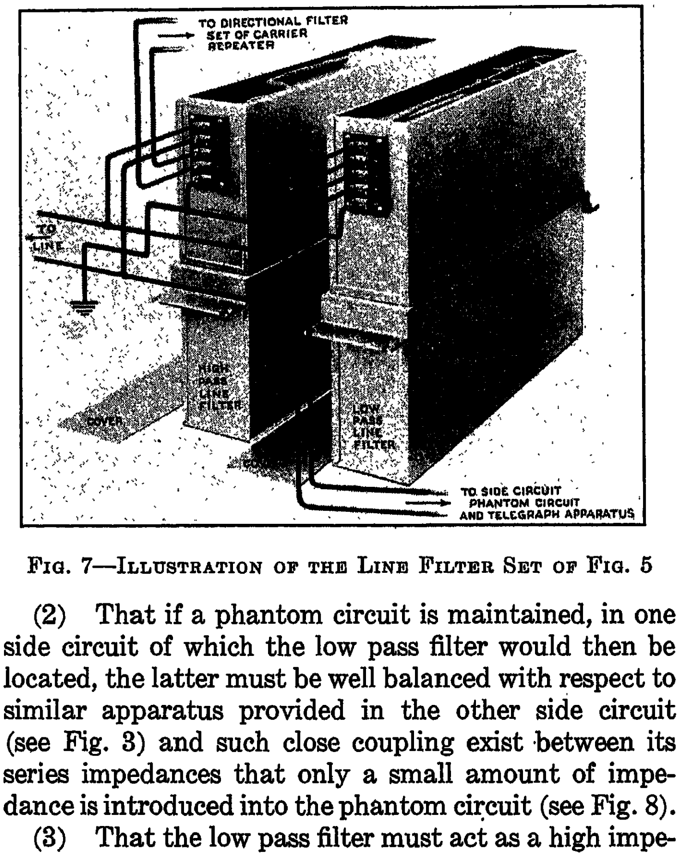
TO DIRECTIONAL FILTER SET OF CARRIER REPEATER
TO LINE
COVER
HIGH PASS LINE FILTER
LOW PASS LINE FILTER
TO SIDE CIRCUIT PHANTOM CIRCUIT AND TELEGRAPH APPARATUS

Fig. 7—Illustration of the Line Filter Set of Fig. 5

(2) That if a phantom circuit is maintained, in one side circuit of which the low pass filter would then be located, the latter must be well balanced with respect to similar apparatus provided in the other side circuit (see Fig. 3) and such close coupling exist between its series impedances that only a small amount of impedance is introduced into the phantom circuit (see Fig. 8).

(3) That the low pass filter must act as a high impedance shunt in the transmitting range of the high pass filter, and that, correspondingly, the high pass filter

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[9] Current, voltage, or power (magnitude) ratios may be expressed logarithmically in decibels (*d b*). The number of *d b* corresponding to a ratio of the magnitudes of two currents I_1 and I_2 is $N_{db} = 20 \log_{10} \left| \frac{I_2}{I_1} \right|$. Voltage ratios are expressed similarly. For two powers P_1 and P_2, however, $N_{db} = 10 \log_{10} \left| \frac{P_2}{P_4} \right|$.

[10] The location of the telegraph composite set in Fig. 3 has been changed to eliminate some of these difficulties, so that *the standard location of this composite set is now on the line side of the low pass line filter.*

must act as a high impedance shunt in the transmitting range of the low pass filter, *i. e.*, that the filters must be mutually designed for parallel operation. Impedance characteristics illustrative of this point are given in Fig. 9.

(4) That disturbances of considerable potential, such as those which may occur when telephone lines are exposed to power transmission lines, find the line filter set as the first networks which they would traverse in the system; and the line filter set must have sufficient dielectric strength to withstand potentials not taken care of by standard protection apparatus.

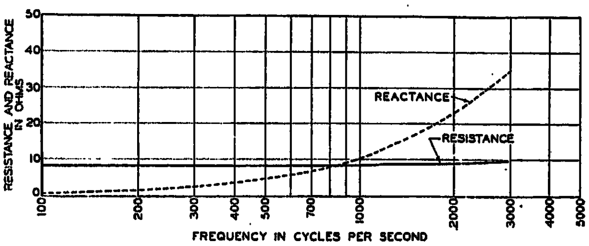

Fig. 8—Impedance Introduced into the Phantom Circuit by a Low-Pass Filter

(5) That a good degree of balance, with respect to ground *G*, between the impedances in the respective halves of the filters must be provided in order to prevent longitudinal currents from entering the carrier system or the normal circuit.

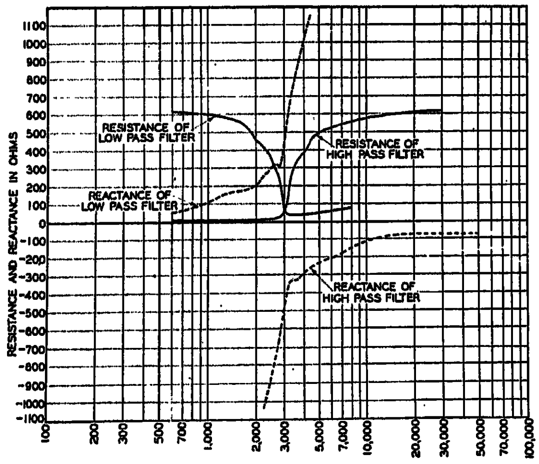

Fig. 9—Impedances of Low- and High-Pass Filters Designed for Parallel Operation

(6) That only a very small amount of transmission loss can be allowed in the transmission range of each filter, because of the number of line filter sets which currents encounter in a system of considerable length.

(7) That only very low distortion is permissible in the transmission bands, for the reason given under (6).

(8) That, because of the rather rapid changes in the transmission loss characteristics in the vicinity of 3000 cycles, precision of location of this frequency as a separational point requires that the filter characteristics be independent of current values.

(9) That from a standpoint of wave reflections at the end of long stretches of line, the line filter set is in a prominent position and the impedance which it presents to the line (this of course depends upon the other apparatus as well as upon the line filter set) must match the impedance of the line, as illustrated in Fig. 10. The difference between the filter and line impedances of Fig. 10 is expressed as a percentage "reflection coefficient" in Fig. 11 to indicate the portion of arriving waves which will be reflected.

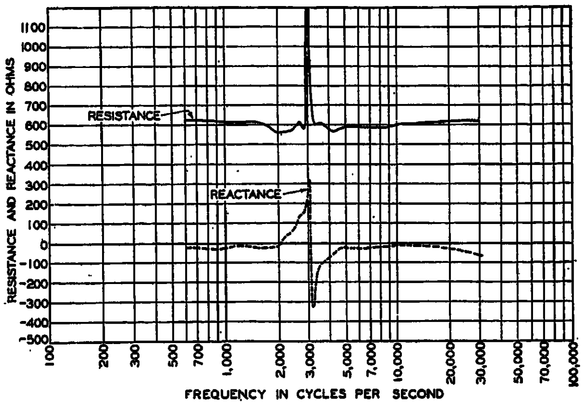

Fig. 10—Impedance of a Long Telephone Line

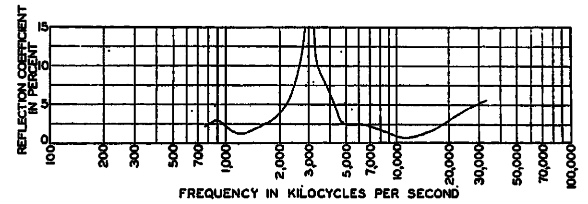

Fig. 11—The Difference Between the Filter and Line Expressed as a Reflection Coefficient

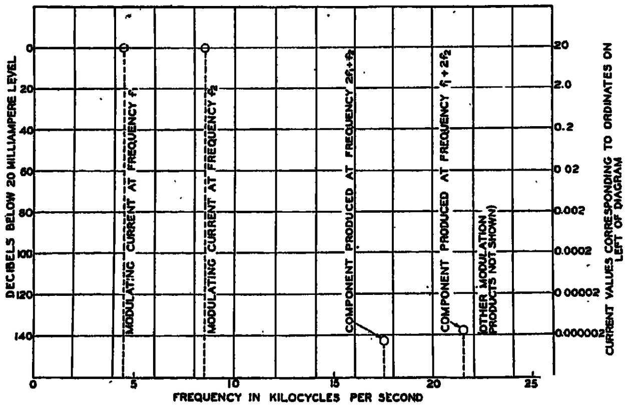

Fig. 12—Illustrating how Modulation May be Generated in the Low-Pass Filter Which Will Penetrate the High-Pass Filter

(10) That modulation occurring due to unstable filter elements can give rise to: (a) high frequencies generated in the low pass filter which will be transmitted

as interference by the high pass filter, and (b) low frequencies generated by intermodulation of carrier frequencies in the high pass filter which will penetrate the low pass filter; thus that an additional requirement exists on the stability of the filter characteristics. Fig. 12 shows how modulation current may be generated in the low pass filter which will penetrate the high pass filter.

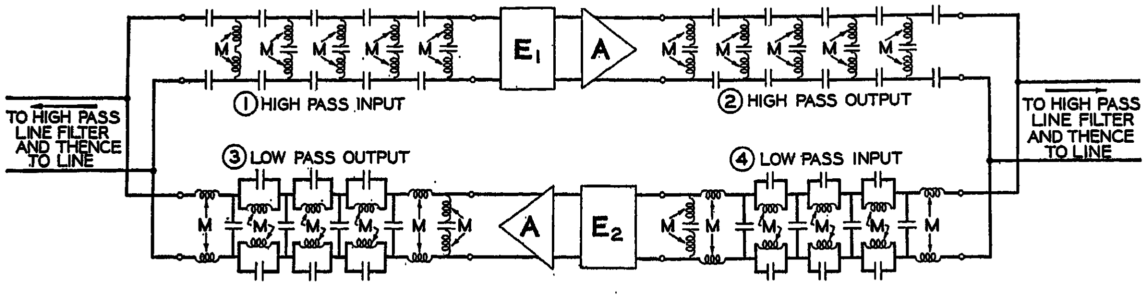
E1
A
M
①HIGH PASS INPUT
②HIGH PASS OUTPUT
③LOW PASS OUTPUT
④LOW PASS INPUT
M2
A
E2
TO HIGH PASS LINE FILTER AND THENCE TO LINE
TO HIGH PASS LINE FILTER AND THENCE TO LINE

FIG. 13—CONFIGURATION OF TYPICAL DIRECTIONAL FILTERS IN A REPEATER CIRCUIT CONTAINING AMPLIFIERS (A) AND EQUALIZERS (E)

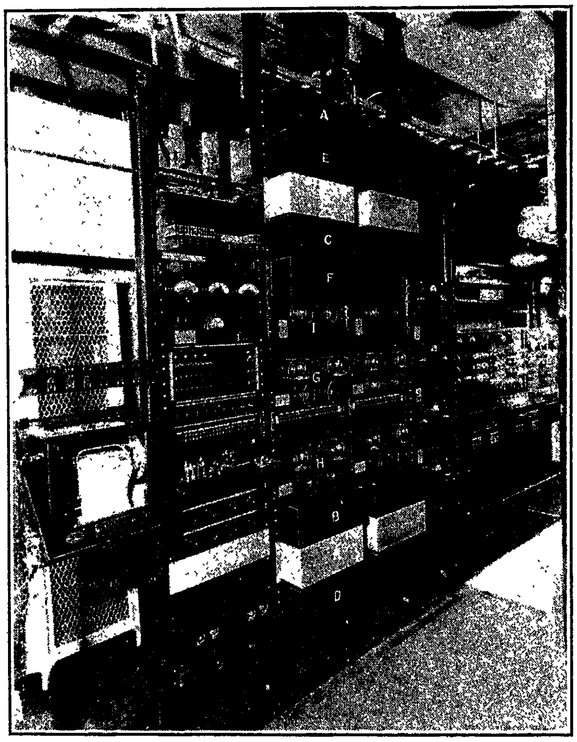

FIG 14—CARRIER REPEATER

A—High-pass input filter
B—High-pass output filter
C—Low-pass input filter
D—Low-pass output filter
E—Equalizer for upper frequency group
F—Equalizer for lower frequency group
G—Amplifier for upper frequency group
H—Amplifier for lower frequency group
I, J, and K—Auxiliary apparatus

(11) That a requirement will exist on the degree of interference permissible between the filters of each set and other apparatus of the same or other systems located at the same station. (In this connection, for example, note from Fig. 4 that interference between high pass line filters at the opposite ends of a repeater provides a return path for amplified currents and tends to cause circulating currents or even "singing".)

(12) That when at repeater points a normal voice repeater exists in the circuit in which the low pass filter is placed, an impedance balance will be required between the line filter set and the corresponding balancing apparatus in the repeater.

Functions of Direction Filter Sets

The functions of directional filters, whether located at terminal or repeater stations, are less numerous in kind than those of the line filters, but a generally more severe degree of performance is asked of them. We need consider only those used in repeater stations, as virtually identical filters are required for the terminal points. Fig. 13 shows the configuration of typical directional filters as employed in a repeater circuit, and Fig. 14 is a photograph of a carrier repeater which includes directional filter sets. The input and output filters are the same in form but differ slightly in values of the network constants.

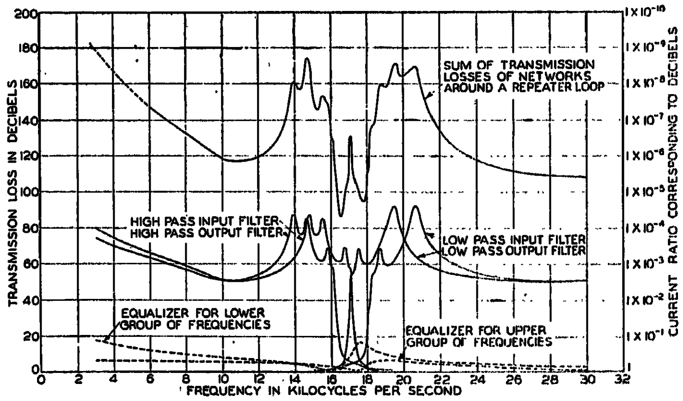

FIG. 15—TRANSMISSION LOSS CHARACTERISTICS OF THE FOUR DIRECTIONAL FILTERS USED IN A REPEATER CIRCUIT. EQUALIZER CHARACTERISTICS ARE SHOWN IN DOTTED FORM

Transmission loss characteristics of the four directional filters are given in Fig. 15. The sharpness of selectivity displayed may be gaged by comparison with the characteristics of Fig. 6 for the line filters, it being remembered that on a frequency percentage basis a 1000-cycle interval at 17 kilocycles would correspond to about 175 cycles at 3 kilocycles. From an economic standpoint, of course, it is the *actual* width of frequency interval given up for separational purposes which is of importance, but from a design standpoint *percentage* width is a measure of difficulty.

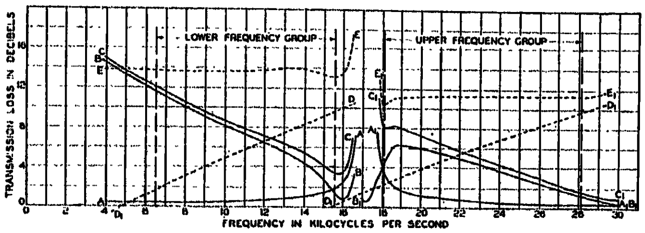

FIG. 16—CHARACTERISTIC OF TYPICAL NETWORK STRUCTURES OF FIG. 20

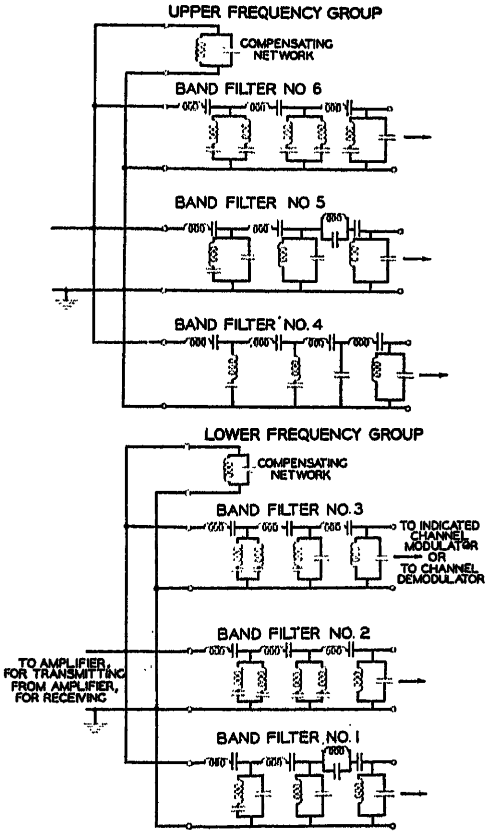

FIG. 17—TYPICAL GROUPS OF BAND FILTERS FOR THE SYSTEM OF FIG. 4

The functions of the directional filters will not be examined in the manner of the line filters, because of lack of space. However, among the most important requirements are those on modulation, interference, distortion, and the combined filter transmission losses around the repeater circuit (safeguarding against "singing").

Functions of Band Filters

Typical groups of band filters for the system of Fig. 4 are exhibited in Fig. 17 and the corresponding transmission loss characteristics in Fig. 18. In Fig. 18 the line filter and terminal directional filter loss characteristics are shown in dotted line form. The lower group of transmission bands lies between the separational point (3000 cycles) of the line filter set and the separational point (17,000 cycles) of the directional filters. The transmission loss in the bands is generally higher than for the directional and line filters, for two reasons; channel currents traverse but two band filters in a system and the loss may be afforded; also it is natural for the loss in filters having fairly narrow transmission bands to be higher than in low pass and high pass filters.

We shall discuss four additional points in connection with the functions of these filters, out of the many which might be selected:

(1) Their parallel operation is aided by certain "compensating networks." In order that each filter may be least affected in its transmission band by the shunting effect of the others, each filter is built out with a special termination at the paralleling end (as if to "load" away the effect of the other filters). Such building-out causes distortion at those *edges* of transmission bands which have no *neighboring* transmission bands in the frequency scale (for example, near 6.5, 15, 18, and 28 kilocycles). To overcome such distortion, the compensating networks are given the impedance values which would be offered by such absent filters in the series.

(2) The band filters are not balanced with respect to a ground system but rather represent only one-half of a

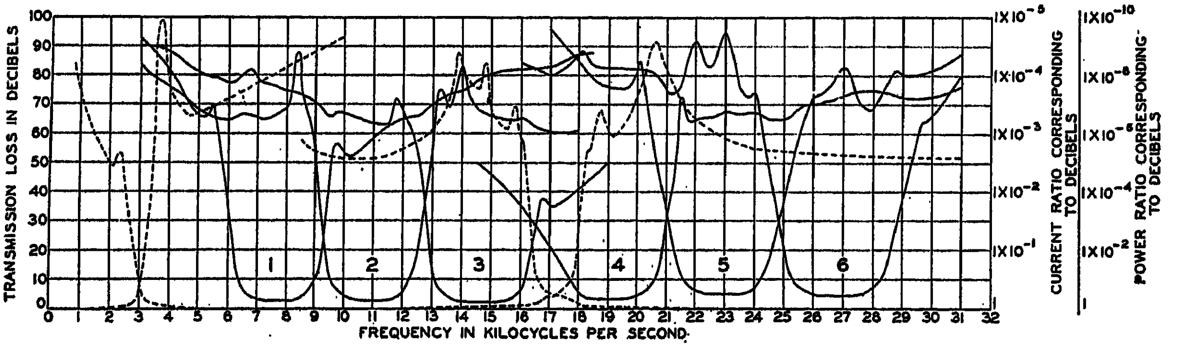

FIG. 18—TRANSMISSION LOSS CHARACTERISTICS OF BAND FILTERS SHOWN IN FIG. 17. LINE FILTER AND TERMINAL DIRECTIONAL FILTER CHARACTERISTICS ARE DOTTED

balanced circuit (a form of "unbalanced" circuit) and a common side of each is maintained at ground potential. This type of circuit is more economical, where it can be employed.

(3) The band filters all possess, at the end remote from paralleling, the same type of termination, *i. e.*, there is in each case an anti-resonant arm, in shunt across the filter. These terminations for band filters correspond to a class of band filter sections which offer desirable impedances in certain frequency ranges to the modulators or demodulators with which, at one end of the system, they work. They do not, however, result in the *least distortion* which might be had in the transmitted bands if modulator and demodulator action had not to be considered. A different type of termination will be found in all of the directional and line filters of Figs. 5 and 13 and the latter termination is chosen from a distortion and reflection coefficient standpoint. This illustrates how two independent requirements both intended to accomplish very desirable objects, may be in conflict. Fig. 19 shows the impedance of a band filter with associated apparatus as seen from a modulator.

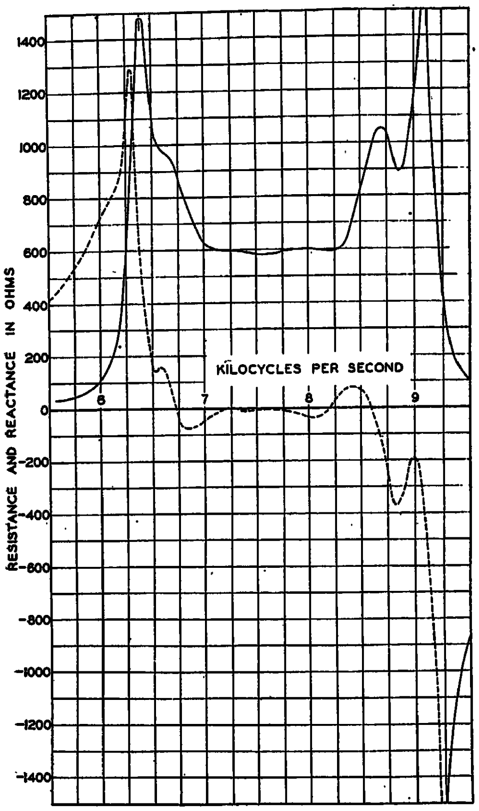

FIG. 19—THE IMPEDANCE OF A BAND FILTER WITH ASSOCIATED APPARATUS AS SEEN FROM A MODULATOR

(4) One frequency which each modulator band filter must suppress is the truly steady-state carrier frequency supplied by its modulator. The carrier frequency in each case will be located just off a transmission band, on a "side" of the loss characteristic. While the voice currents represented by the transmitted sideband may, from instant to instant, change in amplitude and frequency content, the carrier frequency is fixed in size and frequency location. For its suppression without undue distortion of the wanted sideband, the various transmission bands must be very accurately located in the frequency scale at predetermined positions. This is one of the predominating features of carrier band filter characteristics.

Functions of Equalizers

The position of equalizers in a carrier repeater has been shown in Figs. 4 and 13 and typical network structures, connected between transmitting apparatus Z_S and receiving apparatus Z_R, are illustrated in Fig. 20. Loss characteristics are shown in Figs. 15 and 16. We shall merely add to the previous discussion of carrier repeaters, first, that in whichever frequency group the equalizers occur, they add at all frequencies to the "loop loss" of the repeater networks and second, that to the extent that they provide transmission loss, they are of such a type as to provide desirable impedances in which to terminate directional filters and thus have an important function from a reflection coefficient standpoint. Fig. 16 brings out the way in which equalization for line and directional filter distortion occurs.

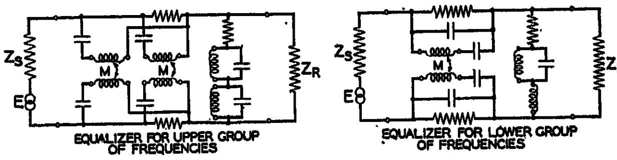
Zs
E
M
M
ZR
EQUALIZER FOR UPPER GROUP OF FREQUENCIES
Zs
E
M
ZR
EQUALIZER FOR LOWER GROUP OF FREQUENCIES

FIG. 20—TYPICAL NETWORK STRUCTURES BETWEEN Z_S AND Z_R

V. ENGINEERING LIMITATIONS ON NETWORK DESIGN AND CONSTRUCTION

To work out theoretically a network of inductances, capacitances, and resistances which will offer certain desirable transmission characteristics over a frequency range, is a matter of following certain theoretical design methods. To build actual networks which will possess and retain these characteristics involves a large number of factors which come into play and which must be balanced against one another.

Accordingly, it is necessary to point out the principal directions in which limitations are to be encountered, for the value of network design methods is limited exactly by the degree to which networks may be physically constructed so as to give desired characteristics.

Between the indicated theoretical performance of a network and its actual performance there enter, in general, four types of discrepancies. They are concerned with the following four questions: (1) How accurately does the indicated theoretical performance correspond to the exact theoretical network chosen? (2) How nearly is the actual form or configuration of the network what it is theoretically supposed to be? (3) How accurately is the network constructed? (4) How stable are the characteristics of the network during operation?

There are two sources of error which affect the theoretical exactness of the computed performance of a network; both have to do with the approximations which are necessary when mechanical aids are used in computation. One lies in computations of the network constants (L's, C's, and R's) from chosen significant frequencies, impedances, or other design bases, and the other in determinations of the characteristics themselves either from the network constants, or from the bases referred to. Discrepancies of either kind are not, when reasonable care is used, ordinarily large enough to be important except in the case of apparatus which needs to be made very accurately.

The form or configuration of a network introduces four important factors leading to discrepancies, *viz.*, (1) interactions between network elements, (2) distributed impedance effects in the elements themselves, (3) admittances from elements to ground, and (4) effects of the wiring system.

Interactions between Network Elements

These arise from the difficulty of confining electrical effects within exact boundaries. Electrostatic and electromagnetic coupling between various pairs of elements give the effect of inserting additional elements in the network. Such effects are most serious when coupling exists between elements differing in potential (relatively) by a considerable amount. An example is the tendency of capacitive coupling between elements at opposite ends of a network of considerable transmission loss to by-pass currents around the network and thus to limit that loss.

Distributed Impedance Effects in Network Elements

It is simplifying from a mathematical standpoint to deal with definite lumped inductances, capacitances, and resistances. But it is only proper to realize that lumped constants do not occur strictly in nature. Rather it is the tendency of circuits, except when specially formed and even then to some extent, to spread out in such a manner as to show distributed impedance effects. Thus the effective impedances of network elements are formed by a summation of differential effects not all of the same kind. What is done, for example, in the design of inductance coils is merely to bring out, or emphasize, inductance in comparison to resistance and capacity, for a certain frequency range and to a first approximation. The character of the impedance, then, must necessarily change with frequency. A coil of wire may be in one frequency range nearly a pure resistance, in another an inductance, and in a third capacity may predominate. By a choice of types of construction, with a view of course to cost, these changes with frequency may be minimized but cannot wholly be eliminated.

Admittances from Elements to Ground

Although interactions between network elements may be reduced by shielding, it should be appreciated that the presence of shields tends to cause admittances to ground from different parts of a network to increase. Even when the best shield has been used there is not, exactly speaking, such a thing as a two-terminal impedance (*i. e.*, one in which current enters and leaves only through a single pair of terminals), unless, of course, one terminal is connected to ground. A condenser, for example, may have not only its main capacity between terminals but a minor capacity from each terminal to ground as well.

Wiring Effects

The impedances of the wiring system of a network are often neglected in theoretical analysis but may be important from several standpoints. Capacity introduced by wiring tends to cause interactions among the network branches, and a ground system of appreciable resistance will not only increase low transmission losses but destroy high transmission losses by providing mutual impedance between input and output terminals of the network.

Accuracy of network construction is dependent, for a given design, primarily on (1) the accuracy of electrical measuring circuits used in conjunction therewith, (2) the fidelity of test conditions, and (3) the care and skill exercised in making adjustments. This assumes that the design is such mechanically as to permit close adjustments to be made. Electrical measuring circuits may be used to measure the constants of individual elements, or the characteristics of groups of elements or of networks as a whole. If the accuracy of the circuits is known, possible deviations in the transmission characteristics may be estimated accordingly. Fidelity of test conditions refers to the condition that elements must be tested with due recognition of the manner in which they are used in networks, and that networks must be measured under the same circuit conditions as used, or allowances made accordingly. How much care and skill may enter into making adjustments is chiefly an economic question.

Stability of network characteristics under operating conditions is largely dependent on the materials employed in construction. Suitable materials are usually limited in number either by economic considerations or by the limitations of engineering knowledge. The chief sources of instability of characteristics are: changes with current variations (either temporary or permanent), changes due to temperature and humidity fluctuations, and a group of changes called "aging" which have to do with releases of strains and fatigue of materials.

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Discussion

S. B. Covey: It is estimated that in the Bell System there are in service more than a quarter of a million of the networks described by Messrs. Shea and Lane. These networks vary from simple resistance windings on small wood spools weighing not more than an ounce to complicated assemblies of coils and condensers occupying space to be reckoned in cubic feet and weighing perhaps as much as 50 lb. Each of the many types of networks must occur at its proper point in the circuit. The providing of a circuit terminal with its full and proper complement of equipment calls for an intricate cross-connection of the office wiring. Changes in circuit layout must be made frequently and the task of executing these changes may be far from simple. Errors in cross-connection contribute definitely to the total number of troubles responsible for improper service, and the rapidly mounting use of these networks cannot but increase this contribution. In connection with the provision of telephone facilities, the present tendencies are toward a greater and greater proportion of the total investment being concentrated in the central office buildings in the form of elaborate equipment. It is estimated that 25 per cent of this equipment is composed of networks. This brings to mind at once that very definite consideration must be given to the matter of space requirements. There are instances where building extensions have been necessary for this purpose alone.

Telephone Circuits for Program Transmission

BY F. A. COWAN[1]

Non-member

Synopsis.—*Networks of telephone circuits which are extensively used in the transmission of programs to broadcasting stations are described in this paper. Certain stages in the development of these networks are considered and the general requirements for satisfactory transmission at the present time are enumerated. The arrangements of the networks as well as the procedures used in setting up and maintaining them are discussed.*

* * * * *

MUCH of the phenomenal growth and present excellence of radio broadcasting has resulted from contributions made by associated branches of the electrical art. Of these contributions perhaps none has had a greater effect than the introduction of the program transmission wire networks, which make chain broadcasting possible. Broadcasting had hardly emerged from the novelty stage before the need for programs presenting music of the highest grade, speeches by prominent people, and descriptions of sports events of sectional or national interest became evident. It was further recognized that in addition to providing programs of this character, it would be desirable to broadcast the programs simultaneously from a number of stations. These conditions established a demand for means of picking up selected program material and transmitting it to broadcasting stations scattered throughout the country. For this transmission, wire lines have proved to be very satisfactory and are in general use for this purpose.

Prior to the general development of radio broadcasting, the Bell System had, incident to other developments, worked out the problems involved in such transmission, and on numerous occasions had set up interconnections of radio telephone stations and public address systems which were substantially the same as those required for broadcast program transmission. The telephone companies were, therefore, able to meet this new requirement for communication service early in the development of radio broadcasting. One of the first occasions of this type was in the fall of 1922, when the description of the Chicago-Princeton football game at Chicago was broadcast through station WEAF, then operated by the American Telephone and Telegraph Company, at New York City. Another of the earlier services was the interconnection of six broadcasting stations, one of which was in Dallas, for the simultaneous broadcasting of President Coolidge's first address to Congress on December 6, 1923. Since that time there has been a steady and continuing growth in this service until there are now furnished by the Bell System more than 30,000 mi. of regularly established program circuits which connect in various combinations and at various times over 125 radio broadcasting stations covering the entire United States. The various stages of this development up to the establishment of a 15 station part-time network in the latter part of 1924 are described in a paper by Messrs. Foland and Rose, published in the January 1925 issue of *Electrical Communications*.

The first multi-station network for which special equipment was provided at key points on a permanent basis was initially called the "red network" for no other reason than that those cities which were to be connected were indicated on a chart by red lines, and a memorandum on the arrangements for the circuits referred to them as the "network shown in red." As other networks came into being, the convenience of a short color name for differentiating between them resulted in a rather general use of this form of designation within the telephone companies, and it was not long before this nomenclature had spread among the various broadcasting companies.

Initially, this network service was confined mainly to evening hours and to sections where telephone message circuits could be obtained but it soon became apparent that more time would be required than could be furnished on this basis, and as rapidly as possible program circuits specially provided for this purpose were made available on a larger scale. The routes of the regularly established program circuits of the Bell System in the United States on March 15, 1929, are shown on Fig. 1. These regular networks are supplemented by special circuits which are established for the transmission to the network control points of programs picked up at the location of events of particular interest or national importance. Also, on certain occasions, several of the chains have been merged into one, thereby forming a network of stations which effectively makes the United States into one large auditorium.

The general transmission requirements of circuits for the satisfactory handling of program material are not greatly different from those for good telephone connections. The nature of the program material is usually such, however, that the specific requirements for a network of program circuits are much more exacting than for the usual types of message circuits.

These differences lie in the fact that for satisfactory transmission of speech from the message traffic stand-

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[1] Long Lines Dept., American Telephone and Telegraph Co., New York, N. Y.

Presented at the Regional Meeting of the South West District of the A. I. E. E., Dallas, Texas, May 7-9, 1929.

point the primary requisite is that the message shall be readily recognizable and intelligible with naturalness of tone as an important but secondary consideration; whereas the satisfactory transmission of programs for broadcasting purposes requires faithfulness of reproduction of speech, music, and incidental details with intelligibility and naturalness of tone of about equal importance. In order to achieve these effects it is necessary to transmit with reasonable uniformity a wider range of frequencies and volumes than is ordinarily required for message traffic.

For example, a frequency band 2500 cycles in width will, if properly utilized, provide facilities for the transmission and ready interchange of ideas through the agency of easily understandable speech whereas for program transmission with the present type of microphones, amplifiers, and loud speakers a frequency band of between 4000 and 5000 cycles in width is usually required. Most program circuits at the present time transmit frequencies above about 100 cycles and below about 5000 cycles. The normal range of volumes in programs is usually much larger than can be successfully handled by the present type of radio broadcast transmitters and receivers with the result that at the pickup point the amplification is adjusted from time to time to reduce the variations in volume. This compression is such that for average broadcast material the momentary volume peaks vary over a range of 20 to 30 decibels. The program transmission circuits are arranged to transmit satisfactory volume ranges of this order of magnitude. Since much of the character of large symphony orchestras is contributed by variations in volume, it would undoubtedly add greatly to the pleasure afforded the radio listener to reproduce these variations even more faithfully. It is to be expected, therefore, that within the next few years improvements in the broadcasting technique will be considered from various points of view and the network circuits will be called upon to keep pace with such developments.

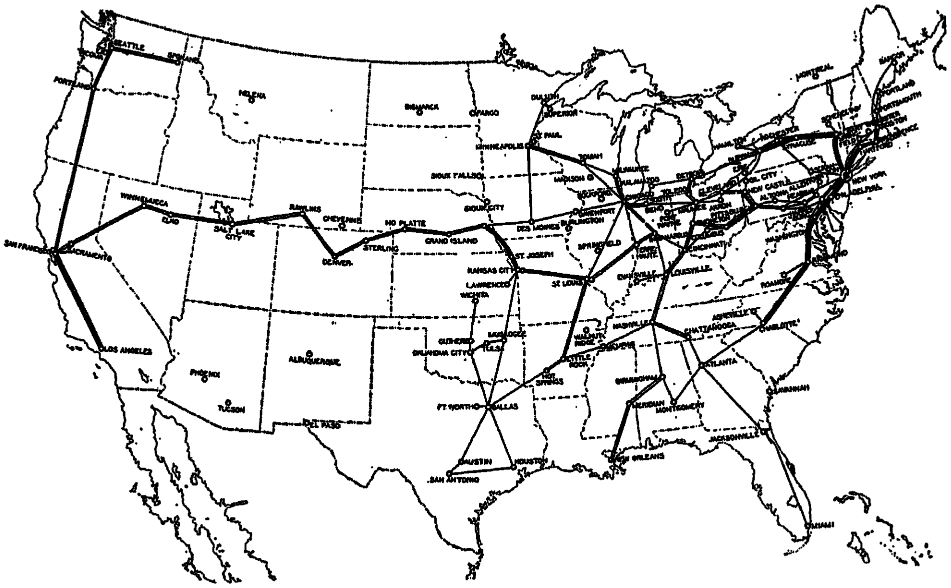

FIG. 1—BELL SYSETM PROGRAM NETWORKS IN THE UNITED STATES ON MARCH 15, 1929

Network	No. of stations	Length in miles
Red — NBC	44	10,800
Blue — NBC	11	3,500
Purple — CBC	42	7,500
Green — ABC	3	1,900
Pink — ABC	5	1,700
Orange — NBC	5	1,700
White — PPA	21	3,800
Brown — DL	3	450
	134	31,350

The wire networks are arranged for the transmission in only one direction in the same manner as the broadcasting stations. This removes the problem of echoes and possible singing which is introduced where two-way transmission is required. It also permits a more simple treatment of the many connections between circuits involved in supplying programs to a large number of points. The networks are set up with main radial circuits over trunk routes from which branches feeding the various stations are taken off. At these branch points the connection is accomplished with the aid of a one-way amplifier arranged to prevent any reaction on the trunk circuit by conditions on the branch circuit.

New York City is the originating point of the red, blue, purple, and white networks and from it the largest number of wire networks radiates. Atlanta, Cincinnati, Kansas City, Chicago, St. Louis, and San Francisco are also large network centers. The work of properly setting up the veritable web of circuits radiating from and interconnecting these points and properly safeguarding the service, requires the constant attention of a large group of men strategically located and provided with means of rapid intercommunication. To increase the effectiveness of this organization the country has been divided into areas with one of the wire centers listed above designated as the control office for the contiguous area. Telegraph wires are provided connecting each of the repeater offices within a given area with the area control point and the area control offices are in turn connected with New York. By the use of these telegraph wires the reports covering the condition of the circuits and instructions regarding measurements and adjustments required in setting up or rearranging the program circuits are handled. Due to the method used in connecting the branches to the trunk circuits, the various area control offices are able to carry on their work practically independently of the other offices.

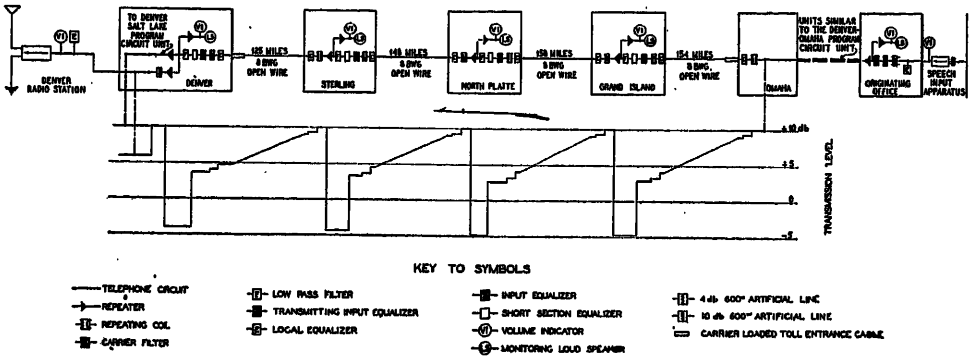

FIG. 2—SCHEMATIC LAYOUT AND TRANSMISSION LEVEL DIAGRAM OF DENVER-OMAHA PROGRAM CIRCUIT UNIT ARRANGED FOR TRANSMISSION FROM OMAHA TO DENVER

To facilitate further the testing and maintenance of the network, the circuits within an area are further subdivided into sections usually about 500 mi. in length. Certain offices are selected and designated as control points for these sections which are known as program circuit units and are established, tested, and maintained as independent units. The units are designated by the names of the two terminal offices and those between the same two points are numbered successively. Complete networks are established by connecting the necessary number of units together. Any two program circuit units over the same route are so designed as to be interchangeable and a spare unit established for emergency or special use may be used to protect the service on several networks.

Information for use by the field forces in establishing and maintaining the networks consisting of data covering the radio stations to be connected and the makeup of the circuits as well as information regarding the transmission features of the set up is prepared and distributed on cards called a "circuit layout record card." The information as to the points at which repeaters are used, as well as the gains to which they should be adjusted, is given on one card while a second card gives detailed information regarding the wires assigned for the circuit unit, the equipment to be connected, and the computed 1000 cycle transmission loss of the line and equipment in decibels. The required transmission equivalent of circuit units of this type is 0 decibel and to obtain this, the sum of the amplifier gains assigned just equals the computed total loss of the line and associated equipment. By making the net loss of all the program units zero the transmission loss of a network is made independent of the number of units entering into its make up.

To illustrate the factors entering into the determination of the proper set up of a circuit unit, the layout of the No. 1 Denver-Omaha program transmission circuit unit which is a typical one, as well as the transmission levels at the various repeater offices, when arranged for transmission Omaha to Denver, is shown in schematic form in Fig. 4. It may be seen from this chart that in addition to making the total gain equal the total loss, the unit is so set up that the gain at each amplifier point just equals the loss in the preceding sections. This results in a uniform level at the output of each amplifier and permits the transmission of the program at a higher average volume level without overloading the amplifiers or introducing interference into adjacent circuits. At each amplifier point there is, in addition to the amplifier transmitting towards the next station, one which is used for monitoring the circuits and making various service observations. An instrument known as a volume indicator is normally connected to this amplifier and is used to check up on the volume of the transmitted program. It is also used sometimes to obtain a quick check of the strength of tones applied for measuring purposes although there is provided for this purpose special precision transmission measuring apparatus.

The line wires in this case are No. 8 B. W. gage copper, and are not loaded, which is the type generally used for program transmission. The incidental cables for entrance into cities are either No. 10 or 13 B. & S. gage and are loaded to have approximately the same characteristic impedance as the line wires, namely, 600 ohms. The cutoff frequency of this cable loading is sufficiently high to permit the operation of the carrier current telephone systems which are connected to these wires. The transmission loss at various frequencies of a typical section of line wires with incidental cables is shown in Fig. 3. The over-all gain at various frequencies introduced at typical repeater points is shown in Fig. 4. It may be seen that within the band which at the present stage of the art the circuits are designed to transmit, the gain is substantially complementary to the loss so that the resultant net loss in each repeater section for the frequencies within this band is practically uniform. The gain of the repeaters at the higher and lower frequencies relative to 1000 cycles is adjustable so that it may be made to conform to the varying conditions likely to be encountered.

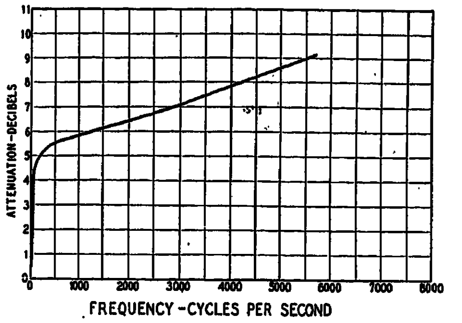

FIG. 3—LINE ATTENUATION OF TYPICAL REPEATER SECTION OF A PROGRAM TRANSMISSION CIRCUIT

In addition to the constant watch which is maintained through the length of the circuit and particularly at all points where broadcasting stations are connected to the circuits during the transmission of programs, frequent tests are required in order to keep the circuits in shape and prepare them for the periods of use. At the present time complete measurements are made on each program circuit at least once each day and periodic tests of a less comprehensive nature are made at frequent intervals throughout the day. On the complete over-all tests a testing tone of the required strength and having a frequency of 1000 cycles per sec. is applied at the originating point of the section of network under test. At each amplifier point in succession beginning with the originating point the level of the testing current is measured and reported back to the control office. Such changes as are necessary to bring the level to the required value are made and when all points have been covered the frequency of the testing tone is changed and the procedure repeated. In general, measurements are made at only about four or five frequencies covering the band which it is desired to transmit on the daily tests although in some cases it may prove desirable to measure at as many as forty different frequencies. After completing the adjustments at all necessary frequencies the testing tone is removed and the circuit is observed for any noise, crosstalk, or other possible sources of trouble. A test program is then sent over the circuits and reports as to quality are made. To provide for making this last test, the more important control offices are equipped with a high quality phonograph pickup as well as microphones for talking tests.

As has been pointed out in the foregoing, it is necessary for the observers at the amplifier points to be constantly on the lookout for any irregularities which might cause trouble and take such steps promptly as are necessary to clear any troubles observed or forestall any potential ones. It is always essential that this action be coordinated by the control office as the networks are now very complicated. Probably no better example of this could be cited than a situation which arose about a year ago on a network, the trunk circuit of which was routed from New York through Pittsburgh, Akron, and Toledo to Chicago. There were branches at each of the points mentioned. On the particular day under consideration, a very severe storm was moving across Ohio into Pennsylvania. As is customary the New York control office was advised of the progress of the storm by the repeater and sub-control offices. Preparing for an emergency, a special circuit was set up from New York to Toledo through Buffalo and Cleveland. When the storm hit the line between New York and Pittsburgh in the Pennsylvania mountains the regular circuit began to fail and the one routed through Buffalo and Cleveland was placed into service. To care for the Pittsburgh and Akron radio stations, a branch of the network going south from Toledo was extended on orders from New York, back east to Pittsburgh where it was connected to the section of the trunk circuit which was still intact between Pittsburgh and Toledo. The ever watchful observer at Toledo then found the program coming in satisfactorily on the trunk circuit from the east. Not knowing that this was the very same program that he was receiving over the emergency circuit from New York coming back to him after it had been sent south

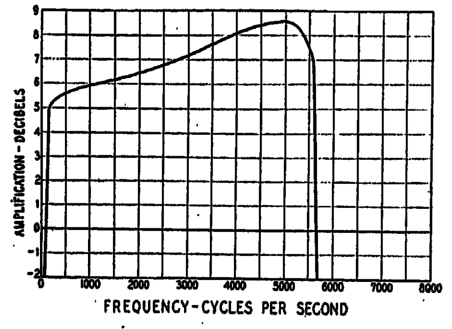

FIG. 4—OVER-ALL AMPLIFICATION OF A TYPICAL REPEATER OFFICE ON A PROGRAM TRANSMISSION CIRCUIT

from Toledo and to Pittsburgh, he advised the New York control office of the condition and asked if he should take the program from this source now that it was again O. K. Needless to say it was not done since the New York control office was in the fortunate possession of all of the facts.

Another condition which requires careful coordination on the part of the repeater and control offices is the practise of splitting a network at a specified time to provide for the transmission of one program to a certain group of the network radio stations while the remainder receive programs from a different source. These rearrangements may involve the creation of a new point as the temporary originating center, or switching a branch of a network feeding one or more stations from one network to another. Program orders are obtained daily from the broadcasting companies to cover the network set up required for each day's program and these orders are transmitted from New York to the area control office using telephone typewriters. The switches necessary for these rearrangements are then made under the directions of the area control offices. The flexibility of layout resulting from setting the network up in units simplifies the work involved in this splitting and rearranging of networks. It is, however, very necessary that the transmission levels on the networks be carefully watched so that the variations introduced by switching may be negligible.

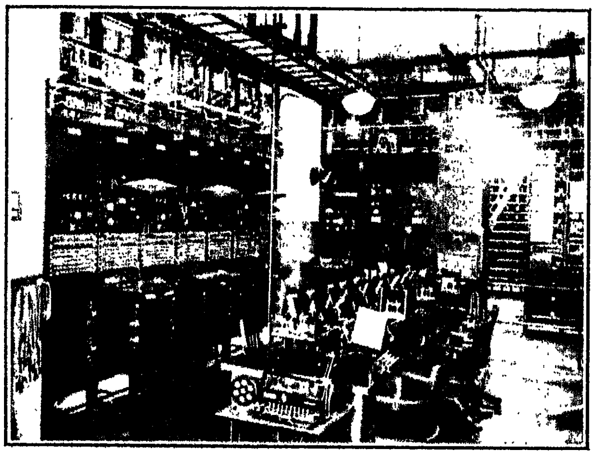

Fig. 5—Part of Equipment at New York for Program Transmission

In the installation of the equipment at repeater stations and distributing centers, care is taken to arrange the circuits so as to provide as great freedom from interruption as practicable and permit the quick rearrangements of circuits which may become necessary during emergencies. The repeaters and associated equipment are grouped together with the telegraph wire terminations and this unit is usually located in a separate room. An example of a part of the equipment arrangements at New York which is typical of the larger wire centers, is shown in Fig. 5. Fig. 6 shows a similar installation at Chicago. One of the features of this installation is a 17-line key switchboard which is used to care for the switching incidental to the rearrangements on networks entering Chicago.

Consideration from an over-all standpoint is essential to the satisfactory performance of any large system. In the case of program networks, this requires among other things that care be taken in making the connections between network and the pick-up as well as the receiving radio station to see that adjustments are made to meet the particular conditions encountered so that the networks may be used to the best advantage. As has been pointed out in the foregoing it is important that the range of volumes in transmitted programs be held within proper limits. The range in volume is a function of adjustments made at the pick-up points but it is necessary to so adjust the network that at the maximum volume, interference will not be introduced into neighboring circuits and that during quiet intervals the program circuits will not be subject to an undue amount of noise or crosstalk from other circuits. This is usually accomplished by adjusting the amplification at the originating point to an average value which has been determined to be correct.

Once having determined the proper set up for a network it is then necessary to obtain a sufficient number of checks upon the over-all performance to ascertain that the service is entirely satisfactory. To assist in this there have been developed a number of indexes by which the program circuit performance is rated.

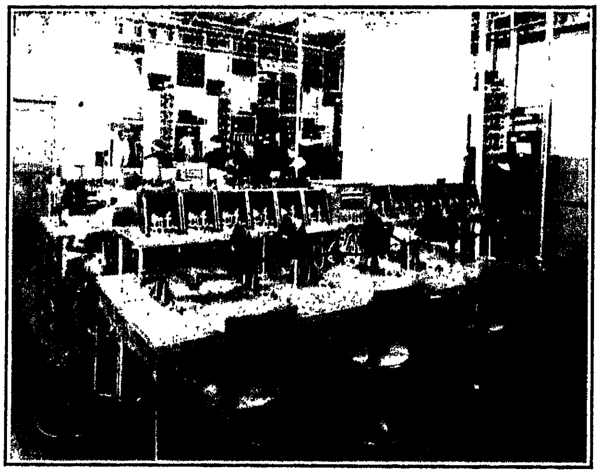

Fig. 6—Program Transmission Equipment in Chicago Office

The foregoing discussion has considered program transmission networks as they are today. At this time any attempt to predict the future course of development would probably require rather extensive recasting at some subsequent time. It seems fairly certain, however, that there will be continual improvements in the general broadcasting art and this factor is kept constantly in mind in the design of telephone circuits for program transmission in the future.

Traveling Waves Due to Lightning

BY L. V. BEWLEY*

Associate, A. I. E. E.

Synopsis.—The purpose of this paper is to describe and analyze the origin and formation of traveling waves on a transmission line induced by lightning discharges, and to investigate their behavior at a transition point where there is an abrupt change of circuit constants. Some of the ground covered is necessarily old and well known, but has been included in the interests of completeness and continuity of treatment. The effect of the rate of cloud discharge and the initial distribution of bound charge on the shape and amplitudes of the traveling waves is brought out. General methods of analysis are formulated and illustrated by practical cases. The assumptions and approximations involved are discussed, and the probable direction of their deviation from fact indicated. Exact and approximate mathematical expressions are derived, and therefrom graphical and tabular methods are outlined. For all of the examples given, line attenuation and distortion are neglected and only the first reflection from a transition point is considered.

I. INTRODUCTION

THIS paper, which deals particularly with the mathematical study of the formation and propagation of lightning waves on transmission lines, is part of an extensive investigation of lightning which has been under way for a number of years under the general direction of F. W. Peek, Jr. Other papers of this series covering laboratory and field studies as well as the effect of lightning on apparatus have already been presented.[1,2,3] While this paper deals essentially with a mathematical analysis of the subject, it has been written in such a way that it is not felt necessary for the reader to go into the mathematics to obtain a knowledge of the results. Such practical aspects as the effects of series induction coils, extra ground wires, etc., on transmission lines are analyzed mathematically and discussed.

II. ORIGIN OF TRAVELING WAVES DUE TO LIGHTNING

When a charged cloud approaches a transmission line, a charge of opposite sign leaks over the insulators and appears on the line as a bound charge fixed in position by the electrostatic field of the cloud. The density σ of bound charge at any point x on the line is proportional to the height h of the line above ground and to the average gradient G between that point and ground, thus:

$$\sigma = \partial Q / \partial x = C \int g \, ds = C h G$$

where

C = capacitance per unit length at point x

g = gradient along the tube of electrostatic flux through x

ds = element along the tube of flux passing through x.

Since the field of the cloud is not uniform, and since the line may sag considerably at the middle of a span, it is evident that the bound charge will not be uniformly distributed, but may have a wide variety of shapes. The situation is shown in Fig. 1, where R is the resistance of the insulator string at the tower, and C_1, C_2, C_3, represent the increasing capacitance of the line elements as the sag brings the conductors nearer to ground. The density of bound charge is shown by the cross-hatched area. However, the corresponding *potential* distribution when this charge is instantaneously released is quite independent of the variation of capacitance, and is $V = \dfrac{1}{C} \dfrac{\partial Q}{\partial x} = h\,G$, thus fully determined by the field of the cloud.

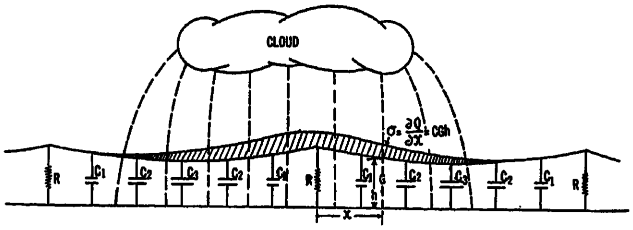

FIG. 1—DISTRIBUTION OF BOUND CHARGE

Measurements by Peek indicate gradients of the order of 100 kv. per foot for heights above ground of the order of a transmission line. However, so far as the author knows, no data are available as to the intensification of the field (if it occurs) during the preliminary or incipient stages of a cloud discharge. Measurements taken on short antennas are not informative. Some idea of the time required for a bound charge to form, may be obtained by calculating the time for a capacitance equal to that of one span of the line to discharge to $(1/\epsilon)$th of its initial potential over a resistance equal to that of a single insulator. Taking $C = 0.003$ mf. and $R = 1000$ megohms, the time is

$$t = R\,C = 3 \text{ seconds}$$

If the preliminary stages of a cloud discharge are completed within 100 microseconds or so, the time is not sufficient for additional bound charge to leak over the insulators.

The measurements taken by Peek[1] on short antennas indicate that the final release of the bound charge sometimes takes place in one or two microseconds. Probably much longer periods may occur, but as shown from the theoretical considerations discussed later, these slower rates of cloud discharge cannot give rise to excessive potentials. Physically, a finite rate of cloud discharge is equivalent to the release of the bound charge by small steps (see Fig. 2), which in the limit coincide with the smooth curve of cloud discharge rate. Each of these blocks forms in succession and immediately begins to move away as a traveling wave. But if the time of release is slow enough, some of the blocks first formed have completely passed out from under the original distribution before all of the bound charge has been released, with the result that only part of the total number of blocks will overlap and add up by superposition.

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*General Transformer Engineering Department, General Electric Company, Pittsfield, Mass.

1. *Lightning—Progress in Lightning Research in the Field and in the Laboratory*, F. W. Peek, Jr., Quarterly TRANS., Vol. 487, April, 1929, p. 436.

2. *Effect of Transient Voltages on Power Transformer Design*, K. K. Palueff, A. I. E. E. Quarterly TRANS., July 1929.

3. *Lightning Studies of Transformers by the Cathode Ray Oscillograph*, F. F. Brand and K. K. Palueff, A. I. E. E. Quarterly TRANS., July, 1929.

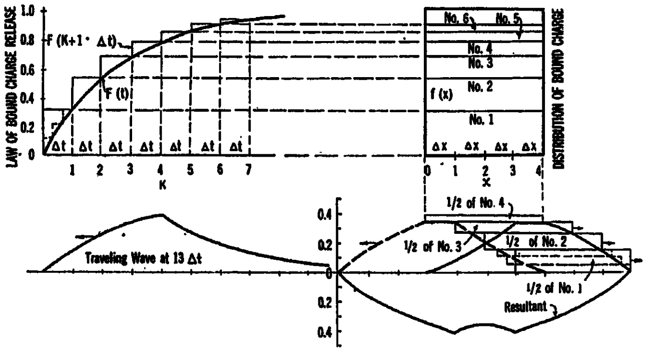


FIG. 2—GRAPHICAL METHOD FOR DETERMINING WAVE SHAPES

Experimental data on the functional rate of cloud discharge are not available. For a strictly mathematical study only the most simple expressions can be handled, but the graphical and tabular methods described elsewhere in this paper are applicable to any rate of cloud discharge which can be drawn as a curve on a piece of paper. The illustrations included are for exponential and straight line discharges. Judging from the meager number of cathode ray oscillograms of potentials induced on short antennas, and working backwards from those of traveling waves caused by lightning, it appears that a fair approximation expressing the release of the bound charge is $(1 - \epsilon^{-at})$. Very rough estimates of the circuit constants of a lightning bolt, also suggest this equation as a possibility. The influence of the function $F(t)$, expressing the law of release of the bound charge, on the shape of the resulting traveling waves is shown in Fig. 3. If $f(x)$ represents the distribution of bound charge, then the curves A, B, C represent the shape of the traveling waves corresponding to the cloud discharging in the same time t, but according to different laws a, b, c.

Neither the shape nor amplitude of a traveling wave is preserved as it moves along the line. This attenuation and distortion may be due in part to the appropriation of a charge from the traveling waves by other cloud fields in the vicinity of a discharge. However, even when such fields do not exist, the line losses soon dissipate the energy of the wave by corona and skin effect. Attempts have been made to formulate mathematically the transient skin effect, but so far as the author knows, nothing has been done in that direction for determining the effects of corona. Although an appendix has been attached to this paper in which the line constants R, L, C, G were assumed to be constant, it is fully realized that they vary over a great range. The resistance increases and the inductance decreases due to skin effect. The conductance increases and there is an effective cyclic increase in the capacitance due to corona. Some degree of approximation may be obtained by arbitrarily attributing different values to the line constants associated with each frequency component, or group of frequencies, in the solution obtained on the assumption of fixed constants. However, the work involved in summing the infinite series to a point of convergency, under such conditions, is prohibitive. There is this value in deriving such expressions,—they show by inspection that the effects of the line loss are to distort the wave shape, reduce the steepness of the front, and depress the crests. Therefore the worst possible conditions are realized by neglecting line losses.

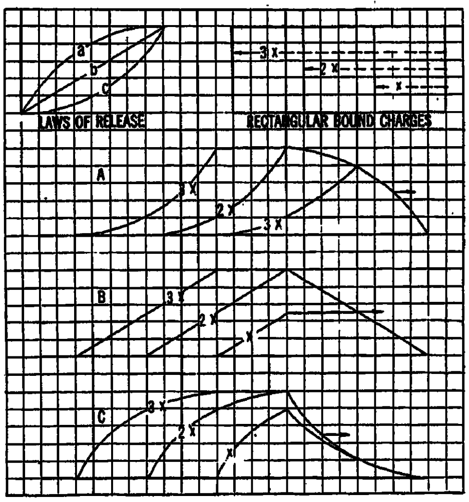
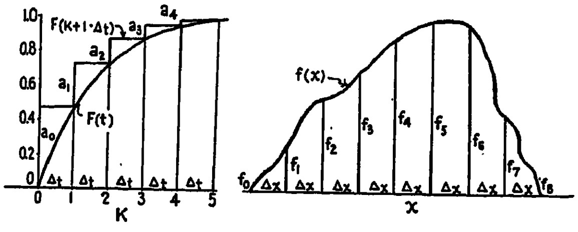


FIG. 3—EFFECT OF THE LAW OF DISCHARGE ON THE SHAPE OF TRAVELING WAVES

For these reasons the attenuation and distortion of traveling waves have been neglected in this paper. Nor was it considered advisable to consider a multiplicity of reflections at a transition point of the circuit, since each successive reflection is rapidly reduced in severity.

The graphical method, Fig. 2, for obtaining the shape of traveling waves from the curves of bound charge release and initial distribution, and the corresponding tabular method, Fig. 4, are described following Equation (**25**). These methods neglect the line losses, but are otherwise arbitrarily as accurate as desired.

The crest values, shapes, and lengths of traveling waves depend upon the initial distribution of bound charge, but to an even greater extent on the rate of cloud discharge. Fig. 5 shows a set of traveling waves developed from rectangular distributions of bound charge of 1000-, 2000-, 3000-, and 4000-ft. lengths, and released according to the law $(1 - \epsilon^{-at})$; so that the cloud discharge was 95 per cent completed in 1, 2, 3, and 4 microseconds. The slower the rate of discharge the lower the crests and the longer the wave. This is a necessary consequence of the fact that all waves originating from a given bound charge must contain exactly the same energy (neglecting line losses) regardless of the rate at which that charge is released. The original bound charge represents the storage of a definite amount of energy, and 25 per cent of this energy must appear in the completely developed traveling potential wave, 25 per cent in the companion current wave, and the other 50 per cent in the pair of potential and current waves moving in the opposite direction. Thus a decrease in crest values must be compensated for by an increase of length. The length of the wave expressed in microseconds (one microsecond being equal to 1000 ft. of wave travel) is roughly equal to that of the bound charge plus the number of microseconds required for the cloud discharge. The steepness of the wave front is greater, the faster the rate of discharge.



K	ΔF_K	$\Delta F_K \cdot f[x-(n-K)\Delta t]$													
											$(n-4)\Delta t$	$(n-3)\Delta t$	$(n-2)\Delta t$	$(n-1)\Delta t$	$n\Delta t$
0	a_0						a_0f_0	a_0f_1	a_0f_2	a_0f_3	a_0f_4	a_0f_5	a_0f_6	a_0f_7	a_0f_8
1	a_1					a_1f_0	a_1f_1	a_1f_2	a_1f_3	a_1f_4	a_1f_5	a_1f_6	a_1f_7	a_1f_8	
2	a_2				a_2f_0	a_2f_1	a_2f_2	a_2f_3	a_2f_4	a_2f_5	a_2f_6	a_2f_7	a_2f_8		
3	a_3			a_3f_0	a_3f_1	a_3f_2	a_3f_3	a_3f_4	a_3f_5	a_3f_6	a_3f_7	a_3f_8			
4	a_4		a_4f_0	a_4f_1	a_4f_2	a_4f_3	a_4f_4	a_4f_5	a_4f_6	a_4f_7	a_4f_8				
·	·						------- etc. -------								
Σ =					$\sum_0^n \Delta F_K \cdot f[x-(n-K)\Delta t]$										

FIG. 4—TABULAR METHOD FOR DETERMINING WAVE SHAPES

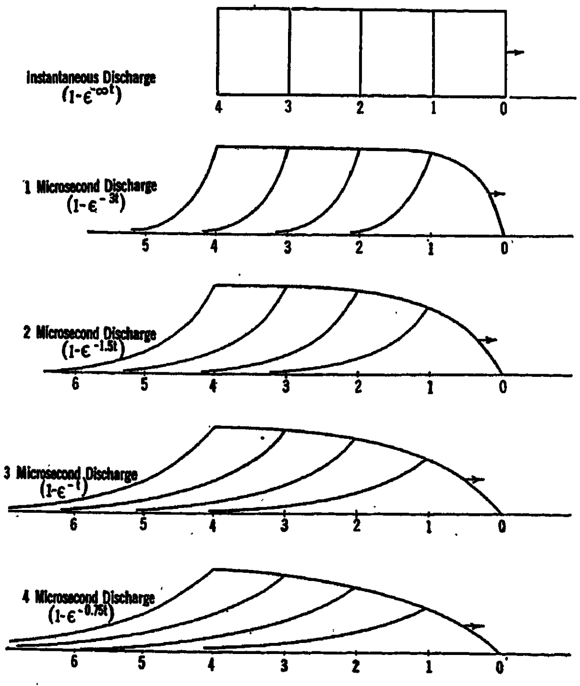


FIG. 5—TRAVELING WAVES FROM RECTANGULAR DISTRIBUTION OF BOUND CHARGE

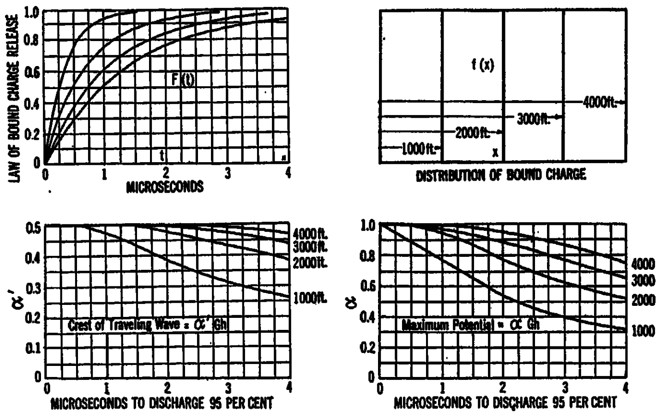


FIG. 6—INFLUENCE OF THE RATE OF DISCHARGE AND LENGTH OF CHARGE ON THE VOLTAGE CRESTS

When the length of a rectangular bound charge (in microseconds) is longer than the period of discharge, the crests of the traveling waves reach a value equal to (1/2 $G\,h$). The maximum potential on the line may reach a value of ($G\,h$) due to the superposition of the two traveling waves. However, in general these crest values depend on the bound charge distribution and the rate of its release. They are:[1,4]

$$\text{Crest of traveling wave} = \alpha' \, G\, h$$
$$\text{Maximum potential} = \alpha \, G\, h$$

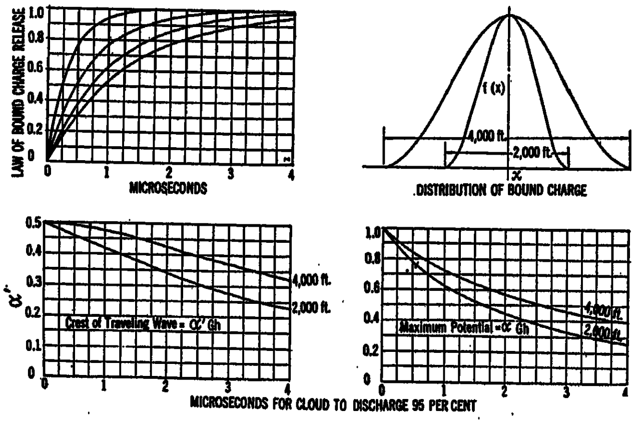


FIG. 7—INFLUENCE OF THE RATE OF DISCHARGE AND LENGTH OF CHARGE ON THE VOLTAGE CRESTS

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4. "Lightning," F. W. Peek, Jr., *Franklin Inst. Jl.*, Feb. 1925.

The factors α and α' corresponding to rectangular distributions and exponential release, are shown in Fig. 6. It will be noticed that 100 per cent potential ($\alpha = 1.0$, $\alpha' = 0.5$) occurs only for an instantaneous discharge, or for very long clouds, and falls off rapidly as the time of release is increased, or the length of the cloud shortened. An interesting point is that ultimately the maximum potential and the crest of the traveling wave have the same value. Fig. 7 is a similar set of curves for a bound charge distributed as one loop of 1/2 $(1 - \cos \theta)$. The values of α and α' are somewhat lower, but exhibit the same tendencies.

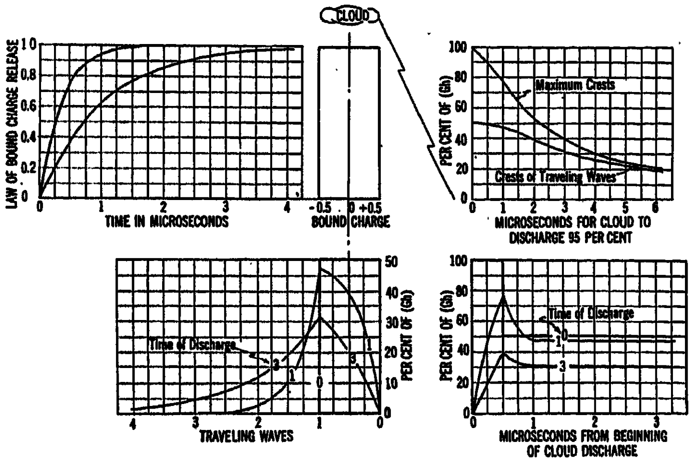

FIG. 8—WAVE SHAPES DERIVED FROM RECTANGULAR BOUND CHARGE RELEASED AT DIFFERENT RATES

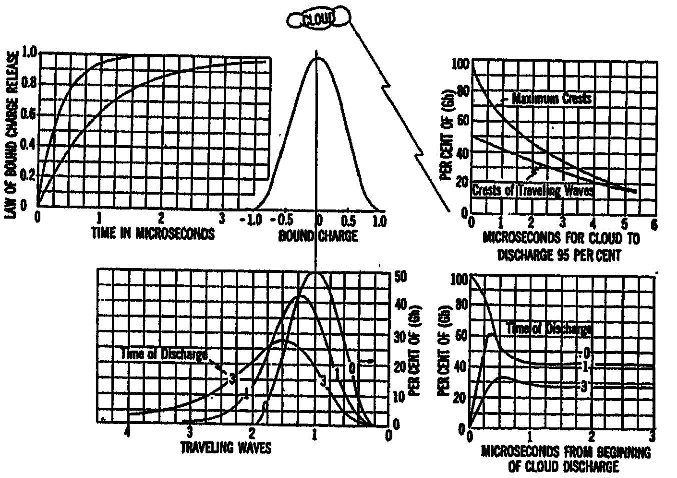

FIG. 9—WAVE SHAPES DERIVED FROM PEAKED BOUND CHARGE RELEASED AT DIFFERENT RATES

Figs. 8 and 9 show the actual wave shapes, and the maximum potential at any instant after the beginning of discharge for a given bound charge released at different rates. While the bound charge distributions in these two figures are quite different, there is a remarkable similarity between the traveling waves which they cause. These traveling waves have been superimposed in Fig. 10. It is this similarity which led the author to state that the rate of cloud discharge was a more influential factor in determining wave shape than the initial distribution of bound charge.

Fig. 11 shows the formation of traveling waves at different stages of their development. This particular set originated from the 2000-ft. peaked bound charge distribution of Fig. 10, released in one microsecond. During the first half microsecond or so, the traveling waves have not fully formed, but are moving out and spreading apart at the same time that they increase in magnitude. Eventually, however, the moving wave crests can be distinguished as they separate out and acquire identity. It is because of this simultaneous growth and separation, that the maximum potential is less than twice that of the traveling wave crests, and it is because of the spreading action that the crests are always less than they would be for an instantaneous cloud discharge.

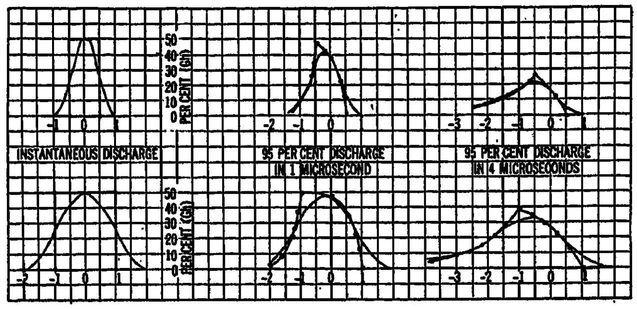

FIG. 10—SIMILARITY OF WAVE SHAPES FROM DIFFERENT DISTRIBUTIONS OF BOUND CHARGE

The mathematical work associated with the foregoing is given in the following section and in the Appendix of this paper.

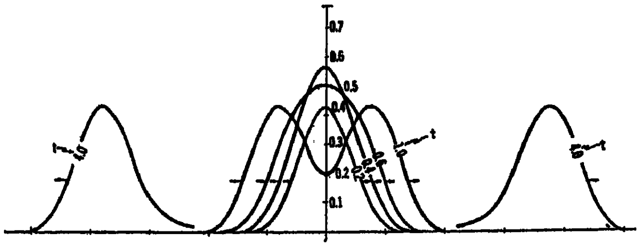

FIG. 11—FORMATION OF TRAVELING WAVES

III. MATHEMATICAL ANALYSIS

General. The differential equations for the potential and current at an unconstrained point on a transmission line are

$$-\frac{\partial i}{\partial x} = (G + C p)\, e = Y e \qquad (1)$$

$$-\frac{\partial e}{\partial x} = (R + L p)\, i = Z i \qquad (2)$$

where $p = \partial / \partial t$. Eliminating i between these two simultaneous differential equations, there is

$$\frac{\partial^2 e}{\partial x^2} = Z\,Y\,e = C\,L\left\{ p^2 + \left(\frac{R}{L} + \frac{G}{C}\right)p + \frac{R}{L}\,\frac{G}{C} \right\} e \qquad (3)$$

$$i = Y \int e\,d\,x = (G + C\,p) \int e\,d\,x \qquad (4)$$

Solving (3) as an ordinary linear differential equation in x,

$$e = \epsilon^{x\sqrt{ZY}} f_1(t) + \epsilon^{-x\sqrt{ZY}} f_2(t) \qquad (5)$$

$$i = Y \int e\,d\,x = \sqrt{\frac{Y}{Z}} \left\{ \epsilon^{x\sqrt{ZY}} f_1(t) - \epsilon^{-x\sqrt{ZY}} f_2(t) \right\} \qquad (6)$$

where $f_1(t)$ and $f_2(t)$ are integration constants with respect to x only, and therefore possible functions of t. Equations (5) and (6) are general operational solutions to (1) and (2), but in this form are not readily evaluated because of the radical $\sqrt{Z\,Y}$. However, as special cases

$$\sqrt{Z\,Y} = \sqrt{C\,L}\sqrt{p^2 + \left(\frac{R}{L} + \frac{G}{C}\right)p + \frac{R}{L}\,\frac{G}{C}}$$

$$= \sqrt{C\,L}\left(p + \frac{R}{L}\right) \text{ if } R\,C = G\,L \qquad (7)$$

$$\sqrt{Z\,Y} = \sqrt{C\,L}\,p \text{ if } R = G = 0 \qquad (8)$$

If the conditions of Equation (7) obtain, the line is said to be *distortionless*, and admits of easy solution. If conditions (8) hold, the line is called a *No-loss* line, and is easily solved. By Taylor's theorem, there is the operational equivalent

$$f(t + a) = f(t) + a f'(t) + \frac{a^2}{\underline{|2}} f''(t) + \ldots.$$

$$= \left(1 + a\,p + \frac{a^2}{\underline{|2}} p^2 + \ldots.\right) f(t) = \epsilon^{ap} f(t) \qquad (9)$$

Hereby, for the special case of a distortionless line, (5) and (6) become

$$e = \epsilon^{\sqrt{CL}\frac{R}{L}x} f_1(t + \sqrt{C\,L}\,x) + \epsilon^{-\sqrt{CL}\frac{R}{L}x} f_2(t - \sqrt{CL}\,x) \qquad (10)$$

$$i = \sqrt{\frac{C}{L}} \left\{ \epsilon^{\sqrt{CL}\frac{R}{L}} f_1(t + \sqrt{C\,L}\,x) - \epsilon^{-\sqrt{CL}\frac{R}{L}x} f_2(t - \sqrt{C\,L}\,x) \right\} \qquad (11)$$

Thus the potential and current distributions are propagated as traveling waves, and each may consist of a forward wave f_2 moving in the direction of positive x, and a reverse wave f_1 moving in the direction of negative x. Corresponding values of x and t which define the same point on the waves are given by

$$\left.\begin{array}{l} (t - \sqrt{C\,L}\,x) = \sqrt{C\,L}\,\lambda_2 \text{ for the forward wave} \\ (t + \sqrt{C\,L}\,x) = \sqrt{C\,L}\,\lambda_1 \text{ for the reverse wave} \end{array}\right\} \qquad (12)$$

The velocity of propagation therefore is

$$\left.\begin{array}{l} v = \dfrac{d\,x}{d\,t} = \dfrac{+1}{\sqrt{C\,L}} \text{ for the forward wave} \\[2ex] v = \dfrac{d\,x}{d\,t} = \dfrac{-1}{\sqrt{C\,L}} \text{ for the reverse wave} \end{array}\right\} \qquad (13)$$

Thus each wave has the same velocity $v = 1/\sqrt{C\,L}$. For parallel wires in air whose distance apart is large compared to their radii, this velocity is practically that of light, or 186,000 mi. per second. This is roughly 1000 ft. per microsecond. It is evident from (10) and (11) that the following relationships exist between corresponding potential and current waves:

$$\left.\begin{array}{l} e_1 = \sqrt{L/C}\,i_1 \text{ or } i_1 = \sqrt{C/L}\,e_1 \text{ for reverse waves} \\ e_2 = -\sqrt{L/C}\,i_2 \text{ or } i_2 = -\sqrt{C/L}\,e_2 \text{ for forward waves} \end{array}\right\} \qquad (14)$$

The quantity $\sqrt{L/C}$ is called the *surge impedance* of the line. Its reciprocal $\sqrt{C/L}$ is the *surge admittance*. They are measured directly in ohms and mhos respectively. The per-unit-length factor does not enter into their description.

Using (12) for a transformation of coordinates, Equations (10) and (11) become

$$e = \epsilon^{-\frac{R}{L}t} \left\{ \epsilon^{\sqrt{CL}\frac{R}{L}\lambda_1} f_1(x - v\,t) + \epsilon^{-\sqrt{CL}\frac{R}{L}\lambda_2} f_2(x - vt) \right\}$$

$$= \epsilon^{-\frac{R}{L}t} \{ f_3(x + v\,t) + f_4(x - v\,t) \} \qquad (15)$$

$$i = \sqrt{\frac{C}{L}}\,\epsilon^{-\frac{R}{L}t} \{ f_3(x + v\,t) - f_4\} (x - v\,t) \} \qquad (16)$$

The energy content of a corresponding pair of potential and current waves is, using (14),

$$W = W_e + W_i = \frac{C}{2}\int e^2\,d\,x + \frac{L}{2}\int i^2\,d\,x$$

$$= \frac{C}{2}\int e^2\,d\,x + \frac{L}{2}\int \frac{C}{L}\,i^2\,d\,x$$

$$= C\int e^2\,d\,x = L\int i^2\,d\,x = \sqrt{L\,C}\int e\,i\,d\,x \qquad (17)$$

Or if preferable to integrate with respect to t

$$W = \int e\,i\,d\,t = \sqrt{\frac{C}{L}}\int e^2\,d\,t = \sqrt{\frac{L}{C}}\int i^2\,d\,t \qquad (18)$$

Equations (17) and (18) are continuations of each other through the relationship $d\,t = \sqrt{C\,L}\,d\,x$ from (13). These equations show that the total energy of a pair of traveling waves is divided equally between the electrostatic and magnetic fields. They are useful expressions for checking purposes.

The equations for the *No-loss* line are obtained from those derived above for the distortionless line, by making $R = 0$, thus causing the decrement factors to disappear. In what follows, the losses will be neglected.

Traveling Waves Induced by Lightning. Suppose that the bound charge is distributed as $2\,C\,f\,(x)$, and that the cloud discharges instantaneously. This is equivalent to suddenly impressing on the line a potential $2\,f\,(x)$ at the instant $t = 0$. The current at this first instant is zero. It then follows by Equations (15) and (16) that

$$i = \sqrt{\frac{C}{L}}\,\{\,f_3\,(x) - f_4\,(x)\,\} = 0 \tag{19}$$

$$e = \{\,f_3\,(x) + f_4\,(x)\,\} = 2\,f\,(x) \tag{20}$$

Therefore $\quad f_3\,(x) = f_4\,(x) = f\,(x) \quad$ (21)

Thus the sudden release of a bound charge distributed as $2\,C\,f\,(x)$ initiates a pair of exactly similar forward and reverse traveling waves having the same shape, but one-half the amplitude, of the potential corresponding to the fixed distribution of bound charge. The equations for the traveling waves under the conditions of instantaneous cloud discharge, and up until the instant when a transition point is reached, are therefore

$$e = f\,(x + v\,t) + f\,(x - v\,t) \tag{22}$$

$$i = \sqrt{\frac{C}{L}}\,\{\,f\,(x + v\,t) - f\,(x - v\,t)\,\} \tag{23}$$

However, the bound charge is not released instantaneously but over a finite time, according to the rate of cloud discharge. Moreover, while it is probable that all parts of the electrostatic field do not collapse proportionately, it will be assumed that such is the case in the following treatment. When more reliable experimental data are available, as to the actual process of a cloud discharge, it will be appropriate to make the necessary change in the mathematical analysis to take care of the way in which the field collapses. As a matter of fact, such a generalization is given in Equation (41) of Appendix I.

If the solution to the differential equations $\psi\,(p)$ specifying the behavior of a conservative system is given by $\phi\,(t)$ when the impressed force is Heaviside's unit function 1, then the solution corresponding to any impressed force $F\,(t)$ is given by Duhamel's theorem (also called the Boltzman-Hopkinson principle of superposition). It is

$$\psi\,(p)\,.\,F\,(t) = \phi\,(t)\,F\,(o) + \int_0^t \phi\,(t - \tau)\,\frac{\partial\,F\,(\tau)}{\partial\,\tau}\,d\,\tau \tag{24}$$

where $\quad \phi\,(t) = \psi\,(p)\,1$

For a derivation and discussion of this equation, as well as of five other equivalent expressions obtained by various transformations, see "Electric Circuit Theory," by J. R. Carson. Applying (24) to (22) there results

$$e = \int_0^t \{\,f\,[x + v\,(t - \tau)] + f\,[x - v\,(t - \tau)]\}\,\frac{\partial\,F\,(\tau)}{\partial\,\tau}\,d\,\tau$$

$$= \lim_{\Delta t \to 0} \sum_{\kappa=0}^{n} \{f\,[x + v\,(n - \kappa)\,\Delta\,t] + f\,[x - v(n - \kappa)\,\Delta\,t]\,\}$$

$$\{\,F\,[(\kappa + 1)\,.\,\Delta\,t] - F\,[\kappa\,.\,\Delta\,t]\} \tag{25}$$

Where $n\,.\,\Delta\,t = t$, $\kappa\,.\,\Delta\,t = \tau$, and $F\,(t)$ is the time function by which the bound charge is released.

The application of the integral in Equation (25) is limited to analytic expressions for f and F, whose integrals in the above combinations are known. But the summation, the limiting case of which is the exact solution, is immediately applicable to any functions f and F whose graphs are known or assumed. Ultimately, since both of these functions must be found from experimental data, it is advisable to deal directly with the finite summation as an approximation of arbitrary exactness. From it, both graphical and tabular methods can be developed. As a simple and obvious example of the graphical method, refer to Fig. 2, in which the bound charge initially has a rectangular distribution, and is released according to the law $F\,(t)$. Divide the base of $F\,(t)$ into equal time increments $\Delta\,t$ and the base of $2\,f\,(x)$ into corresponding equal space increments $\Delta\,x = v\,\Delta\,t$. For ordinary purposes, when dealing with parallel wires in air, the space and time increments may be taken as equal, if time is measured in microseconds and distance in thousands of feet. At the end of $n = 3$ time increments, one-half of block No. 1 has formed and moved to the right a distance $3\,\Delta\,t$ and the other half to the left a distance $3\,\Delta\,t$. Block No. 2 has moved to the right and left a distance $2\,\Delta\,t$, and block No. 3 a distance $1\,\Delta\,t$. Block No. 4 has just started to form and move out. Only the half blocks which have moved to the right are shown in the figure. It is advisable to subdivide the increments $\Delta\,t$ for the larger steps, as indicated for block No. 1. The traveling component is found by drawing a curve through the feet of the rectangular blocks of which it is composed. The forward and reverse traveling waves at $n = 3$ are shown as heavily drawn full and dashed lines respectively. Their instantaneous resultant has been plotting upside down to avoid interference. This graphical method is very awkward when applied to other than rectangular bound charges. A tabular method, shown in Fig. 4, has been devised as a substitute applicable to complicated bound charges. The schedule is carried through for only one of the pair of traveling waves. The resultant is found by superimposing two such waves displaced by $2\,n\,\Delta\,x$. For periods of time in excess of that required for the cloud to completely discharge, the traveling waves are fully formed, and thereafter may be moved out as rigid distributions. Formations and waves determined by these graphical and tabular methods have been described in the first part of this paper.

Empirical Equation for Approximate Wave Shape. The combined influence of the initial distribution of bound charge and the rate of cloud discharge, imparts to the traveling waves a characteristic shape. The shapes obtained by the previous analysis check those which have been measured in the field and in the laboratory. An empirical equation which represents this characteristic shape quite accurately, and at the same time is remarkably simple for analytic purposes, is given by

$$e = E\left(\epsilon^{-a\lambda} - \epsilon^{-b\lambda}\right) \quad \text{for } t > 0. \tag{26}$$

As special cases of this equation there are the various wave shapes illustrated in Fig. 12. Still greater variety of form may be obtained by using

$$e = E_1\,\epsilon^{-a\lambda} - E_2\,\epsilon^{-b\lambda} \tag{27}$$

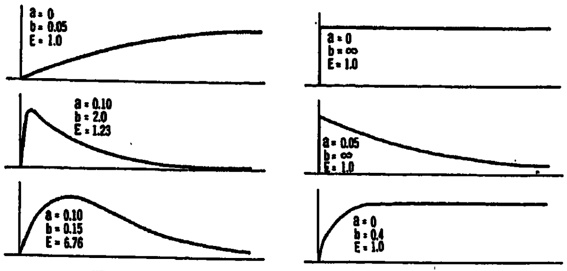

FIG. 12—EMPIRICAL WAVE SHAPES

Given by $e = E\left(\epsilon^{-at} - \epsilon^{-bt}\right)$

but for the purposes of this paper only (26) will be applied. The three parameters E, a, b, are sufficient to determine uniquely the *crest*, *wave length*, and *front* of the wave. The parameters a and b may be *real*, *imaginary*, or *complex*. The crest, or maximum value of e, occurs when

$$\frac{d\,e}{d\,t} = 0 = -\,a\,\epsilon^{-at} + b\,\epsilon^{-bt}$$

hence at that value of t for which

$$t = t_1 = \frac{\log b/a}{b - a}$$

The instant $t = t_2$ at which the wave has $(100\,p)$ per cent of e is found from the equation

$$e = p\,e_{max} = E\left(\epsilon^{-at_2} - \epsilon^{-bt_2}\right)$$

Hence the crest, wave length, and front of the wave are determined by the three simultaneous equations

$$\left.\begin{aligned} t_1 &= \frac{\log b/a}{b - a} \\ e_{max} &= E\left(\epsilon^{-at_1} - \epsilon^{-bt_1}\right) \\ p\,e_{max} &= E\left(\epsilon^{-at_2} - \epsilon^{-bt_2}\right) \end{aligned}\right\} \tag{28}$$

where (e_{max}, p, t_1, t_2) are specified and (a, b, E) are to be solved for. Practically, since (28) is in transcendental form, it is easier to plot a family of curves directly from (26) and to choose the nearest approximation to the desired wave shape by inspection.

Behavior of a Traveling Wave at a Transition Point. When a traveling wave reaches a transition point at which there is an abrupt change of circuit constants, as an open or short-circuited terminal, or a junction with another line, etc., a part of the wave is reflected back, and a part may pass on to other sections of the circuit. The impinging wave is called an *incident* wave, and the two waves to which it gives rise at a transition point are called the *reflected* and *transmitted* waves respectively. Such waves are formed at the transition point in accordance with Kirchhoff's Laws. They satisfy the differential equations of the transmission line, and are consistent with the principle of the Conservation of Energy.

Suppose that the line is closed at the transition point by a general impedance consisting of any arrangement of inductances, resistances, capacitances, and other lines. Let the operational equation specifying this general impedance be written as $Z_0(p)$. Let the transition point be taken as the origin of coordinates, and distance along the line away from the point be counted as positive, so that an approaching wave is traveling in the negative direction. By Equation (14) the potential and current incident waves will then have the same sign. Denote the incident waves by (e) and (i), the reflected waves by (e') amd (i'), and the transmitted waves, if they exist, by (e'') and (i''). Then the total potential at the transition point is, using (14),

$$e_0 = e + e' = (i + i')\,Z_0(p) = (e - e')\,Y Z_0(p) \tag{29}$$

where $Y = 1/Z = \sqrt{C/L}$ = surge admittance.

Solving this equation for e' there is

$$e' = \frac{Z_0(p) - Z}{Z_0(p) + Z}\,e = \text{reflected potential wave} \tag{30}$$

The total resultant wave at the transition point is the sum of the incident and reflected waves,

$$e_0 = e + e' = \frac{2\,Z_0(p)}{Z_0(p) + Z}\,e$$

$$i = i + i' = Y(e - e') = \frac{2}{Z_0(p) + Z}\,e \tag{31}$$

In general, $Z_0(p)$ may consist of any number of branches in parallel. If one of these branches consists of another line of surge impedance Z_2 connected through a concentrated impedance network $Z_m(p)$, then the potential wave transmitted through this impedance to the line is

$$e'' = e_0 - Z_m(p)\,i'' = e_0 - Z_m(p)\,\frac{e_0}{Z_m(p) + Z_2}$$

$$= \left\{ 1 - \frac{Z_m (p)}{Z_m (p) + Z_2} \right\} e_0$$

$$= \left(\frac{Z_2}{Z_m (p) + Z_2} \right) \left(\frac{2\, Z_0 (p)}{Z_0 (p) + Z} \right) e \qquad (32)$$

Thus if e is known at the transition point as a function of time, then e_0, e', and e'' are determined by solving the above differential equations. In particular, if e is a rectangular wave with an infinite tail, it may be taken as Heaviside's *unit function* 1, and the solution obtained by means of the expansion formula. The solution for a finite wave of any shape is then found from Duhamel's theorem, Equation (**24**).

The application of these general relationships are illustrated in this paper for the wave of Equation (**26**), with the parameters adjusted to give both a rectangular wave with an infinite tail, and a characteristic lightning wave having a 7-microsecond front and a 13-microsecond tail. The differences in behavior of these two waves is sometimes quite striking, and conclusions based on one of them are not always generally applicable to the other. Only two cases will be worked out here in detail. But exactly the same procedure has been followed in deriving all of the results.

1. Two Lines Connected by an Inductance (Fig. 22).

$Z_1 = 1/Y_1$ = surge impedance of line No. 1

$Z_2 = 1/Y_2$ = surge impedance of line No. 2

L_0 = connecting inductance

$$Z_m (p) = L_0\, p$$

$$Z_0 (p) = Z_2 + L_0\, p \qquad \text{and let}$$

$$\alpha = (Z_1 - Z_2)/L_0$$

$$\beta = (Z_1 + Z_2)/L_0$$

Then by Equation (**30**) the reflected potential wave is

$$e' = \frac{Z_0 (p) - Z_1}{Z_0 (p) + Z_1}\, e = \frac{(Z_2 + L_0\, p) - Z_1}{(Z_2 + L_0\, p) + Z_1}\, e = \frac{p - \alpha}{p + \beta}\, e$$

If e is a rectangular wave with an infinite tail, the solution by the expansion theorem is

$$e' = \left[\left(1 + \frac{\alpha}{\beta} \right) \epsilon^{-\beta t} - \frac{\alpha}{\beta} \right] \text{if } e = 1$$

and by Equation (**24**), if $e = E\, (\epsilon^{-at} - \epsilon^{-bt})$

$$e' = E \left[\frac{a + \alpha}{a - \beta}\, \epsilon^{-at} - \frac{b + \alpha}{b - \beta}\, \epsilon^{-bt} + \frac{(\alpha + \beta)\, (a - b)}{(a - \beta)\, (b - \beta)}\, \epsilon^{-\beta t} \right]$$

From Equation (**32**)

$$e'' = e_0 - Z_m (p)\, i'' = e_0 - Z_m (p)\, Y_1\, (e - e')$$

$$= E \left[\frac{\alpha - \beta}{a - \beta}\, \epsilon^{-at} - \frac{\alpha - \beta}{b - \beta}\, \epsilon^{-bt} + \frac{(a - b)\, (\alpha - \beta)}{(a - \beta)\, (b - \beta)}\, \epsilon^{-\beta t} \right]$$

If $\alpha \to \beta$, $e'' \to 0$ and there is no transmitted wave. Taking $Z_1 = Z_2 = 500$ ohms and $L_0 = 0.000033$ henry, there results for the 20-microsecond wave

$\alpha = 0$, $\beta = 30 \times 10^6$, $a = .1 \times 10^6$, $b = .2 \times 10^6$ and

$$e'' = E \left[\frac{30.0}{29.9}\, \epsilon^{-at} - \frac{30.0}{29.8}\, \epsilon^{-bt} + \frac{.1 \times 30}{29.9 \times 29.8}\, \epsilon^{-30 \times 10^6 t} \right]$$

$$\cong E\, (\epsilon^{-at} - \epsilon^{-bt}) = e$$

Thus such a small series inductance is entirely ineffective except for very short waves. The entire incident wave is transmitted with negligible change of shape.

2. Impedance Network of Fig. 25. The variations shown in (*a*), (*b*), (*c*), and (*d*) of Fig. 25 are in fact mutually convertible combinations, and therefore, under the conditions of a proper interpretation, the same equations apply to each. Moreover, by letting the different constants become infinite or vanish as required, this single set-up contains 24 special cases, of which the majority are described and illustrated in this paper.

Since the surge impedance of a connected line, as Z_2, enters in the generalized impedance $Z_0 (p)$ in exactly the same way as a resistance to ground R_2, it is evident that Fig. 25B is the analytic equivalent of Fig. 25A, so that the same equations apply to each when R_2 and Z_2 are exchanged. In Fig. 25C the grounding resistance $R_2 = 0$, and R_0 has been expressed as the resultant of two resistances R' and R'' in parallel. But now again, the form of the generalized impedance is not altered if R'' be replaced by the surge impedance $Z'' = R''$ of an outgoing line. Fig. 25D then follows as a natural consequence.

While this equivalence is quite interesting and allows many combinations to be expressed by a single general equation, yet under many conditions it is quicker, and there is less chance for error, if the result is derived directly from the differential equations of the particular case. In performing a reduction from the general equation applying to Fig. 25, great care must be taken in evaluating the indeterminates which appear for limiting values (zero or infinity) of the constants, and in transforming functions of imaginary variables to function of real variables, etc.

Referring to Fig. 25A, there is

$$Z_0 (p) = Z_2 + Z_m (p) = Z_2 + \frac{L_0\, R_0\, p}{R_0 + L_0\, p + R_0\, L_0\, C_0\, p^2}$$

$$e' = \frac{Z_0 (p) - Z_1}{Z_0 (p) + Z_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \left(\frac{p^2 + 2\, \alpha\, p + \omega_0^2}{p^2 + 2\, \beta\, p + \omega_0^2} \right) e$$

where

$$\alpha = \gamma\, (Z_2 - Z_1 + R_0)/(Z_2 - Z_1)$$

$$\beta = \gamma\, (Z_2 + Z_1 + R_0)/(Z_2 + Z_1)$$

$$\gamma = 1/2\, R_0\, C_0$$

$$\omega_0^2 = 1/L_0\, C_0$$

$$\omega^2 = \omega_0^2 - \beta^2$$

when $e = 1$, the solution by the expansion formula is

$$e' = \frac{Z_2 - Z_1}{Z_2 + Z_1} \left\{ 1 - \frac{2}{\omega} (\beta - \alpha)\, \epsilon^{-\beta t} \sin\, \omega\, t \right\} \text{if } e = 1.$$

If the incident wave is given by

$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$, then by Equation (24)

$$e' = \frac{Z_2 - Z_1}{Z_2 + Z_1} E \left\{ \frac{a^2 - 2\,a\,\alpha + \omega_0^2}{a^2 - 2\,a\,\beta + \omega_0^2}\,\epsilon^{-at} \right.$$

$$- \frac{b^2 - 2\,b\,\alpha + \omega_0^2}{b^2 - 2\,b\,\beta + \omega_0^2}\,\epsilon^{-bt} + \frac{2}{\omega}\,(\beta - \alpha)\,\epsilon^{-\beta t}$$

$$\left[\sqrt{\frac{a^2}{a^2 - 2\,a\,\beta + \omega_0^2}}\,\sin\,(\omega\,t - \phi_1) \right.$$

$$\left.\left. - \sqrt{\frac{b^2}{b^2 - 2\,b\,\beta + \omega_0^2}}\,\sin\,(\omega t - \phi_2) \right] \right\}$$

where $\phi_1 = \tan^{-1}\left(\dfrac{\omega}{a - \beta}\right)$ and $\phi_2 = \tan^{-1}\left(\dfrac{\omega}{b - \beta}\right)$

The differential equation for the corresponding transmitted wave by Equation (32) is

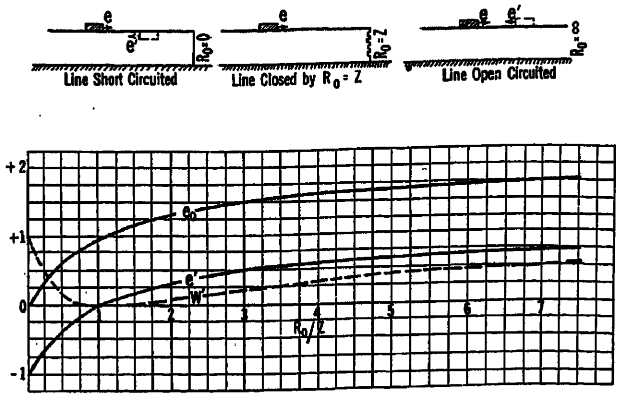

FIG. 13—LINE CLOSED BY A RESISTANCE R_0

$$e = f\,(t) \quad e' = \left(\frac{R_0 - Z}{R_0 + Z}\right) e \quad e_0 = e + e' = \left(\frac{2\,R_0}{R_0 + Z}\right) e$$

$$e'' = \left(\frac{Z_2}{Z_m\,(p) + Z_2}\right)\left(\frac{2\,Z_0\,(p)}{Z_0\,(p) + Z_2}\right) e$$

$$= \frac{2\,Z_2}{Z_0\,(p) + Z_2}\,e = \frac{2\,Z_2}{Z_2 + Z_1}\left(\frac{p^2 + 2\,\gamma\,p + \omega_0^2}{p^2 + 2\,\beta\,p + \omega_0^2}\right) e$$

This differential equation has the same form as that for e', except that in the outside multiplier $(2\,Z_2)$ has replaced $(Z_2 - Z_1)$, and γ has replaced α. The solution, by comparison with that for e', therefore is

$$e'' = \frac{2\,Z_2}{Z_2 + Z_1} E \left\{ \frac{a^2 - 2\,a\,\gamma + \omega_0^2}{a^2 - 2\,a\,\beta + \omega_0^2}\,\epsilon^{-at} \right.$$

$$- \frac{b^2 - 2\,b\,\gamma + \omega_0^2}{b^2 - 2\,b\,\beta + \omega_0^2}\,\epsilon^{-bt} + \frac{2}{\omega}\,(\beta - \gamma)\,\epsilon^{-\beta t}$$

$$\left[\sqrt{\frac{a^2}{a^2 - 2\,a\beta + \omega_0^2}}\,\sin\,(\omega\,t - \phi_1) \right.$$

$$\left.\left. - \sqrt{\frac{b_2}{b^2 - 2\,b\,\beta + \omega_0^2}}\,\sin\,(\omega\,t - \phi_2) \right] \right\}$$

Figs. 13 to 24 inclusive illustrate in a general sort of way the variation in the shape of the reflected and transmitted waves caused by different circuit conditions

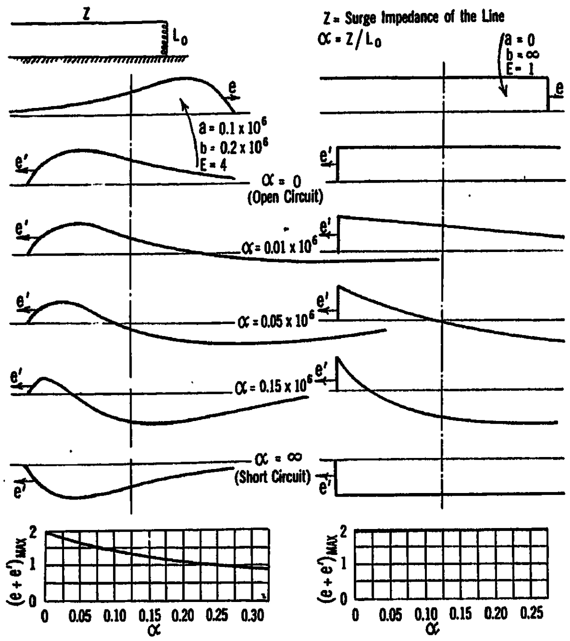

FIG. 14—LINE CLOSED BY AN INDUCTANCE L_0

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = E\left[\frac{a + \alpha}{a - \alpha}\,\epsilon^{-at} - \frac{b + \alpha}{b - \alpha}\,\epsilon^{-bt} + \frac{2\,\alpha\,(a - b)}{(a - \alpha)\,(b - \alpha)}\,\epsilon^{\,\alpha t}\right]$$

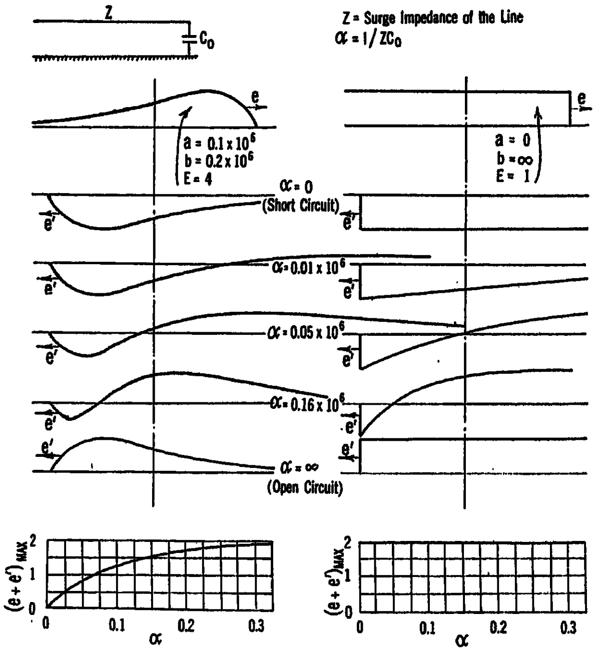

FIG. 15—LINE CLOSED BY A CAPACITANCE C_0

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = -\,E\left[\frac{a + \alpha}{a - \alpha}\,\epsilon^{-at} - \frac{b + \alpha}{b - \alpha}\,\epsilon^{-bt} + \frac{2\,\alpha\,(a - b)}{(a - \alpha)\,(b - \alpha)}\,\epsilon^{-\alpha t}\right]$$

at the transition point. Each figure shows a sketch of the circuit under consideration, defines the variable parameters, and indicates the range in shape of the reflected and transmitted waves corresponding to both an infinite rectangular incident wave and a characteristic 20-microsecond incident wave. The equations of the incident, reflected, and transmitted waves are given below each figure as e, e', and e'' respectively. While these cases are fairly self-explanatory, a few pertinent remarks about each are given below.

Fig. 13. Line Closed by a Resistance R_0. When an incident wave of any shape strikes the end of a line of surge impedance Z closed by a pure resistance, a reflected wave of exactly the same shape will start back on the line. But this reflected wave changes sign as the ratio R_0/Z passes through the critical value $R_0/Z = 1$. If $R_0 = 0$, the line is short-circuited and the reflection is of the same size as the incident wave, but reversed in sign. At $R_0 = Z$ there is no reflection, and the energy of the incident wave is entirely consumed in the resistance. At $R_0/Z = \infty$ the line approaches the open circuit condition, and the reflection is of the same size and sign as the incident wave. These three limiting conditions are illustrated in the sketch. The curves show the way in which the reflected voltage e', the total voltage e_0, and the energy content W' of the reflected wave vary with the ratio R_0/Z.

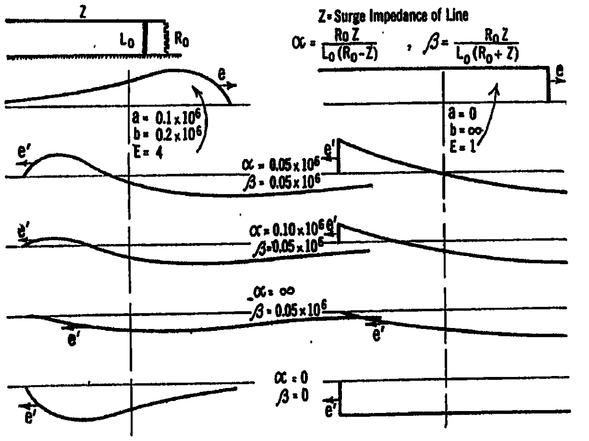

FIG. 16—LINE CLOSED BY R_0 AND L_0 IN PARALLEL

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = E\,\frac{\beta}{\alpha}\left[\frac{a+\alpha}{a-\beta}\,\epsilon^{-at} - \frac{b+\alpha}{b-\beta}\,\epsilon^{-bt} + \frac{(\alpha+\beta)\,(a-b)}{(a-\beta)\,(b-\beta)}\,\epsilon^{-\beta t}\right]$$

Fig. 14. Line Closed by an Inductance L_0. As L_0 passes through the intermediate values between zero and infinity, the reflected waves are first elongated, and then contracted. Each reflected wave changes sign as it develops. Except for an infinite value of L_0, the total voltage e_0 at the transition point is never double for a finite incident wave.

Fig. 15. Line Closed by a Capacitance C_0. The form of the equations is the same as for the case of the inductance given above, but the sign is reversed.

A capacitance acts as a short-circuit at the first instant.

Fig. 16. Line Closed by R_0 *and* L_0 *in Parallel.* While it is impossible by such a combination to completely dissipate the incident wave, yet for values of R_0 of the order of the line surge impedance Z, the reflected wave is considerably reduced in value and spread out.

Fig. 17. Line Closed by R_0 *and* C_0 *in Parallel.* The reflections have the same characteristics as for Fig. 15 but the amplitudes have been reduced by the resistance.

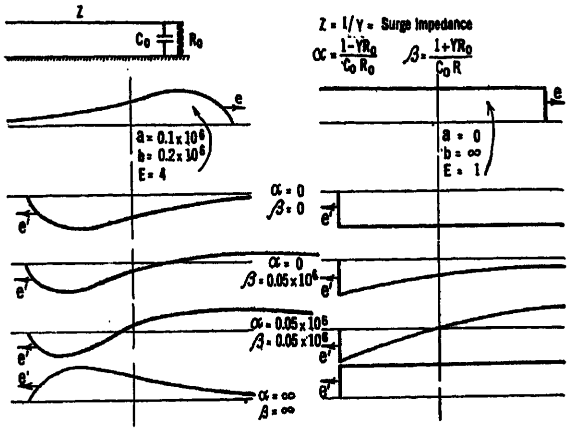

FIG. 17—LINE CLOSED BY R_0 AND C_0 IN PARALLEL

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = E\left[-\frac{a-\alpha}{a-\beta}\,\epsilon^{-at} + \frac{b-\alpha}{b-\beta}\,\epsilon^{-bt} + \frac{(\alpha-\beta)\,(a-b)}{(a-\beta)\,(b-\beta)}\,\epsilon^{-\beta t}\right]$$

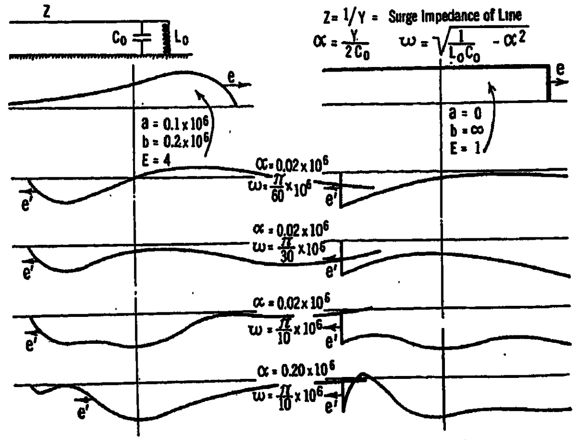

FIG. 18—LINE CLOSED BY L_0 AND C_0 IN PARALLEL

$$e = E\,(\epsilon^{-at} - \epsilon^{-bt})$$

$$e' = E\left\{-\frac{(a+\alpha)^2+\omega^2}{(a-\alpha)^2+\omega^2}\,\epsilon^{-a} + \frac{(b+\alpha)^2+\omega^2}{(b-\alpha)^2+\omega^2}\,\epsilon^{-bt} + 4\,\frac{\alpha}{\omega}\,\epsilon^{-\alpha t}\right.$$
$$\left.\left[\left(\frac{b\,(b-\alpha)}{(b-\alpha)^2-\omega^2} - \frac{a\,(a-\alpha)}{(a-\alpha)^2+\omega^2}\right)\sin\omega t - \left(\frac{b\,\omega}{(b-\alpha)^2+\omega^2} - \frac{a\,\omega}{(a-\alpha)^2+\omega^2}\right)\cos\omega t\right]\right\}$$

Fig. 18. Line Closed by L_0 *and* C_0 *in Parallel.* There is an oscillation in the reflected waves, caused by the oscillatory circuit. By such an arrangement the reflected wave may be either compressed or lengthened out. The development of the wave front in the reflected wave may be made to lag behind several

microseconds,—approximately 6 microseconds in the example shown by the last wave in the figure.

Fig. 19. Junction of Two Lines. When the incident wave reaches the junction, a part is reflected back and a part passes on to the other line. The relative division depends on the ratio of the surge impedances of the two lines. The waves shown are drawn on a *time axis.* If drawn on a *distance axis* the transmitted waves will be contracted in the ratio of the velocities of propagation of the two lines, as shown in the next figure. While this case has been illustrated with particular waves, yet incident waves of any form are reflected from and transmitted across a junction between two lines, without change of shape.

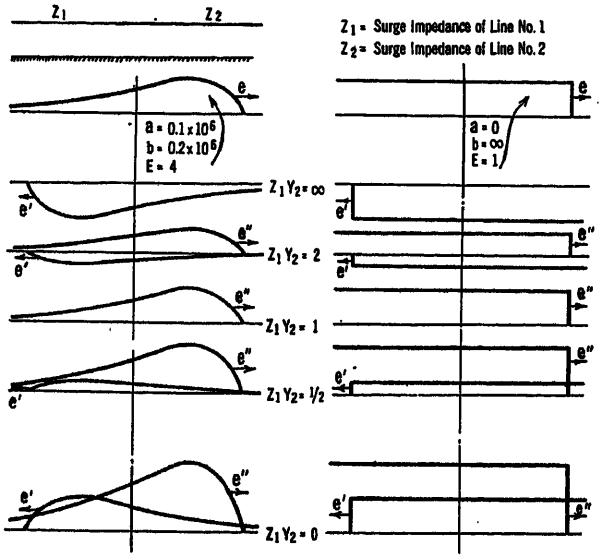

FIG. 19—JUNCTION OF TWO LINES

$$e = E\left(\epsilon^{-at} - \epsilon^{-bt}\right) \qquad e' = \frac{1 - Z_1 Y_2}{1 + Z_1 Y_2}\, e \qquad e'' = \frac{2}{1 + Z_1 Y_2}\, e$$

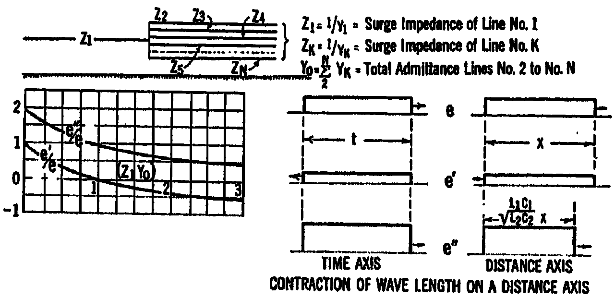

FIG. 20—JUNCTION OF *N* LINES

$$e = f\,(x + v_1 t)$$

$$e' = \left(\frac{1 - Z_1 Y_0}{1 + Z_1 Y_0}\right) f\,(x - v_1 t)$$

$$e'' = \left(\frac{2}{1 + Z_1 Y_0}\right) f\,(x + v_k t)$$

Fig. 20. Junction of "N" Lines. Incident waves of any shape are reflected and transmitted at the junction without change of shape. Incident, reflected, and transmitted waves have the same length on a *time axis,* but are contracted on a *distance axis* in the ratio $\sqrt{C_1 L_1} : \sqrt{C_2 L_2}$. This contraction is illustrated for rectangular waves, but applies to waves of all shapes.

Fig. 21. Two Lines Connected by a Resistor R_0. As in the previous two cases there is no distortion of wave shape due to impact, but the resistor consumes part of the energy. By making ($R_0 = Z_1 - Z_2$) it is possible to wipe out the reflected waves. These curves show how the waves and their energy content vary with R_0, Z_1, and Z_2.

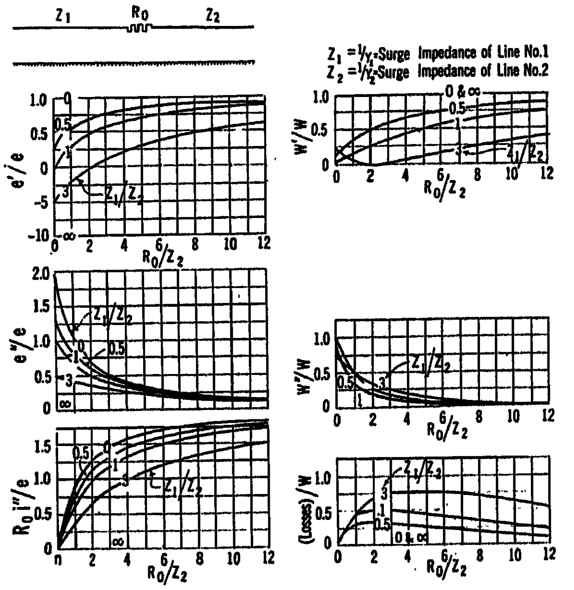

FIG. 21—TWO LINES CONNECTED BY A RESISTOR R_0

$$e = f\,(t) \quad e' = \left(\frac{1 + R_0/Z_2 - Z_1/Z_2}{1 + R_0/Z_2 + Z_1/Z_2}\right) e \quad e'' = \left(\frac{2}{1 + R_0/Z_2 + Z_1/Z_2}\right) e$$

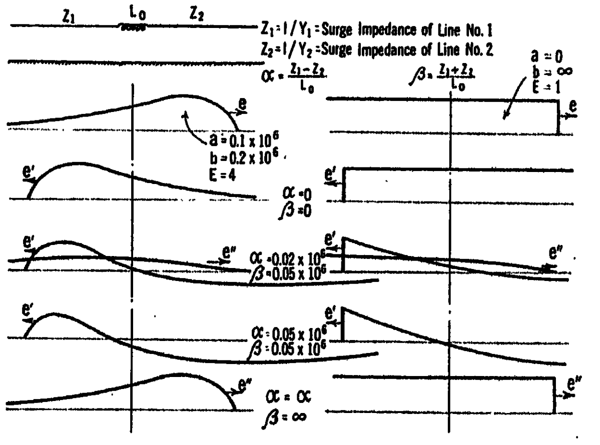

FIG. 22—TWO LINES CONNECTED BY AN INDUCTANCE L_0

$$e = E\left(\epsilon^{-at} - \epsilon^{-bt}\right)$$

$$e' = E\left[\frac{a + \alpha}{a - \beta}\,\epsilon^{-at} - \frac{b + \alpha}{b - \beta}\,\epsilon^{-bt} + \frac{(\alpha + \beta)(a - b)}{(a - \beta)(b - \beta)}\,\epsilon^{-\beta t}\right]$$

$$e'' = E\left[\frac{\alpha - \beta}{a - \beta}\,\epsilon^{-at} - \frac{\alpha - \beta}{b - \beta}\,\epsilon^{-bt} + \frac{(\alpha - \beta)(a - b)}{(a - \beta)(b - \beta)}\,\epsilon^{-\beta t}\right]$$

Fig. 22. Two Lines Connected by an Inductance L_0. The reflected wave exhibits the same characteristic as in the case of Fig. 14. If L_0 is large enough, the major

portion of the incident wave can be reflected back, and only a small transmitted wave passed through.

Fig. 23. Two Lines with Shunt Inductance at their Junction. The equations and curves for the incident and reflected waves of this case hold identically, when Z_2 is replaced by R_0, to ground.

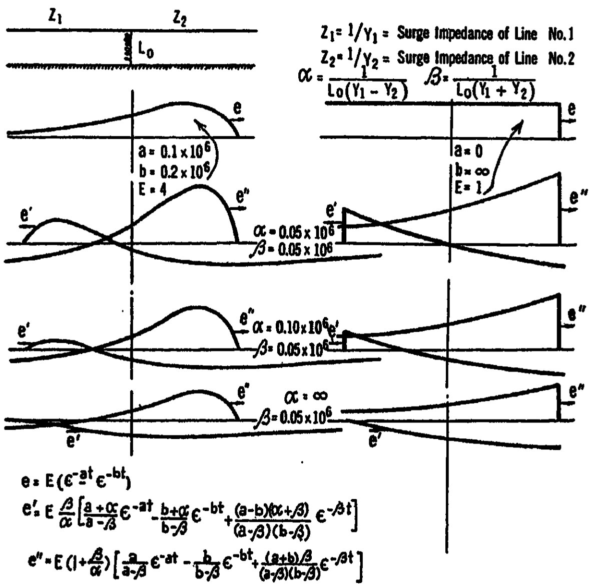

FIG. 23—TWO LINES WITH SHUNT INDUCTANCE AT THEIR JUNCTION

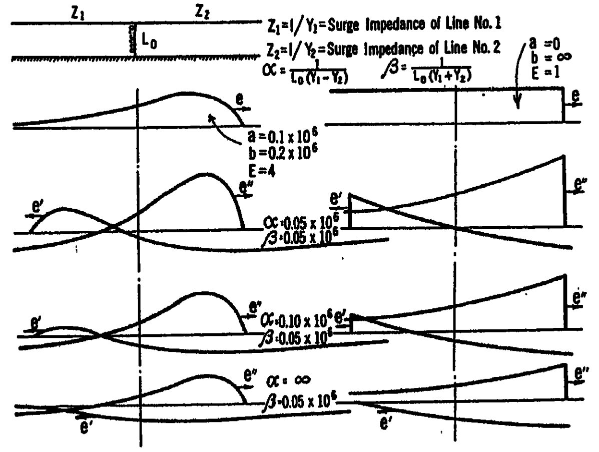

FIG. 24—TWO LINES WITH SHUNT CAPACITANCE AT THEIR JUNCTION

$$e = E\left(\epsilon^{-at} - \epsilon^{-bt}\right)$$

$$e' = E\frac{\beta}{\alpha}\left[\frac{a+\alpha}{a-\beta}\epsilon^{-at} - \frac{b+\alpha}{b-\beta}\epsilon^{-bt} + \frac{(a-b)(\alpha+\beta)}{(a-\beta)(b-\beta)}\epsilon^{-\beta t}\right]$$

$$e'' = E\left(1+\frac{\beta}{\alpha}\right)\left[\frac{a}{a-\beta}\epsilon^{-at} - \frac{b}{b-\beta}\epsilon^{-bt} + \frac{(a-b)\beta}{(a-\beta)(b-\beta)}\epsilon^{-\beta t}\right]$$

and reflected waves of this case hold identically, when Z_2 is replaced by R_0, to ground.

Fig. 24. Two Lines with Shunt Capacitance at their Junction. The effect of a capacitance in shunt is similar to that of an inductance in series, Fig. 22. The equations and curves for the incident and reflected waves of this case hold identically, when Z_2 is replaced by R_0, to ground.

ACKNOWLEDGMENTS

The author acknowledges with pleasure the encouragement and the many helpful suggestions of Mr. F. W. Peek, Jr., under whose direction this investigation was carried out. The curves, numerical computations, and planimeter work were done by Mr. W. A. Carman.

Appendix

Formation of traveling waves when R, L, G, and C, are all included. The release of a bound charge on a transmission line initiates potential and current waves whose shape and behavior are independent of the terminal conditions at the end of the line, up until the time when the advancing wave fronts have reached those ends. However, the form of the equations expressing these waves must take cognizance of the terminal conditions if they are to remain valid over all time. They will therefore be the more complicated, the greater the complexity of the terminal conditions. Therefore if the purpose of an analysis is merely to study the effects on wave shape of the rate of release and initial distribution of a bound charge, it is advisable in the interests of mathematical simplicity to arbitrarily choose such terminal conditions as will lead to the most easily handled equations, for regardless of terminal conditions the functions have identical graphs over the time period required for the waves to reach a transition point. Accordingly, for the purposes of this Appendix the line will be considered as short-circuited at each end.

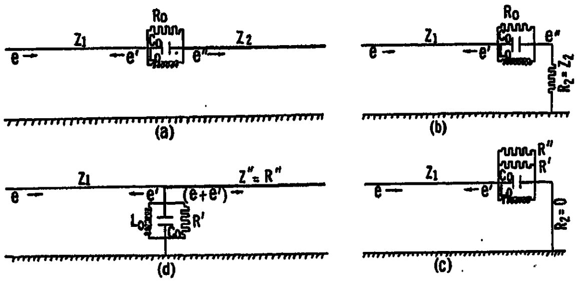

FIG. 25—MUTUALLY CONVERTIBLE NETWORKS

While any mathematical analysis must almost necessarily regard the line constants as the same for all waves, yet these "constants" vary over a wide range depending on the amplitudes and shapes of the traveling waves, the resistance and inductance due to skin effect, and the capacitance and shunt conductance due to corona. The equations derived in this Appendix are based on the assumption that the resistance, shunt conductance, inductance, and capacitance per unit length are perfectly constant. In the final solution, which is an infinite series, it may be feasible to segregate out from the series certain groups of terms and to apply different values of the line parameters to each of these separate groups. As an approximation, such a procedure is justifiable only when it can be shown that the line constants are primarily functions of the frequency or other parameter with respect to which a segregation has been made. The principal value in deriving equations with the loss factors R and G included, is to be able to notice by inspection of the equations, the direction of their influence on wave shape.

Consider the point charge Q of Fig. 26 suddenly placed at a point x on the transmission line of length l, short-circuited at its ends. The fact that Q is suddenly released may be expressed by Heaviside's unit function $Q\,1$. Let that part of the line to the right of x be designated as Line 1 and that to the left as Line 2. Subscript all associated symbols to correspond. Then the solution to Equation (3), consistent with these conditions is[5]

$$V_1 = \frac{\sinh n\,x}{\sinh n\,l} \sinh n\ (l - y)\ \frac{n}{Y}\ p\,Q\,1 \qquad (33)$$

$$V_2 = \frac{\sinh n\,(l - x)}{\sinh n\,l} \sinh n\,y\ \frac{n}{Y}\ p\,Q\,1 \qquad (34)$$

where $n^2 = Z\,Y = (R + p\,L)\,(G + p\,C)$

Solving (33) and (34) by a direct application of the expansion formula, there is

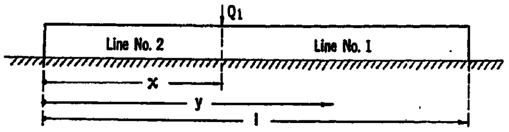

FIG. 26—POINT CHARGE SUDDENLY PLACED ON A LINE

$$V_1 = V_2 = Q\,\phi\,(t) = \frac{2\,Q}{l\,C}\,\epsilon^{-\alpha t}$$

$$\sum_{s=1}^{\infty} \sin\frac{s\,\pi\,x}{l}\,\sin\frac{s\,\pi\,y}{l}\left(\cos\beta\,t - \frac{\gamma}{\beta}\sin\beta\,t\right) \qquad (36)$$

where

$$\alpha = \frac{1}{2}\left(\frac{G}{C} + \frac{R}{L}\right) = \text{attenuation factor} \qquad (37)$$

$$\gamma = \frac{1}{2}\left(\frac{G}{C} - \frac{R}{L}\right) = \text{distortion factor} \qquad (38)$$

$$\beta = \sqrt{\frac{s^2\,\pi^2}{l^2\,L\,C} - \gamma^2} = \text{wave length factor} \qquad (39)$$

Suppose that the charge Q instead of being a point charge suddenly applied, is distributed along a section of the line and is applied gradually over a period of time. The charge on the line is then distributed as a space and time function $Q\,(x, t)$. The amount of charge on the line element $d\,x$ is $Q\,(x, t)\,d\,x$. By Duhamel's theorem, Equation (24), the voltage due to this point charge $Q\,(x, t)\,d\,x$ is

$$d\,V = \left\{ Q(o, x)\,.\,\phi(t) + \int_0^t \phi(t - \tau)\,.\,\frac{\partial\,Q\,(x, \tau)}{\partial\,\tau}\,d\,\tau \right\} dx \qquad (40)$$

The voltage due to the total space distribution is

$$V = \int_0^l Q\,(o, x)\,\phi\,(t)\,d\,x$$

$$+ \int_0^l \int_0^t \phi\,(t - \tau)\,\frac{\partial\,Q\,(x, \tau)}{\partial\,\tau}\,d\,\tau\,d\,x. \qquad (41)$$

If $Q\,(x, t)$ is due to the release of a bound charge by the discharge of a cloud, a finite time is required for $Q\,(x, t)$ to appear. Therefore $Q\,(o, x) = 0$. Substituting (36) in (41)

$$V = \frac{2}{l\,C} \sum_{s=1}^{\infty} \sin\frac{s\,\pi\,y}{l} \int_0^l \sin\frac{s\,\pi\,x}{l}$$

$$\int_0^t \epsilon^{-\alpha(t-\tau)}\left[\cos\beta(t - \tau) - \frac{\gamma}{\beta}\sin\beta(t - \tau)\right]\frac{\partial\,Q\,(x, \tau)}{\partial\,\tau}\,d\,\tau\,d\,x \qquad (42)$$

This equation is general, in that the bound charge at different points along its distribution may be released at different rates.

Suppose that all parts of the bound charge are released at the same rate, proportioned to the function $F\,(t)$. Then

$$Q\,(x, t) = Q\,.\,f\,(x)\,.\,F\,(t) \qquad (43)$$

Under this condition, Equation (42) becomes

$$V = \frac{2\,Q}{l\,C} \sum_{s=1}^{\infty} \sin\frac{s\,\pi\,y}{l} \int_0^l f\,(x)\,\sin\frac{s\,\pi\,x}{l}\,d\,x$$

$$\int_0^t \epsilon^{-\alpha(t-\tau)}\left[\cos\beta\,(t - \tau) - \frac{\gamma}{\beta}\sin\beta\,(t - \tau)\right]\frac{\partial\,F\,(\tau)}{\partial\,\tau}\,d\,\tau \qquad (44)$$

As an example take

$$f\,(x) = \frac{1}{2}\left[1 + \cos\left(\frac{l}{2} - x\right)\frac{\pi}{b}\right]$$

for

$$\left(\frac{l}{2} - b\right) \leq x \leq \left(\frac{l}{2} + b\right) \qquad (45)$$

$$F\,(t) = (1 - \epsilon^{-at}) \qquad (46)$$

These functions are shown in Fig. 7. Substituting (45) and (46) in (44) and performing the integrations, these results

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[5] "Heaviside's Operational Calculus as Applied to Engineering and Physics," by E. J. Berg, Equations (99) and (100), *General Electric Rev.*, July 1928, Vol. 31, No. 7.

$$V = \frac{2Qa}{lC} \sum_{s=1}^{\infty} \sin \frac{s\pi y}{l} \left\{ \frac{\frac{\pi^2}{b^2} \sin \frac{s\pi b}{l} \, . \sin \frac{s\pi}{2}}{\frac{s\pi}{l}\left(\frac{s^2\pi^2}{l^2} - \frac{\pi^2}{b^2} \right)} \right\}$$

$$\frac{a - \alpha + \gamma}{(a-\alpha)^2 + \beta^2} \Big[- \epsilon^{-at} + \epsilon^{-\alpha t} \cos \beta t$$

$$- \frac{\gamma (a - \alpha) - \beta^2}{(a - \alpha + \gamma)\beta} \epsilon^{-\alpha t} \sin \beta t \Big] \qquad (47)$$

Let the line be measured in units of length such that $b = 1$. Replace y by $y = (l/2 \pm z)$, that is, measure distance from the center of the disturbance. Combine the trigonometric functions of space and time angles. After simplifying and rearranging the equations there is

$$V = \frac{2Q}{lC} \sum_{s=1}^{\infty} \frac{a \left(\sin \frac{s\pi}{2} \right)^2}{a - \alpha - \gamma} C(s,k) \Big\{ - \epsilon^{-at} \cos \theta z$$

$$+ \frac{\epsilon^{-\alpha t}}{2} [S(\lambda_1) + S(\lambda_2)] \Big\} \qquad (48)$$

where

$C(s, \kappa) = \dfrac{\pi^2 \sin \theta}{\theta (\theta^2 - \pi^2)} \dfrac{\kappa^2}{\kappa^2 + \theta^2}$ = convergency factor

$S(\lambda_1) = \cos(\lambda_1) - \dfrac{\gamma(a - \alpha) - \beta^2}{(a - \alpha + \gamma)\beta} \sin(\lambda_1)$ = forward wave component

$S(\lambda_2) = \cos(\lambda_2) - \dfrac{\gamma(a - \alpha) - \beta^2}{(a - \alpha + \gamma)\beta} \sin(\lambda_2)$ = reverse wave component

$\theta = s\pi/l \qquad \kappa^2 = LC[(a - \alpha)^2 - \gamma^2]$

$\lambda_1 = \beta t - \theta z \qquad \lambda_2 = \beta t + \theta z$

If the cloud discharges instantaneously, $a \to \infty$, and (48) reduces to

$$V = \frac{2Q}{lC} \sum_{s=1}^{\infty} \left\{ \frac{-\pi^2 \sin\theta}{\theta(\theta^2 - \pi^2} \right\} \left(\sin \frac{s\pi}{2} \right)^2 \frac{\epsilon^{-\alpha t}}{2}$$

$$\left[\cos \lambda_1 - \frac{\gamma}{\beta} \sin \lambda_1 + \cos \lambda_2 - \frac{\gamma}{\beta} \sin \lambda_2 \right] \qquad (49)$$

It is of some interest to check Equation (48). The terminal conditions are satisfied as follows:

At $t = 0$, $V = 0$ because the term inside {brackets} vanish

At $t = \infty$, $V = 0$ because the decrement factors vanish

At $z = \dfrac{-l}{2}$, $V = 0$ because the term inside {brackets} vanish.

At $z = \dfrac{l}{2}$, $V = 0$ because the term inside {brackets} vanish

Substituting (48) in the differential equation of the transmission line, there is, after simplification

$$- \frac{\partial^2 V}{\partial y^2} + RG + (RC + GL) \frac{\partial V}{\partial t} + LC \frac{\partial^2 V}{\partial t^2}$$

$$= \frac{2Q}{lC} \sum_{s=1}^{\infty} \frac{a\,\epsilon^{-at}}{a - \alpha - \gamma} \frac{k^2}{k^2 + \frac{s^2\pi^2}{l^2}} \sin \frac{s\pi y}{l}$$

$$\left[\frac{s^2\pi^2}{l^2} + RG - a(RC + GL) - a^2 LC \right]$$

$$\left\{ \frac{-\pi^2 \sin \frac{s\pi}{2} \sin \frac{s\pi}{l}}{\frac{s\pi}{l}\left(\frac{s^2\pi^2}{l^2} - \pi^2 \right)} \right\} = \frac{2Q}{lC} a(a - \alpha + \gamma) LC \epsilon^{-at}$$

$$\int_0^l \sum_{s=1}^{\infty} \sin \frac{s\pi y}{l} \sin \frac{s\pi x}{l} f(x)\,dx \qquad (50)$$

since the term in {brackets} appeared from the integral:

$$\int_0^l f(x) \sin \frac{s\pi x}{l}\,dx$$

of Equation (44), where $f(x)$ is the bound charge distribution. But the term under the integral of Equation (50) is the function $f(x)$ multiplied by the Green's function

$$G(x, y) = \sum_{s=1}^{\infty} \sin \frac{s\pi y}{l} \sin \frac{s\pi x}{l} \,. \qquad (51)$$

A characteristic property of this function is that

$$\int_0^l \sum_{s=1}^{\infty} \sin \frac{s\pi y}{l} \sin \frac{s\pi x}{l} f(x)\,dx = f(y) \qquad (52)$$

Therefore

$$- \frac{\partial^2 V}{\partial y^2} + RG + (RC + GL) \frac{\partial V}{\partial t} + LC \frac{\partial^2 V}{\partial t^2}$$

$$= \frac{2Q}{lC} a(a - \alpha + \gamma) LC \epsilon^{-at} f(y) \qquad (53)$$

But $f(x)$ was the initial distribution of bound charge. Hence, over that portion of the line for which $f(y)$ exists, the potential is constrained, and the differential equations do not hold. But beyond the distribution of bound charge the line is not under an artificial constraint, and obeys the differential equation, since $f(y) = 0$ for such points. After infinite time, or for an instantaneous cloud discharge, the right hand side of (53) is zero and then the differential equation is satisfied at all points. But while the discharge is going on, the differential equation does not hold except for points external to the distribution $f(x)$.

Equation (48) shows that the potential distribution at any instant may be regarded as consisting of *forward* and *reverse* traveling waves $S(\lambda_1)$ and $S(\lambda_2)$ respectively, multiplied by the same attenuation factor $\epsilon^{-\alpha t}$, and a term $(-\epsilon^{-\alpha t}\cos\theta Z)$ which exists only over that time interval during which the cloud is discharging. This latter term is fixed in space and vanishes as the cloud discharges. At $t = 0$ it exactly cancels the contributions of the traveling waves, so that there is no potential on the line at that first instant. Thereafter, this fixed distribution of potential falls in value while the traveling waves move out, and its cancellation effect diminishes as the cloud discharge reaches completion. This term has no real or physical existence. It is simply a mathematical compensating function which accounts for the growth and development of the traveling waves during their formation period.

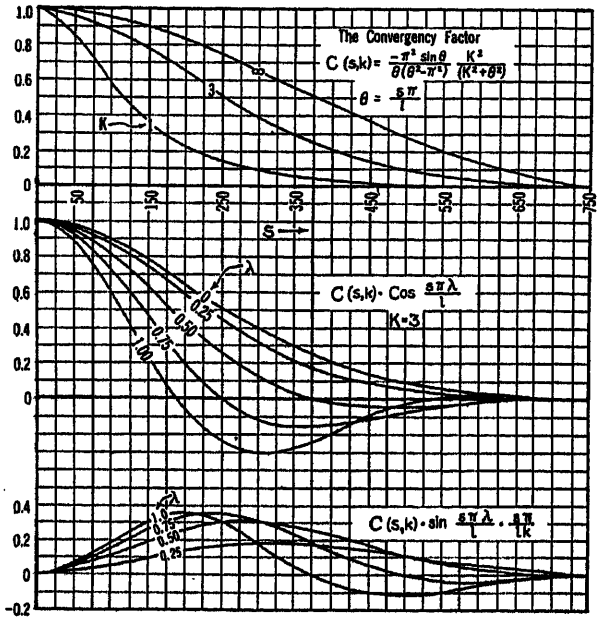

FIG. 27—HARMONIC FACTORS OF THE POTENTIAL DISTRIBUTIONS

Numerical Solution of Equation (48). Equation (48) converges so slowly, that all odd harmonics up to $(s \cong 2\,l)$, where l is the length of the line, must be included, before the series may be terminated. This means that a point by point substitution is out of the question. However, the convergency factor $C(s, k)$ is a smooth curve which remains positive over this range of s, and the trigonometric terms do not give an excessive number of oscillations before their contributions become negligible. If $v_{(2s-1)}$ is an individual harmonic of Equation (48), and if the magnitudes of successive harmonics are near enough together to allow the construction of a smooth continuous curve, then as approximation

$$V = \sum_{1}^{\infty} v_{(2s-1)} \cong \sum_{1}^{s \cong l} v_{(2s-1)}$$

$$= \frac{l}{2}\,(v \text{ average}) = \frac{l}{2}\left(\frac{\text{Area under curve}}{\text{Length of base}}\right) \qquad (54)$$

The procedure then is:

1. Plot curves of $C(s, k) \sim s$ with κ as parameter, Fig. 27.

2. Plot curves of $[C(s, k)\,.\cos\theta z] \sim s$ with z as parameter, Fig. 27. For each z find the area under the curve with a planimeter, and the average ordinate from (54). Multiply these ordinates by $\epsilon^{-\alpha t}$ for required values of t, and plot the family of curves shown as $D(z, t)$ in Fig. 28.

3. Plot curves of $[C(s, k)\,.\,S(\lambda)] \sim s$ with λ as parameter, and find average ordinate as in 2, above. These values are plotted as $W(\lambda)$ in Fig. 28. This is the fully developed traveling wave (neglecting attenuation).

The potential distribution at any instant is then given by

$$V = \frac{2\,Q}{l\,C}\,\{-D(z, t) + W(\lambda_1) + W(\lambda_2)\} \qquad (55)$$

Combining the waves $W(\lambda_1)$ and $W(\lambda_2)$ at their proper space phase, with the term $D(z, t)$, wave formations such as shown in Fig. 11 were obtained.

It is quite evident that solutions such as (48) are of little value for obtaining numerical results. Their application is limited to bound charge distributions and cloud discharge rates which can be expressed by very simple analytic functions, such that the integrations required by Equation (44) can be performed. Even when these integrals can be found, and the equation brought down to the form of Equation (48), it is a long tedious task to calculate and plot the curves, find their areas with a planimeter, and finally combine the components to obtain the wave formations.

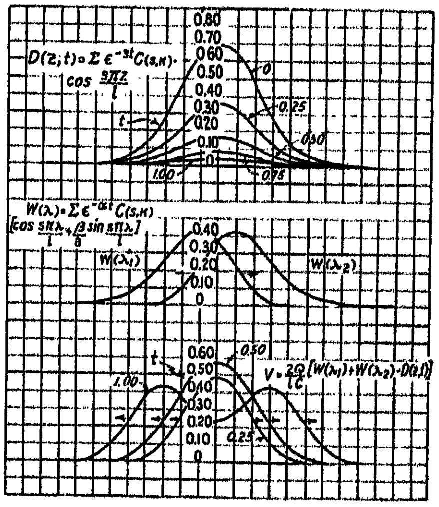

FIG. 28—THE STATIONARY AND TRAVELING COMPONENTS OF THE POTENTIAL DISTRIBUTION

Report of the Board of Directors

FOR THE FISCAL YEAR ENDING APRIL 30, 1929

The Board of Directors of the American Institute of Electrical Engineers presents herewith to the membership its Forty-fifth Annual Report, for the fiscal year ending April 30, 1929. A general balance sheet showing the condition of the Institute's finances on April 30, 1929, together with other detailed financial statements, is included herein. The following is a brief summary of the principal activities of the Institute during the year, more detailed information having been published from month to month in the Institute JOURNAL.

Directors' Meetings.—The Board of Directors held seven meetings during the year, five in New York, one at Denver, Colorado, and one at Cincinnati, Ohio. The Executive Committee acted upon various matters during intervals between Board Meetings.

Information regarding the more important activities of the Institute which have been under consideration by the Board of Directors, the committees, and the various officers is published each month in the section of the JOURNAL devoted to "Institute and Related Activities."

President's Visits.—President Schuchardt has attended various national conventions and regional meetings, also Section and Student Branch meetings, during the past year.

The following is a list of places visited in addition to many trips to Institute headquarters at New York: Spokane, Wash. (Pacific Coast Convention); St. Louis, Mo.; Madison, Wis.; Atlanta, Ga., (Regional Meeting); Washington, D. C.; Schenectady, N. Y.; Milwaukee, Wis.; Chicago, Ill.; Toronto, Ont.; Cincinnati, Ohio (Regional Meeting); Ithaca, N. Y.; Urbana, Ill. (Joint meeting of Urbana Section and Purdue University and Rose Polytechnic Institute Branches). During May and June the president will attend the Dallas Regional Meeting, Section meetings at Cleveland and Akron, Ohio, and the annual Summer Convention at Swampscott, Mass.

Meetings.—Three national conventions of the Institute were held during the year, namely, the Summer, Pacific Coast, and Winter. Regional meetings under the auspices of the Geographical Districts were held in New Haven, Conn., District No. 1; Atlanta, Ga., District No. 4; Cincinnati, O., District No. 2.

Annual Meeting.—The Annual Business Meeting was held at the Cosmopolitan Hotel, Denver, Colo., on Tuesday morning, June 26, 1928, during the annual Summer Convention. The Annual Report of the Board of Directors for the fiscal year ending April 30, 1928 was presented. The Tellers Committee made its report upon the election of officers for the administrative year beginning August 1, 1928 and upon the ballots cast for the proposed constitutional amendments as submitted to the membership, under date of April 2, 1928. There was an overwhelming vote in favor of the amendments which related to change in date of annual meeting and various matters of policy and administration.

Summer Convention.—The forty-fourth annual Summer Convention was held at Denver, Colorado, June 25-29, 1928. Four technical sessions were held at which nineteen papers were presented. In addition other sessions were held as follows: the Annual Business Meeting of the Institute, a lecture on "Geophysical Methods of Prospecting," by Dr. C. A. Heiland, and two sessions at which fifteen technical committee reports were presented. The Annual Conference of Officers and Delegates under the auspices of the Sections Committee was held on Monday, June 25; forty-six Sections being represented. The entertainment features included golf and tennis tournaments, a dinner, and an all-day sightseeing trip. About five hundred members and guests attended.

Pacific Coast Convention.—The seventeenth Pacific Coast Convention was held at Spokane, Washington, August 28-31, 1928. At five technical sessions nineteen technical papers and an address were presented. Two student sessions were held at which nine papers were presented. Various social events completed the program. Attendance about two hundred and fifty.

Winter Convention.—The seventeenth Winter Convention was held in New York, N. Y., January 28–February 1, 1929. At nine sessions forty-two technical papers were presented. At another session the Edison Medal was presented to Dr. Frank B. Jewett and a lecture on "Electricity and Chemistry, Team-mates in Progress," by Dr. Harrison E. Howe was given. Numerous inspection trips, a smoker, and a dinner-dance were held. Attendance about two thousand.

Regional Meetings.—Three Regional Meetings were held during the year:—the first in District No. 1 at New Haven, Conn., May 9-12, 1928. Five technical sessions comprising eighteen papers were held, and also a student session with nine papers presented by students. Attendance about five hundred. Second, at Atlanta, Ga., District No. 4, October 29-31, 1928. Five technical sessions were held at which thirteen papers were presented. A general session was held at which two lectures were given and a student session at which five papers were presented. Attendance about six hundred. Third, at Cincinnati, Ohio, District No. 2, March 20-22, 1929. Four technical sessions were held at which seventeen papers were presented; a student session was also held. Attendance two hundred and seventy.

Sections.—While maintaining their interest in purely technical and moderately technical subjects, the Sections have given more attention than heretofore to methods of encouraging their members to participate in the meetings and in public affairs involving engineering. Their responsibilities toward the students and recent graduates have received much attention.

President Schuchardt's deep interest in these phases of Section activity resulted in the arrangement of a considerable number of special programs, in many of which he participated, and his messages in the JOURNAL dealing with the duties and opportunities of Sections attracted wide attention.

The Sections within five geographical districts cooperated in the planning and conduct of the Pacific Coast Convention and three Regional meetings.

The New York Section decided to establish a number of groups in order to extend the activities to a larger number of members, particularly the younger men. The Power Group was organized and others under consideration include Transportation, Communication, and Illumination Groups. The Chicago Section is contemplating the establishment of several groups.

Copies of the first annual report on Section and Branch activities were distributed at the conference of officers and delegates held in Denver during the Summer Convention. It was decided that a further trial of the present plan of preparing these reports should be continued for at least two more years.

As a result of the recommendations of a special committee appointed by President Gherardi early in 1928, the Sections were requested in a letter dated October 31, 1928, to encourage their members who were qualified for transfer to a higher grade to submit their applications.

Many of the Sections held joint meetings with neighboring Student Branches, some programs being supplied entirely by students, and others including both students and practising engineers. Notable among the latter type were several programs designed to assist the students to learn what the industry will expect of them after graduation. Several very successful student conventions were sponsored by Sections.

New Sections were organized at Dallas and Houston, and both have been very active. The formation of a North Carolina Section was authorized by the Board of Directors on March 21, 1929.

Student Activities.—The Branches have shown great activity during the year both in the holding of individual meetings and in participation in special meetings with other Branches and with the Sections.

Conferences on Student activities involved the Committees on Student Activities and Branch Chairmen of seven geographical districts. Also, conferences involving smaller groups of Branches were held in Pittsburgh and at Ohio State University.

Seven Student Conventions were held, three sponsored by District Committees on Student activities, three by Sections, and one by a Student Branch.

The number of talks delivered at Branch meetings by students was considerably greater than during the previous year.

President Schuchardt's messages in the JOURNAL included several of deep interest to students and recent graduates.

Special efforts were made during the year to encourage young men whose periods of enrolment as Students expired and other recent graduates to apply for admission as Associates.

All Counselors were invited to send to headquarters lists of names and addresses of graduates of their schools in the last three classes. A large number of names and addresses was received and sent to the Sections in the territories in which the men are now located. Suggestions regarding methods of forming contacts with these men were sent to the Section officers.

By means of special letter and form, efforts were undertaken to ascertain the principal reasons which prevent former Enrolled Students from applying for admission as Associates immediately after the expiration of their enrolment.

A revised edition of the Suggested By-laws for Student Branches was distributed. Upon recommendations of various District Committees on Student Activities and the Committee on Student Branches, the Board of Directors took favorable action on several matters of importance to the Branches.

New Branches were authorized at the University of Detroit, University of South Carolina, and Cornell University, and all have been active.

Section and Branch Statistics.—

	For Fiscal Year Ending			
	April 30 1923	April 30 1925	April 30 1927	April 30 1929
SECTIONS				
Number of Sections......	46	49	52	54
Number of Section meetings held..............	344	386	431	460
Total Attendance........	46,672	49,029	60,708	73,254
BRANCHES				
Number of Branches.....	68	82	95	100
Number of Branch meetings held.............	503	548	842	940
Total Attendance........	26,893	27,603	42,650	47,408

Meetings and Papers Committee.—The arrangement of programs for national conventions and assistance with programs of regional meetings have been the most important duties of the Meetings and Papers Committee during the year 1928-29. The committee also has reviewed many papers submitted for publication only and given recommendations on their acceptability. The committee members have furthermore assisted the Committee on Award of Institute Prizes in selection of papers for prize awards.

Three national conventions and three regional meetings were held during the year. The technical program of the 1929 Summer Convention is completed; the programs for the Pacific Coast Convention and the Regional meeting in Chicago are taking definite form and consideration has been given to scheduling certain papers and topics at the 1930 Winter Convention.

One hundred and forty-three papers have been presented during the year including fifteen annual reports of Technical Committees. Many excellent contributions to the information on the theory and practical applications of electricity are contained in these papers. Through the year a large number of high grade papers has been available for use at the various meetings.

The Code of the Meetings and Papers Committee, which is intended as a guide for the activities of the Committee and its members, has been revised. The revised edition includes many policies which have been adopted as a result of the Committee's experience during the last few years.

Publication Committee.—The plan of publishing the TRANSACTIONS quarterly and printing papers in abridged form in the JOURNAL, with complete copies available upon request of those desiring more detailed information, has now been in effect for sixteen months. This period has been long enough to fully demonstrate that the new plan has more completely met the requirements of the membership and further, that it costs less to publish than under the previous plan.

While the new plan has solved many of the problems of the Committee, it was felt by some that the JOURNAL would be more widely read and referred to if in addition to the present contents, special features were added that would be of general interest to the entire membership. For the purpose of obtaining the views of the Institute members on this subject, the Committee published in the January 1929 issue of the JOURNAL, an article outlining in a general way, the proposed changes that have been suggested, and asked the members to express their opinions as to what changes, if any, should be made in the Institute publications.

A large number of replies has been received and many more are expected. From the nature of those received thus far, the Committee believes that the survey will be of material value in that it will not only determine whether any consensus of opinion in regard to JOURNAL improvements exists, but will also give further information as to the attitude of the members towards the new plan in effect since January 1928. Before making an analysis of the results of this survey, the Committee feels that it would be desirable to hear from more of our members, especially from those active in Institute affairs.

Beginning with the Quarterly TRANSACTIONS for 1929 an authors' index of the papers and discussors is being included in the first three issues in order to facilitate the work of looking up references. The last Quarterly of the year will contain the complete authors' index for the year in addition to the usual classified subject index.

Coordination Committee.—The Committee on Coordination of Institute Activities has considered and made recommendations to the Board of Directors upon various matters coming within its jurisdiction under the By-laws. The Committee corresponded with the officers of all Districts and Sections and obtained their views regarding national and regional meetings during the year 1930, resulting in the recommendation to, and adoption by, the Board of Directors, in January 1929, of a complete schedule of national and regional meetings to be held during 1930.

Other matters considered by the committee included qualifications for Student enrolment and organization of Student Branches, procedure in reference to appointment of Counselors of Student Branches, conditions of award for student prize papers, appropriation for duplicating student papers, and proposed revision of various by-laws.

Standards.—The Standards committee has continued its work along recognized lines. There have been no important changes in organization nor in methods of conducting its work during the past Institute year. The Standards Committee has received valuable assistance from a number of the technical committees which have initiated new Standards work principally in revising and amplifying existing Standards. In this connection, the work of the Transformer Sub-committee of the Electrical Machinery Committee deserves special mention. An important new contribution of this subcommittee has been its work on an appendix to Transformer Standards, covering the important subject of service recommendations, with reference to operation at loads greater than rated load when air or water temperatures are lower than standard. This is still in report status, as outlined in the August 1928 JOURNAL.

A new project first called to the attention of the Institute members in the February 1929 JOURNAL, and officially recommended to the Board by the Standards Committee by action at its meeting held May 9, 1929, is the preparation of Test Codes for electrical apparatus. This is an undertaking that may prove equal, in magnitude, to the preparation of the present Institute Standards. The Institute Standards in their present form contain some information and rules for testing machinery and apparatus. There has developed a need, which the Institute has now undertaken to supply, for more complete and detailed Standards of practise covering methods of making tests similar in scope to the Power Test Codes prepared by the Power Test Code Committee of the A. S. M. E.

These two new projects—the transformer service recommendations and test codes—represent important service that, it is hoped, will prove acceptable to the industry.

The more routine work of the Standards Committee is represented by a number of new and revised Standards adopted and additional Standards approved as American Standards by the American Standards Association. Since May 1, 1928 there have become available eight new or revised Standards pamphlets covering standards and six specifications. Three reports on proposed standards have also appeared.

Three Standards have been approved as American Standards by the A. S. A. There is now a total of sixteen Standards approved as American Standards.

The Institute is at the present time sole sponsor for ten projects and joint sponsor for eighteen projects under A. S. A. procedure. In addition to the direct responsibility represented by these sponsorships, the Institute is co-operating with A. S. A. by being represented on twenty-six additional sectional committees.

Among new Institute Standard projects under way, the following are of unusual interest and importance: Mercury Rectifiers, Switchboards and Switchgear, and Relays.

During the year the Standards Committee has continued its close cooperation in the technical work of the U. S. National Committee of the I. E. C. Following the plenary meeting of the I. E. C. at Bellagio, Italy, in the fall of 1928, the I. E. C. submitted for approval a revision of Publication 34 covering Standards of machinery and transformers. The Institute Standards Committee cooperated in the preparation of the statement of the U. S. N. C., disapproving the adoption of this document by the I. E. C. and in the preparation of the statement of the American proposals to be submitted to a meeting of the Rating Advisors of the I. E. C. to be held in London in the coming summer.

U. S. National Committee of the I. E. C.—The activities of this committee during the past year have been chiefly of a routine character. Important Advisory Committee Meetings have been held during the year. A meeting of the Subcommittee of the Advisory Committee on Nomenclature, for which the U. S. National Committee exercises the secretariat, was held in Paris in April 1928. At this meeting considerable progress was made on the vocabulary which the committee has in preparation.

A meeting of the Subcommittee on Symbols for Telegraphy and Telephony which was held in Berne in April 1928 in conjunction with a corresponding committee of the International Consultative Committee on Long-Distance Telephony resulted in a revised list of graphical symbols for telegraphs and telephones being prepared for submission to the National Committees.

A very important meeting of the Advisory Committee on Steam Turbines (Secretariat—U. S. National Committee) was held at The Hague in May 1928. There were twenty-three individuals from seven nations other than the U. S. and Holland in attendance. Great progress was made in drafting Rules for Acceptance Tests of Steam Turbines. A further meeting of this committee is to be held during the coming summer and it is expected that a set of international rules covering steam turbines will be ready for adoption at the Plenary Meeting in Stockholm in 1930.

It is expected that meetings of the Advisory Committee on Rating of Electrical Machinery and of the Advisory Committee on Instruments will be held next summer. Preparations for these meetings are under way.

U. S. National Committee of the International Commission on Illumination.—The last year has been a very important one for this committee inasmuch as during it occurred the American Meeting of the Commission. The committee was fortunate in enlisting the good offices of the Illuminating Engineering Society in making arrangements for this meeting. The meeting was attended by about sixty delegates from twelve different nations and was held in Saranac, New York, from September 22nd to 28th.

Subsequent to the meeting, the delegates were taken on a tour which included New York, Boston, Philadelphia, Cleveland, Detroit, Chicago, Niagara Falls, concluding at Toronto where the delegates attended the annual convention of the Illuminating Engineering Society.

The Commission has some fifteen technical committees and considered recommendations and reports from the various national committees with a view to formulating recommendations for adoption by the Commission.

The technical papers at this meeting covered a wide variety of subjects, the practical details of lighting practise receiving a large amount of attention. Discussions of street lighting, automobile headlighting, factory and school lighting, residence lighting, aviation field lighting, show window lighting, of a very thorough-going and valuable character and interspersed with other discussions of more specialized topics of interest to the photometrists, etc. To the other technical committees for which the United States National Committee exercises the secretariat were added two new ones; *viz.*, the Technical Committee on Lighting for Aerial Navigation and the Technical Committee on General Practical Applications of Illumination.

The Proceedings of the meeting, now being printed, will constitute a very notable summary of the present state of the art and science of illumination as it exists not only in this country but throughout the world.

The next meeting of the Commission is to be held in 1931 in England.

Committee on Safety Codes.—No new rules for the protection of persons or property against fire accidents and other hazards in connection with electrical installations and equipments were referred to this committee during the year, either from other technical committees within the Institute or from bodies outside of the Institute.

The Institute was represented by one of the members of the Committee on Safety Codes at the annual Safety Congress held in New York City, October 1 to October 5, 1928. The particular committee whose activity seemed to be most directly related to the scope of the A. I. E. E. activity was the Low-Voltage Electrical Committee of the Engineering Section of the National Safety Council.

The National Electric Light Association, which was represented at the meeting, reported that it was tabulating statistics with respect to accidents caused by low-voltage equipment.

The Committee on Safety Codes was represented at the regular annual meeting of the Electrical Committee of the National Fire Protection Association on February 18th and 19th in New York City. Probably the outstanding feature of the meeting was the general agreement reached with respect to certain controversial questions on grounding and wiring methods.

The committee was represented also at the annual meeting of the National Fire Waste Council held in Washington on March 26th, 1929.

Technical Committees.—Reports of Technical Committees embracing an outline of the year's work and a summary of progress in the industry will be presented at the annual Summer Convention and printed in the TRANSACTIONS.

Membership—The additions to, and deductions from, the membership are shown in the following table. In accordance with the established policy of the Institute, emphasis has continued to be placed on the qualifications of candidates. The total membership as of April 30 is 18,133; this indicates a small net loss during the year, which is due principally to the deductions as shown. There is every indication that the development of section, student, and other activities, now in progress, will result in a steadily increasing list of applicants during the coming years.

	Honorary	Fellow	Member	Associate	Total
Membership on April 30, 1928..	4	659	3,163	14,439	18,265
Additions:					
Transferred....	4	38	219		
New Members qualified.....	1	4	101	1,238	
Reinstated.....		1	13	85	
Total............	9	702	3,496	15,762	19,969
Deductions:					
Died...........	1	9	22	46	
Resigned.......		1	20	328	
Transferred....		3	32	226	
Dropped.......		2	43	1,103	
Membership April 30, 1929..	8	687	3,379	14,059	18,133

Deaths.—The following deaths have occurred during the year.

Honorary Member: Ferdinand Foch.

Fellows: William Esty, Valeré A. Fynn, Edwin B. Katte, William Weaver, Jr., William N. Motter, William Pestell, Samuel G. Rhodes, Allen M. Schoen, Edgar E. Stark.

Members: Johan M. Anderson, William F. Bennington, Frederick S. Blackall, Adam Bosch, Charles S. Bradley, George Broughall, Thomas Duncan, Rufus T. Gent, John C. Hatzel, Basil F. Howard, Benjamin N. Jones, Fred A. Jones, Robert C. Jones, Herbert Lloyd, John J. Murphy, John G. Pertsch, Jr., Theodore C. Roberts, Frank R. Shepherd, William L. Simpson, Robert I. Todd, Francis R. Weller, Charles A. Wright.

Associates: Leo O. Ambrose, Philip V. Berger, Harley S. Berry, Paul T. Brady, Wallace L. Brooks, V. Frank Buddeke, John J. Burke, Lester W. Cooper, E. Preston Clifford, Robert L. Crawford, John H. Cuntz, Orin D. Dausman, Earle N. Dillard, Alex. C. Dogherty, Charles N. Fenstermacher, Earl R. Filkins, Arthur L. Freyer, Joseph E. Frisbee, Francis M. Garretson, Jr., George F. Harrington, William S. Hearding, Francis R. Howe, Griffith O. Hughes, Harold B. James, Gunnar Jensen, Samuel J. Keese, James F. Kimbell, Harry R. Knettles, Joseph W. Legg, Clarence R. Lininger, Roy B. MacPherson, Harry C. Martell, Seizo Misaki, Gilbert Mitchell, E. J. Morrissey, Alfred G. Place, Donald K. Prossor, William M. Stratton, Harold P. Thomas, Walter D. Uptegraff, Lauro G. Villanueva, Frank C. Wagner, Charles S. Winston, Cecil F. Wilson, Kyojiro Yosugi, Nelson T. Young.

Board of Examiners.—The Board of Examiners held twelve meetings during the past year averaging about three and one-half hours. A total of 3115 cases were considered, divided as shown in the following table:

APPLICATIONS FOR ADMISSION

Recommended for grade of Associate.....	1064	
Not recommended......................	20	1084
Recommended for grade of Member......	97	
Not recommended for admission to this grade...........................	22	119
Recommended for grade of Fellow.......	3	
Not recommended for admission to this grade...........................	2	5
Recommended for enrolment as Students..		1612

APPLICATIONS FOR TRANSFER

Recommended for grade of Member......	229	
Not recommended for transfer to this grade	19	248
Recommended for grade of Fellow........	45	
Not recommended for transfer to this grade	2	47
Total number of applications considered..		3115

Institute Prizes.—Four National Prizes and eleven Regional Prizes for papers were awarded during the year, as announced through the Institute JOURNAL. These prizes were for papers presented at Institute meetings during 1927. The National Prizes were presented at the 1928 Summer Convention and the Regional Prizes at various meetings in the respective Districts. The National Prizes consist of $100.00 each and a certificate, while the Regional Prizes consist of $25.00 cash and a certificate.

Scholarships.—The governing bodies of Columbia University have placed at the disposal of the Institute each year a scholarship in electrical engineering for each class. The awards are made annually by an Institute committee. The scholarships pay $350 toward annual tuition with provision for reappointment. An award was made for the academic year 1928-9 to Walter L. Lawrence.

Complete details governing prizes and scholarships may be obtained by applying to the National Secretary of the Institute.

Edison Medal.—The Edison Medal, founded by associates and friends of Thomas A. Edison, is awarded annually by a committee consisting of twenty-four members of the Institute "for meritorious achievement in electrical science, electrical engineering, or the electrical arts." The medal for 1928 was awarded to Frank B. Jewett, "for his contribution to the art of electrical communication," The presentation took place at the Winter Convention of the Institute, January 30, 1929.

John Fritz Medal.—The John Fritz Medal Board of Award, which is composed of representatives of the national societies of Civil, Mining, Mechanical, and Electrical Engineers, awarded the twenty-fifth medal to Herbert Clark Hoover, "Engineer, Scholar, Organizer of Relief to War-Stricken Peoples, Public Servant." The medal was presented by the Board of Award at the Executive Mansion, Washington, D. C., Tuesday, April 25, 1929.

Lamme Gold Medal.—The Lamme Medal was founded as a result of a bequest in the will of the late Benjamin G. Lamme, Chief Engineer of the Westinghouse Electric and Manufacturing Company, who died on July 8, 1924. The bequest provides for the award by the Institute of a gold medal (together with a bronze replica thereof) annually to a member of the A. I. E. E. "who has shown meritorious achievement in the development of electrical apparatus or machinery" and for the award of two such medals in some years if the accumulation from the funds warrants.

The Lamme Medal Committee of the Institute has awarded the first (1928) Lamme Medal to Mr. Allan Bertram Field "for the mathematical and experimental investigation of eddy current losses in large slot-wound conductors in electrical machinery." Arrangements are being made for the presentation of the Medal at the annual Summer Convention at Swampscott, Mass., June 24-28.

Commission of Washington Award.—The Washington Award for 1929 was made to Bion J. Arnold "for pioneering work in the engineering and economics of electrical transporation." Mr. Arnold is the second past-president of the A. I. E. E. to receive this honor.

The award is made annually "to an engineer whose work in some special instance, or whose services in general have been noteworthy for their merit in promoting the public good," by a committee composed of nine representatives of the Western Society of Engineers and two each from the A. S. C. E., the A. I. M. E., the A. S. M. E., and the A. I. E. E.

Employment Service.—The employment service which the Institute has maintained for many years is conducted as a cooperative bureau in conjunction with a similar service maintained by the National Societies of Civil, Mining, and Mechanical Engineers under the title, "Engineering Societies Employment Service." In addition to the main office in the Engineering Societies Building, New York, offices have been opened in Chicago in cooperation with the Western Society of Engineers and in San Francisco under similar arrangements with the California Section, American Chemical Society, and Engineers Club of San Francisco. It is hoped to continue this development from year to year as conditions warrant. The service is supported by the joint contributions of the societies and their individual members who are benefited. As in the past it consists principally in acting as a medium for bringing together the employer and the employe. In addition to the publication of the "Employment Service Bulletin" in the monthly JOURNAL, weekly subscription bulletins are issued for those seeking positions.

American Engineering Council.—Since the organization of American Engineering Council nearly nine years ago, Council has experienced a steady normal growth. It now comprises twenty-five national, state, and local engineering societies which have an individual membership of over 44,000 professional engineers. Last December the American Society of Civil Engineers voted to become a member of Council, and has since been admitted.

Council represents its constituents at conferences, conventions, and on important national committees, and at Congressional hearings upon matters involving engineering. It also extends valuable individual service to its member organizations. On matters of concern to the entire engineering profession Council has conducted several notable surveys. During 1928 it completed a report on Street Traffic Signs, Signals, and Markings. It is now engaged in a movement through the engineering organizations to have the report presented for consideration of municipal authorities.

Council also, in cooperation with the National

Bureau of Economic Research, completed its portion of the survey on Recent Economic Changes. It has actively supported measures providing for an inventory of our water resources; measures providing for equitable status of sanitary engineering; a more extensive appropriation for topographic mapping; the unification and coordination of governmental public works functions; the establishment of a national hydraulic research laboratory, etc.

Council has continued the publication of a quarterly known as "A. E. C. Bulletin" and also supplies to national member organizations a monthly engineering news letter. The following are some of the committees in active operation: Public Affairs; Publicity and Publications; Flood Control; Government Reorganization; Patents; Power; Research Program; Reforestation; Street and Highway Safety; Communications; Finance; Uniform State Mechanics Lien Act.

United Engineering Society.—This Society performs for the national societies of Civil, Mining, Mechanical, and Electrical Engineers, certain specific acts which are governed by contracts; the primary function of the United Engineering Society being to hold in trust and to administer for these societies the Engineering Societies Building, in which the headquarters of the national societies are located.

Extracts from the annual financial report of the United Engineering Society were published in the March 1929 JOURNAL.

Engineering Societies Library.—The library of the Institute is combined with the libraries of the national societies of Civil, Mining, and Mechanical Engineers, administered as the "Engineering Societies Library" under the direction of the Library Board of the United Engineering Society; this board is composed of representatives of each of the four societies referred to above.

In order to place the facilities of the library at the disposal of persons residing at a distance from New York, a Library Service Bureau has been established, and a staff of expert searchers and translators is employed to cover almost any engineering topic, in the following manner: abstracting, translating, bibliographing, statistical searches and reports, searches for patent purposes, copying, preparing reference cards, etc. A lending department is also maintained.

A copy of the annual report of the Engineering Societies Library covering the calendar year 1928 may be obtained by applying to Institute headquarters.

Engineering Foundation.—Engineering Foundation is a trust fund established in 1914 by Ambrose Swasey, of Cleveland, Ohio, by gifts to United Engineering Society as a nucleus of a large endowment "for the furtherance of research in science and in engineering, or for the advancement in any other manner of the profession of engineering and the good of mankind." The fund has been generously increased through the gifts of Edward D. Adams and others, and also through bequest under the will of the late Henry R. Towne. It is administered by the Engineering Foundation Board upon which the Institute and other national engineering societies are represented. The Board is a department of United Engineering Society.

The Foundation has made appropriations for various research projects and has cooperated in others.

An abstract of the annual report of the Foundation was published in the JOURNAL for May 1929 and a complete report is available in printed form.

Representatives.—The Institute has continued its representation upon various national committees and other local and national bodies with which it has been affiliated in past years, and has accepted sponsorship and appointed representatives upon a number of new Sectional Committees of the American Standards Association. A complete list of representatives is published frequently in the JOURNAL.

Finance Committee.—During the year the committee has held monthly meetings, has passed upon the expenditures of the Institute for various purposes, and otherwise performed the duties prescribed for it in the Constitution and By-laws.

Haskins and Sells, certified public accountants, have audited the books, and their report follows:

Respectfully submitted for the Board of Directors,

F. L. HUTCHINSON,

National Secretary.

New York, May 22, 1929.

HASKINS & SELLS
CERTIFIED PUBLIC ACCOUNTANTS
OFFICES IN THE PRINCIPAL CITIES OF
THE UNITED STATES OF AMERICA
—AND IN—
LONDON, PARIS, BERLIN, SHANGHAI, MANILA,
MONTREAL, HAVANA, MEXICO CITY

NEW YORK CENTRAL BUILDING
75 EAST 45TH STREET

May 20, 1929.

American Institute of Electrical Engineers,
33 West 39th Street,
New York.

Dear Sirs:

We have made a general audit of your accounts for the year ended April 30, 1929, and submit the following exhibits and schedule:

Exhibit

A—Balance Sheet, April 30, 1929.
Schedule 1—Reserve Capital Fund—Securities.

B—Summary of Income and Profit and Loss for the Year Ended April 30, 1929.

WE HEREBY CERTIFY that in our opinion Exhibits A and B, respectively, correctly set forth the financial condition of the Institute at April 30, 1929, and the results of its operations for the year ended that date.

Yours truly,

HASKINS & SELLS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

EXHIBIT A.

BALANCE SHEET, APRIL 30, 1929

ASSETS

REAL ESTATE:			
One-fourth interest in United Engineering Society's Land, Building, and Building Equipment, 25 to 33 West 39th Street, (depreciation carried on books of the United Engineering Society)			$493,352.60
EQUIPMENT:			
Library—Volumes and fixtures		$ 41,452.30	
Works of art, paintings, etc		3,001.35	
Office furniture and fixtures	$28,003.45		
Less reserve for depreciation (including $7,431.79 funded)	12,243.65	15,759.80	
Total equipment			60,213.45
WORKING ASSETS:			
"Transactions," etc		$ 4,796.00	
Text and cover paper		3,700.10	
Badges		1,931.54	
Total working assets			10,427.64
CURRENT ASSETS:			
Cash		$ 23,273.56	
Note receivable		437.50	
Accounts receivable:			
Members—For dues		22,664.01	
Advertisers		1,179.50	
Miscellaneous		8,081.55	
Accrued interest on investments		2,529.89	
Total current assets			58,166.01
FUNDS:			
Reserve Capital Fund:			
Securities—Schedule 1		$171,370.63	
Life Membership Fund:			
Cash	$ 6,525.41		
Chicago, Burlington & Quincy Railroad Company 4% bonds, 1958, registered, par value, $5,000.00.	$ 4,868.75		
Accrued interst	33.33	11,427.49	
International Electrical Congress of St. Louis—Library Fund:			
Cash	$ 1,097.89		
New York City 4½% corporate stock, 1957, par value, $2,000.00	2,204.05		
New York Telephone Company 4½% bond, 1939, registered, par value, $1,000.00	878.75		
Accrued interest	67.50	4,248.19	
Mailloux Fund:			
New York Telephone Company 4½% bond, 1939, registered, par value, $1,000.00	$ 1,000.00		
Accrued interest	22.50	1,022.50	
Lamme Medal Fund:			
Cash	$ 456.07		
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series "C" bond, 1995, registered, par value, $4,000.00	4,330.00		
Accrued interest	100.00	4,886.07	
Depreciation of Furniture and Fixtures Fund:			
Cash	$ 3,421.79		
Cleveland Union Terminals Company 5% sinking fund series "B" gold bonds, 1973, registered, par value, $4,000.00	4,010.00	7,431.79	
Midwinter Convention Fund:			
Cash		26.70	
Total funds			200,413.37
Total			$822,573.07

LIABILITIES

CURRENT LIABILITIES:		
Accounts payable	$ 9,905.27	
Dues received in advance	2,823.17	
Entrance fees and dues advanced by applicants for membership	519.85	
Subscriptions for "Transactions" received in advance	24.00	
Total current liabilities		$ 13,272.29
FUND RESERVES (NOT INCLUDING DEPRECIATION RESERVE):		
Reserve Capital Fund	$171,370.63	
Life Membership Fund	11,427.49	
International Electrical Congress of St. Louis—Library Fund	4,248.19	
Mailloux Fund	1,022.50	
Lamme Medal Fund	4,886.07	
Midwinter Convention Fund	26.70	
Total fund reserves (not including depreciation reserve)		192,981.58
SURPLUS, Per Exhibit "B"		$616,319.20
Total		$822,573.07

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

SUMMARY OF INCOME AND PROFIT AND LOSS FOR THE YEAR ENDED APRIL 30, 1929

EXHIBIT B.

INCOME:			
Dues		*$227,274.03	
Students' fees		10,792.00	
Entrance fees		13,356.00	
Transfer fees		1,315.00	
Advertising		84,790.80	
Journal subscriptions		10,152.92	
"Transactions" subscriptions		20,737.41	
Miscellaneous sales		11,086.45	
Badges sold	$ 4,073.75		
Less cost	3,599.93	473.82	
Interest on securities in reserve capital fund		7,942.29	
Interest on bank balances		1,487.46	
Interest on securities in depreciation reserve fund.		250.00	
Total			$389,658.18
EXPENSES:			
Publications:			
Journal	$84,832.06		
"Transactions"	37,316.57		
Year Book	8,668.32		
Miscellaneous	5,537.64	$136,354.59	
Institute meetings		21,300.52	
Administrative expenses		60,803.98	
Sections		34,516.31	
Membership		7,323.40	
Standards		9,826.74	
Finance		400.00	
Code committees		60.00	
Technical committees		163.30	
Edison Medal		156.44	
Geographical districts' expenses:			
Traveling expense:			
Executive committee	$ 1,958.73		
Vice-Presidents	673.46		
Speaker's bureau	107.24		
Best Paper Prize	100.00		
First Paper Prize	100.00	2,939.43	
Branches		10,387.38	
American Engineering Standards Committee		1,500.00	
International Electrotechnical Commission		1,152.18	
Forward		$286,884.27	$389,658.18

TOTAL INCOME—FORWARD		$389,658.18
EXPENSES—(FORWARD)	$286,884.27	
United States National Committee of International Commission on Illumination	300.00	
Louvain Memorial	200.00	
President's special appropriation	668.58	
Board of Directors, traveling expenses	4,454.23	
National Nominating Committee, traveling expenses	893.47	
Institute representatives, traveling expenses	109.73	
Honorary Secretary	4,000.00	
Engineering Societies Library, maintenance	9,334.71	
United Engineering Society assessment	5,237.43	
American Engineering Council	17,673.50	
Engineering Societies Employment Service	1,635.00	
Institute prizes	400.00	
Total		331,790.92
NET INCOME		$ 57,867.26
PROFIT AND LOSS CREDITS:		
Adjustment of "Transactions" inventory	$ 458.59	
Profit on redemption of Cleveland Union Terminals Company 5% sinking fund series "B" gold bond, 1973, registered, par value, $1,000.00 (Depreciation Reserve Fund)	47.50	
Total		506.09
GROSS SURPLUS FOR THE YEAR		$ 58,373.35
PROFIT AND LOSS CHARGES:		
Uncollectible dues and members charges written off	$ 11,628.68	
Furniture, fittings, and equipment scrapped or sold—Net	130.25	
Provision for depreciation of furniture and fixtures	2,092.91	
Total		13,851.84
SURPLUS FOR THE YEAR		$ 44,521.51
SURPLUS, MAY 1, 1928	$599,472.69	
Less transferred to Reserve Capital Fund in accordance with resolution of the Board of Directors	27,675.00	571,797.69
SURPLUS, APRIL 30, 1929		$616,319.20

*Includes $91,325.00 allocated to JOURNAL subscriptions.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

RESERVE CAPITAL FUND—SECURITIES

APRIL, 30, 1929

EXHIBIT A.
SCHEDULE No. 1.

	Par Value	Book Value
RAILROAD BONDS:		
Baltimore and Ohio Railroad Company 4½% convertible gold bonds, due 1933, registered	$ 10,000.00	$ 9,387.50
Baltimore and Ohio Railroad Company 6% refunding and general mortgage series "C" bonds, due 1995, registered	8,000.00	8,940.00
Central of Georgia Railway Company 5½% refunding and general mortgage series "B" bonds, due 1959, registered	5,000.00	5,283.75
Chicago and Erie Railroad Company 5% first mortgage gold bonds, due 1982, registered	1,000.00	1,105.00
Chicago, Burlington & Quincy Railroad Company 5% first and refunding mortgage series "A" gold bond, due 1971, registered	1,000.00	1,010.00
Chicago, Terre Haute & Southeastern Railway Company 5% first and refunding mortgage gold bonds, due 1960	8,000.00	7,940.00
Florida East Coast Railway Company 5% first and refunding mortgage series "A" gold bonds, due 1974, registered	10,000.00	9,818.75
Great Northern Railroad Company 5½% general mortgage series "B" gold bonds, due 1952, registered	10,000.00	9,847.50
St. Louis, San Francisco Railway Company 5% prior lien mortgage series "B" bonds, due 1950, registered	6,000.00	5,497.50
Southern Railway Company 5% first consolidated mortgage gold bonds, due 1994, registered	1,000.00	980.00
The New York Central Railroad Company 5% refunding and improvement mortgage series "C" bonds, due 2013, registered	6,000.00	5,742.50
The Pennsylvania Railroad Company 4½% general mortgage series "A" gold bonds, due 1965, registered	5,000.00	5,130.00
Total (Forward)	$ 71,000.00	$ 70,682.50

	Par Value	Book Value
Total (Forward)	$71,000.00	$ 70,682.50
PUBLIC UTILITY BONDS:		
Consolidated Gas Company of New York 5½% gold debentures, due 1945, registered	$ 5,000.00	$ 5,187.50
Duquesne Light Company, 4½% first mortgage series "A" gold bonds, due 1967, registered	3,000.00	2,970.00
New York Telephone Company debenture 6% gold bond, due 1949	5,000.00	5,525.00
Northern States Power Company 5½% first lien and general mortgage gold bond, series "B," due 1950	10,000.00	10,400.00
Pacific Gas & Electric Company 5½% first and refunding mortgage gold bonds, due 1952, registered	5,000.00	5,137.50
Philadelphia Company secured 5% gold bond, series "A," due 1967	10,000.00	10,000.00
The American Telephone and Telegraph Company 5% gold debentures sinking fund, due 1960, registered	15,000.00	14,625.00
The Detroit Edison Company 6% first and refunding mortgage series "B" gold bonds, due 1940, registered	5,000.00	5,178.13
United Light and Power Company 5½% first lien and consolidated mortgage gold bond, due 1959	5,000.00	4,975.00
Total	$ 63,000.00	$ 63,998.13
INDUSTRIAL BONDS:		
American Smelting and Refining Company 5% first mortgage 30-year series "A" gold bonds, due 1947, registered	$ 9,000.00	$ 9,085.00
Bethlehem Steel Company 5% purchase money and improvement mortgage 20-year sinking fund gold bonds, due 1936, registered	5,000.00	5,033.75
New York Steam Corporation 6% first mortgage gold bonds, due 1947, registered	10,000.00	10,837.50
The Western Electric Company 5% bonds, due 1944, registered	10,000.00	9,818.75
United States Rubber Company 5% first and refunding mortgage series "A" bonds, due 1947, registered	2,000.00	1,915.00
Total	$ 36,000.00	$ 36,690.00
Total	$170,000.00	$171,370.63

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